Leapfrogging, Growth Reversals and Welfare*

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Abstract:

We show that leapfrogging and growth reversals entail sizeable welfare gains and losses, respectively, in an AK economy that cannot credibly commit to investment when borrowing from international financial markets. Small no-commitment delays originate a trade-off that has an ambiguous effect on welfare: they reduce the long-run consumption growth rate but increase the initial level of consumption that is optimally chosen. Essentially, the larger the delay, the tighter the borrowing constraint and the weaker the incentives to accumulate capital, so that smaller growth and larger initial consumption follow. We show under logarithmic utility and small delays that the short-run effect dominates the long-run effect and that welfare improves, provided that the economy has historically been growing fast enough, and

[*] Boucekkine, Fabbri and Pintus [6] is a technical companion paper which gathers some results and proofs that we use in this paper. First draft: December 2010.

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numerical examples suggest that this benchmark result extends to CRRA utility. When relative risk aversion is larger than one, it follows that there exists a positive welfare-maximizing delay associated with slower growth relative to the no-delay case. We then apply our results to show that leapfrogging in consumption level typically imply large welfare gains. In contrast, growth reversals occur for large delays and lead to significant welfare losses. Finally, financial integration, as measured by the credit multiplier given the no-commitment delay, is welfare-improving only for economies that have historically been growing fast enough.

Keywords: Growth Reversals, Leapfrogging, International Borrowing, Open Economies, Welfare

Journal of Economic Literature Classification Numbers: F34, F43, O40

1 Introduction

In this paper, we show that leapfrogging and growth reversals entail sizeable welfare gains and losses, respectively, in an AK economy that cannot credibly commit to investment when borrowing from international financial markets. We extend the analysis of Boucekkine and Pintus [5] by considering optimal growth and we show that small no-commitment delays originate a trade-off that has an ambiguous effect on welfare: they reduce the long-run consumption growth rate but increase the initial level of consumption that is optimally chosen. Essentially, the larger the delay, the tighter the borrowing constraint and the weaker the incentives to accumulate capital, which in turn imply smaller growth but larger initial consumption. The (long-run) growth effect reduces welfare whereas the (short-run) level effect improves it when the no-commitment delay increases from zero.

We show under logarithmic utility that small delays improve welfare (i.e., the short-run effect dominates the long-run effect) provided that the economy has historically been growing fast enough, and numerical examples suggest that this benchmark result extends to CRRA utility. When relative risk aversion is larger than one, numerical examples show that there exists a positive welfare-maximizing delay that is associated with slower growth relative to the no-delay case.

We next apply our results in two directions. First, we show that leapfrogging occurs, in the sense that economies that have historically been poorer but growing faster end up enjoying a larger long-run consumption level. We show that leapfrogging typically entail welfare gains due to significant consumption gains. In particular, the gains from leapfrogging are substantially larger for economies that have been either declining or growing slowly in the past. Second we show that, for large delays, the economy experiences sudden breaks such that the growth rate goes either from below to above trend or from above to below trend. These growth reversals lead to sizeable welfare losses that arise because the negative growth effect dominates the positive level effect on consumption. Finally, we also derive results showing how financial integration, as measured by the credit multiplier, affects welfare in a non-trivial way under small no-commitment delays: it is welfare-improving only for economies that have historically been growing fast enough.

Although there is documented evidence that growth reversals are ubiquitous (Hausmann, Pritchett and Rodrik [11], Jones and Olken [12], Cuberes and Jerzmanowski [8]), there is to our knowledge no assessment of whether such episodes are associated with significant welfare changes. Our paper aims at providing a first step in this direction. It is also connected to the large and growing literature showing that credit and collateral constraints may trigger macroeconomic volatility and sudden stops (see, among many others, Paasche [17], Aghion *et al.* [1], Mendoza [16], Devereux and Yetman [9]). In contrast with our study, however, this research body abstracts away from welfare analysis. Therefore, an open question is whether or not those models give rise to business cycles that are costly in terms of welfare, in view of the fact that they rely on the assumption that growth is exogenous (Barlevy [3]). Different from this literature, our paper shows that large welfare gains and losses originate from leapfrogging and growth reversals, respectively, under small no-commitment delays. In particular, growth reversals are particularly costly in terms of consumption utility, relative both to autarky and to the no-delay case configuration.

Our analysis underlines that some parameter constellations do accord with the empirical evidence emphasizing that finance promotes growth (King and Levine [13, 14], Rajan and Zingales [18]) and that strong growth fueled by international borrowing may occasionally lead to sudden downturns (Rancière, Tornell and Westermann [19]). Related to this strand of literature also is one corollary of our results pointing at the fact that the debt-to-GDP ratio may be a poor indicator of how financial integration affects growth and welfare. Our analysis shows that two economies that have the same debt-to-GDP ratio but face different values for the credit multiplier and the no-commitment delay (both determine how imperfect are international credit markets) may end up experiencing very different patterns of consumption, capital and output, and ultimately very different welfare levels. Last but not least, our results contrast with that of Uribe [21] and they provide some theoretical ground to the often expressed view that economies relying too much, in the short-run, on international financial markets may suffer from an overborrowing syndrome. Our paper stresses that economies that have been growing too slowly may not reap the benefits of financial integration but suffer instead from welfare losses, in contrast with economies that have been more successful in terms of their past growth performances.

