An open economy forward looking monetary policy rule model for Japan and its policy implications

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Abstract

This paper has suggested an open economy forward looking threshold monetary policy rule model for Japan. This model assumes that, in addition to inflation rate and real output deviations, the short term nominal interest rate of the central bank (CB) of Japan responds to nominal (or real) exchange rate deviations from their target levels. This happens only when the economy lies in the recession regime. This result means that depreciation in Japan's currency will tend to offset decreases in the short term interest rate of the CB due to negative deviations in inflation and real output. A small scale open economy model simulated by the paper shows that the above offsetting effects of exchange rate deviations on interest rates help to reduce the volatility of real exchange rates (the terms of trade) coming from exogenous shocks in domestic real productivity, foreign output and inflation.

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1. Introduction

Despite the plethora of studies estimating monetary policy rules for closed economies and simulating their policy effects, they are a few studies which conduct analogous studies for open economies (see, eg., Clarida *et al.* (1998)). These studies assume that central banks (CBs), in addition to stabilizing inflation and output deviations from their target levels, set the nominal interest rate so as to stabilize also the nominal exchange rate and the terms of trade, or, at least, to maintain these variables within reasonable bounds. Thus, deviations in nominal (or real) exchange rates from their target levels may have an effect on CBs' interest rates policy independently on inflation and/or output gap deviations from their corresponding target levels, as assumed by standard, closed economy monetary policy rules. The policy properties of these extended by the above exchange rate variables monetary policy rule models have studied in a number of recent papers (see, e.g., Smets and Wouters (2002), Devereux and Engle (2003), Gali and Monacelli (2005), Lubik and Schorfheide (2007), Justiniano and Preston (2010).

The evidence provided by the above studies implies that the reaction of nominal interest rates to exchange rate deviations (nominal or real) is rather weak and small for many countries examined, or there is not exist at all (see, e.g., Clarida *et al*. (ibid), Lubik and Schorfheide (2007), Justiniano and Preston (2010)). For instance, for Japan, Clarida *et al*. find that a one appreciation of the yen relative to the US dollar induces a 9 points increase of Japan's CB nominal interest rate, which is quite small. In this paper, we will investigate if evidence of small (or insignificant) effects of exchange rate on CBs interest rate can be associated to changes in the business conditions of the economy. That is, shifts between the expansion or recession regimes of the economy. This investigation is very useful since there are many studies which show that CBs monetary policy respond asymmetrically to different business conditions and, in particular, that it becomes less active (or completely passive) during recessions (see, e.g., Davig and Leeper (2007)). Evidence that this monetary policy behavior also characterizes exchanges rate movements will be very important for economic policy reasons, as it means that exchange rate targeting is not an ultimate target of CBs. Furthermore, it will unveil if CBs of open economies adopt monetary policy measures to improve their terms of trade (or their foreign competitiveness) during recessions. This can be done by approving further reductions to nominal interest rates than those predicted by falls in inflation rates or negative output gap deviations.

The paper answers the above questions for Japan, which is an export oriented economy and thus, deterioration of terms of trade are expected to influence monetary policy. In so doing, the paper estimates a forward looking threshold monetary policy rule interest rate model where its slope coefficients with respect to inflation, real output and nominal (or real) exchange rate deviations depend on the state of business cycle which is captured by a threshold variable. As such variable, we consider real output gap. This has been suggested in the literature as a direct measure of business cycle conditions (see, e.g. Stock and Watson (2003)). To estimate the above model, the paper adopts a new econometric technique avoiding any possible biases on the structural coefficient estimates of the model due to the endogenous nature of the threshold variable. This method has been suggested recently by Kourtelos *et al.* (2008). It extends Canner and Hansen's (2004) two-stage least squares estimation method of forward looking threshold models allowing for a possible contemporaneous correlation between the threshold variable and the disturbance term of the monetary policy rule structural equation.

The estimation results of our paper lead to a number of very interesting conclusions. First, it shows that weak evidence of exchange rate effects on CBs interest rates can be attributed to ignoring changes in business conditions of the economy. Our results clearly indicate that Japanese monetary policy authorities consider nominal (or real) exchange rate deviations in their monetary policy rule only during recessions for Japan, since under this regime are concerned more about deteriorations in the terms of trade. During expansions, the Japanese CB does not seem to target nominal or real exchange rates. To investigate the effects of the above regime-switching of the Japanese monetary authorities on the economy, the paper simulates a New Keynesian macroeconomic model and, then, it derives its impulse response functions for inflation, output gap, nominal or real exchange rates and the short term nominal interest rate following home labor productivity, foreign output, foreign inflation or foreign nominal interest rate exogenous shocks. The results of this exercise clearly indicate that, under the recession regime, Japanese monetary authorities are interested in reducing the volatility effects of an exogenous shock to nominal or real exchange rates (the terms of trade) coming from domestic real productivity, foreign output and inflation.

The plan of the paper is as follows. Section 2 presents our threshold monetary policy rule model for Japan, which considers exchange rate deviations. Section 3 presents the econometric methodology of the paper and carries out its estimation. Section 4 conducts the simulation exercise of the New Keynesian model based on the estimates of the model provided in Section 4. Section 5 concludes the paper.

2. Model Set Up

2.1 Open economy linear monetary policy rules

For an open economy, forward looking linear monetary policy rule models assert that the short term nominal interest rate, denoted as i_t, which is the main operating instrument of monetary policy of the CB, is targeted within any operating period t as follows:[1]

$$i_t^* = \bar{i} + \beta\left[E_t(\pi_{t+n}) - \pi^*\right] + \gamma E_t(\tilde{y}_{t+k}) + \delta E_t(\tilde{z}_{t+l}) \qquad (1)$$

where i_t^* is the desired level of interest rate i_t, i.e. the nominal interest rate when inflation, output and the additional variables are at their target levels, $E_t(\cdot) \equiv E(\cdot \mid \Omega_t)$ is the conditional expectation on the current information set of the economy at time t, denoted as Ω_t, $\bar{i}$ is a constant which denotes the long run equilibrium level of short interest rate i_t^*, π_{t+n} is the inflation rate n-periods ahead, where π^* stands for the CB's target level for inflation, $\tilde{y}_{t+k}$ denotes the real output gap rate k-periods ahead

[1] See, e.g., Clarida *et al* (1998).

and, finally, $\tilde{z}_{t+l}$ is a variable which captures changes in nominal (or real) exchange rates.

If we take into account the CB's tendency to smooth out changes in interest rates i_t over time according to the partial equilibrium model,[2]

$$i_t = (1-\rho)i_t^* + \rho i_{t-1} + \varepsilon_t \tag{2}$$

where $\rho \in [0,1]$ reflects the degree of smoothness and ε_t is an $i.i.d.(0,\sigma^2)$ monetary policy shock, then the monetary policy rule given by (1) can be written as

$$i_t = (1-\rho)\left(\bar{i} + \beta\left[E_t(\pi_{t+n}) - \pi^*\right] + \gamma E_t(\tilde{y}_{t+k}) + \delta E_t(\tilde{z}_{t+l})\right) + \rho i_{t-1} + \varepsilon_t . \tag{3}$$

2.2 An open economy threshold monetary policy rule model

The open economy monetary policy rule given by (3) assumes that the monetary authorities respond symmetrically to inflation rate, output and nominal (or real) exchange rate deviations from their desired (targeted) levels. However, this may not be true in practice. In the literature, there are many recent studies which show that the above responses are asymmetric and change with the business cycle conditions, reflecting the expansion and recession regimes of the economy.[3] These studies imply that the policy rule parameters of model (3), namely β (beta), γ (gamma) and δ (delta) depend on the state of the economy. To capture shifts in these parameters of this type, next we suggest the following forward looking threshold monetary policy rule model:

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[2] Interest rates smoothing can stem from various reasons such as the fear of disrupting capital markets, the loss of credibility from sudden large policy reversals or the need for consensus building to support a policy change. Moreover, the central bank may regard the interest rate smoothing as a learning device due to imperfect information (see, e.g., Clarida *et al*. (1998)).

[3] This evidence is mainly provided for closed economy monetary policy rules (see, e.g., Bec *et al*. (2002), Surico (2003)). But, there are recently studies which indicate significant effects of exchange rate changes on interest rates in a multivariate framework used to estimate open economy New

$$
i_t = \begin{cases} (1-\rho_1)\left(a_1 + \beta_1\left[E_t(\pi_{t+n}) - \pi^*\right] + \gamma_1 E_t(\tilde{y}_{t+k}) + \delta_1 E_t(\tilde{z}_{t+l})\right) + \rho_1 i_{t-1} + \varepsilon_t, \ \text{if } q_{t\text{-}d} \le \overline{q} \\ \\ (1-\rho_2)\left(a_2 + \beta_2\left[E_t(\pi_{t+n}) - \pi^*\right] + \gamma_2 E_t(\tilde{y}_{t+k}) + \delta_2 E_t(\tilde{z}_{t+l})\right) + \rho_2 i_{t-1} + \varepsilon_t, \ \text{if } q_{t\text{-}d} > \overline{q} \end{cases} \quad (4)
$$

where q_t denotes the threshold variable indicating the current business conditions state of the economy upon which shifts in policy rule parameters depend, and $\overline{q}$ denotes the threshold parameter. As noted in the introduction, a nature choice of threshold variable which can be used to reflect the above changes in policy parameters is the real output gap deviations variable, $\tilde{y}_t$. Thus, we will henceforth denote $q_t \equiv \tilde{y}_t$.

Threshold model (4) belongs to the class of regime-switching monetary policy models (see, e.g., Davig and Leeper (2007)). This identifies the following two monetary policy regimes: "1" and "2". The first of the captures recessionary conditions in the economy and it is defined by condition $q_t \equiv \tilde{y}_t \le \overline{q}$. The second reflects expansionary, defined by $q_t \equiv \tilde{y}_t \le \overline{q}$. The specification of these two regimes through threshold model (4) has some attractive features, compared to other regime-switching models introduced in the literature or the intervention dummy-variables analysis often used in practice to distinguish different monetary policy regimes. Compared to the last approach, our threshold model captures regime-switching endogenously from the data. It does not rely on exogenous information based on certain events. In contrast to Markov regime switching models (MRS) where regime shifts are driven by a latent variable following a Markov chain (see, e.g., Davig and Leeper (2007)), threshold model (4) directly links regime-switching to changes in the business cycle conditions measured by an observable variable, such as the output gap. The latent variable assumed by MRS models may not always reflect business conditions. Finally, compared to logistic smooth transition regression (STR) models (see, e.g., Milas and Naraidoo (2010)), it can better capture abrupt (or aggressive) adjustments of nominal interest rate i_t in response to inflation or any other explanatory variable entering the RHS of model (4), often observed in practice.

Keynsian dynamic stochastic general equilibrium (DSGE) models (see, e.g. Lubik and Schorfheide (2007), Justiniano and Preston (2010), Liu and Mumtaz (2010)).

