

On the construction of two-country cointegrated VAR models with an application to the UK and US

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Abstract

In this paper we introduce a cointegrated VAR modelling approach for two-country macro dynamics. In order to tackle the curse of dimensionality resulting from the number of variables in multi-country models, we investigate the applicability of the approach by Aoki (1981) frequently used in economic theory. Aoki showed that for a system of linear differential equations, the assumption of country symmetry allows to decouple the dynamics of country averages and country differences into two autonomous subsystems. While this approach can not be applied straightforwardly to economic time series, we generalize Aoki's approach and demonstrate how it can be utilized for the determination of the long-run properties of the system. Symmetry is rejected for the short-run, thus for the given cointegration vectors the final modelling stage is based on the full two-country system. The econometric modelling approach is then enhanced by a general-to-specific model selection procedure, where the VAR based cointegration analysis is combined with a graph-theoretic search for instantaneous causal relations and an automatic general-to-specific reduction of the vector equilibrium correction model. As an application we build up a macro-econometric two-country model for the UK and the US. The empirical study focusses on the effects of monetary policy on the $\$/\pounds$ exchange rate. We find interest rate shocks in the UK cause much stronger exchange rate effects than an unanticipated interest rate change by the Fed.

Keywords: Two-country model; Cointegration; Structural VAR; *Gets* Model Selection, Monetary Policy, Exchange Rates.

JEL classification: C22; C32; C50.

1 Introduction

In empirical macroeconomics, the vector autoregressive (VAR) model is the most commonly used modelling approach. A persistent issue with this approach is the problem of dimensionality. With every additional variable included in a VAR model, the number of parameters to be estimated rises strongly, leading to an inflation of estimation uncertainty. This problem of being restricted to a small number of macroeconomic variables is more severe in a two-country setup, with the need of including time series for both countries plus an exchange rate. The cointegrated VAR model (CVAR) is an adequate model to study carefully the long-run and the short-run properties of macroeconomic time series. When using two-country Cointegrated VAR models the limitation of the number of variables is even more binding,

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because it is getting very difficult, with an increase in dimension of the model, to impose long-run meaningful structure on the unrestricted cointegration relations.¹ To overcome this problem we introduce a modelling approach suggested by Aoki (1981) for dynamic Macroeconomic modelling in empirical research. Aoki showed for a system of linear differential equations that, when assuming symmetry on the two-country model, the variables can be transformed into a set of country averages and country differences and these two sets being orthogonal to each other, can be analysed separately. We apply this idea to determine the long-run properties of our empirical model. This solution divides the size of the sets of variables for the cointegration part into two, offering the possibility to study much larger Macro dynamics. We assume advanced economies to behave similar in the long-run. There is no reason why large economies like the UK or the US should differ in their aggregate behaviour in a systematic way. In contrast the speed of adjustments to equilibria may vary markedly, due to unequal sizes of the countries or structural differences. We allow for different contemporaneous effects, different short-run dynamics, and different speeds of adjustment for the two economies, but determine the long-run equilibria to be symmetric over the two countries, like the international parity conditions are established. The symmetry assumption in the long-run allows to apply the method, proposed by Aoki (1981) and applied by others, for example Turnovsky (1986), to determine the long-run properties of the model in two smaller subsystems. It makes the cointegration analysis in smaller subsystems feasible. Symmetry is rejected for the short-run, thus for the given cointegration vectors further modelling of the short-run is based on the full two-country system.

The method with the aim of breaking the analysis of a system down into submodels can be compared to the integrated model approach of Juselius (2006) or also to the GVAR approach of Pesaran, Schuermann and Weiner (2004). In the integrated model of Juselius (2006), see also Tuxen (2007), the long-run structures of different sectors are analysed separately and the results are combined in a complete model. In Juselius (2006) inflation is modelled by combining submodels from a money market, an external sector and a labour market and this is extended in Tuxen (2007) with a public sector. In the GVAR approach country-specific models, including domestic variables and country-specific global variables, are estimated. A large number of individual country models are linked together in a global model via a trade weighted matrix.

We support our modelling approach by an appropriate econometric model selection procedure. In the econometric model selection, we follow a data-driven approach, which emphasizes three economically and econometrically important aspects:

- (i) To carefully study the long-run and short-run properties of the macroeconomic time series under consideration, we are commencing from an unrestricted cointegrated VAR model and are developing a parsimonious structural vector equilibrium correction model, which is the adequate $I(0)$ representation of a system, containing $I(1)$ variables. An econometric model with a well-defined long-run equilibrium imposes important data-coherent constraints on impulse response functions, which are critical when assessing the effects of macroeconomic stabilization policies.
- (ii) There has been an intense discussion about arbitrary assumptions leading to the identification of the direction of instantaneous causality. Many of the proposed schemes are based on theoretical ad-hoc assumptions. In this paper, we seek to overcome these limitations by taking advantage of recent advances in graph theory and its application to the search for causality among variables.
- (iii) Highly parameterized unrestricted VAR or just-identified structural VAR models require the estimation of a irrelevant large number of parameters and suffer from the curse of dimensionality: as

¹Experienced researchers in the area of cointegration analysis like Katarina Juselius usually limits the number of variables in a system to not more than 7.

the degrees of freedom are being exhausted and estimation uncertainty is inflated with a growing number of variables or lags, so do the impulse responses functions become inconclusive due to a growing width of confidence intervals, which will eventually include the zero line. To avoid this problem we will make use of the breakthrough in automatic general-to-specific model reduction procedures in reducing the complexity of the model while preserving the characteristics of the data.

Our data-driven econometric model selection combines the VAR based cointegration analysis of Johansen (1995) and Juselius (2006) with the graph-theoretic approach of Spirtes, Glymour and Scheines (2001) implemented in TETRAD for the search for instantaneous causal relations (see Demiralp and Hoover, 2003, for its application to econometrics) and the automatic general-to-specific model selection algorithm implemented in *PcGets* of Krolzig and Hendry (2001) and Krolzig (2003) for the selection of a congruent parsimonious structural vector equilibrium correction model.

We develop in this paper a small economy-wide macroeconomic two-country model for the UK and the US. Using monthly data from 1972M3 to 2010M8, the system consists of nine variables: inflation rates, output growth rates, the 3-month interest rates, the 10-year government bond yields, and the nominal $\$/\pounds$ exchange rate.

The model is applied to analyse the dynamic reaction of the exchange rate to monetary policy shocks in the form of variations of the short-term interest rates. While in the standard overshooting models of Dornbusch (1976) and Frankel (1979)² the exchange rate jumps instantaneously in response to an interest rate shock in order to depreciate over time and thereby restoring the uncovered interest rate parity (UIP). There is a growing body of empirical evidence suggesting that exchange rates do not tend to jump instantaneously as predicted by the theory, but rather appreciate steadily for several months before finally depreciating. Whether or not such a ‘delayed overshooting puzzle’ is present in the case of the $\$/\pounds$ exchange rate is the question our model application seeks to answer.

The paper follows up on Heinlein and Krolzig (2011), in which also international transmission of monetary policy and especially the effects of monetary policy on the $\$/\pounds$ exchange rate is analysed. In the former paper a symmetric two-country model is estimated, that means country differences of variables are included into the model, with the consequence that a positive interest rate shock in the UK has the same effect on the exchange rate, as a negative interest rate shock in the US with the same size. In the present paper there is an assumption of symmetry only in the long-run of the modelling process, the symmetry assumption is relaxed in the short-run and for the adjustment to the long-run. The long-run is in the present paper not only analysed in respect to country specific features in a country difference model, but also in respect to the global features in a country average model. The role of the country average model can be compared to the global factor in a dynamic factor model (see Forni, Hallin, Lippi and Reichlin, 2000, about dynamic factor models). While the model in the former paper is estimated with quarterly data, in the present paper monthly data is used. Central results of the cointegration section could be replicated with monthly data.

Vector autoregressive (VAR) models have long served as the workhorse for studying the empirical reaction of exchange rates to monetary policy shocks.³ In the seminal paper of Eichenbaum and Evans (1995), the effects of US monetary policy shocks on five exchange rates were analyzed in a VAR framework with Cholesky-type causal ordering. Three different measures of shocks were considered: shocks to the federal funds rate, shocks to the ratio of non-borrowed to total reserves and changes in the Romer

²See Rogoff (2002) for a survey.

³There has been some criticism in the literature about the limited information set of a small-scale VAR approach. For example, Mumtaz and Surico (2009) applied a factor augmented VAR with the UK as the domestic country and 17 other industrialized countries as the foreign block. For the period 1974Q1 to 2005Q1, they find no delayed overshooting.

and Romer (2004) monetary policy index. For the period from 1974M1 to 1990M5, Eichenbaum and Evans (1995) found the considered exchange rates to appreciate for several months after an expansionary US monetary policy shock until reaching a peak from which they then start to decline in value. The detected delay in overshooting was 2 to 3 years, with Japan having the shortest and the UK the longest delay. Pronouncedly shorter delay estimates were produced by Grilli and Roubini (1995, 1996), who discussed the delayed overshooting puzzle within the framework of the ‘liquidity model’ where, in contrast to sticky price models, goods prices are flexible while asset markets only adjust gradually.⁴

Following up on the Eichenbaum and Evans (1995) approach, recent contributions including Cushman and Zha (1997), Faust and Rogers (2003), Kim (2005) and Scholl and Uhlig (2008) have all used vector autoregressions with superimposed exclusion, sign or shape identification restrictions usually derived from economic theory to overcome the ad-hoc nature of recursive orderings in a Cholesky approach. Commencing from a small open-economy assumption, Cushman and Zha (1997) considered a structural VAR model with imposed block exogeneity, such that the non-domestic block of US variables were not affected by domestic Canadian variables. Allowing the CAD-USD exchange rate to react contemporaneously to (domestic) monetary policy shocks via an information market equation, no puzzles were found for the period 1974 to 1993. Also Kim and Roubini (2000) found no delayed overshooting for non-US G-7 exchange rates from 1974M7 to 1992M12, when identifying the contemporaneous effects with zero restrictions derived from economic theory nonrecursively. These conflicting empirical results were underpinned by Faust and Rogers (2003), who demonstrated the delayed overshooting result can be sensitive to questionable assumptions, such that the peak appreciation could be within one month after the monetary policy shock when allowing for simultaneity. Seeking to avoid ‘dubious identifying assumptions’, Faust and Rogers (2003) identified the VAR only partly, but used informal restrictions to calculate the impulse responses following the approach in Faust (1998). 7 and 14-variable models of the US-UK and US-German bilateral exchange rate from 1974M1 to 1997M12 showed that monetary policy shocks, while not the main source of exchange rate variability, generate large UIP deviations.

The effects of monetary policy on exchange rates have recently been revisited by Scholl and Uhlig (2008) using an identification procedure with sign restrictions. Analyzing bilateral exchange rate data from 1975M07 to 2002M07 for US-Germany, US-UK, US-Japan and US-G7, they found evidence for delayed overshooting with a delay of around 2 years. The delay in the response of the US-UK exchange rate was with 17 months the shortest. Even when the possibility of delayed overshooting was excluded by construction, a ‘sizeable’ positive forward premium remained. It was shown that these deviations from UIP can be exploited by hedging strategies with Sharpe ratios greater than those in equity markets. Combining short and long-run restrictions, i.e., allowing for simultaneity between interest rates and exchange rate, but assuming no long-run effects of monetary policy on exchange rates, Bjørnland (2009) rejected a delayed overshooting puzzle for the real exchange rates of Australia, Canada, New Zealand and Sweden with the US in the period from 1983Q1 to 2004Q4. Finally, using an identification method which exploits breaks in the heteroscedasticity of the structural innovations, Bouakez and Normandin

⁴Some recent papers have revived the interest in finding an explanation for the delayed-overshooting phenomenon. According to Gourinchas and Tornell (2004), the puzzle is caused by systematic distortion in investors’ beliefs about the interest rate process. Suppose investors overestimate the relative importance of transitory interest rate shocks. Confronted with a higher than expected interest rate in the next period, investors revise their beliefs. This ‘updating effect’ has been suggested as a cause of the forward premium effect and the delayed overshooting puzzle. Kim (2005) proposed that foreign exchange rate interventions of the central bank as driving factors of the delayed overshooting puzzle for the Canadian-US bilateral exchange rate. Exchange rate appreciation on impact might be counteracted by policy interventions in the foreign exchange market. According to Bacchetta and van Wincoop (2010) infrequent foreign currency portfolio decisions of agents can explain the forward discount and the delayed overshooting puzzle. They suggest that the infrequent portfolio decisions can be optimal as the welfare gain from active currency management may be smaller than the corresponding fees.

(2010) obtained a delay of about 10 months for US-G7 bilateral exchange rates.⁵

The structure of the paper is as follows. In §2 we present the methodology of the economic modelling approach and the econometric model selection procedure. In §3 we introduce the data set and provide a brief overview of the UK-US macro history since the breakdown of Bretton Woods in light of the international parity conditions. This will give valuable insights for the formation of the two-country model to be discussed in §4. §5 investigates the effects of a monetary policy shock with focus on the presence of a delayed overshooting puzzle, violations of UIP and the question of a symmetric response of the exchange rate across countries. Finally §6 concludes.

2 Methodology

2.1 Economic modelling approach

We propose a symmetric linear two-country model with dimension $2K + 1$ of the following way. To simplify the writing the model is presented here with one lag only, but can be extended to several lags easily:

$$\begin{aligned} \mathbf{y}_t &= \mathbf{A}\mathbf{y}_{t-1} + \mathbf{A}^*\mathbf{y}_{t-1}^* + \boldsymbol{\alpha}e_{t-1} + \boldsymbol{\varepsilon}_t \\ \mathbf{y}_t^* &= \mathbf{A}\mathbf{y}_{t-1}^* + \mathbf{A}^*\mathbf{y}_{t-1} - \boldsymbol{\alpha}e_{t-1} + \boldsymbol{\varepsilon}_t^* \\ e_t &= \gamma e_{t-1} + \boldsymbol{\Gamma}(\mathbf{y}_{t-1} - \mathbf{y}_{t-1}^*) + \eta_t \end{aligned} \quad (1)$$

where $\boldsymbol{\varepsilon}_t \sim \text{NID}(\mathbf{0}, \boldsymbol{\Sigma})$, $\boldsymbol{\varepsilon}_t^* \sim \text{NID}(\mathbf{0}, \boldsymbol{\Sigma})$, $\boldsymbol{\eta}_t \sim \text{NID}(0, \sigma^2)$ are mutually independent Gaussian white noise processes.