The paper is organized as follows. Section 2 presents the open AK economy model and section 3 contains our main analytical and numerical results regarding welfare. In section 4, we show that leapfrogging and growth reversals entail large welfare gains and losses, respectively, while section 5 investigates the condition under which financial integration is welfare-improving. Finally, section 6 gathers some concluding remarks.

2 The Open AK Economy Without Investment Commitment

The economy produces a tradeable good Y by using physical capital K, according to the following technology:

$$Y = AK, \tag{1}$$

where $A > 0$ is total factor productivity. Whereas output is tradeable, labor and capital are not.[1] The Ramsey households are defined by their utility:

$$\int_0^\infty e^{-\rho t} \frac{C(t)^{1-\sigma} - 1}{1-\sigma} dt, \tag{2}$$

where $C > 0$ is consumption, $\sigma \geq 0$ is relative risk aversion, and $\rho \geq 0$ is the discount rate. The budget constraint is:

$$\dot{K}(t) - \dot{D}(t) = AK(t) - \delta K(t) - rD(t) - C(t), \tag{3}$$

where D is the amount of net foreign debt and the initial stocks $K(0) > 0$, $D(0)$ are given to the households.

We focus on collateral-constrained borrowing without commitment to investment and, following Boucekkine and Pintus [5], we posit that the creditor lends up to some fraction of the past value of collateral $\lambda K(t-\tau)$, for some exogenous (no-commitment) delay $\tau \geq 0$ and credit multiplier $\lambda > 0$.

Assumption 2.1 *Foreign borrowing is subject to a limit such that $D(t) = \lambda K(t-\tau)$, with the credit multiplier $\lambda > 0$ and the no-commitment delay $\tau \geq 0$.*

[1] Our results are virtually unchanged under capital mobility, which sets the net marginal product of capital to the world interest rate.

Replacing D by its expression from Assumption 2.1, the budget constraint (3) can be written as:

$$\dot{K}(t) = \lambda \dot{K}(t-\tau) + \varepsilon K(t) - r\lambda K(t-\tau) - C(t), \tag{4}$$

where $\varepsilon \equiv A - \delta > 0$. The budget constraint (4) is a non-autonomous, linear *Neutral delay* Differential Equation (NDE for short), as both K and $\dot{K}$ are delayed (see Bellman and Cooke [4, chap. 6]). The simplest case of a constant savings rate is analyzed in Boucekkine and Pintus [5]. Here we study the dynamics arising when households optimally choose consumption by maximizing (2) subject to (4). Note that under autarky, the economy does not borrow - that is, $\lambda = 0$ - and one goes back to the standard, closed-economy AK model such that the autarkic growth rate is $g_a \equiv (\varepsilon - \rho)/\sigma$, which is assumed to be positive. The remaining part of this section is devoted to the analysis of the dynamics and asymptotic properties when the credit multiplier $\lambda > 0$.

In a technical companion paper, Boucekkine, Fabbri and Pintus [6], we solve the optimal control problem and we refer to that article for details on the dynamic programming approach which provides a closed-form solution. In particular, we show there that: (i) under the assumptions that $\varepsilon > r$ and $\lambda < 1$, there exists a unique stable balanced growth path (BGP for short) that is associated with the unique positive characteristic root ξ, say, of $z - \lambda z e^{-z\tau} - \varepsilon + r\lambda e^{-z\tau} = 0$ (see Proposition 4.1 in Boucekkine et al. [6]), such that $d\xi/d\lambda > 0 > d\xi/d\tau$ (see Boucekkine and Pintus [5] for some comparative statics); (ii) if $\rho > (1-\sigma)\xi$, then there exists a unique (closed-form) solution to the optimal control problem (see Proposition 4.3 in Boucekkine et al. [6]); (iii) consumption jumps at $t = 0$ to the BGP, that is, $C(t) = C_0 e^{gt}$ for $t \geq 0$ (see Proposition 4.4 in Boucekkine et al. [6]), where $g = (\xi - \rho)/\sigma$ and:

$$C_0 = \left\{ \frac{\rho - (1-\sigma)\xi}{\sigma} \right\} \left\{ K_I(0) - \lambda K_I(-\tau) + (\xi - \varepsilon) \int_{-\tau}^{0} e^{-\xi s} K_I(s) ds \right\}, \tag{5}$$

where $K_I(t)$ is the initial function that is given over $t \in [-\tau, 0]$.

