Analysts' recommendations and managers' disclosing behavior

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Abstract

This paper analyzes how the monitoring activity by sell-side analysts who produce trading recommendations affects the managers' incentives to provide truthful information to the market. We show that the managers' communication strategy depends on the market prior belief on firm performance. A perfectly informed manager reveals a truthful (resp. uninformative) signal to the market when the firm performance is believed to be sufficiently bad (resp. good). In case the report of the manager does not contain any information additional to the publicly available one, the analysts do not have incentives to provide meaningful recommendations if their private signal precision is low. Taking this behavior into account, the manager is more likely to report non-informative signals if analysts poorly informed compared to the situation in which his firm is not monitored by any analyst. Our result then casts doubts over the role of poorly informed analysts in enhancing markets efficiency.

1 Introduction

Over every economic quarter, managers of publicly traded firms regularly disclose information to the financial analysts and to the financial markets; then analysts make forecasts about the firm's current and future earnings. At the end of each quarter, the managers report firms' earnings. When the next quarter starts, the next round of disclosing and forecasting is repeated again. Do managers reports always contain accurate and meaningful information? How do the analysts process the information obtained from the managers? Even more crucially, how do analysts' forecasts affect managers' reporting strategy in the first place? Many questions are worthwhile to investigate in this disclosure/forecast/reporting cycle. Despite a large literature focuses either on analysts' incentives and forecasting strategy (see for example Scharfstein and Stein 2000, Zwiebel 1995, Banerjee 1992), or managers' manipulating behaviors (e.g. Hong and Kubik 2003, Beyer 2008), to the best of our knowledge theoretical research on the interplay between managers disclosure behavior and analysts forecasts is scarce.[3]

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[3]For complete literature surveys see Lambert (2001), Kothari (2001) and Ramnath, Rock and Shane (2008).

Our paper tries to fill this gap and studies how the coverage of analysts affects the incentives for a perfectly informed manager to disclose truthful information. In order to address this point, we first characterize the equilibrium stock price and the optimal signaling strategy of a manager in a benchmark one-period model without analysts. Then we assume the firm is covered by informed analyst(s) and analyze whether their monitoring activity improves the informational content of managers' messages. Throughout the analysis we assume that managers report the quarterly earnings truthfully at the end of each period, abstracting then from earnings manipulation behavior.

Our main result is the following. Consider a manager whose compensation package is related to the stock price and who exactly knows the firm current performance. When disclosing information to the market, he takes into account his own private information, i.e. the firm's underlying performance, and the market reaction to his announcement. The presence of privately informed traders on the market guarantees that the order flow, hence the stock price, contains some information even if the message disclosed by the manager is void. When the market is sufficiently pessimistic about the firm performance it is too costly for the manager to change the public belief, so that bad news are disclosed. However, if the market belief is very positive, bad news are concealed from the market, due to the expected high stock price when doing so. When the informed traders operating on the market rely on analysts' recommendations for their trades, these latter indirectly affect the stock price through the informed trades. In providing their recommendations, analysts face a trade-off between the benefit of trading commissions, which increase with the informativeness of the recommendation, and the cost of inaccuracy. When all the information received by the market is coming from analysts recommendations, the cost of inaccuracy is high if the analysts are relatively poorly informed themselves. Hence, in such a case, recommendations do not add information and the market overall has a less precise estimate of the firm performance than in the case analysts where not covering the firm. In such a situation, the incentive for the manager not to report a bad news when the market does not expect one is even higher: analysts' coverage then does not improve market efficiency.

Our paper is closely related to Guttman, Kadan and Kandel (2006), and Beyer and Guttman (2011). Guttman, Kadan and Kandel (2006) explicitly analyzes how stock-based compensation scheme affects the manager's optimal reporting strategy. Beyer and Guttman (2011) focuses on how incentives to generate trading commissions induces analysts to bias their forecasts. Other models related to ours includes Beyer and Guttman (2010), who study how managers bias their voluntary reports prior to issuing equity; Demski and Dye (1999), Fischer and Verrecchia (2000), and Beyer (2008) who study how managers manipulate the firm's earnings in order to meet analysts' forecast.

The paper is organized as follows. Section 2 presents the benchmark model without analyst. In Section 3 we add financial analysts scrutinizing the firm in the model. In Section 4 we provide some testable implications of the model, while Section 5 concludes.

2 The benchmark model: a market without analysts

As a benchmark case, we first study a one-period model without financial analysts. Consider a publicly traded firm with end-of-period value which is represented by a random variable $\tilde{v}$. For simplicity, we assume the realization of $\tilde{v}$ can be either $\overline{v}$ or $\underline{v}$, with $\bar{v} > \underline{v}$.The probability that $\tilde{v}$ takes the high value is equal to p, with $p \in (0,1)$ known to all market participants. Let us denote the expected value of $\tilde{v}$ by $v^e = p\overline{v} + (1-p)\underline{v}$.

The firm is run by a risk-averse manager, whose compensation is positively related to the firm stock price P:

$$I^M = wP \tag{1}$$

where w denotes the total pay-for-performance sensitivity of the managerial contract. The manager knows the realization of $\tilde{v}$ so that when the realized value is $\overline{v}$ (resp. $\underline{v}$), we refer to the manager as high (resp. low) type. Before the firm value becomes publicly known, the manager sends a public signal y to the market.[4] The manager may reveal his type, i.e. play a perfectly separating strategy where $y(\overline{v}) \neq y(\underline{v})$, or hide his private information, playing a pooling strategy, $y(\overline{v}) = y(\underline{v})$. Although the manager is not confined to tell the truth, ex-post he bears a cost if he sends a signal that proves to be false. Finally, we assume that the manager is risk averse and his utility performs a constant degree of absolute risk-aversion β. The manager expected utility can be written then as:

$$EU^M(\tilde{v}; y) = wE[P|\tilde{v}; y] - \beta w^2 var(P|\tilde{v}; y) - k(y - \widetilde{v})^2 \tag{2}$$

where k reflects the cost of sending a false signal.[5]

To model the stock trading, we follow Glosten and Milgrom (1985) and assume that two types of traders operate on the market. More precisely, a quota $i \in [0,1]$ of the traders are informed, while the remaining $1-i$ are uninformed traders who trade for liquidity reasons. In addition to the publicly observed managerial signal y, the informed traders observe a private signal s which is correct (i.e. equal to the realized $\tilde{v}$) with probability $g \in [\frac{1}{2}, 1]$.[6] The informed

[4] Here we have to specify the sets of possible signals.

[5] The cost of sending a false signal intentionally reflects the reputation or litigation cost explained in Kasznik (1999). A symmetric quadratic cost function is used for the sake of tractability, see Fischer and Verrecchia (2000), Beyer (2005), and Guttman et al. (2006). The main intuition of the result follows from the convexity of the cost function instead of its specific functional form.