Symmetry means, that both countries are reacting with the same autoregressive coefficient, \mathbf{A} , to its own past, with the same coefficient, \mathbf{A}^* , to the other country and with opposite sign but the same size to the exchange rate. The exchange rate is driven by its own past and the difference between the countries. Following Aoki (1981) the set of domestic-foreign variables is transformed into a set of country-average-difference variables:

$$y_{j_t}^a = \frac{y_{j_t} + y_{j_t}^*}{2} \quad \text{and} \quad y_{j_t}^d = y_{j_t} - y_{j_t}^*$$

for $j = 1, \dots, K$.

The average-difference representation of the model looks like follows:

$$\begin{aligned} \mathbf{y}_t^a &= (\mathbf{A} + \mathbf{A}^*)\mathbf{y}_{t-1}^a + 0.5(\boldsymbol{\varepsilon}_t + \boldsymbol{\varepsilon}_t^*) \\ \mathbf{y}_t^d &= (\mathbf{A} - \mathbf{A}^*)\mathbf{y}_{t-1}^d + 2\boldsymbol{\alpha}e_{t-1} + (\boldsymbol{\varepsilon}_t - \boldsymbol{\varepsilon}_t^*) \\ e_t &= \gamma e_{t-1} + \boldsymbol{\Gamma}\mathbf{y}_{t-1}^d + \eta_t \end{aligned} \quad (2)$$

with uncorrelated error terms. The domestic-foreign system (1) and the country-average-difference system (2) are observationally equivalent. In the average-difference representation of the symmetric model the dynamics can be decomposed into two autonomous subsystems: a system of country averages of dimension (K) and a system of country differences and the exchange rate of dimension ($K + 1$).

The separate solution of the two autonomous subsystems simplifies the analysis and gives additional insights. Subsequently the system is re-transformed into domestic-foreign variables.

⁵The omitting of multilateral spillover effects was criticized by Binder, Chen and Zhang (2010), who proposed a Global VAR model for the analysis of the effects of US monetary policy shocks. For a sample from 1978 to 2006, no delayed overshooting was found.

2.2 Econometric model selection procedure

For the econometric model selection we follow a data-driven approach that combines the VAR based cointegration analysis of Johansen (1995) and Juselius (2006) with the graph-theoretic approach of Spirtes *et al.* (2001) implemented in TETRAD for the search for instantaneous causal relations (see Demiralp and Hoover, 2003, for its application to econometrics) and the automatic general-to-specific model selection algorithm implemented in *PcGets* of Krolzig and Hendry (2001) for the selection of a congruent parsimonious structural vector equilibrium correction model.

(i) Cointegration in subsystems to determine the long-run

- *Specification of the general unrestricted system.*

We commence from a p -th order reduced-form vector autoregressive (VAR) model without any equation-specific restrictions to capture the characteristics of the data:

$$\mathbf{y}_t^s = \boldsymbol{\nu}^s + \sum_{j=1}^p \mathbf{A}_j^s \mathbf{y}_{t-j}^s + \mathbf{u}_t^s, \quad (3)$$

where $\mathbf{u}_t^s \sim \text{NID}(\mathbf{0}, \boldsymbol{\Sigma})$ is a Gaussian white noise process and $s = a, d$. This step involves the specification of the deterministic terms, selection of the lag length p and misspecification test to check the validity of the assumptions made.

- *Johansen cointegration tests and identification of the cointegration vectors.*

The Johansen procedure for determining the cointegration rank, r , is then applied to the system (3) mapped into its vector equilibrium-correction mechanism (VECM) representation:

$$\Delta \mathbf{y}_t^s = \boldsymbol{\nu}^s + \boldsymbol{\Pi}^s \mathbf{y}_{t-1}^s + \sum_{j=1}^{p-1} \boldsymbol{\Gamma}_j^s \Delta \mathbf{y}_{t-j}^s + \mathbf{u}_t^s, \quad (4)$$

For a cointegrated vector process, the reduced-rank matrix, $\boldsymbol{\Pi}^s$, can be decomposed into loading matrix, $\boldsymbol{\alpha}^s$, and cointegration matrix, $\boldsymbol{\beta}^s$, containing the information of the long-run structure of the model. The Johansen procedure delivers unique estimates of $\boldsymbol{\alpha}^s$ and $\boldsymbol{\beta}^s$ as a result of requiring $\boldsymbol{\beta}^s$ to be orthogonal and normalized. These estimates provide a value for the unrestricted log-likelihood function to be compared to the log-likelihood under economically meaningful overidentifying restrictions, $\boldsymbol{\beta}^{s,r}$:

$$\Delta \mathbf{y}_t^s = \boldsymbol{\nu}^s + \boldsymbol{\alpha}^s \boldsymbol{\beta}^{s,r'} \mathbf{y}_{t-1}^s + \sum_{j=1}^{p-1} \boldsymbol{\Gamma}_j^s \Delta \mathbf{y}_{t-j}^s + \mathbf{u}_t^s, \quad (5)$$

with $\boldsymbol{\Sigma}^s = \text{E}[\mathbf{u}_t^s \mathbf{u}_t^{s'}]$. The empirical modeling procedure for finding the cointegration relations follows Juselius (2006).

(ii) Graph-theoretic search for instantaneous causal relations

The determination of the contemporaneous causal links between the variables has been advanced by modern graph-theoretic methods of searching for causal structure based on relations of conditional independence developed by computer scientists (Pearl, 2000) and philosophers (Spirtes *et al.*, 2001). Following Demiralp and Hoover (2003), who introduced this approach to econometrics, we use the PC algorithm implemented in TETRAD 4 (see Spirtes, Scheines, Ramsey and Glymour, 2005 for details). The PC algorithm exploits the information embedded in the residual variance-covariance matrix, $\hat{\boldsymbol{\Sigma}}^s$, of the system in (5). A causal structure is represented by a graph

with arrows from causes to caused variables. To detect the directed acyclic graph, the algorithm starts by assuming that all variables are linked to each other through an undirected link. In the elimination stage, connections are first removed between variables which are unconditionally uncorrelated. Then connections are eliminated for variables which are uncorrelated conditional on other variables. Having identified the skeleton of the graph, the orientation step of the algorithm seeks to orient the undirected edges by logical reasoning. This involves the analysis of indirect connections by taking into account the whole graph, considering every pair of variables, exploiting already directed edges and the acyclicity condition.

If all edges could be oriented, a directed acyclic graph (DAG) results. Based on the identified contemporaneous causal structure of the system, the VECM in (5) can be represented as a *recursive* structural vector equilibrium correction mechanism (SVECM). By a suitable ordering of the variables of the system, the DAG can be mapped to a lower-triangular contemporaneous matrix, \mathbf{B}^r , with units on the diagonal and non-zero lower-off-diagonal elements representing the causal links found by the PC algorithm. In contrast to a traditional orthogonalisation with the help of a Choleski decomposition of $\hat{\Sigma}^s$, this approach results in an overidentified SVECM in the majority of cases. The zero lower-triangular elements of \mathbf{B}^r provide testable overidentifying constraints allowing to verify the validity of the selected contemporaneous structure. Most importantly, as the contemporaneous causal structure captured by \mathbf{B}^r is data determined, it avoids the problems associated with the ad-hoc nature of orthogonalised structural VAR models. In step (iii) we will consider *Gets* reductions of the SVECM to reduce the complexity of the model.

If the PC algorithm finds a link but has insufficient information to identify if, say, ‘A causes B’ or ‘B causes A’, an undirected edge emerge. In this case, there exists a set of contemporaneous causal structures, $\{\mathbf{B}^{(i)}\}$, that are all consistent with the data evidence. An additional modelling stage is then required for the selection of \mathbf{B}^r and, thus, the identification of the direction of causality. Having applied the model reduction step in (iii) to each SVECM associated with one of the found contemporaneous causal structures, the dominant econometric model is finally selected in (iv).

(iii) *System and single-equation reductions of the SVECM*

Starting point is the structural VECM with long-run relations $\beta^{s,r}$ determined by stage (i) and contemporaneous structure \mathbf{B}^r given by the corresponding directed acyclic graph:

$$\mathbf{B}^r \Delta \mathbf{y}_t = \boldsymbol{\delta} + \tilde{\boldsymbol{\alpha}} \begin{pmatrix} \beta^{a,r'} \mathbf{y}_{t-1}^a \\ \beta^{d,r'} \mathbf{y}_{t-1}^d \end{pmatrix} + \sum_{j=1}^{p-1} \boldsymbol{\Upsilon}_j \Delta \mathbf{y}_{t-j} + \boldsymbol{\omega}_t, \quad \boldsymbol{\omega}_t \sim \text{NID}(\mathbf{0}, \boldsymbol{\Omega}), \quad (6)$$

where \mathbf{B}^r is the lower-triangular matrix found by TETRAD and $\boldsymbol{\Omega}$ is a diagonal variance-covariance matrix. A single-equation based *Gets* reduction procedure such as *PcGets* can be applied to the equations in (6) straightforwardly and, as shown in Krolzig (2001), without a loss in efficiency. The parameters of interest are the coefficients collected in the intercept, $\boldsymbol{\delta}$, the adjustment matrix $\tilde{\boldsymbol{\alpha}}$ and the short-run matrices $\boldsymbol{\Upsilon}_j$ in the structural VECM. The result is a parsimonious structural vector equilibrium correction model denoted PSVECM, which is nested in (6) and defined by the selected $\boldsymbol{\delta}^*$, $\tilde{\boldsymbol{\alpha}}^*$ and $\boldsymbol{\Upsilon}_j^*$ with $j = 1, \dots, p-1$.

(iv) *Selection of the dominant PSVECM*

If the graph-theoretical search in (ii) produces an acyclic graph with at least one undirected edge, the determination of the direction of instantaneous causal relations has to rely on the information from the PSVECMs resulting from the *Gets* reduction of the SVECMs as defined by the set of contemporaneous causal structures. As the PSVECMs are mutually non-nested and the union is

usually unidentified, we propose to select the PSVECM with the greatest penalized likelihood. Thus the dominant design of the contemporaneous effects matrix according to information criteria such as Akaike or Schwarz would be used.

3 Time series

3.1 Data

We develop in this paper a macroeconometric two-country model for the UK and the US, consisting of nine variables: inflation rates, output growth rates, the 3-month interest rates, the 10-year government bond yields, and the nominal exchange rate. We are using monthly data from 1972M3 to 2010M8, involving a total of 462 monthly observations. The paper is written from the UK perspective, so we will refer to UK variables as the domestic ones and US variables as foreign ones, marked by a star. Table 1 gives an overview over the macro time series under consideration.

Table 1 Time Series Definitions and Source.

Variable	Description	Source	EcoWin code
P_t	UK Retail Prices, all items excluding mortgage interest payments (RPIX), Index, (1987M1=100), spliced with RPI (before 1975)	ONS	<i>ew</i> : <i>gbr</i> 11815
Y_t	UK Industrial Production, SA, (2005=100), USD	IFS	<i>ifs</i> : <i>s1126600czfm</i>
I_t	UK Treasury bills, Bid, 3 month, Yield, End of Period, GBP	Reuters	<i>ew</i> : <i>gbr</i> 14010
R_t	UK Government Benchmarks, Bid, 10 year, Yield, End of Period GBP	Reuters	<i>ew</i> : <i>gbr</i> 14020
P_t^*	US Consumer Prices, all items, SA, Index, (1982-1984=100)	BLS/Reuters	<i>ew</i> : <i>usa</i> 11970
Y_t^*	US Industrial Production, SA, (2005=100), USD	IFS	<i>ifs</i> : <i>s1116600czfm</i>
I_t^*	US Treasury bills, 3 month, Yield, Close, USD	Reuters	<i>ew</i> : <i>usa</i> 14430
R_t^*	US Government Benchmarks, Bid, 10 Year, Yield, End of Period, USD	Reuters	<i>ew</i> : <i>usa</i> 14021
e_t	Spot rates, GBP/USD transformed to USD/GBP, End of period	Reuters	<i>ew</i> : <i>gbr</i> 19005

The price index P_t is seasonal adjusted with Seats/Tramo. Both Industrial Production series Y_t, Y_t^* are outlier corrected with Seats/Tramo. Variables without a superindex are of the domestic country (UK), a * indicates the foreign country (US), ^a is a country average and ^d indicates a country difference. All financial variables are end-of-period series.

To guarantee the consistency of the parity conditions to be considered in §3.2, variables have been transformed to ensure that each interest rate, bond yield, rate of inflation and currency movement is measured as monthly log return. Table 2 explains in detail how each variable entering the model has been created.

Table 2 Model variables.

Variable	Description
$\pi_t = \Delta \log P_t$	rate of inflation
$\Delta y_t = \Delta \log Y_t$	output growth
$i_t = \log(1 + I_t/1200)$	short-term interest rate
$r_t = \log(1 + R_t/1200)$	long-term interest rate
$e_t = \log E_t$	exchange rate

For the cointegration part, like explained in §2, the system is split into two subsystems, a country difference model with five variables and a country average model with four variables.

3.2 Discussion

In the following, we discuss the properties of the macro time series as far as they are relevant for the econometric modelling to follow in §4.

3.2.1 Inflation and the output growth

Focussing first on the real economy, Figure 1 plots the rates of inflation and output growth in the UK and the US, also the differences and averages between the two countries. It can be seen that, except for the most recent years, the UK macro economy is characterized by a far more volatile output growth and a higher rate of inflation.