Each of the two regimes implied by model (4) can be characterized as anti-inflationary and stabilizing towards real output gap and exchange rate deviations if we have β_i>1, γ_i>0 and δ_i>0, for i={1,2}. The positive sign of δ_i means that monetary authorities are concerned about exchange rate movements. For instance, if variable $\tilde{z}_t$ denotes changes in the nominal exchange rate between the home and a foreign country, where exchange rate is defined as the home country currency per unit of the foreign currency, then the positive sign of δ means that depreciation (appreciation) of the nominal exchange rate is expected to lead to an increase (decrease) of nominal interest rate i_t. In a New Keynesian model (see, e.g., Lubik and Schorfheide (2007)), this will happen to control for a rise of domestic inflation coming from the imported goods price increases due to exchange rate depreciation and/or their corresponding changes in the terms of trade. The latter can be affected by the different home business cycle conditions, especially the recessionary.

3. Empirical Analysis

The main goal of this section is to estimate the threshold monetary policy model given by equation (4). This is done assuming that Japan is the home country, while US is the foreign. As also is well known in the literature (see, e.g. Tachibana (2004)), inflation movements are concerned Japanese monetary authorities since the late of 70's, after nearly a decade that the Japanese economy were suffering by high inflation.

Our analysis starts with estimating the linear monetary policy rule model (3). A comparison of this model with threshold model (4) is useful, as it can indicate if there exist substantial differences in the estimates of the policy rule parameters of these two models which may lead to wrong inference about the objectives of monetary policy authorities. Another reason for estimating standard monetary policy rule model (3) is in order to retrieve a sample average estimate of the target level of inflation rate. This is often used as an estimate of parameter π^* in empirical studies of monetary policy rules (see, e.g. Clarida *et al*. (2000)).

3.1. Data

In our empirical analysis, we use quarterly observations from 1973:I to 2010:II. For Japan, we use the "Call-Money" of the CB as interest rate i_t, while inflation rate π_t is calculated as $\pi_t = \frac{P_t - P_{t-4}}{P_{t-4}} \cdot 100$, where $P_t = \frac{\text{nominal } GDP_t}{\text{real } GDP_t} \cdot 100$ constitutes the GDP deflator. As a measure of the output gap, we employ the percentage change between real GDP and potential real GDP $\left(\tilde{y}_t = \frac{\text{real GDP}_t - \text{real potential GDP}_t}{\text{real potential GDP}_t} \cdot 100 \right)$. The real GDP and the real potential GDP are expressed in annual rates using year 2005 as base year. Consistently with our definition of variable $\tilde{z}_t$ assumed in previous section, nominal exchange rate, denoted as e_t, is defined as the units of home country's currency per unit of the foreign 's (US), while real exchange rate is defined as $s_t = e_t \frac{P_t^{\$}}{P_t^{¥}}$, where $P_t^{\$}$ is US's GDP deflator.

All the above series were downloaded from the OECD database for Japan. The deviations of nominal (or real) exchange rate series used in the estimation of model (4), or (3), are taken as the percentage changes of e_t (or s_t) from their Hodrick-Prescott filter. Note that, apart from the above series, in our empirical analysis we will as instruments variables the US' federal funds rate and the unemployment rate of Japan. The latter is the seasonally adjusted civilian unemployment rate including persons 16 years of age and older at the last month of each year. The Fed rate is taken from the Federal Reserve Bank of st. Louis.

Table 1 presents descriptive statistics, namely the mean and standard deviation, of all variables of monetary policy rule model (4). Graphs of these variables are given in Figures 1A-B. The table also reports values of these statistics for the short term real interest rate series, denoted as r_t. This is calculated as $r_t = \bar{i}_t - \bar{\pi}_t$, where $\bar{i}_t$ and $\bar{\pi}_t$ denote the four quarter moving averages of current and past interest and inflation rates, respectively (see Kim *et al*. (2005)).[4] The results of Table 1 indicate that the

[4] This method of estimating real interest rate r is often used in the literature (see Kim *et al*. (2005)).

average output gap is negative and has substantial volatility (standard deviation). This can be attributed to the fact that that, for our sample, Japanese economy lies in the recession regime for most of the time (see Figure 1A). Very high volatility have also all financial variables of model (4), namely nominal interest rate i_t, inflation rate π_t and nominal (or real) exchange rate changes $\tilde{z}_t$. Finally, note that the mean of nominal or real exchange rate deviations from their long-term trend is negative which means that exchange rate was overvalued for most of the time of our sample. This can be confirmed by the inspection of Figure 1B. The negative value of the mean of real exchange deviations reported in the table means that the terms of trade of Japan's economy were not competitive for most of our sample periods.

Table 1*:* Descriptive statistics

	JAPAN	
Variables	*Mean*	*Std. Dev.*
Nominal Interest Rate	3.84	3.62
Real Interest Rate	1.95	2.70
Inflation rate	1.95	4.54
Output gap deviation	-0.33	2.24
Real exchange rate deviation	-0.17	8.49
Nominal exchange rate deviation	-0.20	8.59

Notes: *St. Dev*. stands for standard deviation. The sample period of our data is from 1973:I to 2010:II.

3.2 Estimation of the linear monetary policy rule model (3)

Replacing the expected values of the variables entering into the RHS of model (3) with their realized values leads to the following regression model:

$$i_t = (1-\rho)\left(\alpha + \beta\pi_{t+n} + \gamma\tilde{y}_{t+k} + \delta\tilde{z}_{t+l}\right) + \rho i_{t-1} + u_t \qquad (3')$$

where

$$u_t = -(1-\rho)\left\{\beta(\pi_{t+n} - E(\pi_{t+n} \mid \Omega_t)) + \gamma(\tilde{y}_{t+k} - E(\tilde{y}_{t+k} \mid \Omega_t)) + \delta(\tilde{z}_{t+l} - E(\tilde{z}_{t+l} \mid \Omega_t))\right\} + \varepsilon_t$$

and $\alpha = \bar{i} - \beta\pi^{*}$. Parameter estimates of this model are reported in Table 2. These are obtained based on the generalized method of moments (GMM) estimation procedure which considers following moment (orthogonality) conditions:

$$E\left[i_t - (1-\rho)\left(\alpha + \beta\pi_{t+n} + \gamma\tilde{y}_{t+k} + \delta\tilde{z}_{t+l}\right) - \rho i_{t-1} \mid \mathbf{h_t}\right] = \mathbf{0}, \qquad (5)$$

where $\mathbf{h_t}$ is a vector of instrumental variables. In our analysis, this vector includes the constant and one up to four lagged values of the following variables: i_t, π_t, $\tilde{y}_t$ and $\tilde{z}_t$, as well as the Japanese unemployment rate. The last variable is used as an instrument which can capture future movements in output gap deviations $\tilde{y}_t$.

The lead intervals *n*, *k* and *l* of the RHS variables of model $(3')$ considered in our GMM estimation procedure are set to *n*=1, *k*=0 and *l*=0 (see, also Clarida *et al* (1998)). These imply that the CB reacts to one quarter ahead deviations of the expected values of π_{t+n}, and the currently observed level of real output gap $\tilde{y}_t$ and exchange rate changes $\tilde{z}_t$. Due to the forward looking specification of model $(3')$ and, hence, the overlapping nature of some of its RHS variables, in our GMM estimation procedure we allow for a moving average variance covariance matrix of the error term with 4 lags based on the Newey-West method.

Table (2) presents two different sets of results. The first assumes that variable $\tilde{z}_t$ reflects changes in nominal exchange rate e_t, while the second in the real, denoted s_t. The results of the table indicate that the CB of Japan does not seem to follow a strong antinflationary policy, given that the estimate of β is found to be less than unity. This is true independently. This result is quite surprising given the strong antinflationary attitude of Japanese monetary authorities since the middle of the seventies (see, e.g., Tachibana (2004)). The estimates of the other structural parameters of model (3) reveal that there are significant effects of output gap deviations and exchange rate on the short term rate i_t. Although the slope coefficients of these variables have the correct sign, their magnitude does not seem to be big enough. In particular, the estimates of γ are far below than unity, while those of δ are very close to zero. The last result means that the CB of Japan responds very weakly to exchange rate changes. This finding is also consistent with evidence provided in other studies of the literature estimating open monetary policy rules for Japan (see, e.g., Clarida *et al*. (1998)). Regarding the estimates of the remaining parameters the model, the results of the table indicate that the CB of Japan is characterized by a strong smoothing attitude of interest rate i_t, given that the value of autoregressive coefficient ρ is found to be close to unity. Finally, note that the sample estimates of inflation target rate π^* implied by the estimates of the table are very close to 2%, given by 1.80%. The values of the long- run equilibrium level of real interest rate, r^*, implied by the estimates of a and π^* (namely $r^* = a - \pi^*$) are very close to those reported in Table 1 (i.e., 2.02% and 2.04% for the cases that nominal and real exchange rate deviations are used as explanatory variable, respectively).

As mentioned above, the result that Japan follows a more accommodating inflation policy is quite striking, given the declared strong antinflationary attitude of the CB and the governments of this country. This can be attributed to ignoring possible structural breaks in the policy parameters of model (3) which may be associated with changes in business conditions allowed by our threshold model (4). To examine this, next we carry out Bai's and Perron (2003) multiple structural breaks test based on the $\sup F_T\left(l+1/l\right)$ statistic.[5] In order to estimate the number of structural breaks considered by this test, we will use the Bayesian Information Criterion (BIC). The results of the above test statistic revealed the existence of three major structural shifts in the structural parameters of model (3). In Table 3, we report the dates of these breaks, with their confidence intervals in parentheses. These are 1978:4, 1984:3 and 1991:2. The recession periods of our sample officially announced by the Japanese government, graphically presented In Figure 1A with the bars, indicate that between these dates there are quite lasting recession periods.

[5] To conduct this test, we consider a reduced, backward looking version of model (3) is given as $i_t = \alpha + b\tilde{\pi}_{t-1} + c\tilde{y}_{t-1} + d\tilde{z}_{t-1} + \rho i_{t-1} + e_t$ where $\tilde{\pi}_t = \pi_t - \pi^*$.

Table 2: Estimates of the standard model

$$i_t = (1-\rho)(a + \beta(E_t \pi_{t+1}) + \gamma E_t(\tilde{y}_t) + \delta E_t(\tilde{z}_t)) + \rho i_{t-1} + \varepsilon_t$$

Parameters	*Nominal exchange rate deviations*	*Real exchange rate deviations*
a	2.23*** (0.43) [5.21]	2.24*** (0.43) [5.26]
ρ	0.93*** (0.02) [58.24]	0.92*** (0.02) [57.74]
β	0.88*** (0.10) [8.80]	0.89*** (0.10) [9.18]
γ	0.79*** (0.23) [3.44]	0.77*** (0.23) [3.39]
δ	0.08* (0.04) [1.92]	0.09** (0.04) [2.22]
R^2	0.97	0.97
J-stat	18.63 (0.29)	18.44 (0.30)
MQLR (p-value)	0.0	0.0
KLM (p-value)	0.0	0.0
JKLM (p-value)	0.003	0.004
M.S.E.	0.387	0.386
Theil	0.119	0.119

Notes: standard errors are included in parentheses and t-statistics in brackets. Newey-West covariance matrix with 4 lags is used for standard errors. ***, **, * denote significant at the 1%, 5%, 10% level, respectively. r^* is obtained from the relationship $r^* = a - \pi^*$. *MQLR*, *KLM* and *JKLM* denote the *LR* test statistic of Moreira (2003), and the *LM* based test statistics of Kleibergen (2005, 2007), respectively. These are robust to weak instruments statistics testing the null hypothesis H_0: $\beta = \gamma = \delta = \rho = 0$.