[6] That is,

$$\begin{aligned}
\widetilde{v} &= \{\overline{v}\} \Rightarrow \begin{cases} s = \overline{v} & \text{prob. } g \\ s = \underline{v} & \text{prob. } 1-g \end{cases} \\
\widetilde{v} &= \{\underline{v}\} \Rightarrow \begin{cases} s = \overline{v} & \text{prob. } 1-g \\ s = \underline{v} & \text{prob. } g \end{cases}
\end{aligned}$$

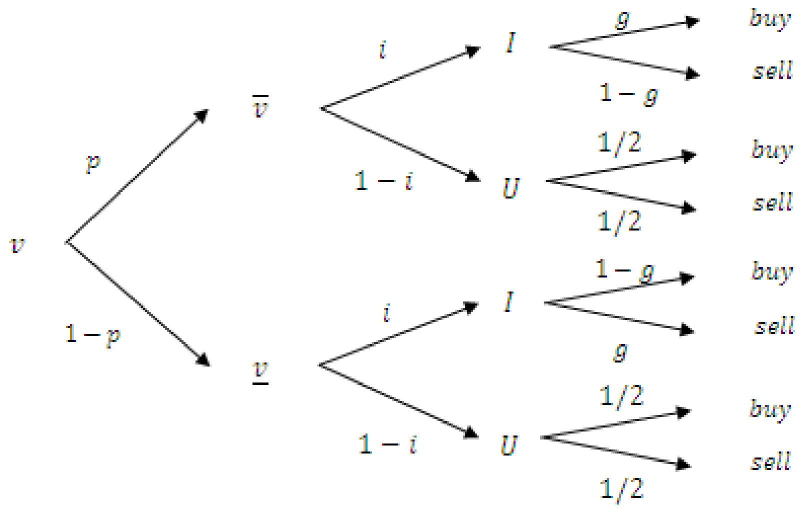

Figure 1:

traders buy a unit of the firm stock if their private signal $s = \overline{v}$ while they sell a unit if $s = \underline{v}$.[7]

If the signal of the manager is uninformative ("pooling" in the following), the uninformed traders buy and sell randomly and on average the probability of both buying and selling a unit of stocks is equal to 1/2. The event tree of the

stock trade is given in Figure 1.

Figure 1: The event tree for the benchmark model.

The market book is cleared by risk-neutral market makers who take the counterpart of the (market) orders submitted by the traders and set the market price P. The equilibrium price contains all public information available at the moment of trading together with the order flow $\widetilde{z}$ observed by the market makers:

$$P = E[\tilde{v}|y,\ \widetilde{z}] \tag{3}$$

Define by q the probability that the market maker observes an excess buy (sell) order given the firm value is high (low), i.e. $q = pr(\overline{z}|v = \overline{v}) = pr(\underline{z}|v = \underline{v})$.

The sequence of the events is as follows. At the beginning of the period, nature decides the realization of $\tilde{v}$, which is privately observed by the manager.

[7] Similar assumptions of market microstructure can be found in Glosten and Milgrom (1985), and Vives (1995). Here we assume informed traders are correct with probability of g, with $1/2 \leq g \leq 1$, whereas in Glosten and Milgrom (1985), the signals of informed traders are correct with probability 1, and in Vives (1995), the signal is assumed to be normally distributed with mean $\tilde{v}$ and variance σ^2.

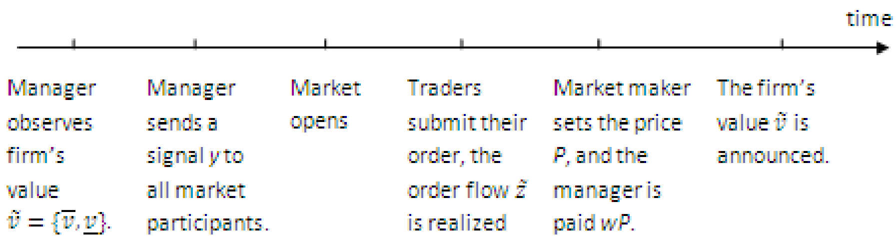

Figure 2:

Knowing the realization of $\tilde{v}$, the manager discloses a public signal to all market participants, i.e. uninformed traders, informed traders and market makers. Traders submit their orders based on their information: for the informed traders I, the information set is $H_I = \{y, s\}$, while for the uninformed traders U, $H_U = \{y\}$. The market maker observes then the order flow $\tilde{z}$ and sets the stock price as in (3). The manager's compensation is paid according to (1). Finally, the firm value is announced and the manager's payoff is realized, i.e. he pays the reputation cost if he sends a false signal previously. The sequence of events is summarized in figure 2.

Figure 2: The timing of events

We proceed now by solving the model backward in time. First, fixing the information contained in the manager signal y, we determine the market trades, the informativeness of the order flow, hence the equilibrium price. Second, given this solution we compute the signal the manager sends in order to maximize his expected utility (2).

The following Lemma determines the stock price in the two cases of a perfectly revealing and a perfectly pooling signal by the manager.[8]

Lemma 1: *i) If the signal of the manager is perfectly revealing, then the stock price is given by:*

$$P = E[\tilde{v}|y = \tilde{v}, \tilde{z}] = \tilde{v}$$

ii) If the signal of the manager is perfectly pooling, then the stock price is given by:

$$P = E[\tilde{v}|\tilde{z}]$$

where $\tilde{z}$ is the realized order flow. In this case, the market maker sets the price based only on the information contained in the order flow:

[8] Here we should then compute also the equilibrium for a partially revealing strategy by the manager (i.e. in mixed strategies).

$$P = E[v|\tilde{z}] = \begin{cases} E[v|\bar{z}] = P(\bar{z}) \\ E[v|\underline{z}] = P(\underline{z}) \end{cases}$$

where

$$P(\bar{z}) = \frac{pq\overline{v} + (1-p)(1-q)\underline{v}}{pq + (1-p)(1-q)}$$

and

$$P(\underline{z}) = \frac{p(1-q)\overline{v} + (1-p)q\underline{v}}{p(1-q) + (1-p)q}$$

Proof of Lemma 1: see the Appendix.

This type of equilibrium is standard in the binary type, asymmetric information models, for example, Copeland and Galai (1983), and Glosten and Milgrom (1985). Using the result of Lemma 1, we can obtain the expected payoff of the manager of type $\tilde{v} = \{\overline{v}, \underline{v}\}$ when he sends a perfectly revealing signal or a perfectly pooling one.

The manager could play two perfectly revealing strategies. One revealing strategy is that both types reveal their types, i.e. $y(\bar{v}) = \bar{v}, y(\underline{v}) = \underline{v}$, in this case the expected payoffs of both types of managers are:

$$EU^M(\bar{v}; y(\bar{v}) = \bar{v}, y(\underline{v}) = \underline{v}) = w\bar{v}$$

$$EU^M(\underline{v}; y(\bar{v}) = \bar{v}, y(\underline{v}) = \underline{v}) = w\underline{v}$$

The other revealing strategy is that both types of managers are lying, i.e. $y(\bar{v}) = \underline{v}, y(\underline{v}) = \bar{v}$, then their expected payoffs are given by:

$$EU^M(\bar{v}; y(\bar{v}) = \underline{v}, y(\underline{v}) = \bar{v}) = w\bar{v} - k\,(\Delta v)^2$$

$$EU^M(\underline{v}; y(\bar{v}) = \underline{v}, y(\underline{v}) = \bar{v}) = w\underline{v} - k\,(\Delta v)^2$$

The manager could also play two perfectly pooling strategy. One pooling strategy is that both types of managers send high signals, in this case their expected payoffs are given by:

$$EU^M(\bar{v}; y(\bar{v}) = y(\underline{v}) = \bar{v}) = wE\,[P|z] - \beta w^2 Var\,[P|z]$$

$$EU^M(\underline{v}; y(\bar{v}) = y(\underline{v}) = \bar{v}) = wE\,[P|z] - \beta w^2 Var\,[P|z] - k\,(\Delta v)^2$$

The other pooling strategy is that both types of managers send low signals, in this case their expected payoffs are given by:

$$EU^M(\bar{v}; y(\overline{v}) = y(\underline{v}) = \underline{v}) = wE[P|z] - \beta w^2 Var[P|z] - k(\Delta v)^2$$

$$EU^M(\underline{v}; y(\overline{v}) = y(\underline{v}) = \underline{v}) = wE[P|z] - \beta w^2 Var[P|z]$$

Given the expected payoff, we can find the conditions under which perfectly revealing equilibria, i.e. $y(\overline{v}) = \overline{v}$ *and* $y(\underline{v}) = \underline{v}$ (Lemma 2) and perfectly pooling equilibria, i.e. $y(\overline{v}) = y(\underline{v}) = \overline{v}$ (Lemma 3) exist.