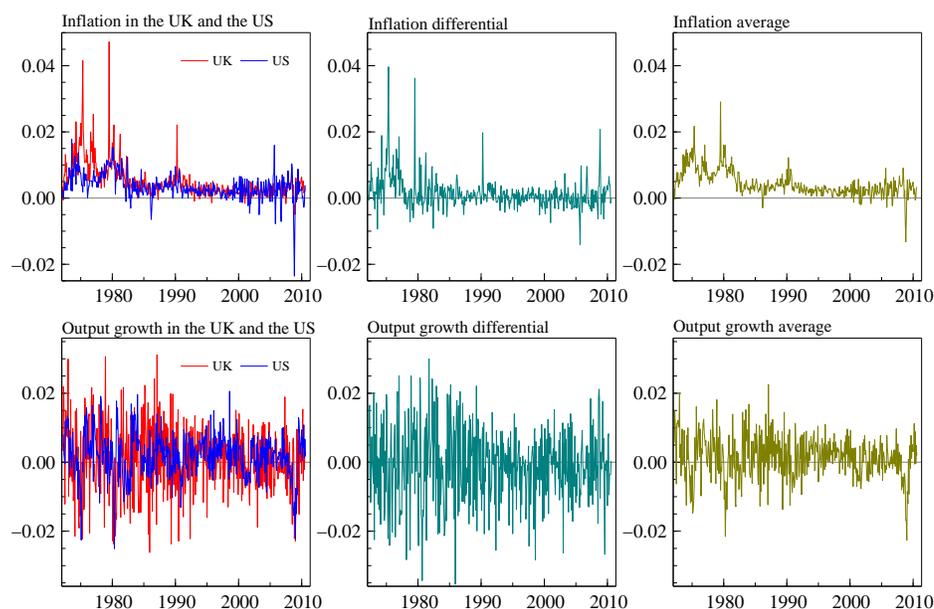


Figure 1 Inflation rates and output growth rates.

Moving to the asset markets, the further discussion is structured along some of the central international parity conditions.

3.2.2 Purchasing power parity

It might have come as a surprise to some readers that we included in our analysis the differences in inflation rates between the UK and US but not the relative price level. In light of the purchasing power parity (PPP) theory, one would have expected that the nominal exchange rate follows the relative price level of the two countries. Thus, the real exchange rate $s_t = e_t + p_t - p_t^*$, which measures the deviation of the nominal exchange rate from the relative price level, should be mean-reverting, such that the law of one price holds at least in the long term.

However, as can be seen in Figure 2, purchasing power parity clearly does not hold for the $\$/\pounds$ exchange rate over the sample period. The Pound Sterling appreciated in real terms by more than 70% from the end of 1984 to the beginning of 2008. In our judgement, the non-stationarity of the real exchange rate can not be explained within the set of macro variables considered here. We therefore leave this issue for further investigations.

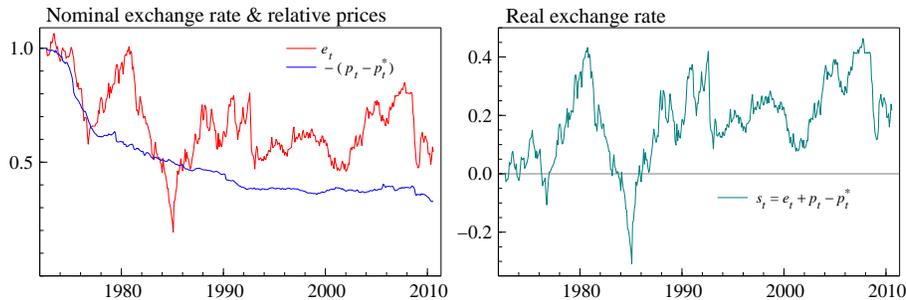


Figure 2 Nominal exchange rate, relative prices and the real exchange rate.

3.2.3 Expectations hypothesis of the term structure

In the expectations model of the term structure, the yield of a zero bond with a maturity of T periods equals the average of the expected one-period interest rates plus a potential risk premium, ϕ_t :

$$r_t = \frac{1}{T} \sum_{j=0}^{T-1} \mathbb{E}_t i_{t+j} + \phi_t. \quad (7)$$

If the short-term interest rate and the risk premium are stationary processes, it follows from (7) that the spread between i_t and r_t is also stationary, $r_t - i_t \sim I(0)$.

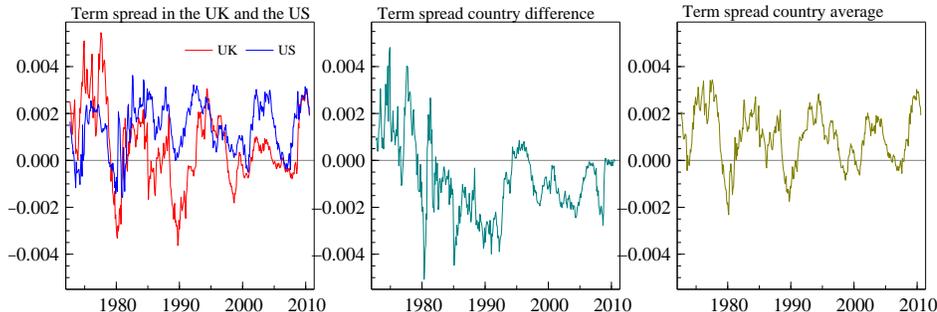


Figure 3 Term structure in the UK and the US, and their differences and averages.

Figure 3 plots the term spread for the UK and the US, as well as their differences and averages. While the term spread appears potentially stationary for the US, this clearly is not the case for the UK term spread. These conjectures were confirmed by ADF tests. We therefore should not expect that short and long-term interest rate differentials cointegrate, but the short and long-term interest rate averages are a candidate.

3.2.4 Nominal interest rate parity

Figure 4 looks at the potential cointegration between the nominal short- and long-term interest rates in the UK and the US. Due to the accommodating UK monetary policy in the 1970s, the long-term interest-rate differential shows clear signs of non-stationarity. As the UK short-term interest rates do not fully reflect the inflation problem of that time period, the short-term interest-rate differential conversely is a potential candidate for a cointegration relation. Both interest rate averages are non-stationary. An ADF

test for the long-term interest rate average rejects marginally a unit root in favour of trend stationarity, what is supposed to be a sample effect, due to the steady decline since 1981.

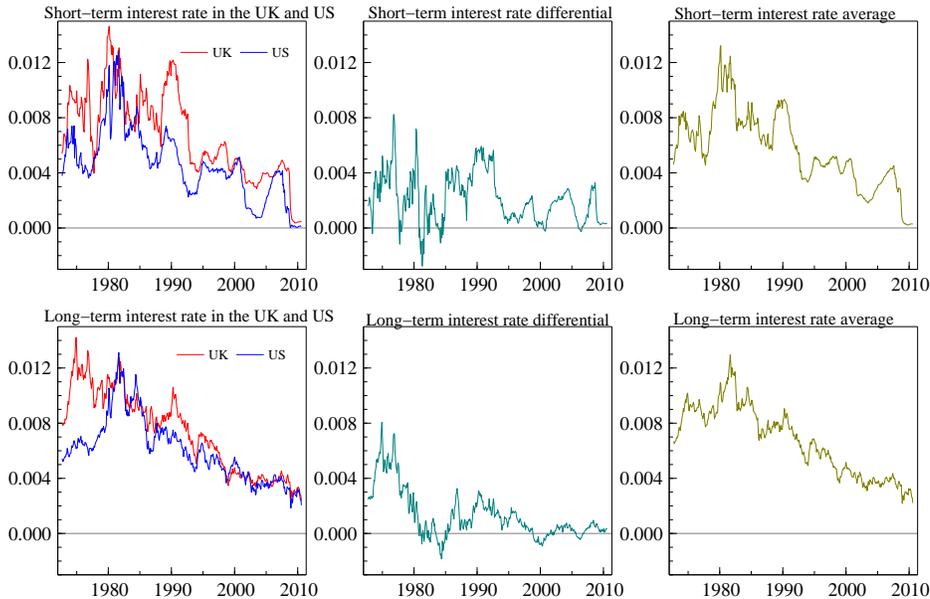


Figure 4 Nominal short- and long-term interest rates, and country differences and averages.

3.2.5 The Fisher hypothesis and the real interest rate parity

Another important relation for our empirical analysis is the Fisher hypothesis. It states that the nominal interest rate equals the real interest rate ρ_t , invariant to monetary policy, plus inflation expectations,

$$i_t = \rho_t + E_t\pi_{t+1}, \quad (8)$$

where the real interest rate is determined by the marginal product of capital and thus expected to be stationary with a low variance.

The Fisher relation motivates the real interest rate parity, according to which the ex-ante real interest rates in home and foreign country should equalize in the long run, i.e.:

$$\rho_t - \rho_t^* = (i_t - E_t\pi_{t+1}) - (i_t^* - E_t\pi_{t+1}^*) \sim I(0). \quad (9)$$

Theoretically, the calculation of ex-ante real interest rate involves future inflation expectations. As those are empirically difficult to measure, we focus here on a naive definition of the real interest rate using the current backward-looking inflation.⁶ These are plotted in Figure 5. Both the short-term and the long-term real interest rates for the UK and the US show a level shift in 1981. Since then a downward trend is present. Overall, the real long-term interest rate differential is more likely to be stationary than the real short-term differential. The level shift in 1981 is also present in the real rate averages.

⁶A common alternative measurement approach would involve the use of realized future inflation rates based on the rational expectations hypothesis, which excludes systematic forecast errors of the agents. This procedure is, however, not compatible with the VAR modelling approach used in this paper.

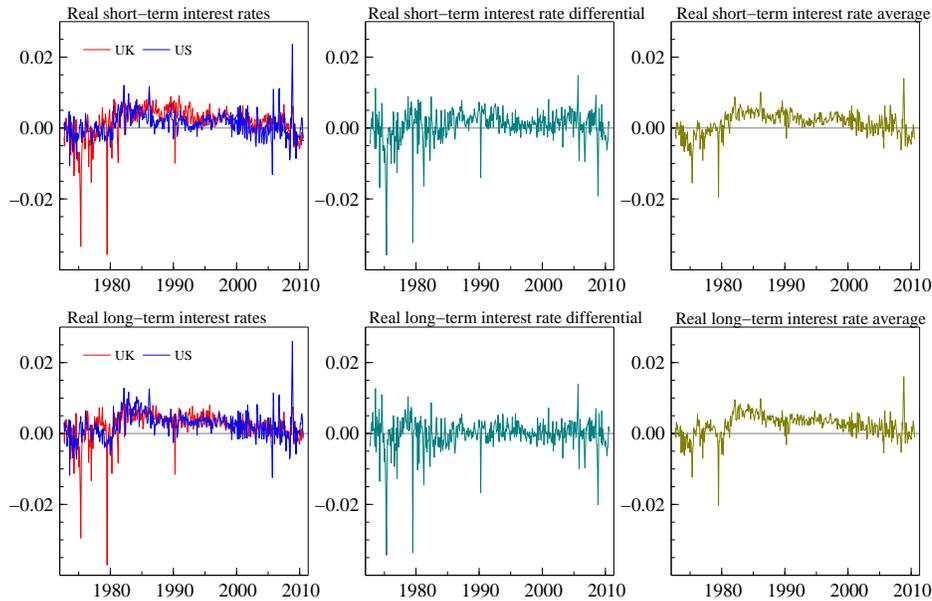


Figure 5 Naive real short- and long-term interest rates, country differences and averages.

3.2.6 Uncovered interest parity

A central parity condition is the uncovered interest rate parity (UIP), which requires that the expected return on the domestic asset is, in equilibrium, equal to expected return, measured in the home currency, on a foreign asset with otherwise identical characteristics. For a one-period bond, this implies:

$$i_t = i_t^* - E_t \Delta e_{t+1}. \quad (10)$$

Under rational expectations, there are no systematic forecast errors and equation (10) can be rewritten as:

$$\xi_t = i_t^d + \Delta e_{t+1}, \quad (11)$$

where ξ_t is a martingale difference sequence and measures the excess return of the UK bond. The realized excess returns over the sample period and their cumulation can be seen in Figure 6.

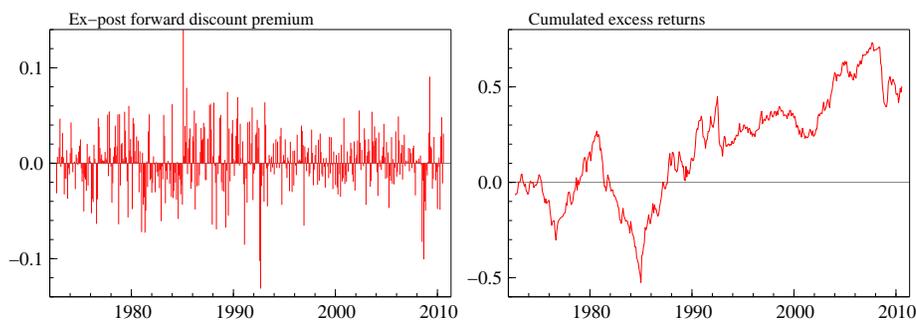


Figure 6 Deviations from UIP: Ex-post excess returns and their cumulation.

The UIP condition in (10) has been formulated for a one-period bond. We now consider its generalization to bonds with multi-period maturities. According to the expectations hypothesis of the term structure, we have that the long-term interest rate, or more precisely the yield of a zero bond of maturity

of T periods, equalizes the expected average return of one-period bonds over T periods:

$$r_t^d = \frac{1}{T} \sum_{j=0}^{T-1} \mathbb{E}_t i_{t+j}^d. \quad (12)$$

Combining (12) with the forward solution of the UIP relation in (10) for e_t ,

$$e_t = \mathbb{E}_t e_{t+T} + \sum_{j=0}^{T-1} \mathbb{E}_t i_{t+j}^d, \quad (13)$$

we get the multi-period form of UIP,

$$e_t = \mathbb{E}_t e_{t+T} + T(r - r^*)_t, \quad (14)$$

which states that the exchange rate is determined by the long-term exchange rate expectation, $\mathbb{E}_t e_{t+T}$, and T times the long-term interest rate differential. Note that this relation will not hold exactly in our data set due to the different type of bonds under consideration, in which case the impact of the bond yield differential is expected to be systematically smaller.

4 The two-country model

4.1 Application of the ‘Aoki’ method

We assume symmetry in the long-run, but allow in the following for different contemporaneous effects, different short-run dynamics and different speeds of adjustment for the two economies. The symmetry assumption in the long-run allows to apply the method, proposed by Aoki (1981) to determine the long-run properties of the model in two smaller subsystems. It makes the cointegration analysis in smaller subsystems feasible.

The variables of the model are transformed from domestic-foreign into country-average-difference variables. The exchange rate is included in the country-difference system. The cointegration analysis is performed in a country difference model of dimension 5 and a country average model of dimension 4, see chapter 4.2 and 4.3. Subsequently the variables are transformed back into the original system, the error correction terms are preserved and the short-run is analysed in the full 9-dimensional system.