Table 3: Sequential Bai-Perron tests for breaks		
MODEL	*BIC*	*Sequential*
Nominal Exchange Rate Gap	1978:4 (1978:4,1979:1)	1991:2 (1991:1,1993:1)
	1984:2 (1984:1,1986:4)	
	1991:2 (1991:1,1992:1)	
Real Exchange Rate Gap	1978:4 (1978:4,1979:1)	1991:2 (1991:1,1993:1)
	1984:2 (1984:1,1987:2)	
	1991:2 (1991:1,1992:1)	

Notes: The table presents the identified dates of the sequential Bai-Perron structural breaks tests for model $i_t = \alpha + b\tilde{\pi}_{t-i} + c\tilde{y}_{t-1} + d\tilde{z}_{t-1} + \rho i_{t-1} + e_t$. In parentheses, we report the confidence dates of these dates.

3.3 Estimation of the threshold monetary policy rule model

3.3.1 Estimation and testing procedure

In this section, we present estimates our threshold monetary policy model (4). To estimate the model, we consider the same values of the lead intervals *n*, *k* and *l* to those assumed for the estimation of model (3), i.e. *n*=1, *k*=0 and *l*=0. After replacing the expected values of the explanatory variables of the model with their realized values, this takes the following regression form:

$$\begin{aligned} i_t \;=\; & \left(c_1 + c_2(\pi_{t+1} - \pi^*) + c_3\tilde{y}_t + c_4\tilde{z}_t + c_5 i_{t-1}\right) I(\tilde{y}_t \leq \overline{q}) \\ & + \left(c_1' + c_2'(\pi_{t+1} - \pi^*) + c_3'\tilde{y}_t + c_4'\tilde{z}_t + c_5' i_{t-1}\right) I(\tilde{y}_t > \overline{q}) + \varepsilon_t, \end{aligned} \qquad (6)$$

where ε_t is the regression error term, $I(\cdot)$ denotes an indicator function which characterizes the monetary policy regime. The policy rule parameters and autoregressive coefficient ρ can be retrieved from the structural parameters of model (4) through the following relationships:

$$c_1 = (1-\rho_1)\alpha_1,\ c_2 = (1-\rho_1)\beta_1,\ c_3 = (1-\rho_1)\gamma_1,\ c_4 = (1-\rho_1)\delta_1, c_5 = \rho_1$$

and

$$c_1' = (1-\rho_2)\alpha_2,\ c_2' = (1-\rho_2)\beta_2,\ c_3' = (1-\rho_2)\gamma_2,\ c_4' = (1-\rho_2)\delta_2, c_5' = \rho_2 .$$

Since the threshold variable of the model $\tilde{y}_t$ can be correlated with interest rate i_t due to the contemporaneous nature of both of these variables, in the estimation procedure of model's (6) parameters we will treat threshold variable $\tilde{y}_t$ as an endogenous variable. Ignoring this will lead to biased estimates of the threshold parameter $\overline{q}$ and, hence, the estimates of the remaining parameters of the model. Then, to estimate the above model will employ the recently suggested method of Kourtelos, Stengos and Tan (2008) (henceforth KST). This can provide a consistent estimate of the threshold parameter $\overline{q}$ allowing for an endogenous threshold variable.[6] Given this estimate, then we can derive consistent estimates of the remaining parameters of our threshold model (4) (or (6)) based on the GMM estimation method. These estimates will be robust to the presence of heteroscedasticity or serial correlation in the disturbance terms ε_t. They can be obtained by splitting the sample into the sub-samples implied by the estimate of $\overline{q}$.

More specifically, to derive consistent estimates of $\overline{q}$ we solve out the following search problem over different possible values of $\overline{q}$ belonging to set Q :[7]

[6] This method constitutes an extension of Caner and Hansen (2004) two-stage least squares (or GMM) method often used in the estimation of forward looking threshold models like (4) which treats the threshold variable as exogenous.

[7] Note that, in practice, to avoid taking very extreme estimates of $\overline{q}$ which may be meaningless and/or leave a very small number of observations in each of the sub-samples implied by threshold model (6), we search for estimates of $\overline{q}$ from the 15th up to 85th percentile of the sample values of q_t .

$$\hat{\bar{q}} = \arg\min_{\bar{q}\in Q} S_T(\bar{q}),$$

where $\hat{\bar{q}}$ denotes the estimator of $\bar{q}$ and

$$\begin{aligned} S_T(\bar{q}) \quad = \quad & \sum_{t=1}^{T}\Big[\left(i_t - c_1 - c_2\hat{\tilde{\pi}}_{t+1} - c_3\hat{\tilde{y}}_t - c_4\hat{z}_t - c_5 i_{t-1} - \kappa\lambda(\bar{q} - d_2' h_t)\right)^2 I(\tilde{y}_t \le \bar{q}) \\ & + \left(i_t - c_1' - c_2'\hat{\tilde{\pi}}_{t+1} - c_3'\hat{\tilde{y}}_t - c_4'\hat{z}_t - c_5' i_{t-1} - \kappa\lambda^*(\bar{q} - d_2' h_t)\right)^2 I(\tilde{y}_t > \bar{q})\Big] \end{aligned}$$

is the sum of squared errors of model (6). This is calculated based on LS based predictions (fitted values) of the expected variables $E_t(\tilde{\pi}_{t+1})$, $E_t(\tilde{y}_t)$ and $E_t(z_t)$, denoted as $\hat{\tilde{\pi}}_{t+1}, \hat{\tilde{y}}_t$, and $\hat{\tilde{z}}_t$, respectively. These are obtained based on the following regressions:

$$\tilde{\pi}_t = \mathbf{d}_1'\mathbf{h}_t + e_t, \quad \tilde{y}_t = \mathbf{d}_2'\mathbf{h}_t + v_t \;\text{ and }\; \tilde{z}_t = \mathbf{d}_3'\mathbf{h}_t + \upsilon_t$$

respectively, where $\mathbf{h}_t$ is a vector of instrumental variables, and e_t, v_t and υ_t are three regression error terms. The two terms $\kappa\lambda(\bar{q} - \mathbf{d}_2'\mathbf{h}_t)$ and $\kappa\lambda^*(\bar{q} - \mathbf{d}_2'\mathbf{h}_t)$ entered into the sum of squared errors function $S_T(\bar{q})$ are bias correction terms of the conditional expectation $E(i_t \mid \mathbf{h}_t)$ which are due to the endogenous nature of the threshold variable q_t implying $\kappa = E(v_t \varepsilon_t) \neq 0$,

$$\lambda(\bar{q} - \mathbf{d}_2'\mathbf{h}_t) = E(v_t \mid \mathbf{h}_t, \tilde{y}_t \le \bar{q}) = \frac{\varphi(\bar{q} - \mathbf{d}_2'\mathbf{h}_t)}{\Phi(\bar{q} - \mathbf{d}_2'\mathbf{h}_t)}$$

and

$$\lambda^*(\bar{q} - \mathbf{d}_2'\mathbf{h}_t) = E(v_t \mid \mathbf{h}_t, \tilde{y}_t > \bar{q}) = \frac{\varphi(\bar{q} - \mathbf{d}_2'\mathbf{h}_t)}{1 - \Phi(\bar{q} - \mathbf{d}_2'\mathbf{h}_t)}.$$

The last two terms are the inverse Mills ratio bias correction terms, where $\varphi(\cdot)$ and $\Phi(\cdot)$ denote the normal and cumulative probability density functions, respectively.[8]

Estimates of model's (6) parameters based on the above KST estimation procedure are reported in Table 4. Namely, these include the estimates of threshold parameter $\overline{q}$ and vector of parameters $\mathbf{\Theta} = (c_1, c_2, c_3, c_4, c_5, c_1', c_2', c_3', c_4', c_5')'$. The table also reports values of policy rule parameters beta, gamma and delta of threshold model (4) under each regime i implied by the point estimates of the above vector. The confidence intervals of the estimates of $\overline{q}$, reported in the table, are calculated based on Caner and Hansen's (2004) procedure. In addition to the above, Table 4 reports values of Chow's optimal test statistic for the following null hypothesis:

$$H_0: \ c_1 = c_1' \text{ and } c_2 = c_2' \text{ and } c_3 = c_3' \text{ and } c_4 = c_4' \text{ and } c_5 = c_5',$$

against its alternative:

$$H_0: \ c_1 \neq c_1' \text{ or } c_2 \neq c_2' \text{ or } c_3 \neq c_3' \text{ or } c_4 \neq c_4' \text{ or } c_5 \neq c_5'.$$

The above null hypothesis means that that the monetary policy rule is linear and is given by the standard model (3), while its alternative supports the specification of our threshold monetary policy rule model (4). Testing the above null hypothesis against its alternative is crucial in investigating whether the monetary policy rule of the CB of Japan changes according to different business conditions, i.e. the expansion and recession regimes of the economy, as is predicted by model (4).