Lemma 2: *An equilibrium where* $y(\overline{v}) = \overline{v}$ *and* $y(\underline{v}) = \underline{v}$ *("perfectly revealing") exists if* $p < \beta w R(p,q) + \frac{k}{w}T(p,q)$*,where* R and T *are defined by*

$$R(p,q) = \frac{[p(1-p)(2q-1)]^2}{[pq + (1-p)(1-q)][p(1-q) + (1-p)q]}$$

and

$$T(p,q) = \frac{[pq + (1-p)(1-q)][p(1-q) + (1-p)q]}{q(1-q)}$$

Lemma 3: *An equilibrium where* $y(\overline{v}) = y(\underline{v}) = \overline{v}$ *("perfectly pooling") exists if*

$$:p > \max\left\{\beta w R(p,q) + \frac{k}{w}T(p,q), 1 + \beta w R(p,q) - \frac{k}{w}T(p,q)\right\}$$

Proof of Lemma 2 and 3: see the Appendix.

The results of Lemma 2 and 3 imply that the manager is better off by revealing his type when the firm's value is believed to be low, i.e. p is small, and he is better off by pooling when the firm's value is believed to be high, i.e. p is large.

We summarize the results we obtained in the benchmark model without analysts in the following proposition.

Proposition 1: *Let* $kT/w > 1/2$*,and consider our benchmark case where no financial analysts are present on the market. Then:*

(i) There exists a fully revealing equilibrium when $p \in [0, \beta wR + kT/w)$*;*

(ii) There exists a perfectly pooling equilibrium when $p \in [\beta wR + kT/w, 1]$*.*

The region of revealing equilibrium increases if i) the cost of sending a false signal is higher (larger k*), ii) the manager is more risk averse (higher* β*), iii) the order flow is more informative (larger* q*), iv) the informed traders' private information is more accurate (larger* g*).*

Proof of Proposition 1: see appendix.

The intuition behind Proposition 1 is as follows. If the cost of sending a false signal is not too small comparing to his wage, it is optimal for the manager to reveal his type when the firm is not performing well (small p). As in this situation, sending a high signal does not change public belief much, whereas the cost for low type of lying is very high comparing to the benefit. Even if the manager sends a high signal, most market participants would doubt whether the manager is telling the truth. The reverse is true when the firm is performing well. It is optimal for the manager to send pooling signals as the benefit from lying exceeds the cost, and market participants are less likely to doubt the truthfulness of the manager's signals.

This result is close to Guttman et al. (2006). They show that in a perfect Bayesian equilibrium, the manager reports truthfully his private information if the firm's earning is either low or high, and sends pooling reports when the earning is in an intermediate range. The optimal strategy of manager when the firm value is high differs in these two models.[9]

The region of revealing equilibrium differs with the parameters of the model. On the one hand, if the manager is more risk averse or the cost of sending a false signal is higher, he is more likely to tell the truth, and thus revealing equilibrium is more likely to exist. On the other hand, if the informed traders can collect more precise private information, then the order flow would be more informative, and the market maker can price the stock more efficiently.

3 The model with financial analysts

Having shown the equilibrium price and the manager's choice of optimal signal in our benchmark model without analysts, we now add financial analysts scrutinizing the firm in the model.

We first assume there is only one analyst following our firm. After the manager sends his public signal, the analyst collects his information signal. To keep the information structure in the model with and without analysts as close as possible, we assume the analyst's signal, denoted by s, is correct (i.e. equal to the realized $\tilde{v}$) with probability g.[10] The analyst sends to his clients (the informed traders in this case, who do not collect information by themselves) a recommendation, denoted by θ, about the position to take on the firm stock. We assume the analyst does not trade for himself.

The analyst's strategy, denoted by $\sigma_A(s)$, is a mapping from his signal s to the space of recommendation θ, which is invertible if the recommendation is informative. When issuing a recommendation, the analyst may play two strategies. He may reveals his signal truthfully, or plays pooling (or partially pooling)

[9] Fischer and Verrecchia (2000) found similar results. They show that the manager's report is less informative if the cost of biasing reports is low, or uncertainty about the manager's objective increases.

[10] That is,

$$\begin{aligned} s &= \{\overline{v}\} \Rightarrow \begin{cases} s = \overline{v} & \text{prob. } g \\ s = \underline{v} & \text{prob. } 1-g \end{cases} \\ s &= \{\underline{v}\} \Rightarrow \begin{cases} s = \overline{v} & \text{prob. } 1-g \\ s = \underline{v} & \text{prob. } g \end{cases} \end{aligned}$$

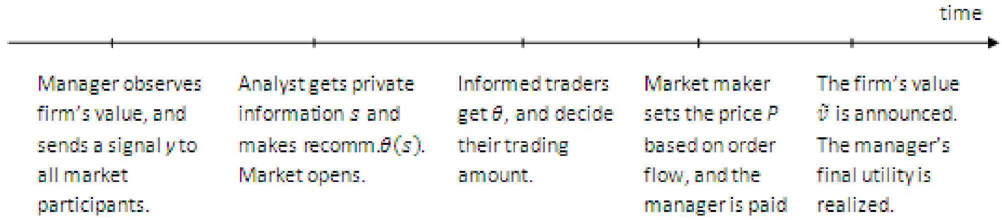

Figure 3:

strategy, given his signal and signal precision.[11] However, he incurs a cost if his recommendation is proved to be inaccurate ex-post. Let the cost of being inaccurate be a positive constant L, which is independent of the recommendation the analyst made.[12]

If the analyst issues an informative recommendation, he receives commission from the trading volume generated by his recommendations from his clients. The informed traders are risk averse as in the benchmark model. Therefore, their trading volume is inversely related to variance of firm value. Then the analyst's expected payoff is given by:

$$E[\pi^A(\theta)|s,g] = \frac{a}{Var[\tilde{v}|\theta]} - (1-g)*L \tag{4}$$

where a is the analyst's per share benefit from trading commission. We further assume that if the recommendation is not informative, i.e. $\sigma_A(s) = \theta$ $\forall s \in S$ and $\forall \theta \in \Theta$, then the analyst will not receive commission, as his clients do not have enough information to trade. However, there is no reputation cost if everyone is wrong. Thus if the analyst issues non-informative recommendation, $\pi^A = 0$.

The sequence of events is now slightly different from the one in Fig. 1. After the manager sends his public signal, the analyst collects his information signal s, and makes a recommendation θ based on it. The informed traders decide their trading amount based on the recommendation. The uninformed traders buy and sell randomly as in the benchmark model. The market maker sets the stock price based on the order flow. The manager's compensation is paid as in (1). Finally the firm value is announced and the manager's utility is realized. The sequence of events is summarized in Fig. 3.

Figure 3: The timing of events

[11] Here we focus on pure strategy equilibria.

[12] The analyst bears a fixed cost of L if his recommendation is later proved to be wrong, and no cost if his recommendation is correct, i.e. $cost \Rightarrow \begin{cases} 0 & prob.g \\ L & prob.1-g \end{cases}$. Thus his expected cost is given by: $g*0+(1-g)*L = (1-g)L$.

