Which Parameters Drive Approximation Inaccuracies?☆

Sebastian Sienknecht⋆

Department of Economics, Friedrich-Schiller-University Jena
Carl-Zeiss-Str. 3, 07743 Jena, Germany

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Abstract

This paper identifies parameters responsible for welfare reversals when the basic New Keynesian model is approximated. In our setting, a reversal occurs when the Ramsey policy under timeless perspective commitment ceases to be dominant against an interest rate rule à la Taylor (1993) after approximating the model, or vice versa.

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⋆*Phone:* ++49-3641-9-43213, *Fax:* ++49-3641-9-43212, *Email:* Sebastian.Sienknecht@uni-jena.de

1 Introduction

Several ways have been proposed to compute a welfare optimizing monetary policy in the New Keynesian model. Kahn et al. (2003) and King and Wolman (1999) derive first-order conditions by maximizing household utility subject to the model economy. Another method by Kahn et al. (2003), Kim and Kim (2006), and King and Wolman (1999) consists in computing a first-order approximation of the exact first-order conditions. While welfare in the nonlinear environment is measured by household utility, the approximated setup requires a second-order loss function along the lines of Benigno and Woodford (2004, 2006), Damjanovic and Nolan (2011), and Woodford (1999, 2003). Alternatively, Benigno and Woodford (2004, 2006) propose the minimization of the second-order loss function using a first-order approximation of the model economy as a constraint.

In a first step, we compute the Ramsey monetary policy under a timeless perspective commitment. We compare the resulting welfare loss to an interest rate rule à la Taylor (1993) after simulating a stagflationary cost-push shock. In the second step, the same comparison is pursued in the approximated model version following Kahn et al. (2003), Kim and Kim (2006), and King and Wolman (1999). We find a potential for contradictory policy recommendations when applying the approximated model. This inconsistency of policy rankings is denominated as a "relative welfare reversal" and it may depend on deep parameters influencing the curvature of the target function or the persistence degree of shocks. An increasing curvature or a higher degree of shock persistence may enhance the possibility of welfare reversals. However, certain parameters can be more influential than others. The aim of this paper is to identify these parameters and to assess the parameter values that cause the welfare reversal effect.

The remainder is as follows. Section 2 presents the model and monetary policy alternatives. Section 3 approximates them. Section 4 presents simulation results and explores parameter regions which cause welfare reversals. Section 5 summmarizes our results and provides concluding remarks.

2 The Model

The standard New Keynesian model for a cashless economy consists of final goods producers, intermediate goods firms, households, and the monetary authority. We introduce adjustment costs in the spirit of Rotemberg (1982) and Hairault and Portier (1993). The following sections present the model in detail.

2.1 Final Goods Producers

Final goods producers demand a continuum of monopolistically offered intermediate goods $Y_t(i)$, which are assembled towards the final product Y_t with a CES production technology. The final product is sold in a perfectly competitive market. A representative final good producer maximizes his profits

$$P_t Y_t - \int_0^1 P_t(i)\, Y_t(i)\, di \tag{1}$$

subject to

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\epsilon_t - 1}{\epsilon_t}} \, di \right)^{\frac{\epsilon_t}{\epsilon_t - 1}} . \tag{2}$$

The elasticity of substitution between input varieties ϵ_t is assumed to vary over time according to a first-order autoregressive process with an i.i.d. shock variable e_t:

$$\left(\frac{\epsilon_t}{\epsilon} \right) = \left(\frac{\epsilon_{t-1}}{\epsilon} \right)^{\rho} \exp\{e_t\} \qquad , \qquad 0 \le \rho < 1, \tag{3}$$

where ϵ is the steady state elasticity and ρ gives the degree of persistence of the shock process. The first-order condition for profit maximization yields the demand schedule for input variety i:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_t} Y_t. \tag{4}$$

Inserting this equation into the CES technology (2) yields the aggregate price index for the bundle Y_t:

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon_t} \, di \right)^{\frac{1}{1-\epsilon_t}} . \tag{5}$$

2.2 Intermediate Goods Producers

The intermediate firm production $Y_t(i)$ requires labor hours $N_t(i)$:

$$Y_t(i) = N_t(i)^{1-\alpha} \quad , \quad 0 \leq \alpha < 1, \tag{6}$$

where $\alpha > 0$ represents decreasing marginal productivity. Total real costs are

$$TC_t(i) = \frac{W_t}{P_t} N_t(i) . \tag{7}$$

Taking the first derivative of (7) subject to the production function (6) yields real marginal costs

$$MC_t(i) = \left(\frac{1}{1-\alpha} \right) \frac{W_t}{P_t} \, Y_t(i)^{\frac{\alpha}{1-\alpha}} . \tag{8}$$

An intermediate firm chooses $P_t(i)$ in order to maximize real profits subject to the demand schedule (4). By doing so, the firm faces real quadratic costs of price adjustment (Rotemberg (1982) and Hairault and Portier (1993)):

$$Q_t(i) = \frac{\psi}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - \pi \right)^2 \quad , \quad \psi \geq 0. \tag{9}$$

The parameter ψ denotes the marginal adjustment cost reaction on deviations of price relations $\frac{P_t}{P_{t-1}}$ from steady state gross inflation π. An intermediate firm chooses a price $P_t(i)$ that maximizes real profits

$$E_t \sum_{k=0}^{\infty} \Delta_{t,t+k} \left[\frac{P_{t+k}(i) Y_{t+k}(i)}{P_{t+k}} - MC_{t+k}(i) Y_{t+k}(i) - Q_{t+k}(i) \right] \tag{10}$$

subject to (4) and (9), where $\Delta_{t,t+k} = \beta^k \frac{\partial U_{t+k} / \partial C_{t+k}}{\partial U_t / \partial C_t}$ is the stochastic discount factor for real profit income flows to the representative household. The first-order condition reads

$$\psi\left(\frac{P_t(i)}{P_{t-1}(i)}-\pi\right)\frac{P_t(i)}{P_{t-1}(i)}=\epsilon_t Y_t(i)\left(MC_t(i)-\frac{1}{\mu_t}\frac{P_t(i)}{P_t}\right) + E_t\left[\psi\frac{\Delta_{t,t+1}}{\Delta_{t,t}}\left(\frac{P_{t+1}(i)}{P_t(i)}-\pi\right)\frac{P_{t+1}(i)}{P_t(i)}\right] \tag{11}$$

The variable μ_t is the time varying markup of monopolistic intermediate firms, which is given by

$$\mu_t=\left(\frac{\epsilon_t}{\epsilon_t-1}\right). \tag{12}$$

Note that setting $\psi=0$ gives the monopolistic price setting without costs of adjustment:

$$P_t\left(i\right)=\mu_t P_t MC_t\left(i\right). \tag{13}$$

2.3 Households

There is a continuum of households $j\in[0,1]$ maximizing the following discounted sum of expected utility streams:

$$E_t\sum_{k=0}^{\infty}\beta^k\left(\frac{C_{t+k}^{1-\sigma}(j)}{1-\sigma}-\frac{N_{t+k}^{1+\eta}(j)}{1+\eta}\right), \tag{14}$$

where $\frac{1}{\sigma}>0$ is the intertemporal elasticity of substitution in consumption and $\frac{1}{\eta}>0$ is the real wage elasticity of labor supply. The household saves in one-period nominal bonds $B_t(j)$ at the gross deposit rate R_t and receives real dividends $Div_t^r(j)$ from intermediate firms. The period-by-period real budget constraint is given by

$$C_t(j)+\frac{B_t(j)}{P_t}=\frac{W_t}{P_t}N_t(j)+R_{t-1}\frac{B_{t-1}(j)}{P_t}+Div_t^r(j). \tag{15}$$

Differentiation of (14) with respect to $C_t(j)$, $N_t(j)$, and $B_j(j)$ subject to (15) gives

$$\lambda_t(j) = C_t(j)^{-\sigma}, \tag{16}$$

$$\frac{W_t}{P_t} = \frac{N_t(j)^{\eta}}{\lambda_t(j)}, \tag{17}$$

and

$$\lambda_t(j) = \beta E_t \left[\lambda_{t+1}(j) \ R_t \frac{P_{t+1}}{P_t} \right], \tag{18}$$

where $\lambda_t(j)$ is the Lagrange multiplier of agent j.

2.4 Equilibrium and Aggregation

We assume a symmetric equilibrium. The market clearing for each intermediate good implies

$$Y_t^d(i) = Y_t^s(i) = Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_t} Y_t, \tag{19}$$

and the symmetry assumption implies that all intermediate firms face the same price setting problem. Therefore, they set the same price, which implies $P_t(i) = P_t$ and $Y_t(i) = Y_t$. Moreover, we neglect the index j since all households are assumed to be identical. Inserting (16) into (18) gives the consumption Euler equation

$$C_t^{-\sigma} = \beta E_t \left[C_{t+1}^{-\sigma} \ R_t \pi_{t+1}^{-1} \right], \tag{20}$$

where $\pi_t = P_t / P_{t-1}$ denotes gross inflation. Combining (16) and (17) leads to the following labor supply:

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\eta}. \tag{21}$$

Using $\Delta_{t,t+1}/\Delta_{t,t} = (C_{t+1}/C_t)^{-\sigma}$, the first-order condition of the intermediate firm (11) can be rewritten as

$$\pi_t\left(\pi_t - \pi\right) = \beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\pi_{t+1} - \pi\right)\pi_{t+1}\right] + \frac{Y_t\epsilon_t}{\psi}\left(MC_t - \frac{1}{\mu_t}\right), \tag{22}$$

which represents a nonlinear New Keynesian Phillips curve. Aggregate real marginal costs and real output are given by

$$MC_t = \left(\frac{1}{1-\alpha}\right)\frac{W_t}{P_t}\,Y_t^{\frac{\alpha}{1-\alpha}} \tag{23}$$

and

$$Y_t = N_t^{1-\alpha} \quad , \quad 0 \leq \alpha < 1. \tag{24}$$

The economy-wide resource utilization is given by the aggregate budget constraint of the household (15), together with the bond market clearing condition $B_t = B_{t-1} = 0$ as

$$C_t = \frac{W_t}{P_t}N_t + Div_t^r, \tag{25}$$

where aggregate real profits of intermediate goods producers are

$$Div_t^r = Y_t - \frac{W_t}{P_t}N_t - Q_t. \tag{26}$$

Inserting (26) into (25) gives the overall resource constraint

$$Y_t = C_t + \frac{\psi}{2}\left(\pi_t - \pi\right)^2 \quad , \quad \psi \geq 0. \tag{27}$$

As can be seen, $\psi > 0$ limits the resources available for aggregate consumption.