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[8] The adjustment of regression model (6) (or sum $S_T(\overline{q})$) with the two bias correction terms $\kappa\lambda(\overline{q} - \mathbf{d}_2'\mathbf{h}_t)$ and $\kappa\lambda^*(\overline{q} - \mathbf{d}_2'\mathbf{h}_t)$ can be easily seen by noticing that, when $E(v_t\varepsilon_t) \neq 0$, the following relationship holds under the assumption that the error terms ε_t and v_t are normally distributed:

$$\begin{aligned} E(i_t \mid \mathbf{h}_t) \;&=\; \left(c_1 + c_2 E_t(\tilde{\pi}_{t+1}) + c_3 E_t(\tilde{y}_t) + c_4 i_{t-1} + \kappa\lambda(\overline{q} - \mathbf{d}_2'\mathbf{h}_t)\right) I(\tilde{y}_t \leq \overline{q}) \\ &+ \left(c_1' + c_2' E_t(\tilde{\pi}_{t+1}) + c_3' E_t(\tilde{y}_t) + c_4' i_{t-1} + \kappa\lambda^*(\overline{q} - \mathbf{d}_2'\mathbf{h}_t)\right) I(\tilde{y}_t > \overline{q}) \end{aligned}$$

For a known value of threshold parameter $\overline{q}$, Chow's test statistic is defined as follows:

$$Wopt(\overline{q}) = T\begin{bmatrix} \hat{\boldsymbol{\theta}}_1(\overline{q}) - \hat{\boldsymbol{\theta}}_2(\overline{q}) \\ \tau\hat{\boldsymbol{\theta}}_1(\overline{q}) + (1-\tau)\hat{\boldsymbol{\theta}}_2(\overline{q}) \end{bmatrix}' \hat{\mathbf{V}}^{-1} \begin{bmatrix} \hat{\boldsymbol{\theta}}_1(\overline{q}) - \hat{\boldsymbol{\theta}}_2(\overline{q}) \\ \tau\hat{\boldsymbol{\theta}}_1(\overline{q}) + (1-\tau)\hat{\boldsymbol{\theta}}_2(\overline{q}) \end{bmatrix}$$

where $\hat{\boldsymbol{\theta}}_1 = \left(\hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{c}_4, \hat{c}_5\right)'$ and $\hat{\boldsymbol{\theta}}_2 = \left(\hat{c}_1', \hat{c}_2', \hat{c}_3', \hat{c}_4', \hat{c}_5'\right)'$ are the two vector of parameters of threshold model (4) corresponding to the two different regimes "1" and "2", respectively, $\tau = \frac{T_1}{T}$, where T_1 is the number of sample observations corresponding to regime "1", and $\hat{\mathbf{V}}$ is a consistent estimator of the variance-covariance matrix of the following vector of coefficient differences $\left[\hat{\boldsymbol{\theta}}_1(\overline{q}) - \hat{\boldsymbol{\theta}}_2(\hat{\overline{q}}), \tau\hat{\boldsymbol{\theta}}_1(\overline{q}) + (1-\tau)\hat{\boldsymbol{\theta}}_2(\overline{q})\right]$ given as follows:

$$\mathbf{V} = \mathbf{R}\left(\mathbf{M}'\mathbf{G}\mathbf{M}\right)^{-1}\mathbf{R}',$$

where

$$\mathbf{R} = \begin{bmatrix} \mathbf{I}_4 & -\mathbf{I}_4 \\ \tau\mathbf{I}_4 & (1-\tau)\mathbf{I}_4 \end{bmatrix}, \ \mathbf{M} = \begin{bmatrix} \frac{1}{T}\mathbf{h}_1'\mathbf{x}_1 & \mathbf{0}_{k\times 4} \\ \mathbf{0}_{k\times 4} & \frac{1}{T}\mathbf{h}_2'\mathbf{x}_2 \end{bmatrix}, \ \mathbf{G} = \begin{bmatrix} \tau\hat{\boldsymbol{\Sigma}}_1 & \mathbf{0}_{k\times k} \\ \mathbf{0}_{k\times k} & (1-\tau)\hat{\boldsymbol{\Sigma}}_2 \end{bmatrix},$$

$$\hat{\boldsymbol{\Sigma}}_1 = \mathbf{h}_1' \begin{bmatrix} \hat{\varepsilon}_{11}^2 & 0 & . & 0 \\ 0 & \hat{\varepsilon}_{12}^2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \hat{\varepsilon}_{1T_1}^2 \end{bmatrix} \mathbf{h}_1, \quad \hat{\boldsymbol{\Sigma}}_2 = \mathbf{h}_2' \begin{bmatrix} \hat{\varepsilon}_{21}^2 & 0 & . & 0 \\ 0 & \hat{\varepsilon}_{22}^2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & \hat{\varepsilon}_{2(T-T_1)}^2 \end{bmatrix} \mathbf{h}_2',$$

$\mathbf{h}_1, \mathbf{h}_2, \mathbf{x}_{1,}\ \mathbf{x}_2$ are matrices whose columns consist of the time series observations of the instrumental variables and regressors used for the estimation of model (6) corresponding to regimes "1" and "2", respectively, and, finally, $\hat{\varepsilon}_{1t}$ and $\hat{\varepsilon}_{2t}$ denote the residuals of the threshold model for the two regimes.

Since the threshold parameter $\overline{q}$ is not known and estimated by the data following the search procedure described before, the Chow's optimal test statistic presented in Table 3 is the supremum of $Wopt(\overline{q})$, for all values of $\overline{q} \in Q$, defined as

$$SupWopt(\overline{q}) \equiv \sup_{\overline{q} \in Q} Wopt(\overline{q}) .$$

This statistic has a non conventional distribution which is the supremum of a chi-square distribution. Since threshold parameter $\overline{q}$ is not identified under the null hypothesis, critical values of the asymptotic distribution of statistic $SupWopt(\overline{q})$ (or p-values) are calculated based on the bootstrap procedure suggested by Hansen (1996). In particular, we define the pseudo dependent variable $i_t^*(\overline{q}) = \hat{\varepsilon}_t(\overline{q})h_t$, where $\hat{\varepsilon}_t(\overline{q})$ are the estimated residuals under the unrestricted model for all $\overline{q} \in Q$ and $h_t \sim N(0,1)$. Then, using this pseudo dependent variable instead of i_t and keeping fixed the values of regressors, we re-estimate model (6) based on the GMM method and calculate test statistic $Wopt(\overline{q})$, for all values of $\overline{q}$. These values $Wopt(\overline{q})$ are used to calculate $SupWopt(\overline{q})$. The above procedure is repeated 1000 times and the p-value of $SupWopt(\overline{q})$, reported in Table 4, is calculated as the relative number of times that the simulated values of $SupWopt(\overline{q})$ exceed its sample estimate.

The results of Table 4 clearly indicate that monetary policy in Japan is subject to regime-switching. The reported values of statistic $SupWopt(\overline{q})$ clearly reject the null hypothesis $c_1 = c'_1$, $c_2 = c'_2$, $c_3 = c'_3$, $c_4 = c'_4$, $c_5 = c'_5$ at a very low p-value (about 1%). This result clearly supports our threshold monetary policy rule model (4) against its linear specification given by equation (3), which does not involve regime-switching. Further support of our threshold model relative to model (3) can be obtained from the values of the mean squared error (MSE) reported in Tables 4 and 2. These are found to be smaller for our model. Finally, note that our model also satisfies the overidentifying restrictions implied by the GMM estimation procedure. These are tested based on Sargan's statistics *J-Stat1 and J-Stat2*, for each regime *i*, reported in

Table 4, for .[9] The values of test statistics show that model (4) constitutes a correct specification of the data for each regime by the estimate of threshold parameter $\overline{q}$.

The estimates of threshold parameter $\overline{q}$ reported in Table 4 indicate that regime "1" (reflecting recessionary conditions) is defined by condition $\tilde{y}_t \leq 0.29$, while regime "2" (reflecting expansionary conditions) by $\tilde{y}_t > 0.29$. The 95% confidence interval of the threshold parameter value is found to be (0.12,0.39) which does not include the value of zero. The last result implies that Japanese monetary authorities are very likely to change their rule towards a more aggressive towards inflation deviations monetary policy (as will be seen below) when the real output gap is clearly bigger than zero. That is, when the economy shows clearer signs of expansion. The econometric implication of this result is that empirical studies using zero as a known value of $\overline{q}$ may lead to bias and inconsistent estimates of the parameters of the threshold model (4).

Inspection of Figure 1A, which presents threshold variable $\tilde{y}_t$ against its threshold value (0.29%), reveals the Japanese economy was found to be in recession over the following periods: 1973-1977, 1980-1988, 1994-1996, 1997-2006 and 2007-2010. These cover some of the periods identified by Bai's and Perron testing procedure, reported in Table 3. They are also very closely related to those recession periods officially announced by the Japanese government. The troughs or peaks of these recession periods are reported Table 5. The graphs of Figure 1B, which presents the nominal (or real) exchange rate series deviations and inflation rate, indicate that periods of recession occurred in Japan since the middle of seventies are associated with very low levels of inflation and substantial currency overvaluation or deterioration in the terms of trade.

[9] The *J-Stat* (referred to as Sargan's test statistic) follow the chi-square distribution. The degrees of freedom (DF) of this distribution are calculated as the number of orthogonality conditions implied in

Table 4: Estimates of threshold monetary policy rule model (6)

$$i_t = \begin{cases} c_1 + c_2\tilde{\pi}_{t+1} + c_3\tilde{y}_t + c_4 z_t + c_5 i_{t-1} + u_t, & \text{if } \tilde{y}_t \le \overline{q} \\ c_1' + c_2'\tilde{\pi}_{t+1} + c_3'\tilde{y}_t + c_4' z_t + c_5' i_{t-1} + u_t, & \text{if } \tilde{y}_t > \overline{q} \end{cases}$$

Parameters	*Nominal exchange rate deviations*	*Real exchange rate deviations*
$c_1 = (1-\rho_1)a_1$	0.38*** ($a_1 = 5.43$) [4.12]	0.38*** ($a_1 = 5.43$) [4.16]
$c_2 = (1-\rho_1)\beta_1$	0.07*** ($\beta_1 = 1.0$) [6.55]	0.07*** ($\beta_1 = 1.0$) [6.79]
$c_3 = (1-\rho_1)\gamma_1$	0.10** ($\gamma_1 = 1.43$) [2.35]	0.09** ($\gamma_1 = 1.29$) [2.37]
$c_4 = (1-\rho_1)\delta_1$	0.01** ($\delta_1 = 0.14$) [2.37]	0.01** ($\delta_1 = 0.14$) [2.48]
$c_5 = \rho_1$	0.93*** [83.25]	0.93*** [80.91]
$c_1' = (1-\rho_2)a_2$	0.67*** ($a_2 = 2.23$) [3.47]	0.69*** ($a_2 = 2.30$) [3.60]
$c_2' = (1-\rho_2)\beta_2$	0.41*** ($\beta_2 = 1.37$) [5.55]	0.42*** ($\beta_2 = 1.40$) [5.79]
$c_3' = (1-\rho_2)\gamma_2$	0.34*** ($\gamma_2 = 1.13$) [6.74]	0.34*** ($\gamma_2 = 1.13$) [6.12]
$c_4' = (1-\rho_2)\delta_2$	-0.01 ($\delta_2 = -0.03$) [1.01]	-0.01 ($\delta_2 = -0.03$) [0.75]
$c_5' = \rho_2$	0.70*** [18.22]	0.70*** [18.97]
J-stat1 (p-value)	9.01 (0.91)	8.93 (0.92)
J-stat2 (*p*-value)	7.07 (0.97)	7.12 (0.97)
SupWopt (*p*-value)	0.003	0.002
Threshold value (95% C.I.)	0.29 [0.12,0.37]	0.29 [0.12,0.37]
MSE	0.349	0.35
Theil	0.115	0.115

Notes: The reported estimates are obtained by the GMM estimation procedure using Newey-West covariance matrix with 4 lags. t-statistics are in brackets. ***, **, * denote significant at the 1%, 5%, 10% level, respectively. The instruments used in the GMM estimation procedure are the constant and the following lagged values of short term interest rate, inflation gap, output gap, additional variable and unemployment rate $c, i_{t-1},..., i_{t-4}, \tilde{\pi}_{t-1},..., \tilde{\pi}_{t-4}, \tilde{y}_{t-1},..., \tilde{y}_{t-4}, z_{t-1},\ldots, z_{t-4}, u_{t-1},\ldots, u_{t-4}$. The grid search space covers the range of $[0.15\cdot T, 0.85\cdot T]$ for all cases.

the estimation procedure minus the number of estimated parameters. In our application, this number of

Regarding the estimates of the policy rule parameters reported in Table 4, namely the betas, gammas and deltas, the results of the table lead to the following conclusions. First, in contrast to those for the linear (standard) monetary policy rule model, given by (3), the estimates of betas and gammas for our threshold model clearly indicate that the Japanese monetary authorities respond aggressively to inflation and real output gap deviations. Their response seems to be stronger to inflation deviations during expansion periods. In contrast, under recession periods it becomes stronger with respect to real output gap deviations. These results imply that there is a slight asymmetry in the preferences of Japanese monetary authorities regarding inflation or real economic activity depending on whether the economy is in expansion or recession, respectively.