3.1 A fully revealing equilibrium

Again, we proceed now solving the model backward in time. First, fixing the information contained in the manager's signal y, we determine the optimal analysts' recommendation, market trades, the informativeness of the order flow, hence the equilibrium price. Second, given this solution we compute the signal the manager sends in order to maximize his expected utility in equation (2).

We start considering the case in which the manager plays a revealing strategy. The equilibrium price is then: $P = E[v|y = \tilde{v}] = \tilde{v}$.

Lemma 4: *A perfectly revealing equilibrium such that the manager sends truthful signals and the analyst passes on the signals to his clients exists when* $p < \beta w R(p, q) + \frac{k}{w} T(p, q)$.

Proof of Lemma 4: see the Appendix.

This perfectly revealing equilibrium is characterized by the coordination between the manager and analyst, such that the manager sends a truthful signal, and the analyst pass on the signal to their clients. If the firm is not performing very well (p is small), the expected cost of lying would be very high comparing the expected gain from sending a pooling signal, thus the manager would rather tell the market the truth. Given the manager reveals his type, the analyst has no incentive to provide a recommendation which is different from the manager's signal, as making wrong recommendation is also costly for the analyst. At this equilibrium, the market maker prices the stock correctly.

3.2 A pooling equilibrium

In the case in which the manager plays a pooling strategy, the signal from the manager is not informative, thus the recommendation of the analyst becomes crucial. If the analyst issues an informative signal, then the informed traders have the same information as in the benchmark model. However, if the analyst issues a non-informative signal, the informed traders do not have any information. The following lemma determines the stock price and market trades in the two cases where the analyst issues informative and non-informative recommendations.

Lemma 5: *i) If the manager sends non-informative signal but the analyst issues informative recommendation, then the equilibrium stock price is the same as in Lemma 1, i.e.:*

$$P(\bar{z}) = \frac{pq\bar{v} + (1-p)(1-q)\underline{v}}{pq + (1-p)(1-q)}$$

$$P(\underline{z}) = \frac{p(1-q)\bar{v} + (1-p)q\underline{v}}{p(1-q) + (1-p)q}$$

Informed traders buy when they receive a buy recommendation, and sell when they receive a sell recommendation.

ii) *If the manager sends non-informative signal and the analyst issues non-informative recommendation, then the stock price is given by* $P = v^e$. *Informed traders do not trade.*

Proof of Lemma 5: see the Appendix.

The analyst decides whether he sends informative or non-informative recommendation based on his expected payoff. When his signal precision is high, his expected gain from trading commission exceeds the cost of being inaccurate, thus he issues informative recommendations; when his signal precision is lower than a threshold value, the expected gain from commission can not cover the cost of inaccuracy, then analyst ignores his private information and issues uninformative recommendations.

The following lemma's determine under which condition it is optimal for the analyst to issue informative (Lemma 6), or non-informative (Lemma 7) recommendations.

Lemma 6: *A perfectly pooling equilibrium such that the manager sends pooling signals and analysts give non-informative recommendations to their clients exists if* $g < g^*(a)$ *and* $p > k/w$.

Proof of Lemma 6: see the Appendix.

The result in the model with one financial analyst is summed up in the following proposition.

Proposition 2: *The equilibria in the signal-sending game of the manager with one analyst are defined as follows.*

i) *If* $p < \dfrac{k}{w}$, *there exists a perfectly separating equilibrium for given market beliefs. The optimal strategy for both types of managers are sending the signal truthfully, i.e.* $y(\bar{v}) = \bar{v}$ *and* $y(\underline{v}) = \underline{v}$. *The optimal strategy for the analyst is to issue informative recommendation which is consistent with the manager's signal, i.e.* $\sigma_A = y$, for all $g \in (\frac{1}{2}, 1]$;

ii) *If* $p > \dfrac{k}{w}$ and $g \in (\frac{1}{2}, g^*(a))$, *there exists a perfectly pooling equilibrium for given market beliefs. The optimal strategy for both types of managers are sending high signals, i.e.* $y(\bar{v}) = y(\underline{v}) = \bar{v}$. *The optimal strategy for the analyst is to issue an uninformative recommendation,* i.e. $\sigma_A(s) = \theta \;\; \forall s \in S$ and $\forall \theta \in \Theta$;

The following corollary collects the difference of the equilibria in the market with one analyst and without analyst.

Corollary 1: *Keeping all the parameter values equal, the size of perfectly pooling interval is larger in the market with analyst.*

Proof of corollary 1: see appendix.

The intuition behind Proposition 2 is as follows. The manager trades off his expected gain from pooling against his reputation cost for lying. If the market believes the firm is not performing well (small p), the benefit of sending pooling signals is not high enough to cover the reputation cost. Hence, the manager is better off by revealing the true firm value to the market. If this is the case, the information becomes publicly known and the analyst has no incentives to issue a recommendation which is inconsistent with the manager's signal, as being inaccurate is costly for him.

If the market believes the firm is performing well (large p), the benefit of sending pooling signals is large enough comparing to the reputation cost,. Hence, the manager has a strong incentive to pretend to be high type when he is actually low type. Given the manager sends a pooling signal, the analyst issues non-informative recommendations when his signal precision is low, as the trading commission generated from recommendation is lower than the expected cost of being inaccurate.

Corollary 1 implies that the manager is more likely to play pooling strategy when there is analyst present on the market. The intuition is as follows. The manager plays pooling strategy if the benefit from pooling exceeds the reputation cost. In the benchmark model, even if the manager does not reveal information, informed traders can collect information by themselves; whereas in the model with analyst, if neither the manager nor the analyst reveals information, the informed traders do not have any information. Fooling the market becomes easier in the sense that the benefit from pooling increases while the cost remains the same. Thus the manager is more likely to play pooling strategy when the analyst colludes with the him.

3.3 The case with more than one analysts

Suppose now that N analysts cover the firm, where each analyst has the same ability and objective function. If $N-1$ analysts issue non-informative recommendations, the last analyst is left in the same situation as in the one-analyst model. He will issue a non-informative recommendation as well, provided that in the one-analyst model it is optimal to do so. The same argument holds for every other analyst, thus the same solution as in Proposition 2 arises.

One may argue that in the real world analysts have more incentives in addition to trading commission and accuracy. For example, competition may arise when there are more than one analysts following a firm; if we relax the assumption that every analyst has the same ability, then less accurate analysts may have incentive to herd with more experienced and accurate analysts. Moreover, analysts may have incentives to upward bias their forecast when they are affiliated to investment banks; maintaining good relationship with management of firms they cover in order to have access to information; and career concern. However, for given model setup and assumptions, increasing the number of analysts does not necessarily lead to higher level of information revelation.

4 Empirical implications

This model provide some empirical implications regarding the manager's signaling and analysts' forecasting behavior and how one party's behavior affect the other's. The first implication is that the manager's signaling strategy depends on the market beliefs about the future firm performance. He tends to send truthful (resp. pooling) signals when the market belief is low (resp. high). The model also implies that analysts pass on the manager's signal when the manager reveals his type: when the manager sends pooling signals, the analysts issue non-informative recommendation provided that their own private information is blurried. Knowing the analysts may collude with him, the manager is more likely to send a pooling signal when his firm is followed by analysts than when informed traders collect their information independently by themselves.