4.2 Cointegrated vector autoregression I - Country difference model

In the following we seek to develop a congruent statistical model for the macro dynamics involving the inflation differential, $\pi_t^d = \pi_t - \pi_t^*$, the output growth differential, $\Delta y_t^d = \Delta y_t - \Delta y_t^*$, the short-term interest rate differential, $i_t^d = i_t - i_t^*$, the long-term interest rate differential, $r_t^d = r_t - r_t^*$, and the exchange rate e_t . The results of Augmented Dickey Fuller tests indicate that the output growth differential Δy_t^d is stationary, the short-term interest rate differential i_t^d is marginally stationary and the other time series were found to be $I(1)$. Thus, the vector process, $\mathbf{y}_t = (\pi_t^d, \Delta y_t^d, i_t^d, r_t^d, e_t)'$ is integrated of order one: $\mathbf{y}_t \sim I(1)$.

As discussed, the first step involves the specification of the deterministic terms, selection of the lag length and misspecification test to check the validity of the assumptions made. The lag structure analysis of the unrestricted VAR, commencing from a maximum lag length of thirteen with consecutive F-tests for excluded individual and joint lags, indicates a lag order of four. An unrestricted constant is included as the only deterministic term. A linear time trend was found to be statistically insignificant.

Table 3 Misspecification tests for equations of the unrestricted VAR(4).

Test		π_t^d	Δy_t^d	i_t^d	r_t^d	e_t
AR 1-13	F(13, 428)	0.947 [0.530]	1.473 [0.124]	1.438 [0.138]	1.167 [0.302]	1.182 [0.289]
Normality	$\chi^2(2)$	149.11** [0.000]	0.27 [0.875]	125.83** [0.000]	30.20** [0.000]	39.70** [0.000]
ARCH 1-13	F(13, 415)	1.591 [0.085]	1.660 [0.067]	7.918** [0.000]	5.371** [0.000]	1.816* [0.039]
Hetero	F(40, 400)	2.300** [0.000]	0.908 [0.634]	2.742** [0.000]	3.394** [0.000]	1.386 [0.065]

** significant at 1% level, * significant at 5% level.

The results of tests for misspecification are displayed in Table 3. There are no problems of autocorrelation in the equations. However, the normality test shows serious non-normality mainly due to excess kurtosis in all but the output growth equation. Also, heteroscedasticity and ARCH effects, mainly in the interest rate equations, are detected. But overall the residuals are sufficiently well behaved to proceed with the system.⁷

We continue by analyzing the long-run properties of the system. The number of stable long-run relations $\beta'y_t$, which is equal to the rank of the matrix $\mathbf{\Pi}$ of the vector equilibrium-correction mechanism in (4), is determined by the Johansen (1995) test for $I(1)$ cointegration. The eigenvalues and trace test results are shown in Table 4. The long-run properties of the system are characterized by four cointegration relations, $r = \text{rank}(\mathbf{\Pi})$ is 4. With dimension $K = 5$ and rank $r = 4$ there is one unit root in the system.

Table 4 Johansen likelihood ratio trace test of $H_0 : \text{rank} \leq r$.

r	eigenvalue	trace test	prob
0	0.263	274.60 **	[0.000]
1	0.189	133.48 **	[0.000]
2	0.042	36.73 **	[0.006]
3	0.030	16.69 *	[0.031]
4	0.005	2.48	[0.115]

** significant at 1% level, * significant at 5% level.

To identify the long-run structure of the system, we continue with the preliminary analysis of testing structural hypotheses regarding α and β . The test results for potential cointegration vectors are shown in Table 5. Here, we impose sequentially restrictions on one cointegration vector while leaving the others unconstrained. Hypotheses H_1 to H_5 test if the inflation spread, the output growth spread, the interest rate spreads or the nominal exchange rate constitute cointegration vectors, i.e., stationary relationships, on their own. According to the likelihood ratio (LR) test statistics, the output growth spread and the short term interest rate spread are possible stationary cointegration relations. Hypotheses H_6 and H_7 state that real interest rates differentials are stationary. This is accepted with a p-value of 0.54 for the long-term but rejected for the short-term differential. H_8 rejects the stationarity of the country differences in the term structure, i.e., the spread between long and short-term interest rates differentials.

In Table 6 we test for long-run weak exogeneity of the variables of the system. Under the null

⁷Some of the non-normalities can be traced back to the reduction in volatility during the Great Moderation as well as outliers, for which dummy variables will be included in the PSVECM in §4.6.

Table 5 Testing simple cointegration relations.

	π_t^d	Δy_t^d	i_t^d	r_t^d	e_t	$\chi^2(1)$	prob
H ₁	1	0	0	0	0	10.6	[0.00]
H ₂	0	1	0	0	0	0.23	[0.63]
H ₃	0	0	1	0	0	3.04	[0.08]
H ₄	0	0	0	1	0	11.2	[0.00]
H ₅	0	0	0	0	1	5.60	[0.02]
H ₆	-1	0	1	0	0	6.67	[0.01]
H ₇	-1	0	0	1	0	0.50	[0.48]
H ₈	0	0	-1	1	0	6.62	[0.01]

hypothesis of a particular zero row in α , the corresponding variable is not adjusting towards the long-run equilibrium. The LR test results of the restrictions on α show that, with a p-value of 0.72, the bond yield differential is the only weakly exogenous variable. Thus, we identified the long-term interest rate differential r_t^d as the unique common stochastic trend in the system.⁸

Table 6 Testing for weak exogeneity.

	π_t^d	Δy_t^d	i_t^d	r_t^d	e_t
$\chi^2(4)$	—	136.14	15.97	2.07	14.84
	[0.00]	[0.00]	[0.00]	[0.72]	[0.00]

Altogether, the tests of hypotheses H₁ to H₈ suggests three linearly independent cointegration vectors. Given a rank of four we will need to identify one further composite cointegration vector. Following the modelling approach suggested by Juselius (2006), the following cointegration vectors were identified by paying attention not only to statistical acceptability but also to consistency with economic theory:

- (i) *Stationary output growth differential.*

$$\Delta y_t^d = \Delta y_t - \Delta y_t^* \sim I(0). \quad (15)$$

The first cointegration vector is the difference between the UK and US output growth rates. Stationarity is expected here due to the stationarity of the output growth rates of both countries.

- (ii) *Stationary nominal short-term interest rate differential.*

$$i^d = i_t - i_t^* \sim I(0). \quad (16)$$

This is somewhat surprising given our previous result that the long-term interest rate differential is nonstationary and constitutes the stochastic trend of the system. In other words, while the nominal interest rate parity holds for the money markets, it is violated for the bond markets. The opposite holds for the real interest rate parity:

- (iii) *Stationary real long-term interest rate differential.*

$$\rho_t^d = r_t^d - \pi_t^d = (r - \pi)_t - (r^* - \pi^*)_t \sim I(0). \quad (17)$$

⁸In the following, we will see that the long-term interest rate differential appears to be driven by long-term inflation expectations as predicted in Fisher hypothesis.

The third cointegrating vector reflects the real interest rate parity and is closely related to the Fisher hypothesis, where the real long-term interest rates are calculated naively with the current rather than the expected future inflation. Since r_t^d is nonstationary this must also hold for the inflation differential, which is driven by the same stochastic trend. It is also worth noting that due to (16) and (17) the UK and US term structures do not cointegrate.

- (iv) *Nominal long-term interest-rate differential based exchange rate determination.* The last cointegration vector is a UIP inspired exchange rate determination relation:

$$e_t - 87.2(r - r^*)_t \sim I(0). \quad (18)$$

This cointegration vector should be interpreted in light of the multi-period form of UIP. For zero bonds with a maturity of 10 years, respectively $T = 120$ months, the formula in (14) results in:

$$e_t = E_t e_{t+120} + 120r_t^d. \quad (19)$$

While, for the type of government bonds analyzed here, the relation above only holds approximately, the estimated multiplier of 87.2 with a 2σ interval of [43.10, 131.33] is consistent with the theory. Furthermore, with sample averages of 8.9 and 7.3 of the yield of 10-year government bonds of the UK and the US, the average duration is only 6.8 and 7.3 years, respectively. Thus, actually, the point estimate of 87.2 is very close to the predicted values of 81.6 and 87.6. According to (18) and (19), the long-term equilibrium movement in the foreign exchange rate can be traced back to the non-stationary long-term interest rate differential, exhibiting long swings, and stable long-term exchange rate expectations.

The system estimation results for the four cointegration vectors and their interaction with the variables of the system are shown in Table 7.

Table 7 Cointegration vectors and loadings, t-values in brackets.

	Cointegration vectors				Loadings			
	β_1	β_2	β_3	β_4	α_1	α_2	α_3	α_4
π_t^d	0	0	-1	0	0.025 (0.62)	0.118 (0.93)	0.736** (9.80)	0.0006 (0.45)
Δy_t^d	1	0	0	0	-1.230** (-12.10)	-0.159 (-0.49)	0.186 (0.98)	-0.0002 (-0.16)
i_t^d	0	1	0	0	0.004 (0.62)	-0.073** (-3.96)	0.006 (0.55)	-0.0002 (-1.26)
r_t^d	0	0	1	-87.2 (4.0)	-0.001 (-0.17)	-0.011 (-1.05)	0.004 (0.64)	0.0001 (0.83)
e_t	0	0	0	1	-0.191 (-0.67)	-0.074 (-0.08)	0.993 (1.89)	-0.031** (-3.20)

** significant at 1% level, * significant at 5% level.

The three over-identifying restrictions on the cointegration space are accepted by the likelihood ratio (LR) test with a statistic of $\chi^2(3) = 3.67$ and a p-value of 0.30. The only unrestricted β -coefficient is precisely estimated. In contrast, only few α -coefficients are statistically different from zero. Altogether we find that the long-term interest rate differential, r_t^d , is of central importance to the system. It constitutes the common stochastic trend, it cointegrates with the inflation differential π_t^d to a stationary

‘real’ long-term rate differential, and it also drives the exchange rate $e_t = 87.2r_t^d$, which is consistent with UIP and stable long-term exchange rate expectations $E_t e_{t+120}$. The output gap y_t^d and the short-term rate differential i_t^d are both self error correcting and weakly exogenous to the other cointegration relations.

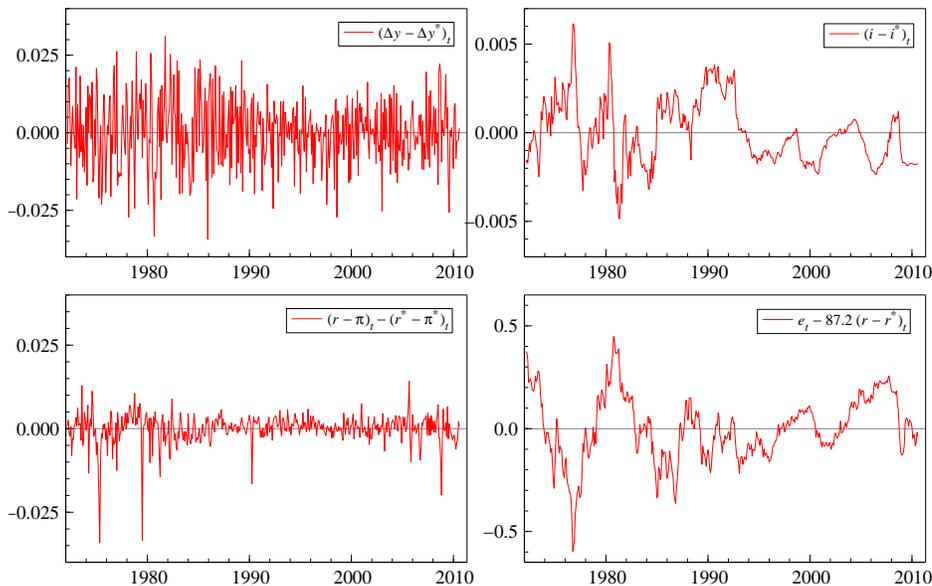


Figure 7 The four cointegrating vectors.

The four cointegrating relations are plotted in Figure 7. The upper panels are just the output growth and the short-term interest rate differentials. In the lower left panel the real long-term interest rate differential can be seen, which is dominated by the pattern of the inflation differential. The only new time series is the last diagram which shows the deviation of the exchange rate from its long-run equilibrium with the bond yield differential.

Table 8 Combined long-run effects $\mathbf{\Pi} = \alpha\beta^{r'}$, standard errors in brackets.

	π_{t-1}^d	Δy_{t-1}^d	i_{t-1}^d	r_{t-1}^d	e_{t-1}
$\Delta\pi_t^d$	-0.736** (0.075)	0.025 (0.040)	0.118 (0.127)	0.682** (0.142)	0.001 (0.001)
$\Delta^2 y_t^d$	-0.186 (0.189)	-1.230** (0.102)	-0.159 (0.321)	0.234 (0.358)	-0.001 (0.003)
Δi_t^d	-0.006 (0.011)	0.004 (0.006)	-0.073** (0.018)	0.028 (0.021)	-0.0002 (0.0002)
Δr_t^d	-0.004 (0.006)	-0.001 (0.003)	-0.011 (0.010)	-0.004 (0.011)	0.0001 (0.0001)
Δe_t	-0.993 (0.526)	-0.191 (0.283)	-0.074 (0.892)	3.667** (0.996)	-0.031** (0.010)

** significant at 1% level, * significant at 5% level.

The adjustment matrix $\mathbf{\Pi} = \alpha\beta^{r'}$ reported in Table 8 determines how the system reacts to the state of the endogenous variables. The negative signs on the diagonal of the matrix indicate stable self-referencing feedback mechanisms for all variable apart from r_t^d , which in Table 6 was found to be weakly exogenous. The only statistically significant cross-equation feedbacks are the inflation differential and

the exchange rate reacting to the bond yield differential, which are driven by the cointegration relations (17) and (18).

4.3 Cointegrated vector autoregression II - Country average model

The country average subsystem is a four dimensional model containing the inflation average, $\pi_t^a = 0.5(\pi_t + \pi_t^*)$, the output growth average, $\Delta y_t^a = 0.5(\Delta y_t + \Delta y_t^*)$, the short-term interest rate average, $i_t^a = 0.5(i_t + i_t^*)$ and the long-term interest rate average, $r_t^a = 0.5(r_t + r_t^*)$. The results of Augmented Dickey Fuller tests indicate that the output growth average Δy_t^a is stationary, the inflation average π_t^a is trend stationary, also the interest rate averages are close to be marginally trend stationary. Altogether the vector process, $\mathbf{y}_t = (\pi_t^a, \Delta y_t^a, i_t^a, r_t^a)'$ is integrated of order one: $\mathbf{y}_t \sim I(1)$.