The bigger differences between the policy rule parameters across the two regimes are observed for delta coefficient, capturing the response of nominal interest rate i_t to nominal (or real) exchange rate deviations, $\tilde{z}_t$. The estimate of this coefficient is found to be significant only in the recession regime independently on whether the nominal, or real, exchange rate deviations is used as explanatory variable in model (4). The estimates of this coefficient in the recession regime are found to be 0.14. The positive sign of this coefficient is consistent with theory (see also Section 3.2). This asserts that an appreciation of the nominal, or real) exchange rate (implying a negative sign of $\tilde{z}_t$) will lead to a decrease in nominal interest rate i_t by the CB in order to capture the effects of a deterioration in the terms of trade and exchange rates on economic activity or inflation. This relationship can be justified by the graphs of i_t, π_t and $\tilde{z}_t$, presented by Figures 1A-B.

degrees of freedom is seventeen.

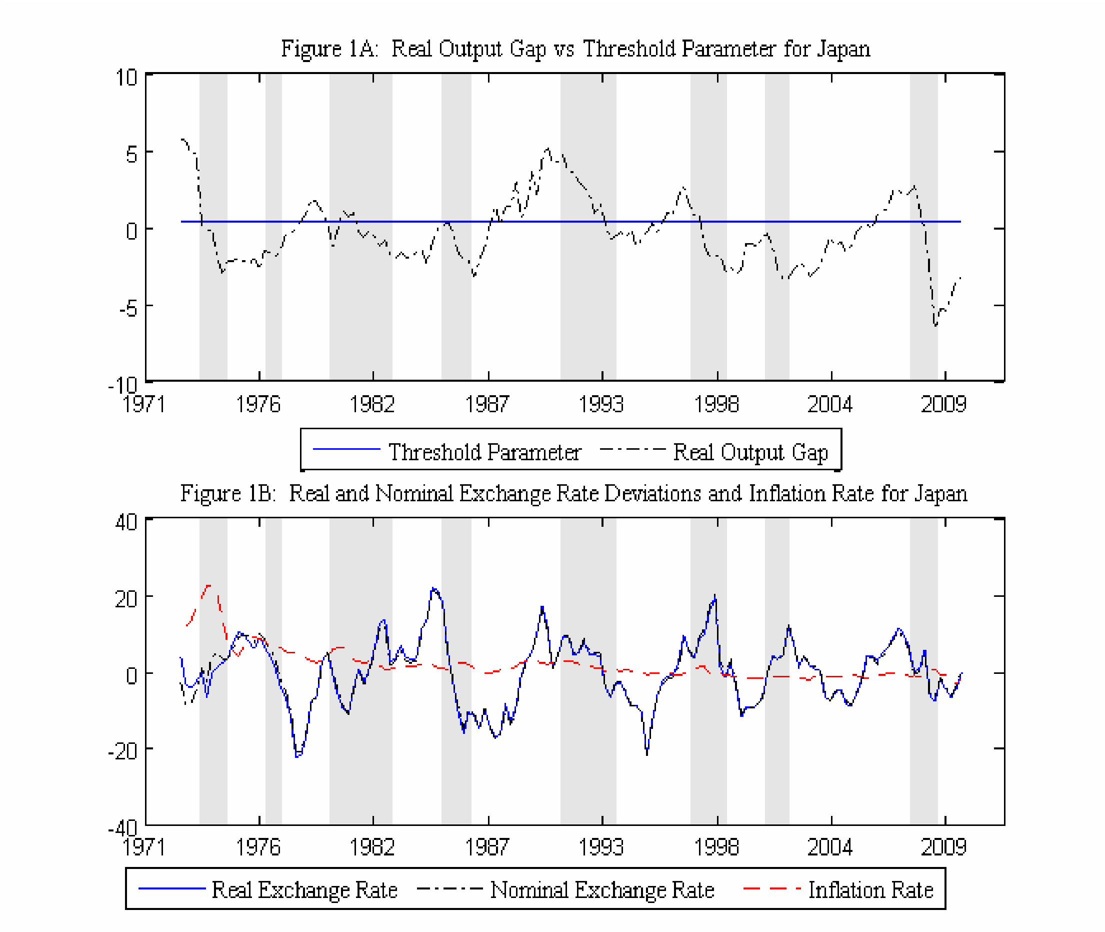


Figure 1A: Real Output Gap vs Threshold Parameter for Japan

Figure 1B: Real and Nominal Exchange Rate Deviations and Inflation Rate for Japan

Notes: Shaded areas indicate recessions.

4. Policy reaction under the threshold monetary policy model

The support of the threshold monetary policy (4) by our data raises a number of important questions about the effects of monetary policy on output and/or inflation fluctuations in the Japanese economy under the different regimes. In particular, it will be interesting to investigate the policy reasons for which short term interest rate i_t reacts to exchange rate deviations $\tilde{z}_t$ under the recession regime. As aptly stated by Davig and Leeper (2007), Liu, Waggoner and Zha (2009), when monetary policy rules are subject to regime-switching, then the effectiveness of the policy rule depends critically on the current policy regime of the economy and/or the expectations formations arising from a regime change. Furthermore, policy evaluation macroeconomic models relied on regime-switching policy monetary rules must be checked out for the determinacy (uniqueness) or stability of their solution before used in practice.

To answer the above questions, in this section we simulate a small open economy version of the New Keynesian (NK) model suggested in the literature by Gali and Monacelli (2005). This model assumes that the economy is driven by exogenous domestic productivity, foreign output, foreign nominal interest rate and foreign inflation shocks. These shocks are assumed that affect the real output gap, inflation, nominal (or real) exchange rate and the short-term interest rate. The monetary policy rule of this NK model is assumed that it is given by threshold model (4).

More specifically, the NK model that we consider is described by the following equations:[10]

$$\pi_t = bE_t(\pi_{t+1}) + \kappa_a \tilde{y}_t, \tag{7.a}$$

$$\tilde{y}_t = E_t(\tilde{y}_{t+1}) - \frac{\omega_a}{\sigma}\left(i_t - E_t(\pi_{t+1}) - r_t^n\right) \tag{7.b}$$

$$r_t^n = -\frac{\sigma(1+\varphi)(1-\rho_a)}{\sigma+\varphi\omega_a}a_t - \varphi\frac{\sigma(1-\omega_a)}{\sigma+\varphi\omega_a}E_t(\Delta y_{t+1}^*) \tag{7.c}$$

$$i_t - i_t^* = E_t(\Delta e_{t+1}) \tag{7.d}$$

$$\Delta s_t = \Delta e_t + \pi_t^* - \pi_t \tag{7.e}$$

and

[10] This model is linearized around a steady state inflation rate and output gap of zero to keep the analysis simple. All variables used in the model are expressed as log deviations from their steady state values.

$$i_t = \left[(1-\rho_1)\left(\beta_1 E_t(\pi_{t+1}) + \gamma_1 E_t(\tilde{y}_t) + \delta_1 E_t(s_t)\right) + \rho_1 i_{t-1}\right] I(\tilde{y}_t \leq \overline{q})$$

$$+ \left[(1-\rho_2)\left(\beta_2 E_t(\pi_{t+1}) + \gamma_2 E_t(\tilde{y}_t) + \delta_2 E_t(s_t)\right) + \rho_2 i_{t-1}\right] I(\tilde{y}_t > \overline{q}) \quad (7.f)$$

The first equation of the above model (i.e., (7.a)) assumes that the percentage change in the aggregate price level (i.e., inflation) of the home country from its target level is a function of its expected future value and the current real output deviations. This equation is known as the New Keynesian Philips curve for a small open economy, where *b* stands for discount factor. The slope coefficient κ_a of this equation is defined as $\kappa_a = \lambda\left(\frac{\sigma}{\omega_a} + \varphi\right)$, where σ is the relative risk aversion coefficient, φ is the elasticity of hours of labor, $\omega_a = 1 + a(2-a)(\sigma\eta - 1) > 0$, where $a \in [0,1]$ is a measure of openness and $\eta > 0$ is the elasticity of substitution between home and foreign goods, and, finally, $\lambda = \frac{(1-\omega \cdot b)(1-\omega)}{\omega}$ is a function of how frequently price adjustments occur (see Calvo (1983)), where ω captures the degree of price stickiness in the economy.

Equation (7.b) is the dynamic IS equation for an open economy which determines the current level of the home country output gap $\tilde{y}_t$ as a function of its expected future output gap level and the difference between the home country real interest rate, given as rate $i_t - E_t(\pi_{t+1})$, and the natural real interest rate denoted by r_t^n. As equation (7.c) indicates, real interest rate r_t^n depends negatively on domestic productivity a_t and the expected change of foreign country's output growth Δy_t^*. That is, an increase in both of these variables will reduce the level of r_t^n. This rate can be thought of as reflecting adverse movements in the terms of trade of the home country due to its own productivity changes or to the expected word real output growth. For a small open economy, a decrease in r_t^n will lead to a fall in economic activity. The mechanism by which this happens will be analysed in more details latter on. The laws of motion of the above two component variables of r_t^n are assumed that are given as

$$a_t = \rho^a a_{t-1} + \varepsilon_{a,t}, \qquad \text{and} \qquad y_t^* = \rho^{y^*} y_{t-1}^* + \varepsilon_{y^*,t},$$

where $\varepsilon_{a,t}$ and $\varepsilon_{y^*,t}$ denotes their structural shocks, with $|\rho^a| < 1$ and $|\rho^{y^*}| < 1$.

Equations (7.d) and (7.e) are our familiar relationships of the uncovered interest rate parity and relative purchasing power deviations in logarithm form, respectively, where i_t^* is the short term nominal interest rate of the foreign country, e_t the nominal exchange rate and s_t is the real exchange rate. As equation (7.e) implies, changes in real exchange rate Δs_t reflect one-to-one changes in the nominal rate Δe_t and the difference of inflation rates between the foreign and the home country, i.e., $\pi_t^* - \pi_t$. The foreign interest and inflation rates, which are taken as exogenous variables in our analysis, are assumed that they are given by the following laws of motion:

$$i_t^* = \rho^{i^*} i_{t-1}^* + \varepsilon_{i^*,t}, \qquad \text{and} \qquad \pi_t^* = \rho^{\pi^*} \pi_{t-1}^* + \varepsilon_{\pi^*,t},$$

where $\varepsilon_{i^*,t}$ and $\varepsilon_{\pi^*,t}$ denote their structural shocks with $|\rho^{i^*}| < 1$ and $|\rho^{\pi^*}| < 1$.