The comparative statics results suggest some additional predictions. Controlling for the volatility of stock price, higher stock-based incentives imply a higher probability of pooling behavior. Thus, controlling for other variables, we expect to observe less pooling signals if the manager is more risk averse. Moreover, a higher stock price informativeness, e.g. higher PIN (probability of informed trading) leads to lower degree of pooling behavior.

5 Conclusions

Managers have some dicretion in the way they disclose information to the financial market. When the firm's current period value is low, the manager is reluctant to tell the truth. However, sending a pooling signal is costly for them. After observing the managers' signal, analysts decide what recommendation they will give to their clients. and the market updates its belief about the firm performance based on the analysts' recommendations. The stock price depends on the market beliefs, which in turn, decide the managers' compensation. Within this structure, our paper shows that the manager tends to send truthful signals when the market belief of the firm's performance is low and pooling signals when the market belief is high. The analysts pass on the manager's signal when the manager reveals his type; when the manager sends pooling signals, the analysts issue non-informative recommendations when their signal precision is low. Knowing the analysts may collude with him, the manager is more likely to send pooling signal when his firm is followed by analysts.

Our paper yields several empirical predictions regarding the manager's signaling and analysts' forecasting behavior, as well as location, size and probability of the pooling interval. We plan to test these predictions as future research.

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Appendix

Proof of Lemma 1:

In case i), if the revelation of the manager is perfectly revealing, the market can learn the manager's type from the signals. The price is given by:

$$P = E[v|y = \tilde{v}] = \begin{cases} E[v|y = \bar{v}] = \bar{v} \\ E[v|y = \underline{v}] = \underline{v} \end{cases}$$

The expected value and variance of stock price are given by:

$$E[P|y = \tilde{v}] = pr(y = \bar{v}) * P(\bar{v}) + pr(y = \underline{v}) * P(\underline{v}) = p\bar{v} + (1-p)\underline{v} \tag{5}$$

$$Var[P|y = \tilde{v}] = pr(\bar{v}) * [P(\bar{v}) - E[P|y]]^2 + pr(\underline{v}) * [P(\underline{v}) - E[P|y]]^2 = p(1-p)\Delta v^2 \tag{6}$$

In case ii), an informed trader's information set $H_i = \{s_i\}$, where s_i is his private signal. Denote by g the probability that informed trader make a right guess, with $g \in (1/2, 1)$.

The informed trader buys if his private signal is high, and sells if it is low. The uninformed traders buy and sell randomly and on average, the probability of buying and selling are equal to 1/2.

Define

$$q = pr(\bar{z}|v = \bar{v}) = pr(\underline{z}|v = \underline{v})$$

From the event tree in Figure 1 we have

$$q = ig + \frac{1-i}{2} = \frac{1}{2} + i(g - \frac{1}{2}) \tag{7}$$

Notice that $q > \frac{1}{2}$ for $g > \frac{1}{2}$.

The market maker sets the price based on the order flow:

$$P = E[v|\tilde{z}] = \begin{cases} P(\bar{z}) = E[v|\bar{z}] \\ P(\underline{z}) = E[v|\underline{z}] \end{cases}$$

where

$$P(\bar{z}) = E[v|\bar{z}] = \bar{v} * pr(\bar{v}|\bar{z}) + \underline{v} * pr(\underline{v}|\bar{z}) \tag{8}$$

and

$$P(\underline{z}) = E[v|\underline{z}] = \bar{v} * pr(\bar{v}|\underline{z}) + \underline{v} * pr(\underline{v}|\underline{z}) \tag{9}$$

Using Bayes' rule:

$$\begin{aligned} pr(\bar{v}|\bar{z}) &= \frac{pr(\bar{z}|\bar{v}) * pr\left(v = \bar{v}|y = \bar{v}\right)}{pr(\bar{z})} = \frac{pr(\bar{z}|\bar{v}) * pr\left(v = \bar{v}|y = \bar{v}\right)}{pr(\bar{z}|\bar{v}) * pr\left(\bar{v}\right) + pr(\underline{z}|\underline{v}) * pr(\underline{v})} \\ &= \frac{pq}{pq + (1-p)\,(1-q)} \end{aligned}$$

$$pr(\underline{v}|\bar{z}) = 1 - pr(\bar{v}|\bar{z}) = \frac{(1-p)\,(1-q)}{pq + (1-p)\,(1-q)}$$

Substituting $pr(\bar{v} \mid \bar{z})$ and $pr(\underline{v} \mid \bar{z})$ into (8) yields:

$$P(\bar{z}) = E[v|\bar{z}] = \bar{v} * pr(\bar{v}|\bar{z}) + \underline{v} * pr(\underline{v}|\bar{z}) = \frac{pq\bar{v} + (1-p)\,(1-q)\,\underline{v}}{pq + (1-p)\,(1-q)}$$

Similarly,

$$\begin{aligned} pr(\bar{v}|\underline{z}) &= \frac{pr(\underline{z}|\bar{v}) * pr\left(v = \bar{v}|y = \bar{v}\right)}{pr(\underline{z})} = \frac{pr(\underline{z}|\bar{v}) * pr\left(v = \bar{v}|y = \bar{v}\right)}{pr(\underline{z}|\bar{v}) * pr\left(\bar{v}\right) + pr(\underline{z}|\underline{v}) * pr(\underline{v})} \\ &= \frac{p\,(1-q)}{p\,(1-q) + (1-p)\,q} \end{aligned}$$

$$pr(\underline{v}|\underline{z}) = 1 - pr(\bar{v}|\bar{z}) = \frac{(1-p)\,(1-q)}{pq + (1-p)\,(1-q)}$$

Substituting $pr(\bar{v}|\underline{z})$ and $pr(\underline{v}|\underline{z})$ into (9) gives the result of $P(\underline{z})$:

$$P(\underline{z}) = E[v|\underline{z}] = \bar{v} * pr(\bar{v}|\underline{z}) + \underline{v} * pr(\underline{v}|\underline{z}) = \frac{p\,(1-q)\,\bar{v} + (1-p)\,q\underline{v}}{p\,(1-q) + (1-p)\,q}$$

Substituting $P(\bar{z})$ and $P(\underline{z})$ into 5 and 6 gives:

$$E\left[P|z\right] = pr(\bar{z}) * P(\bar{z}) + pr(\underline{z}) * P(\underline{z}) = p\bar{v} + (1-p)\underline{v} = v^e \tag{10}$$

$$\begin{aligned} Var\left[P|z\right] &= pr(\bar{z}) * \left[P(\bar{z}) - E\left[P|z\right]\right]^2 + pr(\underline{z}) * \left[P(\underline{z}) - E\left[P|z\right]\right]^2 \qquad (11) \\ &= \left[p(1-p)(2q-1)\Delta v\right]^2 \left[\frac{1}{pq + (1-p)\,(1-q)} + \frac{1}{p\,(1-q) + (1-p)\,q}\right] \end{aligned}$$

■

Proof of Lemma 2:

i) From (2), we know the expected payoff of the manager is:

$$EU^M(\widetilde{v}; y) = wE[P|\widetilde{v}; y] - \beta w^2 var(P|\widetilde{v}; y) - k(y - \widetilde{v})^2$$

Suppose high type plays $y(\bar{v}) = \bar{v}$. Construct the best response for low type. Low type can play two pure strategies, i.e. $y(\underline{v}) = \underline{v}$ (fully revealing), and $y(\underline{v}) = \bar{v}$ (perfectly pooling).