Again we start with the specification of the deterministic terms, selection of the lag length and misspecification test to check the validity of the assumptions made. The lag structure analysis of the unrestricted VAR, commencing from a maximum lag length of thirteen with consecutive F-tests for excluded individual and joint lags, indicates a lag order of three. An unrestricted constant is included as deterministic term. A trend is statistical significant, but not included into the model, because of economic reasons. When a trend is included, the inflation average together with a trend, or also the long-term interest rate average together with a trend, is a possible stationary cointegration relationship. That the inflation rate average is supposed to be deterministically downward trended is a pure sample effect and not supported by economic theory.

Table 9 Misspecification tests for equations of the unrestricted VAR(3).

Test		π_t^a	Δy_t^a	i_t^a	r_t^a
AR 1-13	F(13, 436)	2.877** [0.001]	2.142* [0.011]	2.084* [0.014]	0.932 [0.520]
Normality	$\chi^2(2)$	254.13** [0.000]	16.15** [0.000]	223.49** [0.000]	25.92** [0.000]
ARCH 1-13	F(13, 423)	2.790** [0.001]	1.127 [0.334]	6.007** [0.000]	4.508** [0.000]
Hetero	F(24, 424)	2.325** [0.001]	2.525** [0.000]	7.408** [0.000]	6.747** [0.000]

** significant at 1% level, * significant at 5% level.

The results of tests for misspecification are displayed in Table 9. The autocorrelation test with 13 lags is a demanding test to pass, therefore we are content with a 1% significance level, which is passed in all equations, but the inflation rate equation. However, the normality test shows serious non-normality mainly due to excess kurtosis in all equations.⁹ Also, heteroscedasticity and ARCH effects are detected.

We continue by analyzing the long-run properties of the system. The number of stable long-run relations $\beta' \mathbf{y}_t$, which is equal to the rank of the matrix $\mathbf{\Pi}$ of the vector equilibrium-correction mechanism in (4), is determined by the Johansen (1995) test for $I(1)$ cointegration. The eigenvalues and trace test results for a model with a constant are shown in Table 10. According to this result the rank of the matrix $\mathbf{\Pi}$ is two. The constant is in the unrestricted model, not only jointly with a F-test p-val of [0.82], but also in every equation, highly non-significant. The trace test results for a model without a constant, see Table 11, suggest a rank of three. The three cointegration relationships in a model without constant and mean-adjusted data are also accepted in a model with constant, which we discuss in the following.

⁹Note that excess kurtosis is not affecting the trace test.

To sum up, a rank of 3 is also appropriate in a model with a non-significant constant. So the long-run properties of the system are characterized by three cointegration relations, $r = \text{rank}(\mathbf{\Pi})$ is 3. With dimension $K = 4$ and rank $r = 3$ there is one unit root in the system.

Table 10 Johansen likelihood ratio trace test of $H_0 : \text{rank} \leq r$. Model including a constant.

r	eigenvalue	trace test	prob
0	0.184	149.23 **	[0.000]
1	0.088	55.25 **	[0.000]
2	0.026	12.61	[0.131]
3	0.001	0.65	[0.420]

** significant at 1% level, * significant at 5% level.

Table 11 Johansen likelihood ratio trace test of $H_0 : \text{rank} \leq r$. Model without constant.

r	eigenvalue	trace test	prob
0	0.184	148.04 **	[0.000]
1	0.087	54.10 **	[0.000]
2	0.024	11.91	[0.058]
3	0.001	0.57	[0.517]

** significant at 1% level, * significant at 5% level.

To identify the long-run structure of the system, we continue with the preliminary analysis of testing structural hypotheses regarding α and β . The test results for potential cointegration vectors are shown in Table 12. Here, we impose sequentially restrictions on one cointegration vector while leaving the others unconstrained. Hypotheses H_1 to H_4 test if the inflation average, the output growth average or the interest rate averages constitute cointegration vectors, i.e., stationary relationships, on their own. According to the likelihood ratio (LR) test statistics, the output growth average is a possible stationary cointegration relationship. Hypotheses H_5 and H_6 state that real interest rates averages are stationary. This is accepted with a p-value of 0.38 for the short-term and with a p-value of 0.23 for the long-term average. H_7 tests and accepts the stationarity of the country average in the term structure, i.e., the spread between long and short-term interest rates average.

Table 12 Testing simple cointegration relations.

	π_t^a	Δy_t^a	i_t^a	r_t^a	$\chi^2(1)$	prob
H_1	1	0	0	0	11.31	[0.00]
H_2	0	1	0	0	0.03	[0.86]
H_3	0	0	1	0	9.26	[0.00]
H_4	0	0	0	1	11.31	[0.00]
H_5	-1	0	1	0	0.78	[0.38]
H_6	-1	0	0	1	1.46	[0.23]
H_7	0	0	-1	1	0.007	[0.94]

In Table 13 we test for long-run weak exogeneity of the variables of the system. Under the null hypothesis of a particular zero row in α , the corresponding variable is not adjusting towards the long-run equilibrium. The LR test results of the restrictions on α show that the short-term interest rate average and the long-term interest rate average are not rejected here as weakly exogenous variables. Which variable is the common stochastic trend in the system can not be decided at that stage.

Table 13 Testing for weak exogeneity.

	π_t^a	Δy_t^a	i_t^a	r_t^a
$\chi^2(3)$	36.35	—	7.54	4.76
	[0.00]	[0.00]	[0.06]	[0.19]

Altogether, the tests of hypotheses H_1 to H_7 give a clear idea of cointegration relationships. We use here the long-term real interest rate average instead of the short-term real interest rate average as a cointegration relationship, the decision will be explained after analysing the combined long-run effects in Table 14. A combination of the three cointegration relationships is passing jointly the test and is not only statistically accepted, but economically reasonable.

(i) *Stationary output growth average.*

$$\Delta y_t^a = 0.5(\Delta y_t + \Delta y_t^*) \sim I(0). \quad (20)$$

The first cointegration vector is the average between the UK and US output growth rates. Stationarity is expected here due to the stationarity of the output growth rates of both countries.

(ii) *Stationary real long-term interest rate average.*

$$r_t^a - \pi_t^a = 0.5[(r - \pi)_t + (r^* - \pi^*)_t] \sim I(0). \quad (21)$$

The second cointegration vector is the average between the UK and the US real long-term interest rates. The Fisher relation suggests the stationarity of the real rates, for a country average of real rates, this is even more likely.

(iii) *Stationary term spread average.*

$$r_t^a - i_t^a = 0.5[(r - i)_t + (r^* - i^*)_t] \sim I(0). \quad (22)$$

The third cointegrating vector reflects the stationarity of the spread between long and short-term interest rate average. Since the short-term interest rate average i_t^a is non-stationary this must also hold for the long-term interest rate average r_t^a , which is driven by the same stochastic trend.

Table 14 Cointegration vectors and loadings, t-values in brackets.

	Cointegration vectors			Loadings		
	β_1	β_2	β_3	α_1	α_2	α_3
π_t^a	0	-1	0	-0.022 (-0.80)	0.251** (5.95)	-0.055 (-0.54)
Δy_t^a	1	0	0	-0.619** (-9.45)	0.136 (1.35)	0.838** (3.46)
i_t^a	0	0	-1	0.008 (1.90)	-0.006 (-0.95)	0.023 (1.56)
r_t^a	0	1	1	0.002 (0.70)	-0.009 (-1.93)	-0.011 (-1.07)

** significant at 1% level, * significant at 5% level.

The system estimation results for the three cointegration vectors and their interaction with the variables of the system are shown in Table 14. The three over-identifying restrictions on the cointegration space are accepted by a likelihood ratio (LR) test with a statistic of $\chi^2(3) = 1.54$ and a p-value of 0.67. All β -coefficients are restricted. Only some α -coefficients are statistically different from zero. Both interest rate averages are reacting to cointegration relationships with a significance level of close to 5%. The short-term rate average is likely to adjust to the output growth average, while the long-term rate average is adjusting to the real interest rate average. Both interest rates are not significantly adjusting to the term spread average.

The three cointegrating relations are plotted in Figure 8. The left panel is just the output growth average. The middle panel is the real long-term interest rate average. In the right panel the spread between long and short-term interest rate average can be seen.

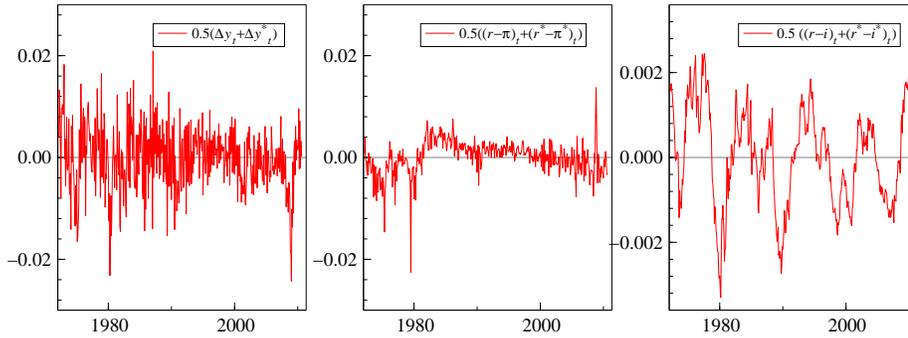


Figure 8 The three cointegrating vectors in the country average model.

The adjustment matrix $\mathbf{\Pi} = \alpha\beta'$ reported in Table 15 determines how the system reacts to the state of the endogenous variables. The negative signs on the diagonal reflect the self correcting effects of the variables. The value for the long-term interest rate average variable is with a t-value of 1.76 marginally significant. The short-term interest rate average is not significantly self correcting. The output growth average is strongly adjusting to the interest rate averages.

Table 15 Combined long-run effects $\mathbf{\Pi} = \alpha\beta'$, standard errors in brackets.

	π_{t-1}^a	Δy_{t-1}^a	i_{t-1}^a	r_{t-1}^a
$\Delta \pi_t^a$	-0.251** (0.042)	-0.022 (0.027)	0.055 (0.101)	0.196 (0.108)
$\Delta^2 y_t^a$	-0.136 (0.101)	-0.619** (0.065)	-0.838** (0.242)	0.974** (0.257)
Δi_t^a	0.006 (0.006)	0.008 (0.004)	-0.023 (0.015)	0.017 (0.016)
Δr_t^a	0.009 (0.004)	0.002 (0.003)	0.011 (0.011)	-0.020 (0.011)

** significant at 1% level, * significant at 5% level.

Because both interest rate averages are adjusting to cointegration relationships, a linear combination out of them is the common stochastic trend. In the moving average representation of the model, a normalized version of the orthogonal complement of the matrix beta, β_{\perp} , is the loading to the common stochastic trend. A possible β_{\perp} is in our case the vector (0, 0, 1, 1). So the stochastic trend is proportional to a linear combination out of the error terms of equation 3 and 4: $trend \propto (\sum \varepsilon_{3t} + \sum \varepsilon_{4t})$.

The long-term real interest rate average is chosen in the second cointegration relationship instead of the short-term real rate because of the following reasons. Due to the term spread cointegration relationship both options are equivalent in a way that they are two different representations of the same model with exactly the same likelihood. But in the following econometric modelling procedure the further restricting of the model leads to different selected parsimonious models for the two options and thus different final models. The decision is made upon the better Akaike and Schwarz information criteria of the final 9-dimensional model, which are both in favour of the model option with the second cointegration relationship to be the stationary long-term real interest rate average.

4.4 Testing for symmetry

We assume symmetry between the two countries in the long-run. Is symmetry also acceptable in the short-run? If yes, the country average and the country difference vector space are orthogonal to each other. Regressors from the country average system would not be significant, when regressed on country difference variables, and the other way around. Thus symmetry can be tested with overidentifying restrictions. In the combined average-difference VECM, see (23), symmetry is tested by testing for significance of the off-diagonal coefficients.

$$\begin{aligned}
\begin{bmatrix} \Delta \mathbf{y}_t^d \\ \Delta \mathbf{y}_t^a \end{bmatrix} &= \begin{bmatrix} \boldsymbol{\nu}^{dd} \\ \boldsymbol{\nu}^{ad} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\alpha}^{dd} & \boldsymbol{\alpha}^{da} \\ \boldsymbol{\alpha}^{ad} & \boldsymbol{\alpha}^{aa} \end{bmatrix} \begin{bmatrix} \boldsymbol{\beta}^{d'} \mathbf{y}_{t-1}^d \\ \boldsymbol{\beta}^{a'} \mathbf{y}_{t-1}^a \end{bmatrix} \\
&+ \begin{bmatrix} \boldsymbol{\Gamma}_1^{dd} & \boldsymbol{\Gamma}_1^{da} \\ \boldsymbol{\Gamma}_1^{ad} & \boldsymbol{\Gamma}_1^{aa} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{y}_{t-1}^d \\ \Delta \mathbf{y}_{t-1}^a \end{bmatrix} + \begin{bmatrix} \boldsymbol{\Gamma}_2^{dd} & \boldsymbol{\Gamma}_2^{da} \\ \boldsymbol{\Gamma}_2^{ad} & \boldsymbol{\Gamma}_2^{aa} \end{bmatrix} \begin{bmatrix} \Delta \mathbf{y}_{t-2}^d \\ \Delta \mathbf{y}_{t-2}^a \end{bmatrix} \\
&+ \begin{bmatrix} \boldsymbol{\Gamma}_3^{dd} & 0 \\ \boldsymbol{\Gamma}_3^{ad} & 0 \end{bmatrix} \begin{bmatrix} \Delta \mathbf{y}_{t-3}^d \\ 0 \end{bmatrix} + \begin{bmatrix} \mathbf{u}_t^d \\ \mathbf{u}_t^a \end{bmatrix}, \\
\begin{bmatrix} \mathbf{u}_t^d \\ \mathbf{u}_t^a \end{bmatrix} &\sim \text{IID} \left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma}^{dd} & \boldsymbol{\Sigma}^{da} \\ \boldsymbol{\Sigma}^{ad} & \boldsymbol{\Sigma}^{aa} \end{bmatrix} \right)
\end{aligned} \tag{23}$$

In table 16 hypotheses H_1 and H_2 show that the cointegration relationships of the country average model have explanatory power in the country difference model and the other way around. From this can be followed, that the adjustments to the long-run equilibria are different in the two countries. Hypotheses H_3 and H_4 test the significance of the off-diagonal short-run dynamics. The short-run dynamics of the country average model has marginally explanatory power in the country difference equations, but the short-run dynamics of the country difference model is clearly not helpful in explaining the country average variables. Hypotheses H_5 and H_6 reject that the adjustment coefficients jointly with the short-run dynamics of one subsystem have no explanatory power in the other subsystem. Symmetry in the short-run and in the adjustment to the long-run is clearly rejected. Therefore in the following we return to the full model of domestic-foreign variables. The cointegration relationships from the subsystems are preserved.