Finally, equation (7.f) of the above NK model is the CB's monetary policy rule which is assumed that is described by our threshold model (4). All the exogenous variables of the model will be collected in vector $\mathbf{z}_t = \left[a_t, y_t^*, i_t^*, \pi_t^*\right]'$, while the endogenous in vector $\mathbf{x}_t = \left[\tilde{\pi}_t, \tilde{y}_t, r_t^n, e_t, s_t, \tilde{i}_t\right]'$. The exogenous structural shocks of the model will be presented by vector $\boldsymbol{\varepsilon}_t = \left[\varepsilon_{a,t}, \varepsilon_{y^*,t}, \varepsilon_{i^*,t}, \varepsilon_{\pi^*,t}\right]'$. These shocks are assumed to be independent white noise processes, with zero mean and constant variance.

Using matrix notation, the NK model (7.a)-(7.f) can be written into the following structural-equation form:

$$\mathbf{B}(\tilde{y}_t \leq \overline{q})\mathbf{x}_t = \mathbf{A}(\tilde{y}_t \leq \overline{q})E_t(\mathbf{x}_{t+1}) + \mathbf{D}(\tilde{y}_t \leq \overline{q})\mathbf{x}_{t-1} + \mathbf{C}\mathbf{z}_t \qquad (8.a)$$

and $$\mathbf{B}(\tilde{y}_t > \overline{q})\mathbf{x}_t = \mathbf{A}(\tilde{y}_t > \overline{q})E_t(\mathbf{x}_{t+1}) + \mathbf{D}(\tilde{y}_t > \overline{q})\mathbf{x}_{t\text{-}1} + \mathbf{C}\mathbf{z}_t, \qquad (8.b)$$

where

$$\mathbf{z}_t = \mathbf{R}\mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t,$$

and the matrices involved have the following definitions:

$$\mathbf{B}(\tilde{y}_t \le \overline{q}) = \begin{bmatrix} 1 & -\kappa_a & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{\omega_a}{\sigma} & 0 & 0 & \frac{\omega_a}{\sigma} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & -(1-\rho_1)\gamma_1 & 0 & 0 & -(1-\rho_1)\delta_1 & 1 \end{bmatrix}$$

$$\mathbf{B}(\tilde{y}_t > \overline{q}) = \begin{bmatrix} 1 & -\kappa_a & 0 & 0 & 0 & 0 \\ 0 & 1 & -\frac{\omega_a}{\sigma} & 0 & 0 & \frac{\omega_a}{\sigma} \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 1 & 0 \\ 0 & -(1-\rho_2)\gamma_2 & 0 & 0 & -(1-\rho_2)\delta_2 & 1 \end{bmatrix}$$

$$\mathbf{A}(\tilde{y}_t \le \overline{q}) = \begin{bmatrix} b & 0 & 0 & 0 & 0 & 0 \\ \frac{\omega_a}{\sigma} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ (1-\rho_1)\beta_1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{A}(\tilde{y}_t > \overline{q}) = \begin{bmatrix} b & 0 & 0 & 0 & 0 & 0 \\ \frac{\omega_a}{\sigma} & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ (1-\rho_2)\beta_2 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\mathbf{D}(\tilde{y}_t \le \bar{q}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_1 \end{bmatrix}, \quad \mathbf{D}(\tilde{y}_t > \bar{q}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_2 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{\sigma(1+\varphi)(1-\rho_a)}{\sigma+\varphi\omega_a} & -\varphi\frac{\sigma(1-\omega_a)}{\sigma+\varphi\omega_a}\left(\rho^{y^*}-1\right) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{R} = \begin{bmatrix} \rho^a & 0 & 0 & 0 \\ 0 & \rho^{y^*} & 0 & 0 \\ 0 & 0 & \rho^{i^*} & 0 \\ 0 & 0 & 0 & \rho^{\pi^*} \end{bmatrix}.$$

Model (8.a)-(8.b) implies the following matrix of transition probabilities between the two regimes “1” and “2” implied by the estimates of threshold model (4) from time *t-1* to *t*:

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} = 1 - p_{11} \\ p_{21} = 1 - p_{22} & p_{22} \end{bmatrix},$$

where

$$p_{11} = \Pr(\beta_t = \beta_1 \wedge \gamma_t = \gamma_1 \wedge \delta_t = \delta_1 \mid \beta_{t-1} = \beta_1 \wedge \gamma_{t-1} = \gamma_1 \wedge \delta_{t-1} = \delta_1)$$

and

$$p_{22} = \Pr(\beta_t = \beta_2 \wedge \gamma_t = \gamma_2 \wedge \delta_t = \delta_2 \mid \beta_{t-1} = \beta_2 \wedge \gamma_{t-1} = \gamma_2 \wedge \delta_{t-1} = \delta_2).$$

Solving out the system of equations (8.a)-(8.b) for vector $\mathbf{x}_t$ we have the following Threshold Regime-Switching Rational Expectations (TRSRE) model:

$$\mathbf{x}_t = \mathbf{B}(\tilde{y}_t \le \overline{q})^{-1}\mathbf{A}(\tilde{y}_t \le \overline{q})E_t(\mathbf{x}_{t+1}) + \mathbf{B}(\tilde{y}_t \le \overline{q})^{-1}\mathbf{D}(\tilde{y}_t \le \overline{q})\mathbf{x}_{t\text{-}1} + \mathbf{B}(\tilde{y}_t \le \overline{q})^{-1}\mathbf{C}\mathbf{z}_t \qquad (9.a)$$

and

$$\mathbf{x}_t = \mathbf{B}(\tilde{y}_t > \overline{q})^{-1}\mathbf{A}(\tilde{y}_t > \overline{q})E_t(\mathbf{x}_{t+1}) + \mathbf{B}(\tilde{y}_t > \overline{q})^{-1}\mathbf{D}(\tilde{y}_t > \overline{q})\mathbf{x}_{t\text{-}1} + \mathbf{B}(\tilde{y}_t > \overline{q})^{-1}\mathbf{C}\mathbf{z}_t. \qquad (9.b)$$

This model is analogous to the Markov Regime-Switching Rational Expectations model (see, e.g., Cho and Moreno (2008), and Cho (2009)). Thus, its general rational expectation equilibrium (REE) solution can be written in the following minimum state variable (MSV) form:

$$\mathbf{x}_t = \mathbf{\Omega}(\tilde{y}_t \le \overline{q})\mathbf{x}_{t\text{-}1} + \mathbf{\Gamma}(\tilde{y}_t \le \overline{q})\mathbf{z}_t \qquad (10.a)$$

and

$$\mathbf{x}_t = \mathbf{\Omega}(\tilde{y}_t > \overline{q})\mathbf{x}_{t\text{-}1} + \mathbf{\Gamma}(\tilde{y}_t > \overline{q})\mathbf{z}_t, \qquad (10.b)$$

where matrices $\mathbf{\Omega}(\cdot)$ and $\mathbf{\Gamma}(\cdot)$ are defined analytically in the Appendix. This REE solution implies that the vector of endogenous variables $\mathbf{x}_t$ depends on the monetary policy regime of the economy at time *t*, as well as its lag values $\mathbf{x}_{t-1}$ and vector $\mathbf{z}_t$. In the Appendix, we present some conditions which guarantee forward convergence, mean square stability and determinacy of this solution. The last property of the model means that its solution is uniquely bounded REE.

The REE solution (10.a)-(10.b) can be used to obtain impulse response functions (IRFs) of the following endogenous variables of the small open economy NK model, namely π_{t+k}, $\tilde{y}_{t+k}$, e_{t+k}, s_{t+k} and i_{t+k}, at time *t*+*k*, to structural shocks $\varepsilon_{a,t}$, $\varepsilon_{y^*,t}$, $\varepsilon_{i^*,t}$ and $\varepsilon_{\pi^*,t}$, for *k* = 0,1,2,3… quarters ahead.[11] To this end, we need to calculate matrices $\mathbf{\Omega}(\cdot)$ and $\mathbf{\Gamma}(\cdot)$. This can be done numerically based on the forward method suggested by Cho (2009). In so doing, we need to assign values of the vector of structural parameters of the NK model (7.a)-(7.f) entered in matrices $\mathbf{B}(\cdot)$, $\mathbf{A}(\cdot)$,

[11] In the Appendix, we show how these IRFs can be obtained from the system of equations (8.a)-(8.b).

$\mathbf{D}(\cdot)$, $\mathbf{C}$ and $\mathbf{R}$ defining matrices $\mathbf{\Omega}(\cdot)$ and $\mathbf{\Gamma}(\cdot)$. Actually, two sets of parameters are required. The first is invariant to monetary policy regime. This involves the subjective discount factor *b*, the relative risk aversion parameter *σ*, the degree of stickiness *ω,* the intertemporal elasticity of substitution *η*, the elasticity of labor *φ*, the degree of openness *α*, and autoregressive coefficients $\rho^a, \rho^{y^*}, \rho^{i^*}$ and ρ^{π^*}.

Following Gali and Monacelli (2005), Pappa (2004) and Davig and Leeper (2007) we set $\sigma = 1$ and $\eta = 1.5$. The discount factor *b* is set to 0.99, implying a riskless annual return of about 4 percent in the steady state. The value of φ is taken to be $\varphi = 3$, which means a labor supply elasticity of 1/3. Parameter ω is set equal to 0.75. This value of *ω* implies that it will take about one year before prices adjust to their new levels. The degree of openness parameter *α* is taken to be 0.10, which is equal to the ratio of imports of goods and services to the GDP of Japan.[12] Finally, as values of autoregressive coefficients $\rho^a, \rho^{y^*}, \rho^{i^*}$, and ρ^{π^*} we consider their LS estimates given in the following regressions:

$$a_t = \underset{(0.06)}{0.68}\, a_{t-1} + \varepsilon_{a,t}, \quad \sigma_\alpha = 0.0089$$

$$y_t^* = \underset{(0.04)}{0.87}\, y_{t-1}^* + \varepsilon_{y^*,t}, \quad \sigma_{y^*} = 0.0074$$

$$i_t^* = \underset{(0.06)}{0.70}\, i_{t-1}^* + \varepsilon_{i^*,t}, \quad \sigma_{i^*} = 0.0126$$

$$\pi_t^* = \underset{(0.03)}{0.88}\, \pi_{t-1}^* + \varepsilon_{\pi^*,t}, \quad \sigma_{\pi^*} = 0.0039$$

These are obtained by fitting the autoregressive model (AR(1)) into the log labor productivity of Japan, the log US real GDP (taken as a proxy for world output), the US nominal interest rate and US inflation rate, respectively:[13] The above least squares values of the autoregressive coefficients $\rho^a, \rho^{y^*}, \rho^{i^*}$, and ρ^{π^*} guarantee that the forward convergence condition (FCC) of the TRSRE model (9.a)-(9.b) hold for a broad set of values of the remaining parameter of the model.