2.5 Monetary Policy

The monetary authority is either following the Ramsey policy under a timeless perspective commitment or a simple interest rate rule. The Ramsey planner aims to maximize household utility subject to the model economy[1]. In our setting, the constraints to be taken into consideration are the New Keynesian Phillips curve (22) and the aggregate resource constraint (27). Substitution of W_t/P_t, MC_t, and Y_t with (21), (23), and (24) gives the constraints only in terms of the control variables C_t, N_t, and π_t:

$$\begin{aligned}\pi_t\left(\pi_t-\pi\right)-\beta E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\left(\pi_{t+1}-\pi\right)\pi_{t+1}\right]\\ -\frac{N_t^{1-\alpha}\epsilon_t}{\psi}\left(\left(\frac{1}{1-\alpha}\right)C_t^{\sigma}\ N_t^{\alpha+\eta}-\frac{1}{\mu_t}\right)=0,\end{aligned} \tag{28}$$

$$N_t^{1-\alpha}-C_t-\frac{\psi}{2}\left(\pi_t-\pi\right)^2=0. \tag{29}$$

The Lagrangian for a given shock process (3) then reads

$$\begin{aligned}\mathcal{L}_t=&\,E_t\sum_{k=0}^{\infty}\beta^k\left(\frac{C_{t+k}^{1-\sigma}}{1-\sigma}-\frac{N_{t+k}^{1+\eta}}{1+\eta}\right)\\ &+E_t\sum_{k=0}^{\infty}\beta^k\lambda_{1,t+k+1}\left(N_{t+k}^{1-\alpha}-C_{t+k}-\frac{\psi}{2}\left(\pi_{t+k}-\pi\right)^2\right)\\ &+E_t\sum_{k=0}^{\infty}\beta^k\lambda_{2,t+k+1}\left(\left(\pi_{t+k}-\pi\right)\pi_{t+k}-\beta\left(\frac{C_{t+k+1}}{C_{t+k}}\right)^{-\sigma}\left(\pi_{t+k+1}-\pi\right)\pi_{t+k+1}\right.\\ &\left.-\frac{N_{t+k}^{1-\alpha}\epsilon_{t+k}}{\psi}\left(\left(\frac{1}{1-\alpha}\right)C_{t+k}^{\sigma}N_{t+k}^{\alpha+\eta}-\frac{1}{\mu_{t+k}}\right)\right),\end{aligned} \tag{30}$$

[1] See Kahn et al. (2003).

where $\lambda_{1,t}$ and $\lambda_{2,t}$ are the respective Lagrange multipliers. The first-order conditions with respect to C_t, N_t, and π_t are given by

$$E_t\Bigg[C_t^{-\sigma} - \lambda_{1,t+1} - \lambda_{2,t+1}\beta\sigma\left(\pi_{t+1} - \pi\right)\pi_{t+1}\frac{1}{C_t}\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \\ -\lambda_{2,t+1}C_t^{\sigma-1}N_t^{1+\eta}\epsilon_t\left(\frac{\sigma}{1-\alpha}\right)\frac{1}{\psi} + \lambda_{2,t}\sigma\left(\pi_t - \pi\right)\pi_t\frac{1}{C_t}\left(\frac{C_{t-1}}{C_t}\right)^{\sigma} = 0\Bigg], \tag{31}$$

$$E_t\Bigg[-N^{\eta} + \lambda_{1,t+1}N_t^{-\alpha}\left(1-\alpha\right) - \lambda_{2,t+1}C_t^{\sigma}N_t^{\eta}\epsilon_t\left(\frac{1+\eta}{1-\alpha}\right)\frac{1}{\psi} \\ +\lambda_{2,t+1}N_t^{-\alpha}\left(\epsilon_t - 1\right)\left(\frac{1-\alpha}{\psi}\right) = 0\Bigg], \tag{32}$$

$$E_t\left[-\lambda_{1,t+1}\psi\left(\pi_t - \pi\right) + \lambda_{2,t+1}\left(2\pi_t - \pi\right) - \lambda_{2,t}\left(2\pi_t - \pi\right)\left(\frac{C_{t-1}}{C_t}\right)^{\sigma} = 0\right]. \tag{33}$$

Note that $\lambda_{1,t}$ is a jump variable, while $\lambda_{2,t}$ is predetermined. The initial value of the latter will be set such that it is non-zero and equalized to its steady state value, which implies that the Ramsey policy is of a timeless perspective nature. This means that the policy maker credibly commits to a time-invariat policy strategy with the disadvantage that aggregate utility is not at its globally optimal level. The alternative strategy is to commit to an interest rate rule with the gross interest rate R_t as the control instrument:

$$\left(\frac{R_t}{R}\right) = \left(\left(\frac{\pi_t}{\pi}\right)^{\delta_\pi}\left(\frac{Y_t}{Y}\right)^{\delta_y}\right)^{1-\phi}\left(\frac{R_{t-1}}{R}\right)^{\phi}. \tag{34}$$

$\delta_\pi > 1$ gives the weight on inflation and $\delta_y > 0$ on output deviations from the steady state. The parameter $\phi > 0$ allows for interest rate smoothing behaviour. Setting $\phi = 0$ gives the nonlinear form of the interest rate rule proposed by Taylor (1993)[2].

[2] Note that the values of the reaction parameters δ_π, δ_y, and ϕ will be set following the literature and are therefore given as *ad hoc*. That is, we will *not* search for optimal reaction parameter values in the sense of optimal simple policy rules.

2.6 Welfare Measure

Absolute ($abs.$) welfare loss at period $t = 0$ is simply measured by

$$V_0^{abs.} = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{N_t^{1+\eta}}{1+\eta} - \frac{C_t^{1-\sigma}}{1-\sigma} \right), \tag{35}$$

where $V_0^{abs.} > 0$ for $\sigma > 1$ and $\eta > 0$. We compare absolute welfare losses between the two policy strategies by computing the following measure of relative ($rel.$) welfare:

$$V_0^{rel.} = \left(\frac{V_0^{abs.}\left(Interest\right)}{V_0^{abs.}\left(Ramsey\right)} \right) \times 100. \tag{36}$$

Therefore, the interest rate rule is superior to the Ramsey policy under timeless perspective commitment if $V_0^{rel.} < 100\%$.

3 Approximations

As an alternative to the nonlinear setting presented so far, we take a first-order Taylor approximation in logarithms around the non-stochastic steady state. In the following, we provide the steady state relationships and the approximated model equations.

3.1 Steady State

We solve for the non-stochastic steady state by dropping all time indices. The optimality condition (21), the production function (24), and the aggregate resource constraint (27) are then

$$\frac{W}{P} = C^{\sigma} N^{\eta}, \tag{37}$$

$$N = Y^{\frac{1}{1-\alpha}}, \tag{38}$$

and

$$C = Y. \tag{39}$$

Inserting (38) and (39) into (37) gives

$$\frac{W}{P} = Y^{\frac{\sigma(1-\alpha)+\eta}{1-\alpha}}. \tag{40}$$

From the inflation curve (22) and the aggregate real marginal costs (23) we obtain

$$MC = \frac{1}{\mu} \tag{41}$$

and

$$MC = \left(\frac{1}{1-\alpha}\right)\frac{W}{P}Y^{\frac{\alpha}{1-\alpha}}. \tag{42}$$

Using (40) and (41) in order to eliminate W/P and MC in (42) and solving for Y delivers

$$Y = C = \left(\frac{1-\alpha}{\mu}\right)^{\frac{1-\alpha}{\sigma(1-\alpha)+\eta+\alpha}}, \tag{43}$$

which is the steady state level of real output depending on model parameters only. Inserting (43) into (38) and into (40) gives the steady state levels

$$N = \left(\frac{1-\alpha}{\mu}\right)^{\frac{1}{\sigma(1-\alpha)+\eta+\alpha}} \tag{44}$$

and

$$\frac{W}{P} = \left(\frac{1-\alpha}{\mu}\right)^{\frac{\sigma(1-\alpha)+\eta}{\sigma(1-\alpha)+\eta+\alpha}}. \tag{45}$$

We assume no trend inflation, which implies $\pi = 1$. From the consumption Euler equation (20), one obtains the steady state gross interest rate

$$R = \frac{1}{\beta}. \tag{46}$$

Using (31), (32), and (33), the steady state values of the costate variables can be computed as

$$\lambda_1 = \left(C^{-\sigma} + \frac{a}{b}N^{\eta}\right)\left(1 + \frac{a}{b}(1-\alpha)N^{-\alpha}\right)^{-1} \tag{47}$$

and

$$\lambda_2 = \frac{1}{a}\left(C^{-\sigma} - \lambda_1\right), \tag{48}$$

where

$$a = \frac{\epsilon}{\psi}\left(\frac{\sigma}{1-\alpha}\right)C^{\sigma-1}N^{1+\eta} \tag{49}$$

and

$$b = \frac{\epsilon}{\psi}\left(\left(\frac{1+\eta}{1-\alpha}\right)C^{\sigma}N^{\eta} - \left(\frac{1-\alpha}{\mu}\right)N^{-\alpha}\right). \tag{50}$$