From Boucekkine et al. [6], we also infer that one gets back to the open-economy version of the standard AK model under investment commitment (that is, when $\tau = 0$), with the corresponding growth rate given by $g = (\xi - \rho)/\sigma > g_a$ with $\xi = (\varepsilon - r\lambda)/(1-\lambda)$. The above results, while technically demanding, are quite rewarding because they enable us to study welfare in a simple way when $\tau > 0$. In particular, the fact that, just as in a standard AK model, consumption jumps at $t = 0$ to the BGP greatly simplifies the analysis because there is no transitional dynamics of consumption (in contrast, the capital stock that solves the NDE (4) is shown to converge asymptotically to the BGP). Therefore, welfare changes are expected to depend on how parameters (most notably the delay τ and the credit multiplier λ) affect both the level of initial consumption C_0 and the growth rate g. In other words, the welfare impact of different parameter values can be divided into a level effect and a growth effect. The next sections are devoted to such a welfare analysis when utility is logarithmic, which we complement by numerical examples in the CRRA case. Then we apply those results to measure the welfare effects of leapfrogging and growth reversals, which can be quite substantial.

3 No-Commitment Delay and Welfare

3.1 Some Analytics of Welfare under Logarithmic Utility and Small Delays

The purpose of this section is to derive analytical results about welfare under logarithmic utility and with delays that are arbitrarily close to zero. Recall that we denote $K_I(t)$ the initial function defined for all $t \in [-\tau, 0]$. Then using the expression of the initial consumption level in equation (5), we now show that, for small delays, C_0 is an increasing function of the delay τ.

Lemma 3.1 (Small Delay and the Initial Consumption Level)

Define $K_I(t) > 0$ *for all* $t \in [-\tau, 0]$ *as the initial function and* $\mu \equiv K_I'(0)/K_I(0)$. *Under the assumption that* $\mu > \underline{\mu}$, *with* $\underline{\mu} \equiv (r - \varepsilon)/(1 - \lambda) < 0$, *and that utility is logarithmic, that is,* $\sigma = 1$, *then* $dC_0/d\tau > 0$ *at* $\tau = 0$.

That is, small no-commitment delays increase the optimal initial consumption level.

Proof: under logarithmic utility, one gets from equation (5) the expression of the optimal initial consumption level and it is then straighforward to show that, evaluated at $\tau = 0$, $dC_0/d\tau = \rho K_I(0)(\lambda\mu + \xi - \varepsilon) > 0$, where $\mu \equiv K_I'(0)/K_I(0)$, using that $\xi = (\varepsilon - r\lambda)/(1 - \lambda)$ when $\tau = 0$, and the assumption that $\mu > \underline{\mu}$ where $\underline{\mu} \equiv (r - \varepsilon)/(1 - \lambda) < 0$. □

From Lemma 3.1 follows the fact that the no-commitment delay has an *ambiguous* effect on welfare. This is because the delay has two opposite effects on the growth rate and on the initial level of consumption. On the one hand, there is a long-run consumption *growth effect*: increasing τ from zero reduces the growth rate $g = \xi - \rho$ (because it decreases the positive characteristic root ξ). In a growing economy, the higher the delay, the more severe the debt constraint, hence the lower the growth rate. On the other hand, there is a short-run consumption *level* effect: a positive τ tends to increase the initial level that is optimally chosen by the infinitely-lived household (as shown in Lemma 3.1 under the mild requirement that the economy is not declining to fast initially, that is, if $\mu > \underline{\mu}$). This is because under the prospect of slower consumption growth, it is optimal for households to increase their initial level of consumption so as to enjoy more consumption in initial periods.

Therefore, when the no-commitment delay increases from zero, a larger C_0 improves welfare (there is a positive short-run, level effect) whereas a lower g has the opposite impact (that is, a negative long-run, growth effect). It perhaps helps intuition to note that a similar trade-off arises in the standard AK model under autarky (that is, when $\lambda = 0$) when the discount rate ρ is made to increase. It can easily be shown that, *ceteris paribus*, a larger ρ translates into slower growth but a larger initial level of consumption so that its impact on welfare is ambiguous *a priori*. Here also, slower growth provides households with stronger incentives to consume more initially, so that a similar trade-off arises and has an ambiguous effect on welfare.[2] In summary, both the discount rate and the delay affect the incentives to accumulate capital in the same way: both larger ρ's and larger τ's mean lower incentives to accumulate, which translate into lower g's but larger C_0's. In the former case, this is because households are more impatient while, in the latter case, it is because of a tighter borrowing constraint.

We now show that the short-run effect dominates the long-run effect, so that welfare increases when the delay goes up from zero, if and only if the initial growth rate μ is positive and large enough. Under logarithmic utility, welfare is given by $W \equiv \int_0^{+\infty} e^{-\rho t} \ln\{C(t)\} dt$. Using $C(t) = C_0 e^{gt}$ and $g \equiv \xi - \rho$, one gets:

$$W = \frac{1}{\rho} \left\{ \ln\{C_0\} + \frac{\xi - \rho}{\rho} \right\} \tag{6}$$

Proposition 3.1 (Small Delay and Welfare)

Under the assumptions of Lemma 3.1, suppose that $\lambda > \underline{\lambda}$, *where* $\underline{\lambda} \equiv \rho/(\rho + \varepsilon - r) > 0$. *Then there exists a threshold* $\overline{\mu} > 0$ *such that* $dW/d\tau > 0$ *at* $\tau = 0$ *if and only if* $\mu > \overline{\mu}$. *That is, small no-commitment delays improve welfare if and only if the economy is initially growing fast enough.*

[2] See also Barlevy [3] for a related discussion of level vs growth effects in a different setting.