If low type also tells the truth, then the signal is revealing, i.e. $y(\bar{v}) = \bar{v}$, and $y(\underline{v}) = \underline{v}$. The market maker can set the stock price based on the manager's signal. For a low type manager, $E[P|y] = \underline{v}$, $Var[P \blacksquare y] = 0$, thus his expected payoff is given by:

$$EU^M(\underline{v}; y(\bar{v}) = \bar{v}, y(\underline{v}) = \underline{v}) = w\underline{v}$$

If low type mimicks high type, the market only observes high signals. As a result, the signal y is not informative. The market maker sets the stock price based on the order flow $\tilde{z}$. For low type manager, the expectation and variance of stock price are given by:

$$\begin{aligned} E[P|z] &= pr(\bar{z}|\underline{v}) * P(\bar{z}) + pr(\underline{z}|\underline{v}) * P(\underline{z}) \\ &= (1-q)\frac{pq\bar{v} + (1-p)(1-q)\underline{v}}{pq + (1-p)(1-q)} + q\frac{p(1-q)\bar{v} + (1-p)q\underline{v}}{p(1-q) + (1-p)q} \end{aligned}$$

$$\begin{aligned} Var[P|z] &= pr(\bar{z}|\underline{v}) * [P(\bar{z}) - E[P|z]]^2 + pr(\underline{z}|\underline{v}) * [P(\underline{z}) - E[P|z]]^2 \\ &= q(1-q)\left[\frac{p(1-p)(2q-1)\Delta v}{[pq + (1-p)(1-q)][p(1-q) + (1-p)q]}\right]^2 \end{aligned}$$

Thus his expected payoff is given by:

$$\begin{aligned} EU^M(\underline{v}; y(\bar{v}) &= y(\underline{v}) = \bar{v}) = wE[P|z] - \beta w^2 Var[P|z] - k(\Delta v)^2 \\ &= w\left[(1-q)\frac{pq\bar{v} + (1-p)(1-q)\underline{v}}{pq + (1-p)(1-q)} + q\frac{p(1-q)\bar{v} + (1-p)q\underline{v}}{p(1-q) + (1-p)q}\right] \\ &\quad -\beta w^2 q(1-q)\left[\frac{p(1-p)(2q-1)\Delta v}{[pq + (1-p)(1-q)][p(1-q) + (1-p)q]}\right]^2 - k(\Delta v)^2 \end{aligned}$$

Normalize $\Delta v = 1$ and define

$$R(p,q) = \frac{[p(1-p)(2q-1)]^2}{[pq + (1-p)(1-q)][p(1-q) + (1-p)q]} \quad (12)$$

$$T(p,q) = \frac{[pq + (1-p)(1-q)][p(1-q) + (1-p)q]}{q(1-q)} \quad (13)$$

Solving for $EU^M(\underline{v}; y(\bar{v}) = \bar{v}, y(\underline{v}) = \underline{v}) > EU^M(\underline{v}; y(\bar{v}) = y(\underline{v}) = \bar{v})$ gives $p < \beta w R + \frac{k}{w} T$. Thus when high type plays $y(\bar{v}) = \bar{v}$, the best response for the low type is to play $y(\underline{v}) = \underline{v}$.

Suppose now low type plays perfect revealing strategy, i.e. $y(\underline{v}) = \underline{v}$, construct the best response for high type. If high type plays $y(\bar{v}) = \bar{v}$, then the market can learn his type from the signal. Thus $P(\bar{v}) = \bar{v}$ and high type manager's expected utility is given by:

$$EU^M(\bar{v}; y(\bar{v}) = \bar{v}, y(\underline{v}) = \underline{v}) = w\bar{v}$$

If high type plays $y(\bar{v}) = \underline{v}$, then the signal is again not informative, the only information comes from the order flow $\tilde{z}$. The high type manager's expected utility is given by:

$$EU^M(\bar{v}; y(\bar{v}) = y(\underline{v}) = \underline{v}) = wE[P|z] - \beta w^2 Var[P|z] - k(\Delta v)^2$$

It is obvious that $EU^M(\bar{v}; y(\bar{v}) = y(\underline{v}) = \underline{v}) < EU^M(\bar{v}; y(\bar{v}) = \bar{v}, y(\underline{v}) = \underline{v})$. Thus when low type plays $y(\underline{v}) = \underline{v}$, the best response for high type is to play $y(\bar{v}) = \bar{v}$. To summarize, if the condition

$$p < \beta w R + \frac{k}{w} T$$

is satisfied, a perfectly separating equilibrium such that $y(\bar{v}) = \bar{v}$, and $y(\underline{v}) = \underline{v}$ exists.

ii) Now we check whether a perfectly separating equilibrium such that $y(\bar{v}) = \underline{v}$, and $y(\underline{v}) = \bar{v}$ exists.

Suppose high type mimicks low type, i.e. he plays $y(\bar{v}) = \underline{v}$, construct the best response for low type.

If low type also mimicks high type, i.e. he plays $y(\underline{v}) = \bar{v}$, then the signal is revealing, and low type's expected payoff is given by:

$$EU^M(\underline{v}; y(\bar{v}) = \underline{v}, y(\underline{v}) = \bar{v}) = w\underline{v} - k(\Delta v)^2$$

If low type tells the truth, his expected payoff is given by:

$$\begin{aligned} EU^M(\underline{v}; y(\bar{v}) = y(\underline{v}) = \underline{v}) &= wE[P|z] - \beta w^2 Var[P|z] \\ &= w\left[(1-q)\frac{pq\bar{v} + (1-p)(1-q)\underline{v}}{pq + (1-p)(1-q)} + q\frac{p(1-q)\bar{v} + (1-p)q\underline{v}}{p(1-q) + (1-p)q}\right] \\ &\quad -\beta w^2 q(1-q)\left[\frac{p(1-p)(2q-1)\Delta v}{[pq + (1-p)(1-q)][p(1-q) + (1-p)q]}\right]^2 \end{aligned}$$

Low type's best response is to mimick high type if $EU^M(\underline{v}; y(\bar{v}) = \underline{v}, y(\underline{v}) = \bar{v}) > EU^M(\underline{v}; y(\bar{v}) = y(\underline{v}) = \underline{v})$, or

$$p < \beta wR - \frac{k}{w}T$$

Suppose low type plays $y(\underline{v}) = \bar{v}$, construct the best response for high type. If high type plays $y(\bar{v}) = \underline{v}$, his expected payoff is:

$$EU^M\left(\bar{v}; y(\bar{v}) = \underline{v}, y(\underline{v}) = \bar{v}\right) = w\bar{v} - k\left(\Delta v\right)^2$$

If high type plays $y(\bar{v}) = \bar{v}$, his expected payoff is given by:

$$EU^M\left(\bar{v}; y(\bar{v}) = y(\underline{v}) = \bar{v}\right) = wE[P|z] - \beta w^2 Var\left[P|z\right]$$

High type's best response is to play $y(\bar{v}) = \underline{v}$ if $EU^M\left(\bar{v}; y(\bar{v}) = \underline{v}, y(\underline{v}) = \bar{v}\right) > EU^M\left(\bar{v}; y(\bar{v}) = y(\underline{v}) = \bar{v}\right)$, or $p < 1 + \beta wR - \frac{k}{w}T$. Thus a separating equilibrium such that both types lying exists if $p < \beta wR - \frac{k}{w}T$ and $p < 1 + \beta wR - \frac{k}{w}T$ both hold.