3.2 Model Approximation

The first-order approximation of the model is of the form $\left(\frac{X_t - X}{X}\right) \approx \log(X_t) - \log(X) \equiv \hat{X}_t$. The core equations (20), (21), (22), (23), (24), (27), and the autoregressive process (3) are then rewritten as

$$\hat{C}_t = E_t\left[\hat{C}_{t+1}\right] - \frac{1}{\sigma}\left(\hat{R}_t - E_t\left[\hat{\pi}_{t+1}\right]\right), \tag{51}$$

$$\hat{W}_t - \hat{P}_t = \sigma\hat{C}_t + \eta\hat{N}_t, \tag{52}$$

$$\hat{\pi}_t = \beta\ E_t\left[\hat{\pi}_{t+1}\right] + \frac{Y(\epsilon-1)}{\psi}\left(\widehat{MC}_t + \hat{\mu}_t\right), \tag{53}$$

$$\widehat{MC}_t = \hat{W}_t - \hat{P}_t + \left(\frac{\alpha}{1-\alpha}\right)\hat{Y}_t, \tag{54}$$

$$\hat{Y}_t = (1-\alpha)\hat{N}_t \quad , \quad 0 \le \alpha < 1, \tag{55}$$

$$\hat{Y}_t = \hat{C}_t, \tag{56}$$

and

$$\hat{\epsilon}_t = \rho\hat{\epsilon}_{t-1} + e_t \quad , \quad 0 \le \rho < 1, \tag{57}$$

where $\hat{\mu}_t = -\hat{\epsilon}_t (\epsilon - 1)^{-1}$. The first-order conditions of the Ramsey planner (31), (32), and (33) are now given by

$$\begin{aligned}
&\sigma \left(\epsilon (1-\sigma) \lambda_2 N^{1+\eta} - \psi (1-\alpha) C^{1-2\sigma}\right) \left(\psi (1-\alpha) \lambda_1 C^{1-\sigma}\right)^{-1} \hat{C}_t - E_t \left[\hat{\lambda}_{1,t+1}\right] \\
&+ \frac{\lambda_1}{\lambda_2} \frac{\sigma}{C} \left(\hat{\pi}_t - \beta E_t \left[\hat{\pi}_{t+1}\right]\right) - \frac{\lambda_1}{\lambda_2} \frac{\epsilon}{\psi} \frac{N^{1+\eta}}{C^{1-\sigma}} \left(E_t \left[\hat{\lambda}_{2,t+1}\right] + \hat{\epsilon}_t + (1+\eta) \hat{N}_t\right) \left(\frac{\sigma}{1-\alpha}\right) = 0,
\end{aligned} \tag{58}$$

$$\begin{aligned}
&-\left(\frac{\alpha (1-\alpha)^2 N^{-\alpha} (\lambda_1 \psi + \lambda_2 (\epsilon - 1)) + \eta N^\eta (\psi (1-\alpha) + \lambda_2 C^\sigma (1+\eta) \epsilon)}{\psi (1-\alpha)}\right) \hat{N}_t \\
&\quad - \left(\frac{\lambda_2 (1+\eta) C^\sigma N^\eta \sigma \epsilon}{\psi (1-\alpha)}\right) \hat{C}_t + \left(\frac{1-\alpha}{\lambda_1^{-1} N^\alpha}\right) E_t \left[\hat{\lambda}_{1,t+1}\right] \\
&\quad + \left(\frac{\lambda_2 \left((\epsilon - 1)(1-\alpha)^2 - C^\sigma N^{\eta+\alpha} (1+\eta) \epsilon\right)}{\psi (1-\alpha) N^\alpha}\right) E_t \left[\hat{\lambda}_{2,t+1}\right] \\
&\quad + \left(\frac{\lambda_2 \left((1-\alpha)^2 - C^\sigma N^{\eta+\alpha} (1+\eta)\right) \epsilon}{\psi (1-\alpha) N^\alpha}\right) \hat{\epsilon}_t = 0,
\end{aligned} \tag{59}$$

and

$$-\lambda_1 \psi \hat{\pi}_t + \lambda_2 \left(E_t \left[\hat{\lambda}_{2,t+1}\right] - \hat{\lambda}_{2,t}\right) + \lambda_2 \left(\hat{C}_t - \hat{C}_{t-1}\right) \sigma = 0. \tag{60}$$

The interest rate rule (34) now reads

$$\hat{R}_t = (1-\phi) \left(\delta_\pi \hat{\pi}_t + \delta_y \hat{Y}_t\right) + \phi \hat{R}_{t-1}. \tag{61}$$

3.3 Welfare Measure Approximation

According to Benigno and Woodford (2004, 2006), Kim and Kim (2006), and Woodford (1999, 2003), welfare could be measured by a second-order approximation of (14). Following Damjanovic and Nolan (2011), the absolute (*abs.*) welfare loss at

$t = 0$ under price rigidities à la Rotemberg (1982) is given by [3]

$$J_0^{abs.} = \frac{1}{2}\, E_0 \sum_{t=0}^{\infty} \beta^t \left(\left(\hat{Y}_t\right)^2 + \Gamma_1 \left(\hat{\pi}_t\right)^2 + \Gamma_2 \left(\hat{\epsilon}_t\right)^2 - \Omega \hat{Y}_t \right), \tag{62}$$

where

$$\Gamma_1 = \frac{\psi}{Y} \left(\Phi \left(\frac{1-\alpha}{1+\eta} \right) \sigma + 1 \right) \left(\frac{1-\alpha}{\eta + \sigma + \alpha\left(1-\sigma\right)} \right), \tag{63}$$

$$\Gamma_2 = \Phi \left(\frac{1-\alpha}{1+\eta} \right) \left(\frac{1-\alpha}{\eta + \sigma + \alpha\left(1-\sigma\right)} \right) \left(\frac{1}{\epsilon - 1} \right)^2, \tag{64}$$

and

$$\Omega = 2\left(1-\sigma\right) \Phi \left(\frac{1-\alpha}{1+\eta} \right) \left(\frac{1-\alpha}{\eta + \sigma + \alpha\left(1-\sigma\right)} \right). \tag{65}$$

The parameter $\Phi \equiv 1 - \frac{\epsilon - 1}{\epsilon}$ gives the monopolistic distortions in the economy at the steady state. Note that setting $\sigma = 1$ reproduces the second-order welfare loss function derived by Damjanovic and Nolan (2011). Relative welfare $(rel.)$ between the two policies is compared with

$$J_0^{rel.} = \left(\frac{J_0^{abs.}\left(Interest\right)}{J_0^{abs.}\left(Ramsey\right)} \right) \times 100. \tag{66}$$

Since $J_0^{abs.} > 0$, the interest rate rule generates a lower welfare loss than the Ramsey policy under timeless perspective commitment if $J_0^{rel.} < 100\%$.

[3] However, Damjanovic and Nolan (2011) set $\sigma = 1$, which implies a logarithmic utility term with respect to consumption. Further, their cost-push shock is due to a direct increase of the monopolistic markup, while in our case this shock is driven by the price elasticity of demand. This is the reason for the algebraically slightly different second-order welfare loss function in our paper. For a detailed derivation under $\sigma \neq 1$ and an elasticity-driven cost-push shock, see the appendix.

4 Simulation

The model parameter values are chosen according to a quarter time unit. In the following, we fix a subset of parameters at constant values for the entire analysis[4]. We set the household subjective discount factor β equal to 0.99, implying an annualized steady state real interest rate of 4 percent. The steady state value of the substitution elasticitiy between intermediate goods is $\epsilon = 6$. This implies a steady state markup on intermediate firms' marginal costs of 20 percent. Considering the interest rate rule (34) (or (61)), the parameters are $\delta_\pi = 1.5$ and $\delta_y = 0.125$. The remaining parameters are varied along the values of the calibration and estimation literature, but such that the parameter regions fulfill the Blanchard-Kahn stability conditions (see Blanchard and Kahn (1980)). We let $\alpha \in [0, 0.5]$ in order to explore the consequence of diminishing returns to labor. Similarly, $\rho \in [0, 0.9]$ represents an increasing degree of persistence in the cost shock process. We also let $\phi \in [0, 0.8]$ (increasing willingness to smooth interest rate settings), $\psi \in [0.001, 500]$ (increasing degree of price rigidity), $\sigma \in [1.001, 2.5]$ (increasing aversion towards intertemporal substitutions of consumption), and $\eta \in [2, 2.9]$ (decreasing real wage elasticity of labor supply). The shock impulse e_t in the autoregressive process (3) (or (57)) leads to a decrease of the elasticity of substitution ϵ_t (or $\hat{\epsilon}_t$) on impact and therefore to an increase of the markup μ_t (or $\hat{\mu}_t$). Thus, we simulate a stagflationary cost-push shock of one percent at $t = 0$. Note that the nonlinear model is deterministic (with perfect foresight), while the approximated framework is stochastic (with $e_t \sim N(0, 1)$). In a first step, we simulate the nonlinear model version (3),(20),(21),(22),(23),(24), and (27) with the interest rate rule (34). In the second step, we replace (34) by the Ramsey policy conditions (31),(32),(33). In the third step, $V_0^{rel.}$ is computed according to (36). These three steps are repeated in the approximated model (51)-(61) with $J_0^{rel.}$ being computed according to (66). A relative welfare reversal occurs if $J_0^{rel.}$ contradicts $V_0^{rel.}$, or vice versa.

[4] The choice of this particular subset of parameters is driven by the fact that their values are almost identical in the calibration and in the estimation literature across most economies. This is not the case for the remaining parameters to be varied in the subsequent exercise.

Table 1 presents relative welfare in the nonlinear model for parameter regions of α and ψ. A graphical representation of table 1 is given by figure 1 and the upper contour plot in figure 7. The interest rate rule (or Taylor rule, given $\phi = 0$) dominates against the Ramsey policy under timeless perspective commitment ($V_0^{rel.} < 100\%$) for the case of near-price flexibility ($\psi = 0.001$), independently of the value of α. The same conclusion can be drawn for low degrees of price rigidity, with the exception of $\psi = 150$ at $\alpha = 0.35$, $\alpha = 0.4$, and $\alpha = 0.5$, where $V_0^{rel.} < 100\%$. However, the Ramsey policy under timeless perspective commitment tends to dominate against the Taylor rule ($V_0^{rel.} > 100\%$) as the degree of price rigidity ψ increases. A glance at table 2 and at the figures 2 and 7 reveals that the policy rankings obtained in the nonlinear model are almost fully reversed in the approximated setting. In the case of near-price flexibility ($\psi = 0.001$), we obtain $J_0^{rel.} > 100\%$ and $J_0^{rel.} < 100\%$ for $\psi \geq 50$ and $\alpha \in [0, 0.35]$. While $J_0^{rel.}$ decreases for an increasing value of ψ at a given value of α, the behavior of $J_0^{rel.}$ when incrasing α at a given value of ψ is less clear-cut[5]. Note that a relative welfare reversal ($J_0^{rel.}$ contradicts $V_0^{rel.}$) often occurs after introducing a higher price persistence at a given value of α[6]. We therefore asses the important role that the degree of price flexibility plays for relative welfare reversals[7].

For the remaining parameter variations, we set $\alpha = 0$ and $\psi = 50$. This is because at these values, we have that $V_0^{rel.} < 100\%$ and $J_0^{rel.} < 100\%$ (see tables 1 and 2)[8]. In this manner, we are able to rule out relative welfare reversals induced by α and ψ and to isolate reversal effects of the other parameters.

Table 3, figure 3, and the upper contour plot in figure 8 show that the interest rate rule remains dominant in the nonlinear model ($V_0^{rel.} < 100\%$), for all combina-

5 For $\psi \in [0.001, 300]$, $J_0^{rel.}$ increases for rising values of α. The opposite can be observed for $\psi \in [350, 500]$.

6 An exception are the three last columns of table 2 ($\alpha = 0.4$, $\alpha = 0.45$,and $\alpha = 0.5$ at $\psi = 50$). We obtain 10 relative welfare changeovers from a varying value of ψ at given values of α and 3 relative welfare changeovers from variations of α at a given value of ψ.