Proof: Using the expressions of both $dC_0/d\tau$ in the proof of Lemma 3.1 and W in equation (6), one computes that:

$$\frac{dW}{d\tau} = \frac{1}{\rho}\left\{\frac{\lambda\mu + \xi - \varepsilon + [\lambda\xi(r-\xi)]/\rho}{1-\lambda}\right\}$$

at $\tau = 0$ so that $dW/d\tau$ and $\psi \equiv \lambda\mu + \xi - \varepsilon + [\lambda\xi(r-\xi)]/\rho$ have the same sign. Using that $\xi = (\varepsilon - r\lambda)/(1-\lambda)$ when $\tau = 0$, it is then easy to show that $\xi + [\lambda\xi(r-\xi)]/\rho < 0$ if and only if $\lambda > \underline{\lambda} \equiv \rho/(\rho + \varepsilon - r)$. It then follows that there exists a threshold value $\overline{\mu} \equiv -\{\xi - \varepsilon + [\lambda\xi(r-\xi)]/\rho\}/\lambda > 0$ such that $\psi > 0$ if and only if $\mu > \overline{\mu}$. □

Note that $\underline{\lambda}$ is bound to be small if ρ takes on reasonable values so that the welfare effect that we describe happens for non-trivial levels of financial integration.

An intuitive interpretation of the result in Proposition 3.1 is as follows: the larger the initial growth rate μ, the larger initial consumption C_0 (more on this in section 4.1 about leapfrogging). This means that large μ's reinforce the level effect. If strong enough, the positive level effect dominates the negative growth effect of a lower g, when $\tau > 0$, and it leads to higher welfare relative to the no-delay case.

3.2 Welfare under CRRA utility: Numerical Examples

We suspect that the above results extend to $\sigma \neq 1$ but the analysis then becomes much more tedious. We now focus on the empirically appealing case $\sigma > 1$ and provide numerical examples when $\tau > 0$ that indeed accord with our conjecture. One important corollary of our numerical analysis is that it allows us, in the next sections, to measure the welfare effect of leapfrogging and growth reversals. To do so, we focus on the following benchmark parameter values:[3]

[3]The value of λ we use falls within the range of estimates provided by Djankov *et al.* [10].

Assumption 3.1 *The benchmark set of parameter values is :* $\lambda = 0.5$, $\rho = r = 0.01$, $\varepsilon = 0.03$ *and* $\sigma = 2$. *In addition, the initial function is exponential, that is,* $K_I(t) = e^{\mu t}$ *for all* $t \in [-\tau, 0]$, *with* μ *real.*

It follows from assumption 3.1 that when $\tau = 0$ (no-delay case), one has that $g = 2\%$ and $C_0 = 0.015$, whereas under autarky (when $\lambda = 0$) the growth rate is $g_a = (\varepsilon - \rho)/\sigma = 1\%$. It is also easily shown that when $\tau > 0$, the growth rate is such that $2\% > g > g_a$. To simplify matters, let us assume that the initial growth rate is $\mu = 3g$. Our benchmark case is therefore such that the economy is initially growing three times faster than the long-run BGP growth rate. In view of Lemma 3.1, we expect that increasing the no-commitment delay τ from zero leads to a smaller g but a larger C_0. The following table and figures show the impact of positive delays on the growth rate loss, the initial consumption level benefit and the overall welfare gain, relative to the case without delay.

Effect of delay τ on:	$\tau = 0.1$	$\tau = 1$	$\tau = 10$	$\tau = 30$	$\tau = 70$
growth rate loss =	0.01 pp	0.09 pp	0.5 pp	0.78 pp	0.94 pp
initial consumption gain =	0.7%	6%	30%	42%	41%
welfare gain =	0.3%	2.7%	**7.9%**	4.6%	−3.2%

Table 1: Effect of no-commitment delay on growth rate loss, initial consumption gain and welfare gain, relative to no-delay case ($\tau = 0$)

Insert Figures 1,2,3 about here

Figures 1, 2 and 3 plot the levels of growth rate, initial consumption and welfare behind the computations in table 1. For small delays, initial consumption goes up whereas the growth rate goes down exponentially fast when the delay increases, and the overall effect is to improve welfare. Eventually though, the level effect weakens and welfare goes down for large delays, because the negative growth effect then dominates.

The same picture turns out to emerge for different parameter values. Not surprisingly, increasing relative risk aversion σ from 2 weakens the incentives to accumulate capital (because the household is now less willing to substitute consumption over time) so that the level effect is stronger and leads to larger welfare gains for small delays. The same effect at work: when the consumption smoothing motive becomes stronger, slower consumption growth and larger initial consumption follow.

Interestingly enough, the fourth column of table 1 and figure 3 show that there exists a welfare-maximizing delay ($\tau_{opt} \approx 10$).[4] Compared to the no-delay case, the optimal τ_{opt} delivers a BGP growth rate that goes down from 2% to 1.5% (from figure 1, hence a loss of 0.5 pp in table 1) and an initial consumption level that goes up by 30% (see figure 2). Due to the large positive effect of a much higher consumption level, which dominates the negative growth effect, the combined effect is a non-trivial welfare increase of about 8% (see figure 3) relative to the no-delay case $\tau = 0$.