However, this separating equilibrium can be eliminated as it is strictly dominated by the separating equilibrium such that both types telling the truth. From the proof of Lemma 2 i), in the revealing equilibrium that both types telling the truth, the expected payoff of both types of managers are given by:

$$EU^M\left(\tilde{v}; y(\bar{v}) = \bar{v}, y(\underline{v}) = \underline{v}\right) = w\tilde{v}$$

whereas in the separating equilibrium such that both types are lying, the expected payoffs of both types of managers are given by:

$$EU^M\left(\tilde{v}; y(\bar{v}) = \underline{v}, y(\underline{v}) = \bar{v}\right) = w\tilde{v} - k\left(\Delta v\right)^2$$

It is obvious that $w\tilde{v} > w\tilde{v} - k\left(\Delta v\right)^2$ for all $k \neq 0$ and $\Delta v \neq 0$. Thus there exists a unique separating equilibrium such that both types send truthful signals. ∎

Proof of Lemma 3:

i) We first construct the pooling equilibrium such that $y(\bar{v}) = y(\underline{v}) = \bar{v}$. Suppose high type plays $y(\bar{v}) = \bar{v}$. Then from Lemma 2, if $p > \beta wR + \frac{k}{w}T$, the best response for low type is to play $y(\underline{v}) = \bar{v}$.

Then suppose low type plays $y(\underline{v}) = \bar{v}$, construct the best response for high type. If high type manager plays $y(\bar{v}) = \bar{v}$, then his expected utility is given by:

$$\begin{aligned}
EU^M(\bar{v}; y(\bar{v}) &= y(\underline{v}) = \bar{v}) \\
&= w\left[q\frac{pq\bar{v} + (1-p)(1-q)\underline{v}}{pq + (1-p)(1-q)} + (1-q)\frac{p(1-q)\bar{v} + (1-p)q\underline{v}}{p(1-q) + (1-p)q}\right] \\
&\quad -\beta w^2 q(1-q)\left[\frac{p(1-p)(2q-1)\Delta v}{\left[pq + (1-p)(1-q)\right]\left[p(1-q) + (1-p)q\right]}\right]^2
\end{aligned}$$

If high type manager plays $y(\bar{v}) = \underline{v}$, then from Lemma 1, $P(\underline{v}) = \bar{v}$ and his expected utility is given by:

$$EU^M(\bar{v}; y(\bar{v}) = \underline{v}) = w\bar{v} - k(\Delta v)^2$$

Set $EU^M(\bar{v}; y(\bar{v}) = \bar{v}) > EU^M(\bar{v}; y(\bar{v}) = \underline{v})$ gives:

$$p > 1 + \beta wR - \frac{k}{w}T$$

If $p > 1 + \beta wR - \frac{k}{w}T$, the best response for high type is to play $y(\bar{v}) = \bar{v}$; otherwise his best response is to play $y(\bar{v}) = \underline{v}$. Thus a perfectly pooling equilibrium such that $y(\bar{v}) = y(\underline{v}) = \bar{v}$ exists if p satisfies both $p > \beta wR + \frac{k}{w}T$ and $p > 1 + \beta wR - \frac{k}{w}T$, or $p > max\{\beta wR + \frac{k}{w}T, 1 + \beta wR - \frac{k}{w}T\}$.

ii) The other possible pooling equilibrium is that both types of managers send low signals, i.e. $y(\bar{v}) = y(\underline{v}) = \underline{v}$. Then their expected utilities are given by:

$$EU^M(\bar{v}; y(\bar{v}) = y(\underline{v}) = \underline{v}) = wE[P|z] - \beta w^2 Var[P|z] - k(\Delta v)^2$$

$$EU^M(\underline{v}; y(\bar{v}) = y(\underline{v}) = \underline{v}) = wE[P|z] - \beta w^2 Var[P|z]$$

We can easily eliminate the strategy pair that both types of managers sending low signals. Since $w\bar{v} > wE[P|z] - \beta w^2 Var[P|z] - k(\Delta v)^2$, when low type plays $y(\underline{v}) = \underline{v}$, high type has incentive to deviate. Thus there is only one possible perfectly pooling equilibrium, which is two types of managers play $y(\bar{v}) = y(\underline{v}) = \bar{v}$. ■

Proof of Proposition 1: From Lemma 2, a fully revealing equilibrium exists if telling the truth is the best response for each other. This is true when $0 \leq p < \beta wR + kT/w$.

From Lemma 3, a perfectly pooling equilibrium such that both types sending high signals exists for $p > max\{\beta wR + kT/w, 1 + \beta wR - kT/w\}$. We do not know which one of the two boundary values, $kT/w + \beta wR$ and $1 + \beta wR - kT/w$, is larger.

If $kT/w + \beta wR < 1 + \beta wR - kT/w$, we would have a problem to characterize the equilibrium when $p \in [kT/w + \beta wR, 1 + \beta wR - kT/w]$. In order to rule out this case, we impose that $1 + \beta wR - kT/w < kT/w + \beta wR$, which implies $kT/w > 1/2$.

Thus when $kT/w > 1/2$, there exists perfectly separating equilibrium in region $[0, kT/w + \beta wR)$, and perfectly pooling equilibrium in region $[kT/w + \beta wR, 1]$. ■

Proof of Lemma 4: If the manager reveals the firm value $\tilde{v}$ with his public signal, this becomes publicly known. The trading volume is not affected by the analyst's recommendation as the firm value is also known to all informed traders. If the analyst's recommendation is consistent with the manager's signal, i.e. $\theta = y = \tilde{v}$, then he bears no cost of being inaccurate. If the analyst's recommendation is different from the manager's signal, then he bears a cost of L. Thus it is optimal for the analyst to issue recommendation which is consistent to the manager's signal. The stock price is $P = E[v|y = \tilde{v}] = \tilde{v}$ (Lemma1).

Given the analyst passes on the manager's signal, we then derive the best response for the manager. If the manager sends a truthful signal, then the manager's expected payoff is given by:

$$EU^M[\tilde{v}; \theta = y = \tilde{v}] = w\tilde{v}$$

If the manager sends pooling signal, then high manager's expected payoff is:

$$EU^M[\bar{v}; \theta = y(\bar{v}) = y(\underline{v}) = \bar{v}] = wE[P|z] - \beta w^2 var[P|z]$$

and low type manager's expected payoff is:

$$EU^M[\underline{v}; \theta = y(\bar{v}) = y(\underline{v}) = \bar{v}] = wE[P|z] - \beta w^2 var[P|z] - k\Delta v^2$$

For high type manager, since $w\bar{v} > wE[P|z] - \beta w^2 var[P|z]$, he is better off by sending $y(\bar{v}) = \bar{v}$. From the result of Lemma 2, low type manager reveals his type, i.e. $y(\underline{v}) = \underline{v}$ if $p < \beta wR(p, q) + \frac{k}{w}T(p, q)$.

Thus a perfectly revealing equilibrium such that the manager sends truthful signal and analyst passes on the signal to his clients exist if $p < \beta wR(p, q) + \frac{k}{w}T(p, q)$. ■

Proof of Lemma 5: We make the following conjecture: the informed

traders buy after receiving a buy recommendaiton, i.e. $x_i(\theta = \bar{v}) > 0$ at equilibrium price; and the informed traderds sell after receiving a sell recommendation, i.e. $x_i(\theta = \underline{v}) < 0$ at equilibrium price. Moreover, if we assume the market maker sets the price based on the order flow, then the information structure is the same as in the benchmark model, when the manager plays pooling strategy. The equilibrium price is as in **Lemma** 1 ii):

$$\begin{aligned} P(\bar{z}) &= E[v|\bar{z}] = \bar{v} * pr(\bar{v}|\bar{z}) + \underline{v} * pr(\underline{v}|\bar{z}) \\ &= \frac{pq\bar{v} + (1-p)(1-q)\underline{v}}{pq + (1-p)(1-q)} \end{aligned}$$

$$\begin{aligned} P(\underline{z}) &= E[v|\underline{z}] = \bar{v} * pr(\bar{v}|\underline{z}) + \underline{v} * pr(\underline{v}|\underline{z}) \\ &= \frac{p(1-q)\bar{v} + (1-p)q\underline{v}}{p(1-q) + (1-p)q} \end{aligned}$$

Given $P(\bar{z})$, is it optimal for informed traders who receive a buy recommendation to buy at price $P(\bar{z})$, i.e. $x_i(\theta^f = \bar{v}) > 0$ if and only if $E[v|\theta^f = \bar{v}, P(\bar{z})] > P(\bar{z})$.