7 Note that a value of ψ near to zero transforms our model into a close version of a Real Business Cycle (RBC) model. Therefore, our results point to potentially misleading normative insights also when applying this kind of models.

8 Note that this reference point is common across all tables and figures for the parameter configuration $\alpha = 0$, $\psi = 50$, $\sigma = 1.001$, $\eta = 2$, $\rho = 0$, $\phi = 0$, $\delta_\pi = 1.5$, and $\delta_y = 0.125$.

tions of ρ and ϕ. That is, an increasing degree of shock persistence and/or interest rate smoothing do not affect the *established* dominance of the interest rate rule[9]. However, table 4, figure 4, and the lower contour plot in figure 8 reveal that this unrestricted dominance is no longer true in the approximated framework. The interest rate rule outperforms the Ramsey policy under timeless perspective commitment *independently* of the smoothing parameter ϕ, but only in the case of uncorrelated or slightly correlated shocks ($\rho = 0$ and $\rho = 0.1$). This relation ($J_0^{rel.} < 100\%$) is preserved as the degree of autocorrelation increases up to $\rho = 0.3$, but only if the degree of interest rate smoothing ϕ also increases[10]. If the degree of interest rate smoothing is not high enough, the Ramsey policy under timeless perspective comitmment arises as the dominant alternative ($J_0^{rel.} > 100\%$) in the case of a slightly persistent cost-push shock. In the case of a higher shock persistence $\rho \in [0.4, 0.9]$, the Ramsey policy under timeless perspective commitment arises as the dominant policy alternative *independently* of the smoothing parameter. A comparison of these results to table 3, figure 3, and the upper contour plot in figure 8 should make clear that the policy recomendations differ across the nonlinear and the log-linear model. The diverging results imply a relative welfare reversal when applying the log-linear model. This effect is primarily induced by high shock persistence values, while low values of ρ induce this effect only if the desire to smooth interest rate movements ϕ is also low[11]. We therefore assess the particular relevance of the persistence degree in the cost-push shock and the minor relevance of the interest rate smoothing degree for relative welfare reversals.

For the analysis of different values for σ and η, we maintain the configuration $\alpha = 0$, $\psi = 50$, $\sigma = 1.001$, $\eta = 2$, $\rho = 0$, $\phi = 0$, $\delta_\pi = 1.5$, and $\delta_y = 0.125$ in order to induce the reference point $V_0^{rel.} < 100\%$ and $J_0^{rel.} < 100\%$. Table 5, figure 5, and the upper

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[9] Again, this dominance is introduced according to the tables 1 and table 2 through $\alpha = 0$, $\psi = 50$, $\sigma = 1.001$, $\eta = 2$, $\delta_\pi = 1.5$, and $\delta_y = 0.125$.

[10] This implies that the central bank is able to lock in welfare gains through moderate interest rate movements if the cost-push shock is slightly persistent. However, this potential welfare gain may require high degrees of interest rate smoothing, even if the persistence degree of the cost-push shock is small (Table 4 indicates that $J_0^{rel.} < 100\%$ for $\phi = 0.7$ and $\rho = 0.3$).

[11] We obtain 10 welfare changeovers by varying ρ at given values of ϕ and 2 welfare changeovers when varying ϕ at given values of ρ.

contour plot in figure 9 establish, again, the unrestricted superiority of the Taylor rule ($\phi = 0$) in the nonlinear model. That is, we obtain $V_0^{rel.} < 100\%$ for all combinations of σ and η. Table 6 and the corresponding figures 6 and 9 reveal that this unrestricted dominance is no longer present in the approximated model. For a near log utility of consumption term ($\sigma = 1.001$ and $\sigma = 1.15$), the Taylor rule dominates independently of the inverse real wage elasticity of labor demand η. However, further increases of the risk aversion parameter σ require lower values of η in order to maintain this dominance ($J_0^{rel.} < 100\%$). Otherwise, we have $J_0^{rel.} > 100\%$ and for higer values of σ this relationship does not depend on the value of η (see the columns for $\sigma = 2.3$ and $\sigma = 2.5$ in table 6.). That is, higher values of σ increase the probability that the Ramsey policy under timeless perspective commitment dominates against the Taylor rule. Most importantly, the parameter region where $J_0^{rel.} > 100\%$ contradicts the relative welfare statement of table 6 (namely $V_0^{rel.} < 100\%$). That is, we obtain a relative welfare reversal. However, to asses which of the two parameters is responsible for this effect is less clear-cut. According to table 6, we obtain 6 relative welfare changeovers generated by changes of η at given values of σ. By keeping the values of η fixed, we obtain 10 relative welfare reversals induced by changes of σ. We therefore assess the particular importance of σ and a minor relevance of η for relative welfare reversals after cost-push shocks.

5 Conclusions

The aim of this paper was to determine the parameters driving relative welfare reversals when approximating the basic New Keynesian model with price adjustment costs following Rotemberg (1982). We measured absolute welfare under an interest rate rule à la Taylor (1993) and under the Ramsey policy under timeless perspective commitment after simulating a cost-push shock. A relative welfare measure was constructed in order to detect the dominance of one policy or the other. Absolute and relative welfare measurement was pursued for given constellations of parameter values across the nonlinear and the approximated model. If relative welfare in the appoximated model contradicted relative welfare in the nonlinear model version (or vice versa), we deducted a counterfactual policy recommendation or a *relative welfare reversal* for the given set of parameter values. We constructed different sets of parameter values in order to identify the most important parameters driving relative welfare reversals.

We find that the assumption of highly flexible prices leads to counterfactual policy recommendations. That is, the interest rate rule generates lower welfare losses than the Ramsey policy under timeless perspective commitment in the nonlinear framework, but not in the approximated model. The same is true for high shares of capital in output if the degree of price rigidity is not high enough. Therefore, the asumption of rigid prices is crucial in order to rule out potential reversals in relative welfare.

A relative welfare reversal effect is always detected when assuming a high persistence degree of the cost-push shock. This is also the case for noncorrelated or slightly autocorrelated cost-push shocks if the central bank decides to smooth interest rate reactions when following an interest rate rule. Since a relative welfare reversal can always be avoided by assuming a noncorrelated or a weakly autocorrelated shock, we conclude that this is a key parameter in order to rule out potential reversals in relative welfare.

The risk aversion parameter of the household and the inverse real wage elasticity of labor supply are both responsible for the relative welfare reversal effect. However, the reversal effect occurs more often if the risk aversion parameter is varied. The ruling out of the relative welfare reversal effect requires lower values of the inverse real wage elasticity of labor supply if the degree of risk aversion increases. However, we find that the relative welfare reversal effect can always be avoided by assuming a logarithmic term of consumption utility.

Our results should help to restrict the parameter spaces that allow for welfare-based policy comparisons in an approximated setting and under stagflationary shocks. Since our results are specific to the type of shock, it could be interesting to verify if the parameter ranges that generate relative welfare reversals remain the same for other shocks (such as technology shocks). It should be interesting to corroborate our results under the widely assumed price rigidity of Calvo (1983) as well.

A Second-Order Welfare Loss Function

We follow the same derivation steps as Damjanovic and Nolan (2011), but in our case for $\sigma \neq 1$ and ϵ_t as the driving variable for the cost-push shock. For the sake of comparability, we assume for the time being that a technology parameter A_t enters the production function (24):

$$Y_t = A_t N_t^{1-\alpha} \quad , \quad 0 \leq \alpha < 1. \tag{A.1}$$

We rewrite household utility (14) as

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\eta}}{1+\eta} \right). \tag{A.2}$$

Inserting (A.1) into (A.2) gives

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\left(A_t^{-1} Y_t\right)^{\frac{1+\eta}{1-\alpha}}}{1+\eta} \right). \tag{A.3}$$

A second-order approximation of (A.3) yields

$$\begin{aligned} U = E_0 \sum_{t=0}^{\infty} \beta^t \Bigg[& Y^{1-\sigma} \hat{C}_t + \frac{1}{2} (1-\sigma) Y^{1-\sigma} \left(\hat{C}_t\right)^2 - \left(\frac{(\epsilon - 1) Y^{1-\sigma}}{\epsilon} \right) \left(\hat{Y}_t - \hat{A}_t \right) \\ & - \frac{1}{2} \left(\frac{(\epsilon-1)(1+\eta) Y^{1-\sigma}}{\epsilon (1-\alpha)} \right) \left(\hat{Y}_t - \hat{A}_t \right)^2 \Bigg] + O_3, \end{aligned} \tag{A.4}$$

where O_3 denotes terms of third and higher order. Note that setting $\sigma = 1$ gives the corresponding equation (5.1) in Damjanovic and Nolan (2011). The second-order approximation of the resource constraint (27) is

$$\hat{Y}_t = \hat{C}_t + \frac{\psi}{2Y} \left(\hat{\pi}_t\right)^2 + O_3. \tag{A.5}$$

Inserting (A.5) into (A.4) gives

$$\begin{aligned} U = {} & \Phi Y^{1-\sigma} E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{Y}_t - \hat{A}_t \right) - \frac{\psi Y^{1-\sigma}}{2Y} E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{\pi}_t\right)^2 \\ & + \frac{1}{2} \frac{\left[(1-\alpha)(1-\sigma) - (1-\Phi)(1+\eta) \right] Y^{1-\sigma}}{1-\alpha} E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{Y}_t - \hat{A}_t \right)^2 + O_3 + tip, \end{aligned} \tag{A.6}$$

where '*tip*' collects terms independent of policy and $\Phi \equiv 1 - \frac{\epsilon - 1}{\epsilon}$ summarizes all distortions present in the economy. Note that this equation corresponds to equation (5.3) in Damjanovic and Nolan (2011). Following the steps of that paper, we proceed to approximate the New Keynesian Phillips curve (22). However, we have to rewrite this equation in terms of Y_t only. To this end, we eliminate MC_t by inserting (23), and W_t/P_t (which enters MC_t) by using (21). The labor variable in (21) is, again, substituted by the production function (A.1). We obtain

$$\begin{aligned} &(\epsilon_t - 1) Y_t - \left(\frac{1}{1-\alpha}\right) \epsilon_t C_t^{\sigma} \left(A_t^{-1} Y_t\right)^{\frac{1+\eta}{1-\alpha}} + \psi \pi_t (\pi_t - 1) \\ &= \psi \beta E_t \left[\left(\frac{C_t}{C_{t+1}}\right)^{\sigma} \pi_{t+1} (\pi_{t+1} - 1)\right], \end{aligned} \tag{A.7}$$

which is equation (4.2) in Damjanovic and Nolan (2011) when assuming $\sigma = 1$. The second-order approximation of (A.7) reads

$$\begin{aligned} &\hat{Y}_t - \hat{T}_t + \frac{\psi}{(\epsilon - 1) Y} \hat{\pi}_t - \frac{\psi \beta}{(\epsilon - 1) Y} E_t [\hat{\pi}_{t+1}] + \frac{1}{2} \left(\left(\hat{Y}_t\right)^2 - \left(\hat{T}\right)^2\right) \\ &+ \frac{3}{2} \frac{\psi}{(\epsilon - 1) Y} E_t \left[(\hat{\pi})^2 - \beta (\hat{\pi}_{t+1})^2\right] - \frac{\sigma \beta \psi}{(\epsilon - 1) Y} E_t \left[\left(\hat{C}_{t+1} - \hat{C}_t\right) \hat{\pi}_{t+1}\right] \\ &+ \left(\frac{\epsilon}{\epsilon - 1}\right) \hat{\epsilon}_t + \frac{1}{2} \left(\frac{\epsilon}{\epsilon - 1}\right) (\hat{\epsilon}_t)^2 = O_3, \end{aligned} \tag{A.8}$$

where

$$\hat{T} = \hat{\epsilon}_t + \sigma \hat{C}_t + \left(\frac{1+\eta}{1-\alpha}\right) \left(\hat{Y}_t - \hat{A}_t\right), \tag{A.9}$$