In contrast, the last column of table 1 and figure 3 show that for $\tau = 70$, the growth rate drops to 1.06% and the initial consumption gain goes up to 41% (see also figure 2 and 3) so that welfare declines. Because the large growth loss now dominates the consumption gain, households suffer a welfare loss of -3.2% compared to the no-delay case. This welfare loss increases to 11.1% relative to the welfare maximum under $\tau_{opt} = 10$. Therefore, for τ's that are much larger than τ_{opt}, the BGP growth rate is low (and indeed close to that prevailing under autarky) and welfare is significantly lower than the no-delay level. For large delays, the associated loss in the growth rate dominates the modest consumption gain and leads to large welfare losses. We now apply our analysis

[4] Robustness analysis shows that this property holds for an open set of parameter values. In particular, τ_{opt} is larger for larger σ's and μ's. For example, other things equal, $\tau_{opt} = 86$ when $\sigma = 5$ and $\tau_{opt} = 2$ when $\mu = 1.5g$.

to investigate the welfare consequences of leapfrogging and growth reversals.

4 Welfare Impact of Leapfrogging and Growth Reversals

4.1 The Large Welfare Gains from Leapfrogging

Following Boucekkine and Pintus [5], we define leapfrogging in consumption as the fact that the larger the initial growth rate μ, the more consumption and the larger welfare that is enjoyed by households. Suppose that economy $\mathcal{A}$, say, had a lower initial growth rate μ and was richer (in terms of capital and output) but growing more slowly prior to $t = 0$ than economy $\mathcal{B}$. In view of our assumption that growth is exponential for $t \in [-\tau, 0]$, both countries end up with the same capital stock, which equals unity, at $t = 0$. Then leapfrogging means that $\mathcal{B}$ gets to enjoy a larger consumption level C_0 at $t = 0$, hence a larger consumption at all dates $t \geq 0$ (because consumption jumps to the BGP at $t = 0$ and the growth rate g does not depend on μ). In contrast, $\mathcal{B}$ had lower capital and output than $\mathcal{A}$, for $t \in [-\tau, 0]$.

To study the welfare effect of leapfrogging, we set $\tau = 10$ so that $g = 1.5\%$ independent of the initial growth rate μ over $[-\tau, 0]$. Therefore, any welfare change due to variations in μ occur because of changes in the initial consumption level. In other words, for a given τ, only the short-run level effect originates welfare changes when μ is made to vary. In fact, it is not difficult to show that, given τ not too large, the level effect is positive on welfare, that is, $dC_0/d\mu > 0$ under CRRA utility. In particular, welfare gains are generated by leapfrogging for $\tau = 10$, as illustrated by table 2 below.

Effect of initial growth rate μ on:	$\mu=-g$	$\mu=0$	$\mu=g$	$\mu=3g$
initial consumption gain =	48%	66%	82%	107%

Table 2: Consumption gain of leapfrogging relative to initial growth rate $\mu=-3g$

Insert Figures 4 and 5 about here

Figure 4 plots C_0 as a function of μ and λ. It shows that given λ, the initial consumption level increases with the initial growth rate. in other words, leapfrogging occurs: the faster growth in the initial time interval, the higher consumption. For the chosen parameter values (see assumption 3.1), there is always leapfrogging for reasonable μ's, which turns out to be also true for larger values of τ's.[5] Note that relative risk aversion σ has no impact on the relative consumption gains because it affects only the scaling factor of C_0 (see equation (5)).

Table 2 shows that consumption gains due to leapfrogging can be very large. For example, suppose that economy $\mathcal{A}$ has been declining at $\mu=-3g=-4.5\%$, implying (in annualized data) that it takes 16 years to halve consumption. In contrast, economy $\mathcal{B}$ has been enjoying fast growth at $\mu=3g=4.5\%$ so that its consumption is expected to double in 16 years. Then $\mathcal{B}$ enjoys an initial consumption that is about twice as large as that of $\mathcal{A}$ (implying that its welfare is about 50% larger). Sensitivity analysis shows that leapfrogging still generates large consumption gains for large delays, for reasonable values of μ. It is also worth noting that the larger λ, the bigger welfare gains due to leapfrogging, as one can see in figure 4. One also notes from figure 4 that, given λ larger than 0.5, C_0 is a concave function of μ, which indicates that the welfare gains from leapfrogging are asymmetric: they are much more important for countries that have initially been either declining or growing very slowly. In other words, an increase of 1 pp in

[5]This is in contrast with what happens in the Solow case studied by Boucekkine and Pintus [5].

the initial growth rate generates larger welfare gains from leapfrogging when μ is small or negative.

Finally, a striking feature in figure 4 is that the effect of the credit multiplier λ on initial consumption C_0 is positive when μ is positive and large enough but reverses for negative μ's. This suggests that, for a given delay, financial globalization may decrease welfare if the economy is initially growing too slowly. We come back to this point in section 5, after we show how growth reversals entail large welfare losses.