Since $P(\underline{z}) \neq P(\bar{z})$ for all $g \in (1/2, 1]$, observing $P(\bar{z})$ is informational equivalent to receiving a recommendation of buy, then $E[v|\theta = \bar{v}, P(\bar{z})] = E[v|\theta = \bar{v}, buy]$, which is given by:

$$\begin{aligned} E[v|\theta &= \bar{v}, buy] = \bar{v} * pr_{buy}(\bar{v}|\theta = \bar{v}) + \underline{v} * pr_{buy}(\underline{v}|\theta = \bar{v}) \\ &= \frac{pgq\bar{v} + (1-p)(1-g)(1-q)\underline{v}}{pgq + (1-p)(1-g)(1-q)} \end{aligned}$$

and similarly, $E[v|\theta = \underline{v}, P(sell)] = E[v|\theta = \underline{v}, sell]$:

$$\begin{aligned} E[v|\theta &= \underline{v}, sell] = \bar{v} * pr_{sell}(\bar{v}|\theta = \underline{v}) + \underline{v} * pr_{sell}(\underline{v}|\theta = \underline{v}) \\ &= \frac{p(1-g)(1-q)\bar{v} + (1-p)gq}{p(1-g)(1-q) + (1-p)gq} \end{aligned}$$

$E[v|\theta = \bar{v}, buy] > P(buy)$ and $E[v|\theta = \underline{v}, sell] > P(sell)$ hold for all $g \in (1/2, 1]$, i.e. $x_i(\theta = \bar{v}) > 0$ and $x_i(\theta = \underline{v}) < 0$, thus what we conjectured at the beginning of the proof is verified.

When the analyst issues a non-informative recommendation, g is equal to 1/2, by definition q is also equal to 1/2. Plug $g = q = 1/2$ into $E[v|\theta = \bar{v}, buy]$ and $E[v|\theta = \underline{v}, sell]$ gives $E[v|\theta = \bar{v}, buy] = E[v|\theta = \underline{v}, sell] = v^e$, and $P(buy) = P(sell) = v^e$, thus the informed traders do not trade when they receive an uninformative recommendation. ■

Proof of Lemma 6: So far we left $var[\tilde{v}|\theta]$ in the analyst's payoff function

unspecified. If the recommendation is informative, i.e. σ_A is invertible, or $\sigma_A^{-1}(\theta) = s$, then $var[\tilde{v}|s] = var[\tilde{v}|\sigma_A^{-1}(\theta)]$, where $s = \begin{cases} v = \tilde{v} & prob.g \\ v \neq \tilde{v} & prob.1-g \end{cases}$.

The ex-post distribution of $\tilde{v} = \{\overline{v}, \underline{v}\}$ is as follows:

$$s_A = \overline{v} => \begin{cases} v = \overline{v} & prob.g \\ v = \underline{v} & prob.1-g \end{cases}$$
$$s_A = \underline{v} => \begin{cases} v = \overline{v} & prob.1-g \\ v = \underline{v} & prob.g \end{cases}$$

Then we have $var[\tilde{v}|s = \overline{v}] = g(1-g)\Delta v^2$ and $var[\tilde{v}|s = \underline{v}] = g(1-g)\Delta v^2$, thus $var[\tilde{v}|s] = g(1-g)\Delta v^2$.

Normalize $\Delta v = 1$, the analyst's expected payoff function in (4) is:

$$E[\pi^A(\theta)|s, g] = \frac{a}{g(1-g)} - (1-g)$$

If the analyst issues an informative recommendation, then his expected payoff is: $\pi^A = \dfrac{a}{g(1-g)} - (1-g)$. If the analyst issues a non-informative signal, then by assumption his expected payoff is $\pi^A = 0$. Solve for $\dfrac{a}{g(1-g)} - (1-g) = 0$ gives

$$g^*(a) = \frac{1}{9\sqrt[3]{\frac{1}{2}a + \sqrt{\frac{1}{4}a^2 - \frac{1}{27}a} - \frac{1}{27}}} + \sqrt[3]{\frac{1}{2}a + \sqrt{\frac{1}{4}a^2 - \frac{1}{27}a} - \frac{1}{27}} + \frac{2}{3}$$

For any positive value of a, there exist an unique $g^*(a)$ such that $\pi^A(g^*(a)) = 0$. The first order derivative of π^A with respect to g is:

$$\frac{\partial \pi^A}{\partial g} = \frac{a}{g^2(1-g)^2}(2g-1) + L$$

which is positive for all $g \in (\frac{1}{2}, 1)$, i.e. the analyst's expect payoff π^A is strictly increasing in g. Thus $\pi^A > 0$ when $g > g^*(a)$, which implies that the analyst is better off by issuing an informative when his precision is high.

Given the analyst issues uninformative recommendations, we can construct the manager's best response in the same way as in Lemma 3. Thus the manager plays a pooling strategy such that $y(\bar{v}) = y(\underline{v}) = \bar{v}$ exists when $p > \beta w R(p,q) + \frac{k}{w}T(p,q)$.

The analyst issues non-informative recommendations if sending informative recommendation gives stictly lower expected payoff, or $g < g^*(a)$. If neither the manager nor the analyst reveals information, the signal precision g is equal to $\frac{1}{2}$. The informed traders do not participate in trading, thus the order flow is not informative, i.e. $q = ig + \dfrac{1-i}{2} = \dfrac{1}{2}$. The stock price is given by: $P(\bar{z}) = P(\underline{z}) = v^e$.

The manager does not deviate if sending pooling signal gives him higher expected utility. Solving the manager's utility maximization problem in Lemma 3 once more with $q = \frac{1}{2}$, gives a new theshold of pooling region, $p > \frac{k}{w}$. Thus a perfectly pooling equilibrium such that neither the manager nor the analyst reveals information exists when $p > \frac{k}{w}$ and $g < g^*(a)$. ■

Proof of corollary 1: The size of perfectly pooling region is larger in the market with analyst if the following condition holds:

$$\frac{k}{w} < \beta w R(p,q) + \frac{k}{w}T(p,q) \tag{14}$$

Rearrange inequality (14) gives

$$-\beta wR(p,q) + \frac{k}{w}\left(1 - T(p,q)\right) < 0 \tag{15}$$

From equation (12), $\beta wR(p,q) > 0$, thus inequality (15) holds if the following condition is satisfied:

$$1 - T(p,q) < 0$$

From equation (13),

$$\begin{aligned} 1 - T(p,q) &= 1 - \frac{[pq + (1-p)(1-q)]\,[p(1-q) + (1-p)q]}{q(1-q)} \\ &= -p(1-p)\frac{(2q-1)^2}{q(1-q)} < 0 \end{aligned}$$

Thus the perfectly pooling region is larger in the case with analyst. ■