Equation (A.8) corresponds to (6.1) and equation (A.9) corresponds to (6.2) of Damjanovic and Nolan (2011). In that paper, the equation (6.3) is the first-order approximation of the New Keynesian Phillips curve, which in our case results in

$$\frac{\psi}{(\epsilon - 1) Y} E_t [\hat{\pi}_t - \beta \hat{\pi}_{t+1}] = (\sigma - 1) \hat{Y}_t + \left(\frac{1+\eta}{1-\alpha}\right) \left(\hat{Y}_t - \hat{A}_t\right) - \left(\frac{1}{\epsilon - 1}\right) \hat{\epsilon}_t + O_2, \tag{A.10}$$

where O_2 denotes terms of second and higher order.

The forward solution of (A.10) can be computed as

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{Y}_t - \hat{T}_t\right) + \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\left(\hat{Y}_t\right)^2 - \left(\hat{T}_t\right)^2\right) = \frac{\sigma\psi\beta}{(\epsilon - 1) Y} E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{C}_{t+1} - \hat{C}_t\right) \hat{\pi}_{t+1}$$
$$- \left(\frac{\epsilon}{\epsilon - 1}\right) E_0 \sum_{t=0}^{\infty} \beta^t \hat{\epsilon}_t - \frac{1}{2} \left(\frac{\epsilon}{\epsilon - 1}\right) E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{\epsilon}_t\right)^2 + O_3 \tag{A.11}$$

Combining this equation with the second-order resource constraint (A.5) and the identity for $\hat{T}_t$ in equation (A.9), we arrive at

$$\left(\frac{1+\eta}{1-\alpha}\right) E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{Y}_t - \hat{A}_t\right) + tip + O_3 = (1-\sigma) E_0 \sum_{t=0}^{\infty} \beta^t \hat{Y}_t - \frac{\sigma\psi}{2Y} E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{\pi}\right)^2$$
$$- \frac{\sigma\psi\beta}{(\epsilon - 1) Y} E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{Y}_t - \hat{Y}_{t+1}\right) \hat{\pi}_{t+1}$$
$$- \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left((1+\sigma) \hat{Y}_t + \hat{\epsilon}_t + \left(\frac{1+\eta}{1-\alpha}\right)\left(\hat{Y}_t - \hat{A}_t\right)\right)\left((\sigma - 1) \hat{Y}_t + \hat{\epsilon}_t + \left(\frac{1+\eta}{1-\alpha}\right)\left(\hat{Y}_t - \hat{A}_t\right)\right), \tag{A.12}$$

which corresponds to (6.4) in Damjanovic and Nolan (2011). As done by these authors, multiplying (A.10) with $\hat{Y}_t$ gives

$$\frac{\psi}{(\epsilon - 1) Y} E_t \left[\hat{\pi}_t \hat{Y}_t - \beta \hat{\pi}_{t+1} \hat{Y}_t\right] = (\sigma - 1) \left(\hat{Y}_t\right)^2 + \left(\left(\frac{1+\eta}{1-\alpha}\right)\left(\hat{Y}_t - \hat{A}_t\right) - \left(\frac{1}{\epsilon - 1}\right) \hat{\epsilon}_t\right) \hat{Y}_t, \tag{A.13}$$

which solved forward gives our version of equation (6.5):

$$\frac{\sigma\psi\beta}{(\epsilon - 1) Y} E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{Y}_t - \hat{Y}_{t+1}\right) \hat{\pi}_{t+1}$$
$$= -E_0 \sum_{t=0}^{\infty} \beta^t \left[\sigma (\sigma - 1) \hat{Y}_t + \left(\frac{\sigma(1+\eta)}{1-\alpha}\left(\hat{Y}_t - \hat{A}_t\right) - \left(\frac{\sigma}{\epsilon - 1}\right) \hat{\epsilon}_t\right)\right] \hat{Y}_t + tip + O_3. \tag{A.14}$$

Combining this equation with (A.12) gives

$$\left(\frac{1+\eta}{1-\alpha}\right) E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{Y}_t - \hat{A}_t\right) = (1-\sigma) E_0 \sum_{t=0}^{\infty} \beta^t \hat{Y}_t - \frac{\sigma\psi}{2Y} E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{\pi}\right)^2$$
$$- \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(-\left(\frac{1}{\epsilon-1}\right)\hat{\epsilon}_t + \left(\frac{1+\eta}{1-\alpha}\right)\left(\hat{Y}_t - \hat{A}_t\right)\right)^2 + tip + O_3, \tag{A.15}$$

which corresponds to equation (6.7) in Damjanovic and Nolan (2011). Combining this equation with the extended second-order utility (A.6) gives

$$U = (1-\sigma)\,\Phi\left(\frac{1-\alpha}{1+\eta}\right)\left(\frac{1-\alpha}{\eta+\sigma+\alpha(1-\sigma)}\right) E_0 \sum_{t=0}^{\infty} \beta^t \hat{Y}_t$$
$$- \frac{1}{2}\frac{\psi}{Y}\left(\Phi\left(\frac{1-\alpha}{1+\eta}\right)\sigma + 1\right)\left(\frac{1-\alpha}{\eta+\sigma+\alpha(1-\sigma)}\right) E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{\pi}_t\right)^2$$
$$- \frac{1}{2}\Phi\left(\frac{1-\alpha}{1+\eta}\right)\left(\frac{1-\alpha}{\eta+\sigma+\alpha(1-\sigma)}\right) E_0 \sum_{t=0}^{\infty} \beta^t \left(-\left(\frac{1}{\epsilon-1}\right)\hat{\epsilon}_t\right)^2 - \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\hat{Y}_t - \hat{A}_t\right)^2, \tag{A.16}$$

which corresponds to (5.4) and (5.6) in Damjanovic and Nolan (2011). Note that setting $\sigma = 1$ delivers the expressions in that paper. However, we do not consider technology shocks (see the production function (24) in our text), which implies $\hat{A}_t = 0$. We can therefore write welfare loss (the negative value of (A.16)) as

$$J_0^{abs.} = \frac{1}{2}\; E_0 \sum_{t=0}^{\infty} \beta^t \left(\left(\hat{Y}_t\right)^2 + \Gamma_1 \left(\hat{\pi}_t\right)^2 + \Gamma_2 \left(\hat{\epsilon}_t\right)^2 - \Omega \hat{Y}_t\right), \tag{A.17}$$

where

$$\Gamma_1 = \frac{\psi}{Y}\left(\Phi\left(\frac{1-\alpha}{1+\eta}\right)\sigma + 1\right)\left(\frac{1-\alpha}{\eta+\sigma+\alpha(1-\sigma)}\right), \tag{A.18}$$

$$\Gamma_2 = \Phi\left(\frac{1-\alpha}{1+\eta}\right)\left(\frac{1-\alpha}{\eta+\sigma+\alpha(1-\sigma)}\right)\left(\frac{1}{\epsilon-1}\right)^2, \tag{A.19}$$

and

$$\Omega = 2(1-\sigma)\,\Phi\left(\frac{1-\alpha}{1+\eta}\right)\left(\frac{1-\alpha}{\eta+\sigma+\alpha(1-\sigma)}\right). \tag{A.20}$$

B Tables

Table 1: Relative welfare in the *nonlinear* model, where $V_0^{rel.} = \left(\frac{V_0^{abs.}(Interest)}{V_0^{abs.}(Ramsey)}\right) \times 100$ for $\psi \in [0.001, 500]$ and $\alpha \in [0, 0.5]$. Parametrization: $\beta = 0.99$, $\sigma = 1.001$, $\eta = 2$, $\phi = 0$, $\rho = 0$, $\delta_y = 0.125$, $\delta_\pi = 1.5$, and $\epsilon = 6$. The shaded areas denote parameter regions implying the superiority of the interest rate rule against the Ramsey policy under timeless perspective commitment ($V_0^{rel.} < 100\%$).

ψ \ α	**0**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**
0.001	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
50	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
100	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
150	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	99.9999	99.9999	99.9999	99.9999
200	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000
250	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000
300	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000
350	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000
400	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000
500	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000	100.0000

Table 2: Relative welfare in the *approximated* model, where $J_0^{rel.} = \left(\frac{J_0^{abs.}(Interest)}{J_0^{abs.}(Ramsey)}\right) \times 100$ for $\psi \in [0.001, 500]$ and $\alpha \in [0, 0.5]$. Parametrization: $\beta = 0.99$, $\sigma = 1.001$, $\eta = 2$, $\phi = 0$, $\rho = 0$, $\delta_y = 0.125$, $\delta_\pi = 1.5$, and $\epsilon = 6$. The shaded areas denote parameter regions implying the superiority of the interest rate rule against the Ramsey policy under timeless perspective commitment ($J_0^{rel.} < 100\%$).