4.2 The Large Welfare Losses from Growth Reversals

We now study the implications of our results for welfare when growth reversals occur. As in Boucekkine and Pintus [5], we define a growth reversal as a sudden break in the growth rate, when the latter goes either from below to above trend or from above to below trend. The next proposition shows that the condition for growth reversals is identical to that derived in Boucekkine and Pintus [5] for the Solow case (the proof is similar and therefore omitted). To derive such a condition, it is more convenient to study the detrended NDE arising when we perform the change of variable $x(t) = e^{-gt}K(t)$.

Proposition 4.1 (Growth Reversals for Large Delays)

Suppose that the initial function of the detrended NDE associated to (4) is $x_I(t) = e^{\phi t}$ *for* $t \in [-\tau, 0]$ *and some* ϕ *real. It follows that* $\phi \dot{x}(0) < 0$*, hence convergence to the BGP is non-monotonic, if and only if:*

$$g_\tau > r + \frac{\phi}{e^{\phi\tau} - 1}, \tag{7}$$

If $\phi \approx 0$*, then condition (7) writes as:*

$$\tau(g_\tau - r) > 1. \tag{8}$$

Insert Figures 6 and 7 about here

Note that the initial growth rate of detrended capital x relates to that of capital K through $\phi = \mu - g$. Condition (8) can be viewed in a simple way as the fact that growth reversals occur only if the no-commitment delay is large enough (see Boucekkine and Pintus [5] for an intuitive discussion of why this is the case). We now provide examples that illustrate Proposition 4.1. Figures 6 and 7 (based on table 1 and figures 1-3) picture the short-run dynamics[6] of detrended capital x for two economies that are similar in all respect except that $\tau = 10$ for economy $\mathcal{A}$ and $\tau = 70$ for economy $\mathcal{B}$. Economy $\mathcal{A}$ goes through negligible growth breaks (figure 6) whereas economy $\mathcal{B}$ experiences sharp growth reversals (figure 7). Because figure 7 plots $\log[x(t)] = \log[e^{-gt}K(t)]$, the kinks that appear at dates that are multiples of $\tau = 70$ indicate growth reversals. For example, figure 7 assumes that the economy grows above the BGP level at $\mu = 3g$ so that the detrended growth rate $\phi = 2g$ is positive prior to $t = 0$. Right after $t = 0$, however, detrended capital declines, which indicates that the growth rate of K is below the BGP level g until $t \approx 25$, when the detrended growth rate becomes positive again. At $t = 70$, a second growth reversal occurs and it leads to growth below trend until $t \approx 125$.

From table 1, we know that $\mathcal{A}$'s welfare is about 11% higher than $\mathcal{B}$'s even though $\mathcal{B}$'s initial consumption is about 9% larger than $\mathcal{A}$'s. The higher welfare in $\mathcal{A}$ comes from the larger BGP growth rate (1.5% vs 1.06% for $\mathcal{B}$; see table 1). Such a difference in annualized growth rates means that while it takes 46 years to double consumption in $\mathcal{A}$, it takes 66 years in $\mathcal{B}$, that is, about one more generation. In addition, figures 6 and 7 tell us that $\mathcal{A}$'s long-run output is about 5% higher than $\mathcal{B}$'s. Therefore, growth reversals are associated with welfare losses because they require large no-commitment delays (that is, τ's that are much larger than τ_{opt}).

[6] Figures 6 and 7 are produced using the MATLAB NDE solver provided by Shampine [20].

It it worth stressing that welfare losses under growth reversals are not due to consumption volatility (as consumption jumps to the BGP at $t = 0$) but rather because the negative growth effects dominates the positive level effect on consumption. Not surprisingly, higher σ's lead to smoother growth reversals, as expected because intertemporal substitution is then less strong. In addition, it is worth mentioning that the dynamics of detrended capital depicted in figures 6-7 is also that of the real exchange rate, defined as the price of non-tradeable good (labor, which is inelastic and normalized to one) in terms of the tradeable good (output), as the latter is proportional to capital in an AK economy. This implies that growth reversals in capital are also associated with growth reversals in the real exchange rate.

5 Is Financial Integration Welfare-improving without Investment Commitment?

A widely used measure of financial integration is the debt-to-GDP ratio. Absent delay (that is, with investment commitment), $D/Y = \lambda/A$ is a measure of how the economy relies on international borrowing and it depends on the credit multiplier λ, which is the only parameter related to credit market imperfections. With no-commitment delay, however, $D(t)/Y(t) = \lambda K(t-\tau)/(AK(t))$ so that the extent of financial globalization depends on both the credit multiplier λ and the delay τ, which both relate to how financial markets are imperfect, and also on the short-run dynamics of K. Note that along the BGP, $D/Y = \lambda/(Ae^{g\tau})$, with $g\tau$ typically increasing with τ so the effect of τ counteracts the effect of λ. This implies that D/Y may be a poor indicator of how financial integration affects growth and welfare because two economies with the same D/Y but different τ's and λ's may experience very different patterns of consumption, capital and output. To illustrate this point, this section takes two steps. We first study, for a given τ, the impact of λ on growth and welfare. Next we give an example of two economies that have a similar debt-to-GDP ratio but experience different time paths, welfare and long-run levels because they have different τ's and λ's.