4.5 Identifying instantaneous causality

The residual correlation matrix of the VECM(3) with $\boldsymbol{\beta}^{s,r}$ is reported in Table 17. Clear statistically significant contemporaneous correlation of shocks is between the short and long-term interest rates of each country. Thus, in the very short term, the term structure is the strongest link between the macroeconomic variables. As the dominant force in transmitting and absorbing macroeconomic shocks,

Table 16 Testing overidentifying restrictions.

$H_o^1 : \alpha^{da} = 0$	$\chi^2(15) = 27.2$	[0.027]
$H_o^2 : \alpha^{ad} = 0$	$\chi^2(16) = 47.7$	[0.000]
$H_o^3 : \gamma_1^{da} = 0, \gamma_2^{da} = 0$	$\chi^2(40) = 55.3$	[0.055]
$H_o^4 : \gamma_1^{ad} = 0, \gamma_2^{ad} = 0, \gamma_3^{ad} = 0$	$\chi^2(60) = 72.4$	[0.130]
$H_o^5 : \alpha^{da} = 0, \gamma_j^{da} = 0$	$\chi^2(55) = 90.4$	[0.002]
$H_o^6 : \alpha^{ad} = 0, \gamma_j^{ad} = 0$	$\chi^2(76) = 134.8$	[0.000]

it will play an important role in the transmission of monetary shocks to the exchange rate. Another large contemporaneous correlation of shocks is between the domestic and foreign bond yields, due to the strong interconnectedness of financial markets.

Table 17 Contemporaneous correlation.

	π_t	π_t^*	Δy_t	Δy_t^*	i_t	i_t^*	r_t	r_t^*	e_t
π_t	1								
π_t^*	0.13	1							
Δy_t	-0.02	-0.06	1						
Δy_t^*	0.01	0.05	0.06	1					
i_t	-0.00	0.15	0.07	0.02	1				
i_t^*	-0.02	0.10	-0.04	0.19	0.13	1			
r_t	0.08	0.10	0.03	0.06	0.52	0.22	1		
r_t^*	0.04	0.17	0.01	0.17	0.12	0.54	0.38	1	
e_t	0.05	0.12	-0.05	-0.06	-0.09	-0.14	-0.10	-0.15	1

For further investigations of these issues, the correlation matrix in Table 17 is subjected to a graph-theoretical search for instantaneous causal relations. The Conservative PC algorithm (CPC) is applied in the following, which is a variant of the PC algorithm, designed to improve arrowpoint orientation accuracy. The CPC algorithm finds, at a 10% significance level the acyclical graph shown in Figure 9. Four directed and six undirected edges are found. The final causal structure is developed in the following by comparing information criteria of final models.

To direct the six undirected edges the information criteria of the $2^6 = 64$ different possible model options are compared. When the model options, which lead to a cyclical directed graph, are removed then 48 model options remain. Because the nine equations of the final model are estimated by OLS and having the present structure, it is possible to calculate the information criteria for three subsystems and add those results together. The first subsystem is the UK output growth equation together with the UK bond rate equation. Both equations are already fully determined with regard to contemporaneous effects. The next subsystem is including the US output growth, the US short-term interest rate, the US long-term interest rate and the exchange rate. Here are four undirected edges to direct, when removing cyclicity, then 12 model options remain. The exchange rate is not significant in a final US short-term interest rate equation, therefore those 6 model options have throughout inferior explanatory power, than the six model options with a US short-term interest rate regressor in the exchange rate equation, which are compared in Table 18. The lowest Akaike and Schwarz criterium has model 3, with the US output growth driving the short- and the long-term US interest rates, the short-term rate driving the long-term rate and a direct effect of the US short-term rate on the exchange rate. All links are reasonable, following economic theory.

The last subsystem to analyse consists of the two inflation rates and the UK short-term interest rate,

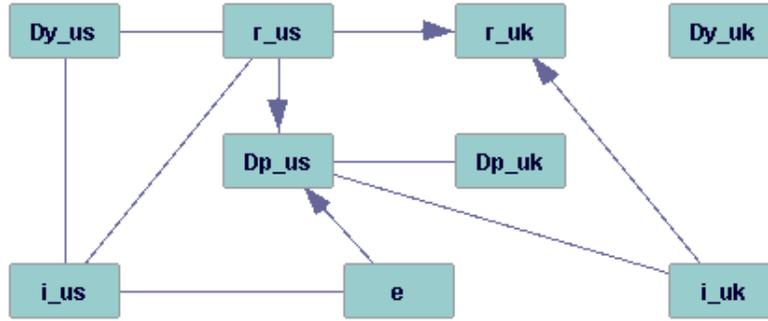


Figure 9 Causal structure, Conservative PC algorithm (10%).

Table 18 Information criteria of final selected models with different causal structure.

	subsystem with $(\Delta y^*, i^*, r^*, e)$	<i>AIC</i>	<i>SC</i>
M_1	$i^* \rightarrow \Delta y^*, i^* \rightarrow r^*, r^* \rightarrow \Delta y^*, i^* \rightarrow e$	-50.924	-50.271
M_2	$i^* \rightarrow \Delta y^*, i^* \rightarrow r^*, \Delta y^* \rightarrow r^*, i^* \rightarrow e$	-50.918	-50.256
M_3	$\Delta y^* \rightarrow i^*, i^* \rightarrow r^*, \Delta y^* \rightarrow r^*, i^* \rightarrow e$	-50.953	-50.309
M_4	$\Delta y^* \rightarrow i^*, r^* \rightarrow i^*, r^* \rightarrow \Delta y^*, i^* \rightarrow e$	-50.869	-50.216
M_5	$\Delta y^* \rightarrow i^*, r^* \rightarrow i^*, \Delta y^* \rightarrow r^*, i^* \rightarrow e$	-50.841	-50.170
M_6	$i^* \rightarrow \Delta y^*, r^* \rightarrow i^*, r^* \rightarrow \Delta y^*, i^* \rightarrow e$	-50.929	-50.266

see Table 19. Although model option 4 has a minimal advantage concerning information criteria, we decide here to proceed with model option 1. The explanation for this is a strong dummy effect in the UK short term interest rate equation on the information criteria. When increasing the information set for the automatic model selection procedure by including an additional regressor to choose from and the regressor is significantly included into the model, also the punished loglikelihood should not decrease after including the additional regressor. In the case of the UK short-term interest rate equation dummies improve the information criteria for the model with the smaller information set.

Table 19 Information criteria of final selected models with different causal structure.

	subsystem with $(\Delta p^*, \Delta p, i)$	<i>AIC</i>	<i>SC</i>
M_1	$\Delta p^* \rightarrow \Delta p, \Delta p^* \rightarrow i$	-39.877	-39.376
M_2	$\Delta p \rightarrow \Delta p^*, \Delta p^* \rightarrow i$	-39.827	-39.326
M_3	$\Delta p \rightarrow \Delta p^*, i \rightarrow \Delta p^*$	-39.831	-39.312
M_4	$\Delta p^* \rightarrow \Delta p, i \rightarrow \Delta p^*$	-39.899	-39.380

After directing all six undirected edges the causal structure is fully determined like shown in Figure 10.

The empirically detected causal structure is mapped into the contemporaneous matrix \mathbf{B}^T , see Table 20¹⁰. Compared with the possible result of a Cholesky decomposition our causal structure is very parsimonious. Only a few non-zero lower-off-diagonal elements are present. Can the causal ordering be

¹⁰The presented causal order is not unique. The choice of one of the possible causal orderings does not influence the further analysis.

approximates in large samples the HannanQuinn (HQ) information criteria (for more about mapping information criteria to significance levels see Campos, Hendry and Krolzig, 2003). The properties of automatic *Gets* selection are discussed in more detail in Hendry and Krolzig (2005).

The final parsimonious model selected by *PcGets* and estimated with OLS is as follows: All coefficients are significant with a t-value of at least 2. The adjusted R^2 of the reduced single equations are from 27% for the exchange rate equation up to 70% for the UK inflation rate equation. Major outliers are corrected by including impulse dummies. The dummies are not reported in detail.

$$\begin{aligned}
\widehat{\Delta\pi_t} = & - 0.0281 (\Delta y + \Delta y^*)_{t-1} + 0.111 [(r - \pi)_{t-1} + (r^* - \pi^*)_{t-1}] \\
& (0.010) \qquad\qquad\qquad (0.017) \\
& - 0.156 (i - i^*)_{t-1} + 0.563 [(r - \pi)_{t-1} - (r^* - \pi^*)_{t-1}] \\
& (0.044) \qquad\qquad\qquad (0.040) \\
& - 0.123 \Delta\pi_{t-1} - 0.0816 \Delta\pi_{t-2} + 0.192 \Delta\pi_t^* - 0.170 \Delta\pi_{t-1}^* \qquad\qquad (24) \\
& (0.037) \qquad\qquad\qquad (0.031) \qquad\qquad\qquad (0.036) \qquad\qquad\qquad (0.045) \\
& - 0.119 \Delta\pi_{t-2}^* - 0.0985 \Delta\pi_{t-3}^* + 0.0251 \Delta^2 y_{t-1} - 0.0112 \Delta e_{t-1}, \\
& (0.041) \qquad\qquad\qquad (0.036) \qquad\qquad\qquad (0.0091) \qquad\qquad\qquad (0.0036) \\
& \hat{\sigma} = 0.00225, \bar{R}^2 = 0.70, 7 \text{ dummies.}
\end{aligned}$$

In the UK inflation rate equation is the highest explanatory power of all nine equations achieved. The speed of adjustment of the UK inflation rate toward the cointegrating real long-term interest rate differential is with 56% per month very high. This suggests that the long-term interest rate differential is a good proxy of differences in inflation expectations in the UK and the US, $\pi_t^{e,d} = \pi_t^e - \pi_t^{*e}$, as predicted by Irving Fisher, such that $r_t^d - \pi_t^d \approx \pi_t^{e,d} - \pi_t^d$. This should, however, give rise to a price puzzle. The contemporaneous effect of the US inflation rate on the UK inflation rate is 19%.

$$\begin{aligned}
\widehat{\Delta\pi_t^*} = & 0.124 [(r - \pi)_{t-1} + (r^* - \pi^*)_{t-1}] - 0.225 [(r - \pi)_{t-1} - (r^* - \pi^*)_{t-1}] \\
& (0.021) \qquad\qquad\qquad (0.033) \\
& - 0.177 \Delta\pi_{t-1}^* - 0.206 \Delta\pi_{t-2}^* - 0.159 \Delta\pi_{t-3}^* + 0.632 \Delta i_{t-1}^* \\
& (0.049) \qquad\qquad\qquad (0.043) \qquad\qquad\qquad (0.040) \qquad\qquad\qquad (0.26) \\
& + 0.790 \Delta i_{t-3}^* + 0.843 \Delta r_{t-1} + 1.40 \Delta r_t^* + 0.0078 \Delta e_t \qquad\qquad (25) \\
& (0.25) \qquad\qquad\qquad (0.35) \qquad\qquad\qquad (0.38) \qquad\qquad\qquad (0.0039) \\
& - 0.00061, \hat{\sigma} = 0.00238, \bar{R}^2 = 0.49, 6 \text{ dummies.} \\
& (0.00015)
\end{aligned}$$

Also in the US inflation rate equation are clear causes for a price puzzle present. The US inflation rate is positively correlated with all US interest rate terms. The effect of the contemporaneous US bond rate is with a size of larger than 1 very strong. The inflation rate is determined by inflation expectations indicated by bond yields.

$$\begin{aligned}
\widehat{\Delta^2 y_t} = & - \underset{(0.037)}{0.391} (\Delta y + \Delta y^*)_{t-1} + \underset{(0.065)}{0.248} [(r - \pi)_{t-1} + (r^* - \pi^*)_{t-1}] \\
& + \underset{(0.18)}{0.430} [(r - i)_{t-1} + (r^* - i^*)_{t-1}] - \underset{(0.041)}{0.731} (\Delta y - \Delta y^*)_{t-1} \\
& + \underset{(0.10)}{0.257} \Delta \pi_{t-2} - \underset{(0.060)}{0.137} \Delta^2 y_{t-2}^* + \underset{(0.82)}{2.36} \Delta i_{t-1} + \underset{(0.83)}{2.25} \Delta i_{t-2} \quad (26) \\
& + \underset{(1.1)}{3.67} \Delta i_{t-3}^* - \underset{(1.6)}{3.95} \Delta r_{t-3}^* - \underset{(0.00062)}{0.00174} , \\
& \hat{\sigma} = 0.00871, \bar{R}^2 = 0.63, 6 \text{ dummies.}
\end{aligned}$$

The UK output growth equation is strongly error correcting with a combined coefficient of minus 1.1. Large bond rates relative to inflation or short-term rates increase output growth in the UK.

$$\begin{aligned}
\widehat{\Delta^2 y_t^*} = & - \underset{(0.023)}{0.237} (\Delta y + \Delta y^*)_{t-1} + \underset{(0.081)}{0.428} [(r - i)_{t-1} + (r^* - i^*)_{t-1}] \\
& + \underset{(0.028)}{0.235} (\Delta y - \Delta y^*)_{t-1} + \underset{(0.06)}{0.118} \Delta^2 p_{t-3} - \underset{(0.04)}{0.222} \Delta^2 y_{t-1}^* \quad (27) \\
& + \underset{(0.47)}{1.16} \Delta i_{t-2} + \underset{(0.54)}{2.40} \Delta i_{t-1}^* + \underset{(0.0077)}{0.0215} \Delta e_{t-2}, \\
& \hat{\sigma} = 0.00484, \bar{R}^2 = 0.51, 11 \text{ dummies.}
\end{aligned}$$