ψ \ α	**0**	**0.1**	**0.15**	**0.2**	**0.25**	**0.3**	**0.35**	**0.4**	**0.45**	**0.5**
0.001	100.0112	100.0103	100.0098	100.0093	100.0087	100.0082	100.0076	100.0071	100.0065	100.0059
50	81.7983	84.4383	86.2588	88.4747	91.1397	94.3119	98.0507	102.4104	107.4276	113.0998
100	74.0288	73.9770	74.2347	74.7180	75.4626	76.5133	77.9265	79.7748	82.1517	85.1785
150	70.1235	69.4343	69.3281	69.4079	69.6997	70.2358	71.0576	72.2179	73.7860	75.8541
200	66.6822	65.6466	65.3505	65.2253	65.2933	65.5824	66.1278	66.9749	68.1831	69.8313
250	63.8066	62.5220	62.0906	61.8207	61.7323	61.8507	62.2079	62.8450	63.8160	65.1934
300	61.4376	59.9579	59.4205	59.0372	58.8264	58.8113	59.0214	59.4949	60.2822	61.4509
350	59.4774	57.8391	57.2154	56.7395	56.4284	56.3038	56.3929	56.7316	57.3672	58.3635
400	57.8389	56.0688	55.3731	54.8199	54.4249	54.2081	54.1952	54.4198	54.9266	55.7760
500	55.2699	53.2931	52.4842	51.8088	51.2806	50.9172	50.7413	50.7825	51.0810	51.6915

Table 3: Relative welfare in the *nonlinear* model, where $V_0^{rel.} = \left(\frac{V_0^{abs.}(Interest)}{V_0^{abs.}(Ramsey)}\right) \times 100$ for $\phi \in [0, 0.9]$ and $\rho \in [0, 0.9]$. Parametrization: $\alpha = 0$, $\psi = 50$, $\beta = 0.99$, $\sigma = 1.001$, $\eta = 2$, $\delta_y = 0.125$, $\delta_\pi = 1.5$, and $\epsilon = 6$. The shaded areas denote parameter regions implying the superiority of the interest rate rule against the Ramsey policy under timeless perspective commitment ($V_0^{rel.} < 100\%$).

ϕ \ ρ	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
0.1	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
0.2	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
0.3	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
0.4	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
0.5	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
0.6	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
0.7	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
0.8	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
0.9	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999

Table 4: Relative welfare in the *approximated* model, where $J_0^{rel.} = \left(\frac{J_0^{abs.}(Interest)}{J_0^{abs.}(Ramsey)}\right) \times 100$ for $\phi \in [0, 0.9]$ and $\rho \in [0, 0.9]$. Parametrization: $\alpha = 0$, $\psi = 50$, $\beta = 0.99$, $\sigma = 1.001$, $\eta = 2$ $\delta_y = 0.125$, $\delta_\pi = 1.5$, and $\epsilon = 6$. The shaded areas denote parameter regions implying the superiority of the interest rate rule against the Ramsey policy under timeless perspective commitment ($J_0^{rel.} < 100\%$).

ϕ \ ρ	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**
0	81.7983	93.8279	108.5212	126.5003	148.4137	174.7072	204.9871	236.4143	260.1588	256.0392
0.1	79.8376	91.1629	104.9564	121.8005	142.3202	166.9981	195.6401	225.9780	250.4035	250.2768
0.2	77.9773	88.6042	101.4961	117.1903	136.2788	159.2658	186.1341	215.1659	240.0136	243.8586
0.3	76.2397	86.1857	98.1889	112.7355	130.3741	151.6115	176.5761	204.0620	228.9970	236.6869
0.4	74.6567	83.9538	95.0998	108.5240	124.7205	144.1758	167.1231	192.8070	217.4053	228.6557
0.5	73.2721	81.9704	92.3146	104.6726	119.4723	137.1550	158.0066	181.6329	205.3749	219.6674
0.6	72.1449	80.3184	89.9485	101.3393	114.8428	130.8291	149.5772	170.9309	193.2163	209.6937
0.7	71.3569	79.1124	88.1614	98.7463	111.1391	125.6177	142.3922	161.3927	181.6273	198.9855
0.8	71.0280	78.5205	87.1899	97.2280	108.8358	122.1958	137.4078	154.3387	172.2598	188.8729
0.9	71.3505	78.8147	87.4213	97.3419	108.7468	121.7728	136.4576	152.6029	169.5080	185.5378

Table 5: Relative welfare in the *nonlinear* model, where $V_0^{rel.} = \left(\frac{V_0^{abs.}(Interest)}{V_0^{abs.}(Ramsey)} \right) \times 100$ for $\eta \in [2, 2.9]$ and $\sigma \in [1.001, 2.5]$. Parametrization: $\alpha = 0$, $\psi = 50$, $\rho = 0$, $\phi = 0$, $\beta = 0.99$, $\delta_y = 0.125$, $\delta_\pi = 1.5$, and $\epsilon = 6$. The shaded areas denote parameter regions implying the superiority of the interest rate rule against the Ramsey policy under timeless perspective commitment ($V_0^{rel.} < 100\%$).

η \ σ	**1.001**	**1.15**	**1.25**	**1.5**	**1.75**	**2.0**	**2.15**	**2.25**	**2.3**	**2.5**
2.0	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
2.1	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
2.2	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
2.3	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
2.4	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
2.5	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
2.6	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
2.7	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
2.8	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999
2.9	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999	99.9999

Table 6: Relative welfare in the *approximated* model, where $J_0^{rel.} = \left(\frac{J_0^{abs.}(Interest)}{J_0^{abs.}(Ramsey)} \right) \times 100$ for $\eta \in [2, 2.9]$ and $\sigma \in [1.001, 2.5]$. Parametrization: $\alpha = 0$, $\psi = 0.001$, $\rho = 0$, $\phi = 0$, $\beta = 0.99$, $\delta_y = 0.125$, $\delta_\pi = 1.5$, and $\epsilon = 6$. The shaded areas denote parameter regions implying the superiority of the interest rate rule against the Ramsey policy under timeless perspective commitment ($J_0^{rel.} < 100\%$).

η \ σ	**1.001**	**1.15**	**1.25**	**1.5**	**1.75**	**2.0**	**2.15**	**2.25**	**2.3**	**2.5**
2.0	81.7983	84.0244	85.3926	88.6648	92.0298	95.6848	98.0459	99.6897	100.5316	104.0204
2.1	83.0968	85.5675	87.0978	90.7569	94.4720	98.4312	100.9500	102.6880	103.5737	107.2144
2.2	84.4288	87.1409	88.8306	92.8687	96.9238	101.1749	103.8433	105.6698	106.5963	110.3770
2.3	85.7874	88.7375	90.5839	94.9929	99.3777	103.9085	106.7184	108.6279	109.5923	113.5017
2.4	87.1665	90.3510	92.3514	97.1228	101.8269	106.6253	109.5689	111.5560	112.5557	116.5831
2.5	88.5604	91.9758	94.1272	99.2525	104.2653	109.3194	112.3891	114.4486	115.4811	119.6165
2.6	89.9644	93.6067	95.9062	101.3764	106.6875	111.9855	115.1740	117.3011	118.3639	122.5982
2.7	91.3741	95.2392	97.6836	103.4898	109.0886	114.6190	117.9194	120.1096	121.2005	125.5254
2.8	92.7855	96.8691	99.4553	105.5881	111.4642	117.2161	120.6218	122.8708	123.9879	128.3957
2.9	94.1951	98.4928	101.2173	107.6676	113.8107	119.7735	123.2782	125.5822	126.7235	131.2076

C Plots

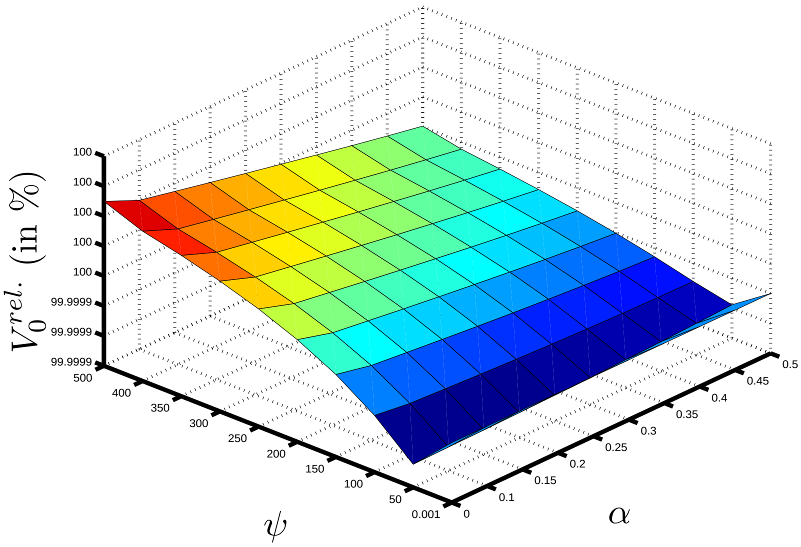

Figure 1: Relative welfare in the *nonlinear* model, where $V_0^{rel.} = \left(\frac{V_0^{abs.}(Interest)}{V_0^{abs.}(Ramsey)}\right) \times 100$ for $\psi \in [0.001, 500]$ and $\alpha \in [0, 0.5]$. Parametrization: $\beta = 0.99$, $\sigma = 1.001$, $\eta = 2$, $\phi = 0$, $\rho = 0$, $\delta_y = 0.125$, $\delta_\pi = 1.5$, and $\epsilon = 6$.

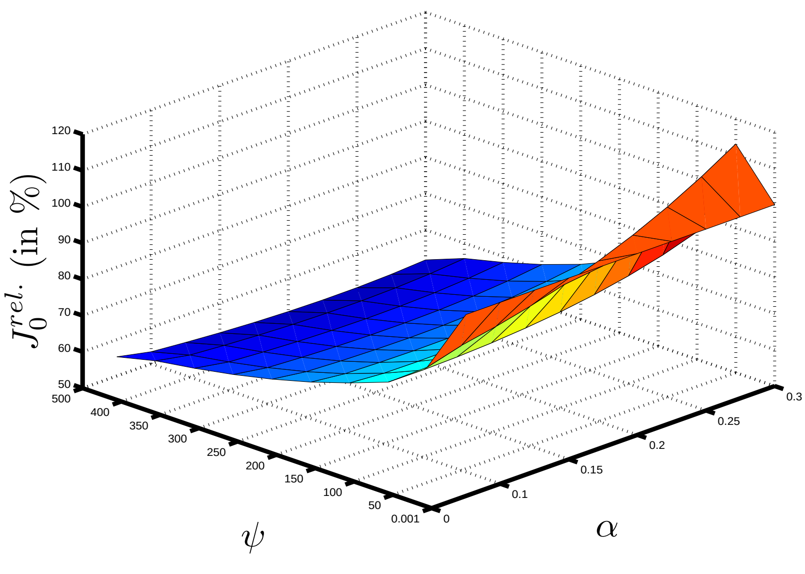

Figure 2: Relative welfare in the *approximated* model, where $J_0^{rel.} = \left(\frac{J_0^{abs.}(Interest)}{J_0^{abs.}(Ramsey)}\right) \times 100$ for $\psi \in [0.001, 500]$ and $\alpha \in [0, 0.5]$. Parametrization: $\beta = 0.99$, $\sigma = 1.001$, $\eta = 2$, $\phi = 0$, $\rho = 0$, $\delta_y = 0.125$, $\delta_\pi = 1.5$, and $\epsilon = 6$.