We first give examples showing that high λ's may lead to welfare losses under the no-commitment delay. Such a conclusion can already be drawn from figures 4 and 5. Figures 4 and 5 reveal that increasing λ is not welfare improving for economies that are growing slowly, when τ is positive. In contrast, when $\tau = 0$, then $dC_0/d\lambda > 0$ and $dg/d\lambda > 0$ so that $dW/d\lambda > 0$, as we now show.[7] The proof is omitted because it follows from straightforward computations.

Proposition 5.1 (Financial Integration and Welfare under Small Delay)

Under the assumptions of Lemma 3.1, $dW/d\lambda > 0$ at $\tau = 0$ if and only if $\varepsilon > r + \rho$. That is, foreign borrowing improves welfare under small no-commitment delays provided that the economy is productive enough.

Proposition 5.1 states that under logarithmic utility and a mild condition ensuring that the economy can afford borrowing, welfare goes up with λ when $\tau = 0$. In essence, this is because both the level effect and the growth effect of consumption on welfare are then positive. Without delay and under logarithmic utility, we get the rosy view of financial globalization: it boosts growth and welfare.

In sharp contrast, for delays that are positive but moderate, both effects may go in opposite direction when μ is too small because a larger λ reduces C_0 (see figures 4 and 5 when $\tau = 10$). When the no-commitment delay is positive and moderate, financial integration may hamper welfare although it boosts growth. To illustrate this, we go back

[7]The comparative statics of the growth rate are studied in Boucekkine and Pintus [5].

to last section's parameter values, setting in particular $\tau = 10$. To make the contrast more dramatic, we compare the effects of $\mu = -g$ and $\mu = 3g$. The following table 3 shows that the welfare impact of financial integration reverses for declining economies.

Welfare change:	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 1$
$\mu = 3g$	2%	18%	51%
$\mu = -g$	−2%	−15%	−155%

Table 3: Welfare gain and losses from financial integration relative to autarky ($\lambda = 0$)

Robustness analysis indicates that the same results hold for larger τ's. The intuitive explanation for this outcome can again be grasped by comparing the level and growth effects. Because of intertemporal consumption substitution, if $\mu >> g$, then growth should slow down to g in the long-run so that large increases of initial consumption and welfare follow (see figures 4 and 5). On the contrary, if $\mu << g$ is small or negative, there is faster consumption growth in the long-run relative to the initial time interval so that initial consumption and welfare fall.

The large welfare losses reported in the last row of table 3 add some skepticism to the rosy view that financial globalization helps stagnating or declining countries to boost both growth and welfare. This is because increasing λ improves g in such a way that the growth effect is strong and adds up to the positive level effect only when μ is large.[8] The overall welfare increase when $\mu = 3g$ is large (51% in the last column of table 3 when $\lambda = 1$), because the growth rate increases by 1.32 pp from 1% (that is, it more than doubles) and C_0 is larger. In contrast, when $\mu < g$, the negative effect dominates

[8]To be more precise, the level effect is negative and small when $\lambda \approx 0$ but becomes positive and large when λ is close to one. In the last column of table 3, the initial consumption increase is about 25%, which adds to the positive growth effect.

because the same increase in g now faces a 76% decline in the initial consumption level.

Some interesting policy implications follow from the above examples. In a world with imperfect financial markets, financial globalization is good for growth and it also improves welfare only for fast-growing countries. In contrast, it may be detrimental to welfare in slowly-growing or declining countries. Because the model is overly simple, it implies that the welfare-maximizing value of λ is either 0 or 1, depending on the initial growth rate. In other words, the model predicts that the economy may be subject to the overborrowing syndrome (in contrast with results in Uribe [21]). This is because for a given no-commitment delay, more borrowing (higher λ's) is worse for welfare than less (lower λ's) when μ is too small, which leads to excessive borrowing from a welfare point of view.

In addition, in our model economy reducing market frictions may take two forms with quite different implications. First, increasing λ unambiguously leads to a larger welfare provided that initial growth is sufficient, as shown above. Increasing the collateral rate relaxes borrowing contraints and is good for welfare because additional financing enhances growth (in line with the evidence on domestic finance documented in, e.g., King and Levine [13, 14] and Rajan and Zingales [18]), even though it may amplify growth reversals and cause episodes of growth deceleration (as indicated in the dataset studied by Rancière *et alii* [19]). A different conclusion arises if reducing market imperfections means reducing the delay τ. Our results above indicate that this is bad for welfare if the economy is growing fast initially and if the delay is such that $\tau_{opt} \geq \tau$. This is because although such an institutional change boosts growth, it reduces initial consumption too much. In contrast, it is welfare-improving to reduce market imperfections when they are quite severe, that is, when the no-commitment delay is very large.