Also the US output growth equation is strongly error correcting. A positive term spread average increases output growth.

$$\begin{aligned}
\widehat{\Delta i_t} = & - \underset{(0.0030)}{0.00670} [(r - \pi)_{t-1} + (r^* - \pi^*)_{t-1}] - \underset{(0.011)}{0.0431} (i - i^*)_{t-1} \\
& - \underset{(0.0044)}{0.0130} \Delta \pi_{t-3} + \underset{(0.0056)}{0.0220} \Delta \pi_t^* + \underset{(0.037)}{0.197} \Delta i_{t-1} + \underset{(0.043)}{0.124} \Delta i_{t-1}^* \quad (28) \\
& + \underset{(0.062)}{0.154} \Delta r_{t-3}^* + \underset{(0.00062)}{0.00162} \Delta e_{t-3} + \underset{(0.00003)}{0.00012} , \\
& \hat{\sigma} = 0.000385, \bar{R}^2 = 0.43, 11 \text{ dummies.}
\end{aligned}$$

The UK short-term interest rate is contemporaneously adjusting to the US inflation rate, as a proxy for the world inflation level.

$$\begin{aligned}
\widehat{\Delta i_t^*} = & \underset{(0.0015)}{0.00681} (\Delta y + \Delta y^*)_{t-1} + \underset{(0.0069)}{0.0215} [(r - i)_{t-1} + (r^* - i^*)_{t-1}] \\
& - \underset{(0.0017)}{0.00712} (\Delta y - \Delta y^*)_{t-1} + \underset{(0.0091)}{0.0356} (i - i^*)_{t-1} \\
& + \underset{(0.00011)}{0.00052} [e_{t-1} - 87.2(r - r^*)_{t-1}] + \underset{(0.0036)}{0.00924} \Delta^2 p_{t-3} + \underset{(0.0044)}{0.0102} \Delta^2 p_{t-1}^* \quad (29) \\
& + \underset{(0.0025)}{0.00823} \Delta^2 y_t^* + \underset{(0.0020)}{0.00490} \Delta^2 y_{t-2}^* + \underset{(0.037)}{0.205} \Delta i_{t-1}^* \\
& - \underset{(0.048)}{0.130} \Delta r_{t-2}^* + \underset{(0.00046)}{0.00108} \Delta e_{t-1} - \underset{(0.00007)}{0.00038} , \\
& \hat{\sigma} = 0.000287, \bar{R}^2 = 0.58, 13 \text{ dummies.}
\end{aligned}$$

Like discussed before the bond rate, as a proxy for inflation expectations, is the driving mechanism. The US monetary policy reacts to inflation expectations and to actual inflation.

$$\begin{aligned} \widehat{\Delta r}_t = & -0.00707 \Delta^2 p_{t-2}^* + 0.311 \Delta i_t + 0.0767 \Delta i_{t-3} + 0.115 \Delta r_{t-1} \\ & (0.0031) \quad (0.020) \quad (0.024) \quad (0.033) \\ & -0.0906 \Delta r_{t-2} - 0.222 \Delta r_{t-3} + 0.349 \Delta r_t^* + 0.136 \Delta r_{t-3}^*, \quad (30) \\ & (0.033) \quad (0.041) \quad (0.034) \quad (0.036) \\ \hat{\sigma} = & 0.000214, \bar{R}^2 = 0.57, 12 \text{ dummies.} \end{aligned}$$

The UK bond rate is mainly reacting to interest rate variables. 31% of a UK monetary policy shock is directly transferred into UK bond yields. One third of a US change in bond yields is copied at once by the UK bond yield.

$$\begin{aligned} \widehat{\Delta r}_t^* = & 0.00591 [(r - \pi)_{t-1} - (r^* - \pi^*)_{t-1}] - 0.00706 \Delta \pi_{t-3}^* \\ & (0.0024) \quad (0.0032) \\ & + 0.00283 \Delta^2 y_t^* - 0.00447 \Delta^2 y_{t-3}^* + 0.382 \Delta i_t^*, \quad (31) \\ & (0.0015) \quad (0.0016) \quad (0.024) \\ \hat{\sigma} = & 0.000226, \bar{R}^2 = 0.45, 8 \text{ dummies.} \end{aligned}$$

The long-term interest rate in the US reacts on impact with 38% to a monetary policy shock. Thus the US central bank can influence long-term rates stronger than the central bank of the UK.

$$\begin{aligned} \widehat{\Delta e}_t = & 1.97 (i - i^*)_{t-1} + 0.755 [(r - \pi)_{t-1} - (r^* - \pi^*)_{t-1}] \\ & (0.57) \quad (0.27) \\ & - 0.0118 [e_{t-1} - 87.2(r - r^*)_{t-1}] + 5.54 \Delta i_{t-3} - 8.79 \Delta i_t^* \\ & (0.0032) \quad (2.4) \quad (2.7) \quad (32) \\ & + 9.56 \Delta r_{t-3}^*, \quad \hat{\sigma} = 0.0255, \bar{R}^2 = 0.27, 8 \text{ dummies.} \\ & (4.0) \end{aligned}$$

Also the exchange rate is error correcting. Three cointegration relationships are driving the exchange rate. The importance of the cointegration approach is obvious. A lot of information would be lost, when the VAR would be specified in differences.

4.7 Testing for the validity and congruency of the model

The efficiency of the single-equation reduction procedure depends on the lack of correlation among the error terms of the model. To investigate the orthogonality assumption we consider the two following statistical tests. Firstly for the exactly identified SVECM, we test for the joint significance of the 26 over-identifying restrictions on the contemporaneous matrix \mathbf{B}^r . In a LR-test the log-likelihood of a SVECM with the contemporaneous matrix \mathbf{B}^r is compared to the log-likelihood of a SVECM with a contemporaneous matrix achieved by applying a Cholesky decomposition. The LR test with a test statistic of $\chi^2(26) = 15.56$ and a p-value of [0.95] does not reject the hypothesis of a diagonal covariance matrix. Secondly, for the selected PSVECM, we can use the likelihood ratio principle to confront our model with its superimposed orthogonal errors against the alternative of an unrestricted variance covariance matrix. To test the involved 36 independent restrictions on $\mathbf{\Omega}$, we compare the log likelihood of the restricted system estimated by OLS with the unrestricted system estimated with Full Information Maximum Likelihood estimation (FIML). Here we utilize the fact that under the condition of a diagonal variance matrix, $\mathbf{\Omega}$, the FIML estimation under normality collapses to OLS. In support of our previous analysis, the LR test with a test statistic of $\chi^2(36) = 32.36$ and a p-value of [0.64] does not reject

the hypothesis of a diagonal covariance matrix. That implies all contemporaneous effects have been captured by the causal search algorithm, the model is valid and can be efficiently estimated by OLS.

The congruency of the model is investigated in Table 21. There are some problems of autocorrelation in the US inflation rate and short-term interest rate equation. With the inclusion of dummies and selection of the model, there is a huge reduction in non-normality, but still some issues of non-normality and heteroscedasticity remain. In some equations are problems of misspecification, documented by the rejected RESET tests. These problems of specification are not due to the model selection, they are also present in the SVECM. Non-linear terms seem to be necessary in the equations.

Table 21 Misspecification tests of the parsimonious SVECM.

Test	π_t	π_t^*	Δy_t	Δy_t^*	i_t	i_t^*	r_t	r_t^*	e_t
AR 1-7	0.828 [0.564]	1.879 [0.071]	1.022 [0.415]	1.136 [0.339]	0.323 [0.943]	0.648 [0.716]	1.056 [0.391]	1.325 [0.237]	0.973 [0.451]
AR 1-13	1.391 [0.160]	3.729** [0.000]	1.192 [0.282]	1.116 [0.343]	1.169 [0.299]	2.110* [0.013]	0.804 [0.656]	1.075 [0.379]	1.174 [0.296]
Normality	42.34** [0.000]	7.262* [0.027]	0.472 [0.790]	0.934 [0.627]	67.33** [0.000]	32.48** [0.000]	4.944 [0.084]	10.26** [0.006]	1.229 [0.541]
ARCH 1-13	8.997** [0.000]	2.665** [0.001]	1.965* [0.022]	0.706 [0.758]	4.918** [0.000]	13.19** [0.000]	3.124** [0.000]	2.462** [0.003]	0.871 [0.585]
Hetero	1.703* [0.012]	3.771** [0.000]	0.698 [0.885]	1.017 [0.445]	1.769** [0.008]	2.926** [0.000]	2.523** [0.000]	1.236 [0.212]	0.464 [0.985]
RESET	7.476** [0.007]	1.835 [0.176]	0.878 [0.349]	3.129 [0.078]	2.006 [0.157]	28.27** [0.000]	8.131** [0.005]	1.088 [0.298]	0.470 [0.493]

** significant at 1% level, * significant at 5% level.

5 The effects of a monetary policy shock

In this section, we consider the dynamic responses to an asymmetric monetary policy shock in form of an unpredicted one percentage-point increase of the nominal short-term interest rate of the UK and respectively of the US.

5.1 An impulse response analysis of a monetary policy shock in the UK

Figure 11 displays the responses of the system variables, i.e., the inflation rates, the output growth rates, the 3-month interest rates, the 10-year government bond yields, and the nominal exchange rate, with regard to an one-percentage point increase in the monthly 3-month treasury bill return of the UK. The 95% confidence bands are Hall (1992) bootstrap intervals with 2000 replications. The computation follows the algorithm of Benkwitz, Lütkepohl and Wolters (2001) in the version of no reestimation of the cointegration relations. Due to the appropriate model selection the impulse responses are highly significant.

After a monetary policy shock in the UK the bond rate of the UK reacts contemporaneously with 31% of the size of the shock. Also the interest rates of the US increase. The gap between the country short-term rates is closed after 4 years. A negative reaction of the country output growth rates is in line with economic theory. On the contrary the inflation rates show a positive impulse, a ‘price puzzle’ is present. The exchange rate appreciates steadily for several month, achieving a peak after 28 month, and finally depreciates thereafter. A clear pattern of delayed overshooting is present.

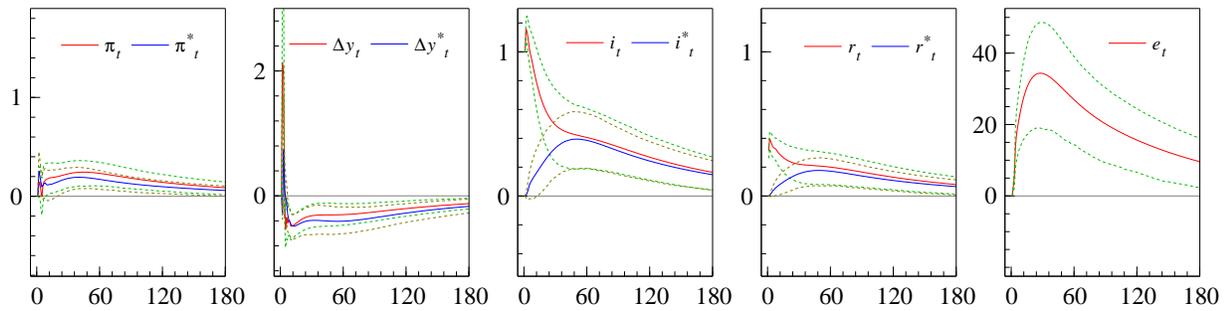


Figure 11 Responses to an asymmetric monetary policy shock of the UK in the parsimonious SVECM, 95% confidence bands.

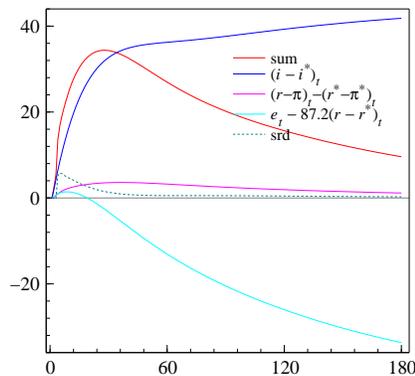


Figure 12 Decomposition of the response of the exchange rate to a UK monetary policy shock: the impulse response of the exchange rate in red colour and four components, three cointegration relationships and the combined short-run dynamics.

A decomposition of the response of the exchange rate, see Figure 12, into contributions of the different terms of (32), gives deeper insights into the dynamics of the adjustments. The short-term interest rate differential term, $(i - i^*)_t$, is responsible for the appreciation. In the long-run after 30 month the exchange rate determination cointegration relation, $e_t - 87.2(r - r^*)_t$, drives the depreciation. The short-term dynamics plays a minor role.

5.2 An impulse response analysis of a monetary policy shock in the US

Figure 13 display the responses of the system variables with regard to an one-percentage point increase in the monthly 3-month treasury bill return of the US.

After a monetary policy shock in the US, the UK short-term interest rate reacts on impact with 1% of the size of the shock, and reaches 54% of the size after 12 month. Thereafter the UK short-term rate is larger than the US short-term rate. This strong mimicry is responsible for a higher bond yield in the UK than in the US. The exchange rate depreciates on impact, then appreciates to a level higher than at the beginning, what is due to the higher bond rate in the UK. In total the reaction of the exchange rate is much smaller in the case of a US monetary policy shock ¹¹.

¹¹ When analyzing the asymmetric effects of US and Australian monetary policy shocks, Voss and Willard (2009) found that only monetary policy shocks caused by the Reserve Bank of Australia are affecting the AUD/USD exchange rate significantly.