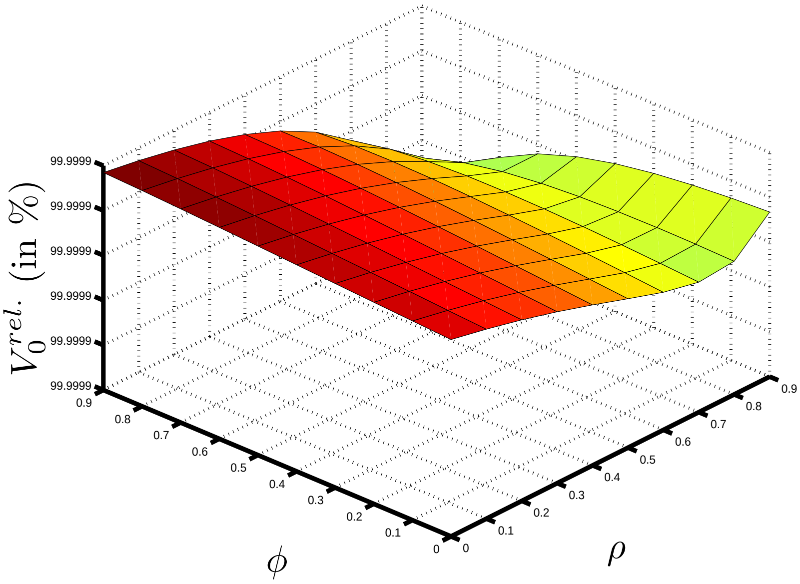

Figure 3: Relative welfare in the *nonlinear* model, where $V_0^{rel.} = \left(\frac{V_0^{abs.}(Interest)}{V_0^{abs.}(Ramsey)}\right) \times 100$ for $\phi \in [0, 0.9]$ and $\rho \in [0, 0.9]$. Parametrization: $\alpha = 0$, $\psi = 50$, $\beta = 0.99$, $\sigma = 1.001$, $\eta = 2$, $\delta_y = 0.125$, $\delta_\pi = 1.5$, and $\epsilon = 6$.

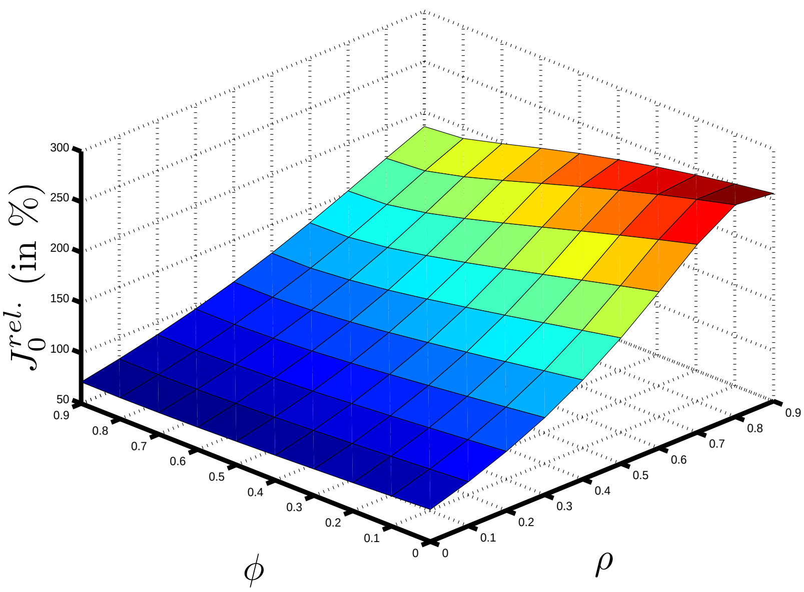

Figure 4: Relative welfare in the *approximated* model, where $J_0^{rel.} = \left(\frac{J_0^{abs.}(Interest)}{J_0^{abs.}(Ramsey)}\right) \times 100$ for $\phi \in [0, 0.9]$ and $\rho \in [0, 0.9]$. Parametrization: $\alpha = 0$, $\psi = 50$, $\beta = 0.99$, $\sigma = 1.001$, $\eta = 2$ $\delta_y = 0.125$, $\delta_\pi = 1.5$, and $\epsilon = 6$.

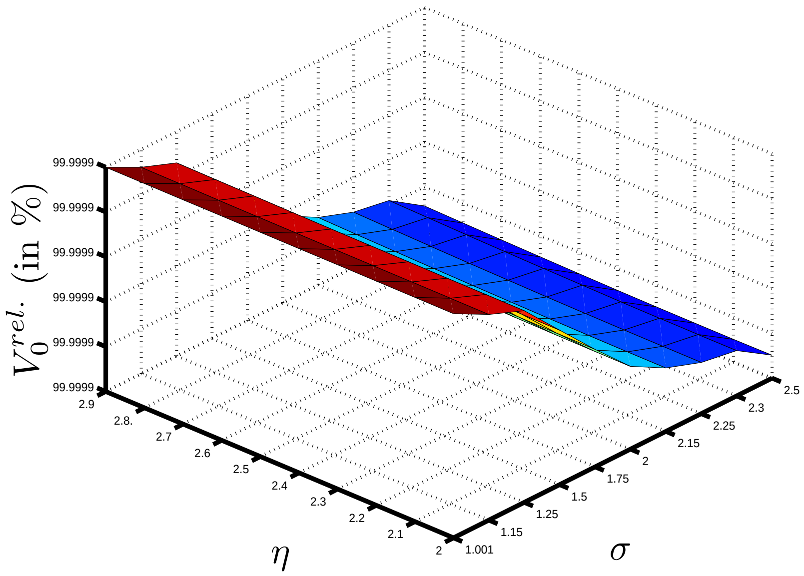

Figure 5: Relative welfare in the *nonlinear* model, where $V_0^{rel.} = \left(\frac{V_0^{abs.}(Interest)}{V_0^{abs.}(Ramsey)}\right) \times 100$ for $\eta \in [2, 2.9]$ and $\sigma \in [1.001, 2.5]$. Parametrization: $\alpha = 0$, $\psi = 50$, $\rho = 0$, $\phi = 0$, $\beta = 0.99$, $\delta_y = 0.125$, $\delta_\pi = 1.5$, and $\epsilon = 6$.

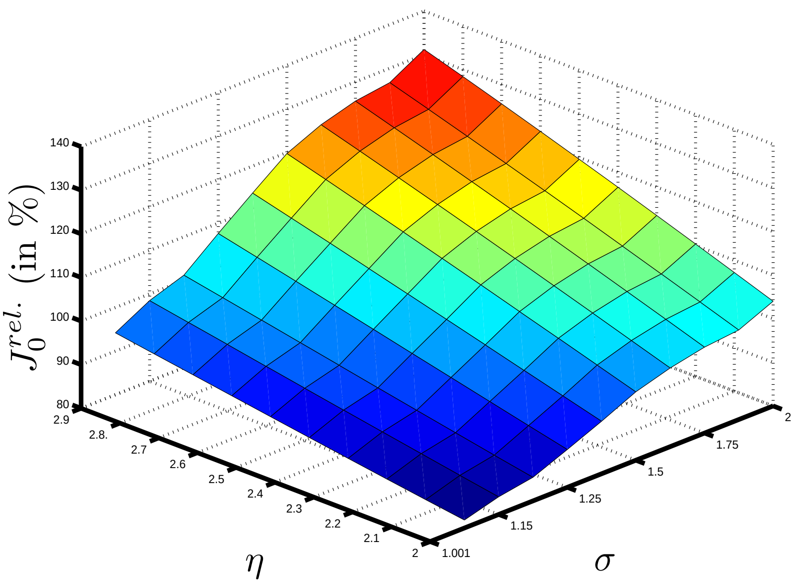

Figure 6: Relative welfare in the *approximated* model, where $J_0^{rel.} = \left(\frac{J_0^{abs.}(Interest)}{J_0^{abs.}(Ramsey)}\right) \times 100$ for $\eta \in [2, 2.9]$ and $\sigma \in [1.001, 2.5]$. Parametrization: $\alpha = 0$, $\psi = 50$, $\rho = 0$, $\phi = 0$, $\beta = 0.99$, $\delta_y = 0.125$, $\delta_\pi = 1.5$, and $\epsilon = 6$.