[12] To calculate this value we have used data from 2000:I to 2010:II.

[13] Standard errors are reported in parentheses. Note that the above four series are taken as percentage deviations from their Hodrick-Prescott filter estimates.

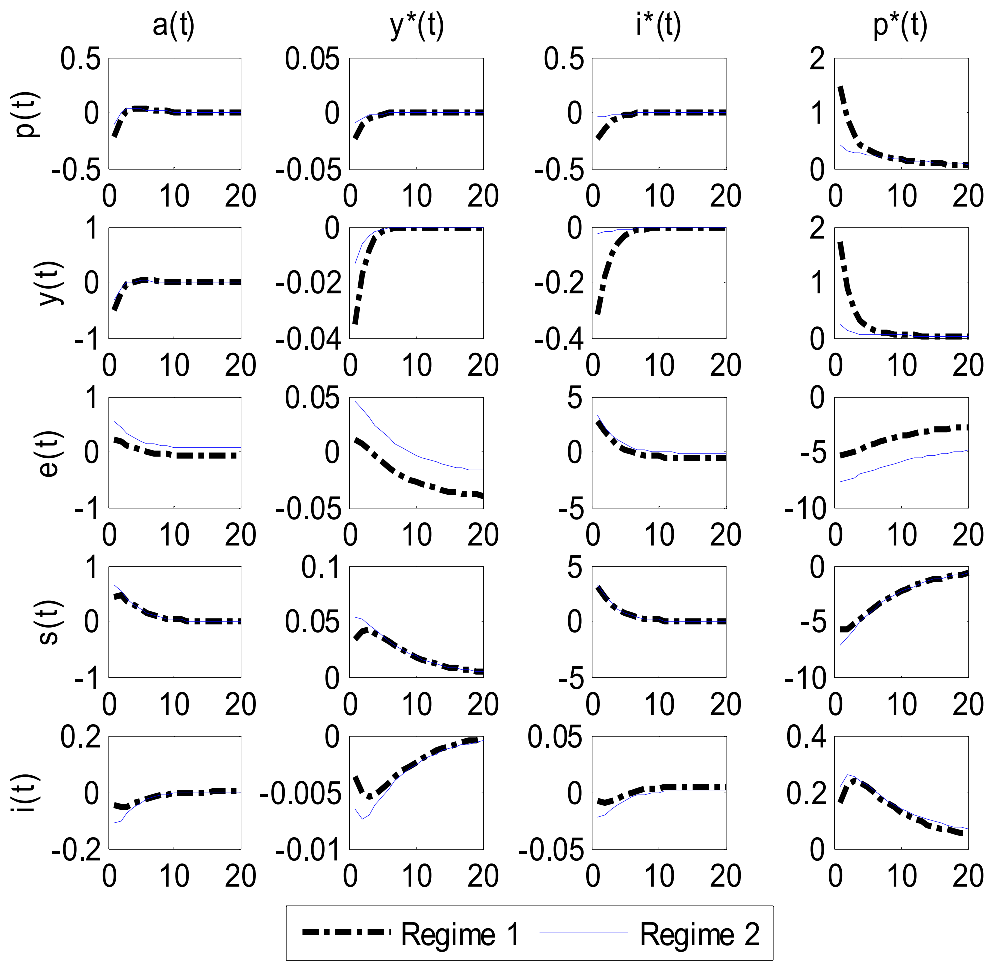
The second set of parameters required to determine the REE solution (10.a)-(10.b) are the policy rule parameters betas, gammas and deltas. These are taken to be equal to the point estimates of our threshold model (4), reported in Table 4, for the case that nominal exchange rate deviations are used as explanatory variable $\tilde{z}_t$. The value of delta parameter in regime "2", which is not found to be different than zero, is set to zero. Finally, the transition probabilities p_{11} and p_{22} implied by our threshold model are calculated ex post based on the number of times that the monetary policy rule stayed in regimes "1" and "2", respectively, over our whole sample. These are found to be $p_{11} = 0.94$ and $p_{22} = 0.87$.

Figure 2 graphically presents the IRFs of the five key variables of the open economy NK model (7.a)-(7.f), namely π_{t+k}, $\tilde{y}_{t+k}$, e_{t+k}, s_{t+k} and i_{t+k}. These present responses to one-percent (i.e. 0.01) standard deviation positive shocks (innovations) in domestic labour productivity a_t, foreign output y_t^*, foreign nominal interest rate i_t^* and foreign inflation π_t^*, respectively.[14] These IRFs are calculated conditionally on each different regime at current time *t*. They allow for regime-switching in the future, according to our threshold model predictions. The parameters values which are used to calculate the above IRFs imply that the REE solution, given by equations (10.a)-(10.b), is determinate (uniquely bounded) and mean square stable.[15] This means that the point estimates of these IRFs are unique.

[14] Note that negative shocks were also tried but these do not changes the main conclusions of our analysis. These produce IRFs which are symmetric to those reported in Figure 2A.

[15] The mean square stability for the REE of the TRSRE model (9.a)-(9.b) requires that the following condition must hold: $r_\sigma(\boldsymbol{\Sigma}_\Omega) < 1$, while determinacy requires $r_\sigma(\boldsymbol{\Sigma}_\mathrm{F}) < 1$, where $r_\sigma(.)$ denotes the maximum eigenvalue of matrices $\boldsymbol{\Sigma}_\mathbf{F}$ or $\boldsymbol{\Sigma}_\Omega$ defined in the Appendix. Necessary conditions for determinacy are to have mean square stability and forward convergence, i.e. ruling out rational bubbles in the solution. The values of the above maximum eigenvalues that we find are as follows: $r_\sigma(\boldsymbol{\Sigma}_\Omega) = 0.98 < 1$ and $r_\sigma(\boldsymbol{\Sigma}_\mathrm{F}) = 0.99 < 1$.

Figure 2: Impulse response functions (IRFs) driven by positive shocks



Inspection of these IRFs leads to a number of interesting conclusions which have important economic policy implications for academics and policy makers. They show that, independently on whether the economy is in the expansion or recession regime, at current time *t*, a positive shock in domestic labor productivity a_t, foreign output y_t^* and foreign nominal interest rate i_t^*, will lead to a fall in domestic real output y_t and a depreciation of both the nominal and real exchange rates, denoted e_t and s_t respectively. The first two effects (i.e. the fall in domestic output and inflation) will lead to decreases in interest rate i_t, according to our threshold monetary policy rule model (4) On the other hand, the depreciation of nominal exchange rate will tend to mitigate these interest rate cuts, by increasing interest rate i_t. However, the last effect will happen only under the recession regime, where our estimates of model (4) indicate that there is a positive relationship between exchange rate changes and interest rate i_t. The opposite to the above all effects will happen when the sign of the shocks considered becomes negative.

The above relationships can be explained as follows. A positive labor productivity shock (a_t) will lead to a fall of the domestic interest rate i_t so as to support a transitory expansion in consumption and potential output as is assumed that prices fully adjust to new productivity levels. Since foreign nominal interest rate i_t^* is assumed constant, the above fall in interest rate i_t will lead to a depreciation of the domestic currency (i.e. an improvement in the terms of trade), according to the uncovered interest rate parity given by equation (7d). This initial currency depreciation will lead to a fall in the level of domestic prices (see (7e)) and, hence, will trigger expectations about of future currency appreciation. Interest rate i_t will return to its steady state level because the above expectations about future currency appreciation will lead to a reduction in the negative output gap (initially triggered by the productivity shock on its potential level) and, hence, to a more contractionary monetary policy, according to monetary policy rule.

Regarding the effect of a positive exogenous demand shock (in y_t^*), the NK model predicts the following. This will lead to a decline in real output gap (as potential output increases) and in inflation rate (see (7a)). By the monetary policy rule, these effects will lead to immediate cuts in nominal interest rate i_t which will offset the expected appreciation of the domestic currency due to the lower domestic prices.

Finally, the effects of positive exogenous nominal interest rate (r_t^*) and inflation rate (π_t^*) shocks on interest rate i_t can be explained as follows. A positive shock in r_t^* increases the demand for foreign assets implying a nominal and real depreciation of the domestic currency. The reduction in the domestic demand will lead to a reduction in the domestic inflation rate and, hence, to the nominal interest rate i_t. A positive shock is π_t^* will appreciate the domestic currency and will increase domestic inflation and real output. This, in turn, will lead to an immediate increase of i_t.

5. Conclusions

In this paper, we have suggested an open economy threshold monetary policy rule model of the interest rate of the CB (central bank) of Japan with the aim of unveiling its policy behavior over the last 40 years. This model allows for regime-shifts of the monetary policy rule parameters triggered by changes in the business cycle conditions of the economy, i.e. its expansion and recession regimes. To capture these kind of shifts, the model employs the real output gap deviations as a threshold variable. As the above model considers an open economy, the CB interest rate is assumed that, in addition to real output gap and inflation rate deviations from their target levels, depends on exchange rate deviations from their long-run levels.

The estimation results of the above threshold switching model indicate that the Japanese monetary authorities follow a stabilizing towards inflation and real output gap deviations interest rate monetary policy rule under both the expansion or recession regimes. This rule is found that it constitutes a better specification of the data than the standard Taylor rule. The CB interest rate of Japan is found to respond positively to nominal (or real) exchange rate deviations from their target level only in the recession regime. That is, it increases the CB interest rate when home currency is depreciated, so as to mitigate the negative effects on interest rates due to real gap negative deviations. This reaction can be attributed to the CB of Japan preference to keep real exchange rates volatility low. This was shown by simulating a small-scale open economy New Keynesian model based on the estimates of our threshold monetary policy rule model found for Japan.

A Appendix

A.1 Solution of TRSRE Model

In this appendix, we present more analytically the rational expectations equilibrium (REE) solution of the TRSRE model (9.a)-(9.b), given by equations (10.a)-(10.b). In particular, we give the definitions of matrices $\mathbf{\Omega}(\cdot)$ and $\mathbf{\Gamma}(\cdot)$ involved in this solution, as well as those of matrices $\mathbf{\Sigma}_{\mathbf{\Omega}}$ and $\mathbf{\Sigma}_F$ whose maximum values determine the mean square stability and determinacy conditions. The above solution can be obtained following the same steps as Cho (2009), for the Markov chain regime-switching model.