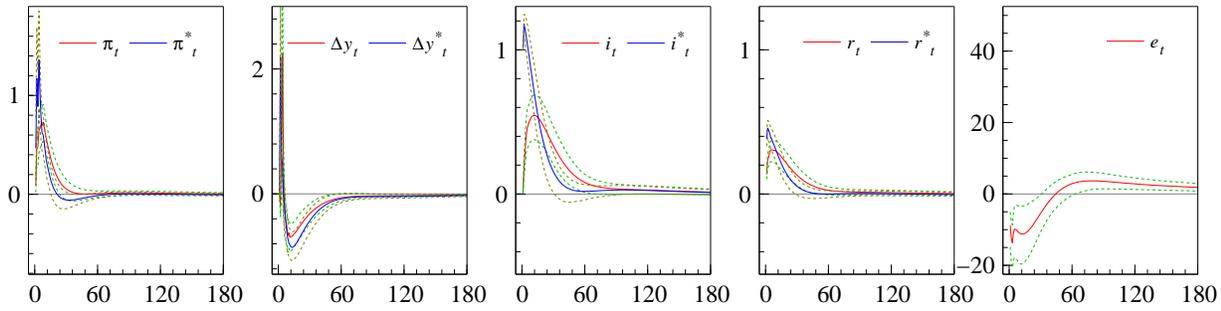


Figure 13 Responses to an asymmetric monetary policy shock of the US in the parsimonious SVECM, 95% confidence bands.

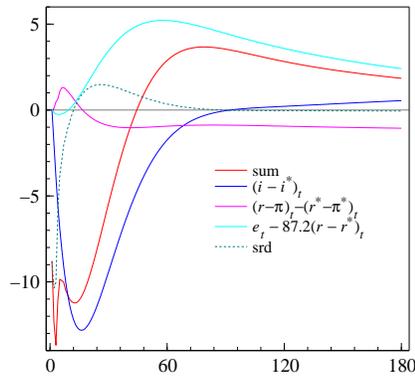


Figure 14 Decomposition of the response of the exchange rate to a US monetary policy shock: the impulse response of the exchange rate in red colour and four components, three cointegration relationships and the combined short-run dynamics.

The decomposition of the response of the exchange rate, see Figure 14, into contributions of the different terms of (32), allows similar conclusions than in the case of the UK shock. The exchange rate is driven first by the short-run dynamics, then by the short-term interest rate differential term, $(i - i^*)_t$, and finally in the long-run by the exchange rate determination cointegration relation, $e_t - 87.2(r - r^*)_t$.

5.3 Delayed overshooting and violations of UIP

Having established solid evidence for the delayed overshooting hypothesis in the UK case, we finally examine its implications for the size and dynamic profile of the violations of UIP during the transmission process of the monetary policy shock. Delayed overshooting generates excess returns violating UIP. This can be seen in the top panels of Figure 15 from the deviation of the response of the exchange rate, ∇e_h , from the line entitled UIP representing the equilibrium response of the exchange rate consistent with the uncovered interest parity hypothesis:

$$\nabla e_h^{\text{UIP}} = \nabla e_T + \sum_{s=h}^{T-1} \nabla i_s^d \quad (33)$$

where in the plots above $T = 250$ was used.

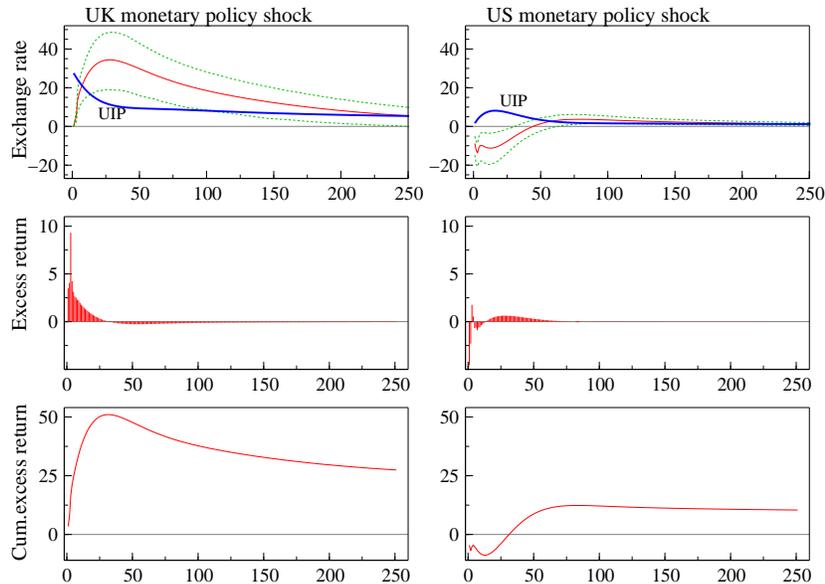


Figure 15 Effects of monetary policy shocks: response of e_{t+h} to a one percentage point increase in i_t , left column, and a one percentage point increase in i_t^* , right column, (with 95% confidence interval).

Approaching the same issue from a different angle, the panels below measure the deviations from UIP with the ex-ante one-period excess return series:

$$\nabla \xi_h = \nabla i_h^d + \Delta \nabla e_{h+1}. \quad (34)$$

The plots reveal excess returns for UK treasury bonds after a tightening of Bank of England policy. In the bottom panels, the cumulated excess returns,

$$\nabla \xi_{0,h} = \sum_{j=0}^h \nabla \xi_j = \sum_{j=0}^h \nabla i_j^d + \Delta_h \nabla e_{h+1}, \quad (35)$$

are plotted.

In the UK case over the first two years excess returns of up to 50 percent are observed constituting major violations of the UIP hypothesis. Altogether, the statistical results discussed in this paper strongly support the presence of a delayed overshooting puzzle for the $\$/\pounds$ exchange rate after a UK monetary policy shock.

6 Conclusion

Two-country cointegrated VAR models suffer from being restricted to a very small number of variables. To overcome this limitation we propose a modelling approach, which allows to split the analysis into two subsystems, a country difference system and a country average system. The necessary condition to be able to decouple the two systems is symmetry between the two countries. This symmetry assumption is to be relaxed for the short-run. The error correction terms from the two subsystems are preserved and included into the full system. For the econometric model selection we propose a data-driven approach combining a VAR based cointegration analysis with a graph-theoretic search for instantaneous causal relations and an automatic general-to-specific approach for the selection of a parsimonious structural

vector equilibrium correction model. Collectively putting these elements together we deliver a strong programme of how to set up empirical two-country models.

In an application we set up a UK - US two-country model and investigate the presence of a ‘delayed overshooting puzzle’ in the response of the $\$/\pounds$ exchange rate to an asymmetric monetary policy shock. We can now conclude by summarizing the main findings of our econometric analysis:

- (i) *Long-run properties.* In the country difference model we found four cointegration relations and one stochastic trend, which could be identified as the long-term interest rate differential, r_t^d , and appeared to be driven by long-term inflation expectations as in the Fisher hypothesis. r_t^d cointegrated with the inflation differential, π_t^d , to a stationary ‘real’ long-term rate differential. It was also found to drive the exchange rate, $e_t = 87.2r_t^d$, which is consistent with UIP and stationary long-term exchange rate expectations, $E_t e_{t+120}$. The output growth rate differential, Δy_t^d , and the short-term rate differential, i_t^d , are error-correcting and weakly exogenous to the other cointegration relations.

In the country average model we found three cointegration relations and one stochastic trend, which could be identified to be a linear combination out of the short-term, i_t^a , and the long-term interest rate average, r_t^a . The long-term interest rate average, r_t^a , cointegrated with the inflation average, π_t^a , to a stationary real long-term rate average and as well with the short-term rate average, i_t^a , to a stationary term spread average. The output growth rate average, Δy_t^a , is error-correcting and weakly exogenous to the other cointegration relations.

- (ii) *The short-run dynamics.* The causal structure is derived empirically and is very parsimonious. All the US variables are ordered before the UK variables. So there are no contemporaneous effects from UK to US variables. In the very short-run a small country model is suitable for the UK. The exchange rate is ordered between the US and the UK short-term interest rates.
- (iii) *Model reduction.* The problem of inconclusive impulse response analysis in the case of an unrestricted (S)VAR, caused by the inherent estimation uncertainty, due to the large number of parameters, can be overcome by general-to-specific model selection procedures employed in this paper.
- (iv) *Monetary policy shock.* Consistently, we found strong evidence for delayed overshooting and violations of UIP in the case of a UK monetary policy shock. In the case of a US monetary policy shock the exchange rate reacts on impact, but altogether considerably less and excess returns are small, due to strong mimicry of the monetary policy authority of the UK.

References

- Aoki, M. (1981). *Dynamic analysis of open economies*. New York: Academic Press.
- Bacchetta, P., and van Wincoop, E. (2010). Infrequent portfolio decisions: A solution to the forward discount puzzle. *The American Economic Review*, **100**, 870–904.
- Benkwitz, A., Lütkepohl, H., and Wolters, J. (2001). Comparison of bootstrap confidence intervals for impulse responses of German monetary systems. *Macroeconomic Dynamics*, **5**, 81–100.
- Binder, M., Chen, Q., and Zhang, X. (2010). On the effects of monetary policy shocks on exchange rates. Working paper 3162, CESifo, Munich.
- Bjørnland, H. C. (2009). Monetary policy and exchange rate overshooting: Dornbusch was right after all. *Journal of International Economics*, **79**, 64 – 77.

- Bouakez, H., and Normandin, M. (2010). Fluctuations in the foreign exchange market: How important are monetary policy shocks?. *Journal of International Economics*, **81**, 139–153.
- Campos, J., Hendry, D. F., and Krolzig, H.-M. (2003). Consistent model selection by an automatic Gets approach. *Oxford Bulletin of Economics and Statistics*, **65**, 803–819.
- Cushman, D. O., and Zha, T. (1997). Identifying monetary policy in a small open economy under flexible exchange rates. *Journal of Monetary Economics*, **39**, 433 – 448.
- Demiralp, S., and Hoover, K. D. (2003). Searching for the causal structure of a Vector Autoregression. *Oxford Bulletin of Economics and Statistics*, **65**, 745–767.
- Dornbusch, R. (1976). Expectations and exchange rate dynamics. *Journal of Political Economy*, **84**, 1161–76.
- Eichenbaum, M., and Evans, C. (1995). Some empirical evidence on the effects of shocks to monetary policy on exchange rates. *The Quarterly Journal of Economics*, **110**, 975–1009.
- Faust, J. (1998). The robustness of identified VAR conclusions about money. *Carnegie-Rochester Conference Series on Public Policy*, **49**, 207 – 244.
- Faust, J., and Rogers, J. H. (2003). Monetary policy's role in exchange rate behavior. *Journal of Monetary Economics*, **50**, 1403.
- Forni, M., Hallin, M., Lippi, M., and Reichlin, L. (2000). The generalized dynamic-factor model: Identification and estimation. *The Review of Economics and Statistics*, **82**, 540–554.
- Frankel, J. A. (1979). On the Mark: A theory of floating exchange rates based on real interest differentials. *American Economic Review*, **69**, 610 – 622.
- Gourinchas, P.-O., and Tornell, A. (2004). Exchange rate puzzles and distorted beliefs. *Journal of International Economics*, **64**, 303 – 333.
- Grilli, V., and Roubini, N. (1995). Liquidity and exchange rates: Puzzling evidence from the G-7 countries. Working paper, Leonard N. Stern School of Business, Department of Economics, New York University.
- Grilli, V., and Roubini, N. (1996). Liquidity models in open economies: Theory and empirical evidence. *European Economic Review*, **40**, 847 – 859.
- Hall, P. (1992). *The Bootstrap and Edgeworth Expansion*. New York: Springer.
- Heinlein, R., and Krolzig, H.-M. (2011). Effects of monetary policy on the $\$/\pounds$ exchange rate. Is there a 'delayed overshooting puzzle'?. Working paper, School of Economics, University of Kent, Canterbury.
- Hendry, D. F., and Krolzig, H.-M. (2005). The properties of automatic GETS modelling. *Economic Journal*, **115**, C62–C98.
- Johansen, S. (1995). *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford: Oxford University Press.
- Juselius, K. (2006). *The cointegrated VAR model*. Oxford: Oxford University Press.
- Kim, S. (2005). Monetary policy, foreign exchange policy, and delayed overshooting. *Journal of Money, Credit and Banking*, **37**, 775 – 782.
- Kim, S., and Roubini, N. (2000). Exchange rate anomalies in the industrial countries: A solution with a structural VAR approach. *Journal of Monetary Economics*, **45**, 561 – 586.
- Krolzig, H.-M. (2001). General-to-specific reductions in vector autoregressive processes. In Friedmann, R., Knüppel, L., and Lütkepohl, H. (eds.), *Econometric Studies - A Festschrift in Honour of*

Joachim Frohn, pp. 129–157. Münster: LIT Verlag.

- Krolzig, H.-M. (2003). General-to-specific model selection procedures for structural vector autoregressions. *Oxford Bulletin of Economics and Statistics*, **65**, 769–801.
- Krolzig, H.-M., and Hendry, D. F. (2001). Computer automation of general-to-specific model selection procedures. *Journal of Economic Dynamics and Control*, **25**, 831–866.
- Mumtaz, H., and Surico, P. (2009). The transmission of international shocks: A Factor-Augmented VAR Approach. *Journal of Money, Credit and Banking*, **41**, 71–100.
- Pearl, J. (2000). *Causality: Models, Reasoning, and Inference*. Cambridge: Cambridge University Press.
- Pesaran, M. H., Schuermann, T., and Weiner, S. M. (2004). Modeling regional interdependencies using a global error-correcting macroeconometric model. *Journal of Business and Economic Statistics*, **22(2)**, 129–162.
- Rogoff, K. (2002). Dornbusch’s overshooting model after twenty-five years. *IMF Staff Papers*, **49**, Special Issue, 1–35.
- Romer, C., and Romer, D. (2004). A new measure of monetary shocks: Derivation and implications. *American Economic Review*, **94**, 1055–84.
- Scholl, A., and Uhlig, H. (2008). New evidence on the puzzles: Results from agnostic identification on monetary policy and exchange rates. *Journal of International Economics*, **76**, 1 – 13.
- Spirtes, P., Glymour, C., and Scheines, R. (2001). *Causation, Prediction, and Search* 2nd edn. Cambridge, MA: MIT Press.
- Spirtes, P., Scheines, R., Ramsey, J., and Glymour, C. (2005). Tetrad 4.3.3-0. Discussion paper, Carnegie Mellon University, <http://www.phil.cmu.edu/tetrad>.
- Turnovsky, S. (1986). Monetary and fiscal policy under perfect foresight: A symmetric two-country analysis. *Economica*, **53**, 139–157.
- Tuxen, C. K. (2007). An empirical relevant model of inflation in the euro area. Working Paper, Nuffield College, Oxford.
- Voss, G., and Willard, L. (2009). Monetary policy and the exchange rate: Evidence from a two-country model. *Journal of Macroeconomics*, **31**, 708 – 720.