D Contour Plots

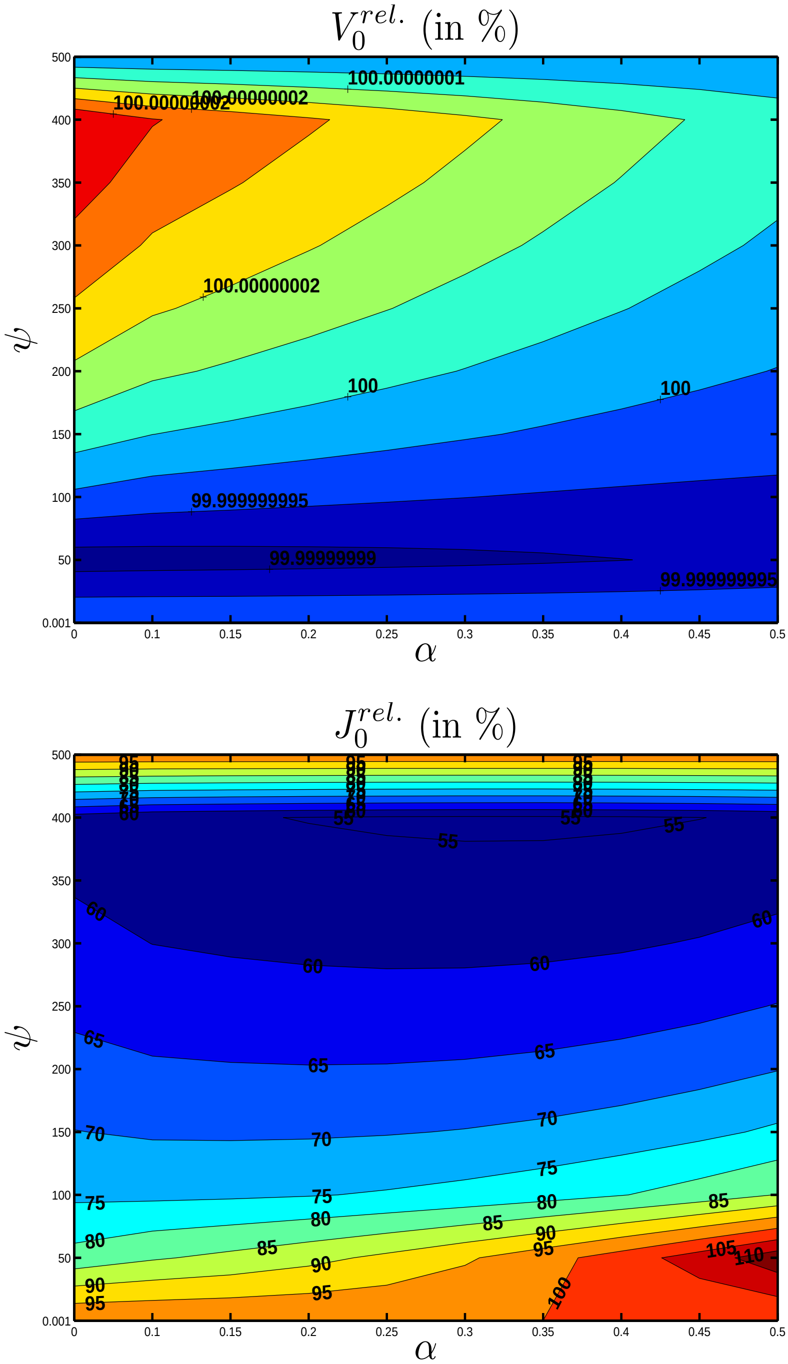

Figure 7: Contour surfaces of figures 1-2. The *upper* figure gives relative welfare $V_0^{rel.} = \left(\frac{V_0^{abs.}(Interest)}{V_0^{abs.}(Ramsey)}\right) \times 100$ in the *nonlinear* model, while the *lower* figure corresponds to the *approximated* model with $J_0^{rel.} = \left(\frac{J_0^{abs.}(Interest)}{J_0^{abs.}(Ramsey)}\right) \times 100$ for $\psi \in [0.001, 500]$ and $\alpha \in [0, 0.5]$. Parametrization: $\beta = 0.99$, $\sigma = 1.001$, $\eta = 2$, $\phi = 0$, $\rho = 0$, $\delta_y = 0.125$, $\delta_\pi = 1.5$, and $\epsilon = 6$.

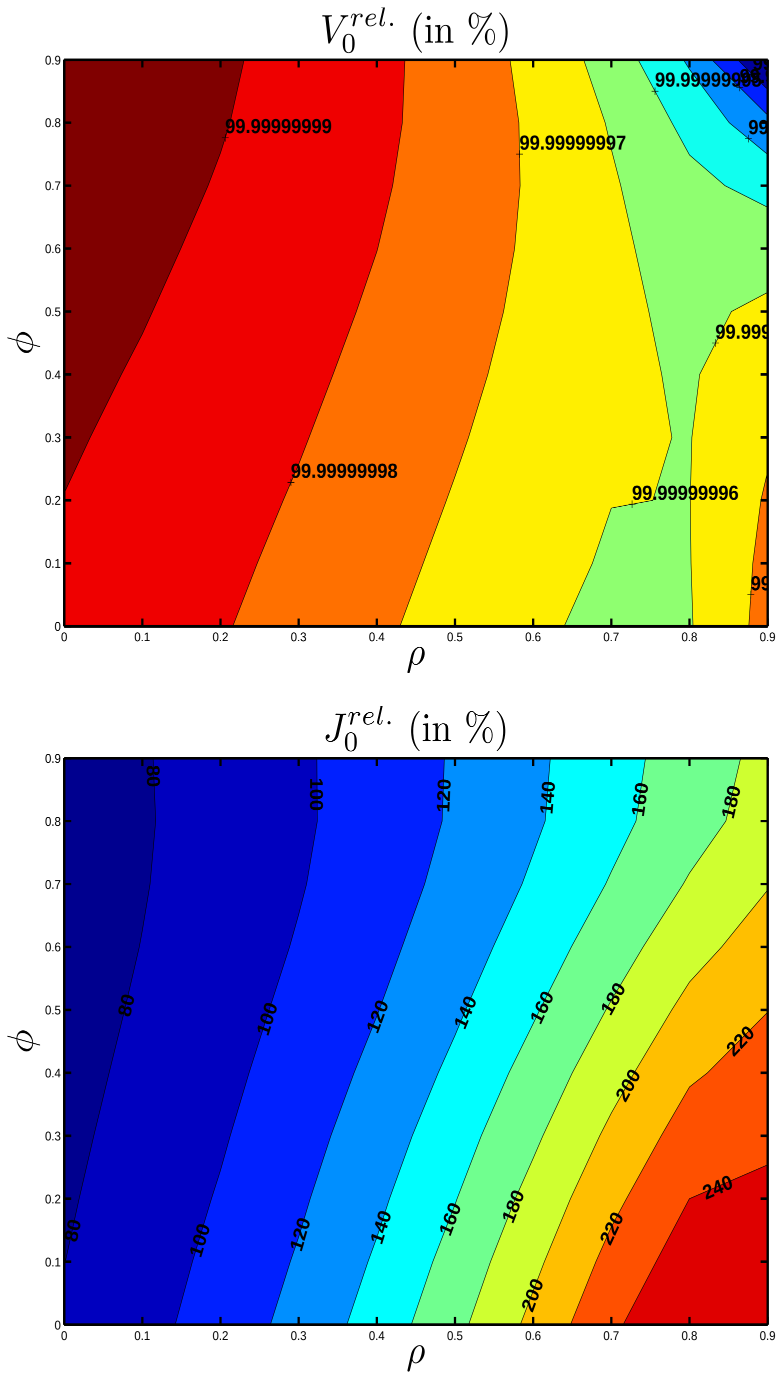

Figure 8: Contour surfaces of figures 3-4. The *upper* figure gives relative welfare $V_0^{rel.} = \left(\frac{V_0^{abs.}(Interest)}{V_0^{abs.}(Ramsey)}\right) \times 100$ in the *nonlinear* model, while the *lower* figure corresponds to the *approximated* model with $J_0^{rel.} = \left(\frac{J_0^{abs.}(Interest)}{J_0^{abs.}(Ramsey)}\right) \times 100$ for $\phi \in [0, 0.9]$ and $\rho \in [0, 0.9]$. Parametrization: $\alpha = 0$, $\psi = 50$, $\beta = 0.99$, $\sigma = 1.001$, $\eta = 2$ $\delta_y = 0.125$, $\delta_\pi = 1.5$, and $\epsilon = 6$.

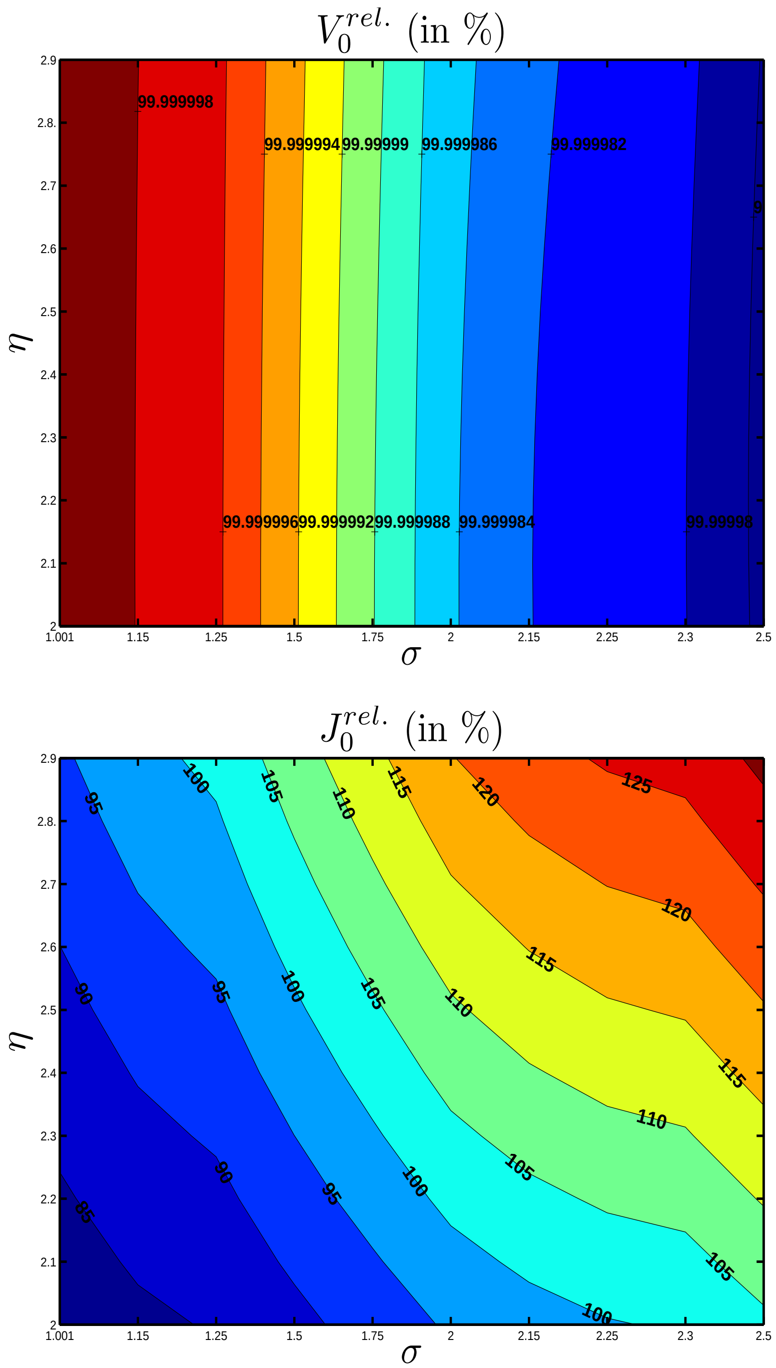

Figure 9: Contour surfaces of figures 5-6. The *upper* figure gives relative welfare $V_0^{rel.} = \left(\frac{V_0^{abs.}(Interest)}{V_0^{abs.}(Ramsey)}\right) \times 100$ in the *nonlinear* model, while the *lower* figure corresponds to the *approximated* model with $J_0^{rel.} = \left(\frac{J_0^{abs.}(Interest)}{J_0^{abs.}(Ramsey)}\right) \times 100$ for $\eta \in [2, 2.9]$ and $\sigma \in [1.001, 2.5]$. Parametrization: $\alpha = 0$, $\psi = 50$, $\rho = 0$, $\phi = 0$, $\beta = 0.99$, $\delta_y = 0.125$, $\delta_\pi = 1.5$, and $\epsilon = 6$.

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