The REE solution (10.a)-(10.b) can be obtained by solving forward the system of equations (9.a)-(9.b) and imposing the forward condition ruling out rational bubbles in equilibrium. This will yield

$$\mathbf{x}_t = \mathbf{\Omega}(\tilde{y}_t \leq \overline{q})\mathbf{x}_{t\text{-}1} + \mathbf{\Gamma}(\tilde{y}_t \leq \overline{q})\mathbf{z}_t$$

and

$$\mathbf{x}_t = \mathbf{\Omega}(\tilde{y}_t > \overline{q})\mathbf{x}_{t\text{-}1} + \mathbf{\Gamma}(\tilde{y}_t > \overline{q})\mathbf{z}_t$$

where

$$\mathbf{\Omega}(\tilde{y}_t \leq \overline{q}) = \lim_{k\to\infty}\mathbf{\Omega}_k(\tilde{y}_t \leq \overline{q}),\ \ \mathbf{\Omega}(\tilde{y}_t > \overline{q}) = \lim_{k\to\infty}\mathbf{\Omega}_k(\tilde{y}_t > \overline{q}),$$

$$\mathbf{\Gamma}(\tilde{y}_t \leq \overline{q}) = \lim_{k\to\infty}\mathbf{\Gamma}_k(\tilde{y}_t \leq \overline{q}) \ \text{ and } \ \mathbf{\Gamma}(\tilde{y}_t > \overline{q}) = \lim_{k\to\infty}\mathbf{\Gamma}_k(\tilde{y}_t > \overline{q})$$

and

$$\mathbf{\Omega}_1\left(\tilde{y}_t \leq \overline{q}\right) = \mathbf{B}(\tilde{y}_t \leq \overline{q})^{-1}\mathbf{D}(\tilde{y}_t \leq \overline{q}),\ \mathbf{\Omega}_1\left(\tilde{y}_t > \overline{q}\right) = \mathbf{B}(\tilde{y}_t > \overline{q})^{-1}\mathbf{D}(\tilde{y}_t > \overline{q}),$$

$$\mathbf{\Gamma}_1(\tilde{y}_t \leq \overline{q}) = \mathbf{B}(\tilde{y}_t \leq \overline{q})^{-1},\ \mathbf{\Gamma}_1(\tilde{y}_t > \overline{q}) = \mathbf{B}(\tilde{y}_t > \overline{q})^{-1},$$

$$\mathbf{\Omega}_k\left(\tilde{y}_t \leq \overline{q}\right) = \mathbf{\Phi}_{k-1}\left(\tilde{y}_t \leq \overline{q}\right)^{-1}\mathbf{B}(\tilde{y}_t \leq \overline{q})^{-1}\mathbf{D}(\tilde{y}_t \leq \overline{q}),$$

$$\mathbf{\Omega}_k\left(\tilde{y}_t > \overline{q}\right) = \mathbf{\Phi}_{k-1}\left(\tilde{y}_t > \overline{q}\right)^{-1}\mathbf{B}(\tilde{y}_t > \overline{q})^{-1}\mathbf{D}(\tilde{y}_t > \overline{q}),$$

$$\boldsymbol{\Gamma}_k(\tilde{y}_t \le \overline{q}) = \boldsymbol{\Phi}_{k-1}\left(\tilde{y}_t \le \overline{q}\right)^{-1} \mathbf{B}\left(\tilde{y}_t \le \overline{q}\right)^{-1} + E_t\left[\mathbf{F}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t \le \overline{q}\right)\boldsymbol{\Gamma}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\right]\mathbf{R},$$
$$\boldsymbol{\Gamma}_k(\tilde{y}_t > \overline{q}) = \boldsymbol{\Phi}_{k-1}\left(\tilde{y}_t > \overline{q}\right)^{-1} \mathbf{B}\left(\tilde{y}_t > \overline{q}\right)^{-1} + E_t\left[\mathbf{F}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t > \overline{q}\right)\boldsymbol{\Gamma}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\right]\mathbf{R},$$

with

$$\boldsymbol{\Phi}_{k-1}\left(\tilde{y}_t \le \overline{q}\right) = \left(\mathbf{I} - E_t\left[\mathbf{B}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t \le \overline{q}\right)^{-1} \mathbf{A}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t \le \overline{q}\right)\boldsymbol{\Omega}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\right]\right),$$
$$\mathbf{F}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t \le \overline{q}\right) = \boldsymbol{\Phi}_{k-1}\left(\tilde{y}_t \le \overline{q}\right)^{-1} \mathbf{B}(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t \le \overline{q})^{-1} \mathbf{A}(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t \le \overline{q}),$$

$$\boldsymbol{\Phi}_{k-1}\left(\tilde{y}_t > \overline{q}\right) = \left(\mathbf{I} - E_t\left[\mathbf{B}(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t > \overline{q})^{-1} \mathbf{A}(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t > \overline{q})\boldsymbol{\Omega}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\right]\right),$$
$$\mathbf{F}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t > \overline{q}\right) = \boldsymbol{\Phi}_{k-1}\left(\tilde{y}_t > \overline{q}\right)^{-1} \mathbf{B}(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t > \overline{q})^{-1} \mathbf{A}(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q} \mid \tilde{y}_t > \overline{q}).$$

Matrices $\boldsymbol{\Sigma}_{\boldsymbol{\Omega}}$ and $\boldsymbol{\Sigma}_F$ are defined as follows

$$\boldsymbol{\Sigma}_{\boldsymbol{\Omega}} = \left[p_{ji}\boldsymbol{\Omega}\left(\tilde{y}_t \le \overline{q}, \tilde{y}_t > \overline{q}\right) \otimes \boldsymbol{\Omega}\left(\tilde{y}_t \le \overline{q}, \tilde{y}_t > \overline{q}\right)\right]$$
$$\boldsymbol{\Sigma}_F = \left[p_{ji}\mathbf{F}\left(\tilde{y}_t \le \overline{q}, \tilde{y}_t > \overline{q}\right) \otimes \mathbf{F}\left(\tilde{y}_t \le \overline{q}, \tilde{y}_t > \overline{q}\right)\right]$$

A.2 Impulse Response Functions of TRSRE Model – IRFs

To see how the IRFs of the REE of the TRSRE model are obtained, first note that the forward solution of the TRSRE model is given as

$$\mathbf{x}_t = \boldsymbol{\Omega}(\tilde{y}_t \le \overline{q})\mathbf{x}_{t\text{-}1} + \boldsymbol{\Gamma}(\tilde{y}_t \le \overline{q})\mathbf{z}_t$$

$$\mathbf{x}_t = \boldsymbol{\Omega}(\tilde{y}_t > \overline{q})\mathbf{x}_{t\text{-}1} + \boldsymbol{\Gamma}(\tilde{y}_t > \overline{q})\mathbf{z}_t,$$

where $\mathbf{z}_t = \mathbf{R}\mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t$, . The one-step ahead prediction of $\mathbf{x}_{t+1}$ conditional on the t-time information set is given as

$E_t\mathbf{x}_{t+1} = \mathbf{F}_1(\tilde{y}_t \le \overline{q})\mathbf{x}_t + \mathbf{G}_1(\tilde{y}_t \le \overline{q})\mathbf{z}_t$, $E_t\mathbf{x}_{t+1} = \mathbf{F}_1(\tilde{y}_t > \overline{q})\mathbf{x}_t + \mathbf{G}_1(\tilde{y}_t > \overline{q})\mathbf{z}_t$

where

$$\mathbf{F}_1(\tilde{y}_t \le \overline{q}) = E\left[\boldsymbol{\Omega}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right) \mid \tilde{y}_t \le \overline{q}\right],$$

$$\mathbf{F}_1(\tilde{y}_t > \overline{q}) = E\left[\boldsymbol{\Omega}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right) \mid \tilde{y}_t > \overline{q}\right],$$

$$\mathbf{G}_1(\tilde{y}_t \le \overline{q}) = E\left[\boldsymbol{\Gamma}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right) \mid \tilde{y}_t \le \overline{q}\right]\mathbf{R},$$

$$\mathbf{G}_1(\tilde{y}_t > \overline{q}) = E\left[\boldsymbol{\Gamma}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right) \mid \tilde{y}_t > \overline{q}\right]\mathbf{R}.$$

The *k*-step ahead prediction of $\mathbf{x}_{t+k}$ is then given as

$$E_t\mathbf{x}_{t+k} = \mathbf{F}_k(\tilde{y}_t \le \overline{q})\mathbf{x}_t + \mathbf{G}_k(\tilde{y}_t \le \overline{q})\mathbf{z}_t \text{ and } E_t\mathbf{x}_{t+k} = \mathbf{F}_k(\tilde{y}_t > \overline{q})\mathbf{x}_t + \mathbf{G}_k(\tilde{y}_t > \overline{q})\mathbf{z}_t,$$

where

$$\mathbf{F}_k(\tilde{y}_t \le \overline{q}) = E\left[\mathbf{F}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\boldsymbol{\Omega}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right) \mid \tilde{y}_t \le \overline{q}\right],$$

$$\mathbf{F}_k(\tilde{y}_t > \overline{q}) = E\left[\mathbf{F}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\boldsymbol{\Omega}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right) \mid \tilde{y}_t > \overline{q}\right],$$

$$\mathbf{G}_k(\tilde{y}_t \le \overline{q}) = E\left[\left(\mathbf{G}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right) + \mathbf{F}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\boldsymbol{\Gamma}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\right) \mid \tilde{y}_t \le \overline{q}\right]\mathbf{R},$$

$$\mathbf{G}_k(\tilde{y}_t > \overline{q}) = E\left[\left(\mathbf{G}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right) + \mathbf{F}_{k-1}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\boldsymbol{\Gamma}\left(\tilde{y}_{t+1} \le \overline{q}, \tilde{y}_{t+1} > \overline{q}\right)\right) \mid \tilde{y}_t > \overline{q}\right]\mathbf{R}$$

for $k = 2,3,\ldots$. For $k = 0$ we define $\mathbf{F}_0(\cdot) = \mathbf{I}_n$ and $\mathbf{G}_0(\cdot) = \mathbf{0}_{n\times m}$ where *n* is the number of endogenous variables and *m* the number of exogenous.

Given the above definitions, the impulse response functions (IRFs) of $\mathbf{x}_{t+k}$ to the *l*-th innovation at time *t* conditional on the state can be calculated by the following expressions:

$$\mathbf{IRF}_k\left(\tilde{y}_t \le \overline{q}\right) = \left(\mathbf{F}_k\left(\tilde{y}_t \le \overline{q}\right)\boldsymbol{\Gamma}\left(\tilde{y}_t \le \overline{q}\right) + \mathbf{G}_k\left(\tilde{y}_t \le \overline{q}\right)\right)\mathbf{e}_l,$$

$$\mathbf{IRF}_k\left(\tilde{y}_t > \overline{q}\right) = \left(\mathbf{F}_k\left(\tilde{y}_t > \overline{q}\right)\boldsymbol{\Gamma}\left(\tilde{y}_t > \overline{q}\right) + \mathbf{G}_k\left(\tilde{y}_t > \overline{q}\right)\right)\mathbf{e}_l,$$

for $k = 0,1,2,3,\ldots$ where $\mathbf{e}_l$ is an indicator vector of which the *l*-th element is 1 and 0 elsewhere.

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