Insert Figures 8,9,10 about here

We end this section by an example suggesting that D/Y may not be a good indicator of how financial integration affects growth and welfare. Suppose we compare economies $\mathcal{A}$ and $\mathcal{B}$ that are similar (in particular $\mu = 3g$ for both), except that $\mathcal{A}$ has $\lambda = 0.5$ and $\tau = 10$ whereas $\mathcal{B}$ has $\lambda = 0.94$ and $\tau = 70$. Both economies have, in the long-run, the same debt-to-GDP ratio which is about 43% (and falls within the range of estimates provided by Lane and Milesi-Ferreti [15]).[9] The time path of $\mathcal{A}$ appears in figure 6 and figure 8 plots that of $\mathcal{B}$, which indicates that whereas the former economy has a quick and monotonic transition towards it detrended long-run level of capital, the latter goes through sharp growth reversals[10] (note that $\mathcal{B}$ enjoys a slightly larger detrended capital stock than $\mathcal{A}$ in the long-run). Moreover, both economies have debt-to-GDP ratios that exhibit very different patterns over time. Figure 9 and 10 plot the debt-to-GDP ratios of $\mathcal{A}$ and $\mathcal{B}$, respectively, against time. In figures 9 and 10, $\mathcal{A}$ enters the period with a large leverage ratio (that is, about 32%), whereas $\mathcal{B}$ starts at $t = 0$ with less than 10% (because it faces a larger delay). In addition, whereas the debt-to-GDP ratio of $\mathcal{A}$ significantly accelerates after $t = 0$ (figure 9), $\mathcal{B}$ has to patiently build up debt and its leverage ratio goes through growth reversals before reaching the same long-run level as that of $\mathcal{A}$ (see figure 10). In other words, despite having eventually the same level of financial integration, both economies have different transitional dynamics. Not surprisingly, $\mathcal{A}$ and $\mathcal{B}$ also have different growth rates, initial consumption and welfare levels. While $\mathcal{B}$ enjoys at $t = 0$ a 14% consumption gain over $\mathcal{A}$, its consumption grows

[9] We now assume $A = 1$, which leads to an unrealistically high value for the depreciation rate δ given that $\varepsilon = A - \delta = 0.03$ according to assumption 3.1. To avoid this problem, we could alternatively suppose that $D(t) = \lambda Y(t - \tau)$, which would leave our analysis unaffected, save for the fact that $\varepsilon = 1 - \delta/A$. Under such an assumption, the scaling factor A would not affect the D/Y ratio.

[10] Compared to figure 7, figure 8 pictures growth reverals that are more pronounced because $\lambda = 0.94$ (instead of $\lambda = 0.5$ in figure 7) so that the economy relies more heavily on external borrowing.

more slowly ($g \approx 1.5\%$ for $\mathcal{A}$ while $g \approx 1.1\%$ for $\mathcal{B}$) so that $\mathcal{B}$ ends up having a lower welfare at $t = 0$. Note that this picture is consistent with the fact that more leverage improves both growth and welfare when μ is large enough: because $\mathcal{A}$ enjoys a larger debt-to-GDP ratio from $t = 0$ onward, it grows more quickly and achieves higher welfare than $\mathcal{B}$ at $t = 0$. Even in the long-run, when both economies have similar D/Y's, $\mathcal{A}$ and $\mathcal{B}$ may have different welfare levels. For example, when $t = 200$; although $\mathcal{A}$ and $\mathcal{B}$ have almost reached their common long-run level of debt-to-GDP, the former economy's consumption is about two times larger than the latter's. This is because $\mathcal{A}$'s growth rate is significantly larger than $\mathcal{B}$'s, which leads to a welfare level that is larger in $\mathcal{A}$ relative to $\mathcal{B}$.

6 Conclusion

In this paper, we have shown that leapfrogging and growth reversals have sizeable effects on welfare, in an open AK economy subject to both a collateral constraint and the inability to commit to an investment level. In particular, the result that sudden reversals in the growth rate may have large adverse effects on consumption well-being accords with intuition. Moreover, our analysis provides still another framework such that, in sharp contrast with the standard AK setting, maximizing growth is not always equivalent to maximizing welfare: this is the case in our model under small no-commitment delays. Our analysis shows that economies that have historically been growing fast are the most likely te reap the benefits from financial integration. More surprisingly, one of our conclusions is that reducing the level of credit market frictions (that is, decreasing the level of the no-commitment delay or increasing the credit multiplier) is not always welfare-improving. Last but not least, our conclusions related to financial integration emphasize that economies relying heavily on international credit markets may indeed suffer from overborrowing, in the sense that a too high debt-to-GDP ratio may be detrimental to welfare. More generally, our analysis of borrowing without investment commitment indicates that both the credit multiplier, that is much stressed by the current literature, and the no-commitment delay, that is a key dimension of this paper's analysis, are important parameters determining how financial integration affects growth and welfare.

Several potential extensions of our analysis appear natural. On the theoretical side, it seems worthwhile to dig deeper into the mechanisms behind growth. In that respect, our results can be build upon to introduce Schumpeterian growth and channels through which external finance affects innovation. On the empirical side, it is an open question whether the no-commitment assumption could explain the patterns of credit flows towards both developed and developing countries and their effects on growth performances. We believe this calls for further research.

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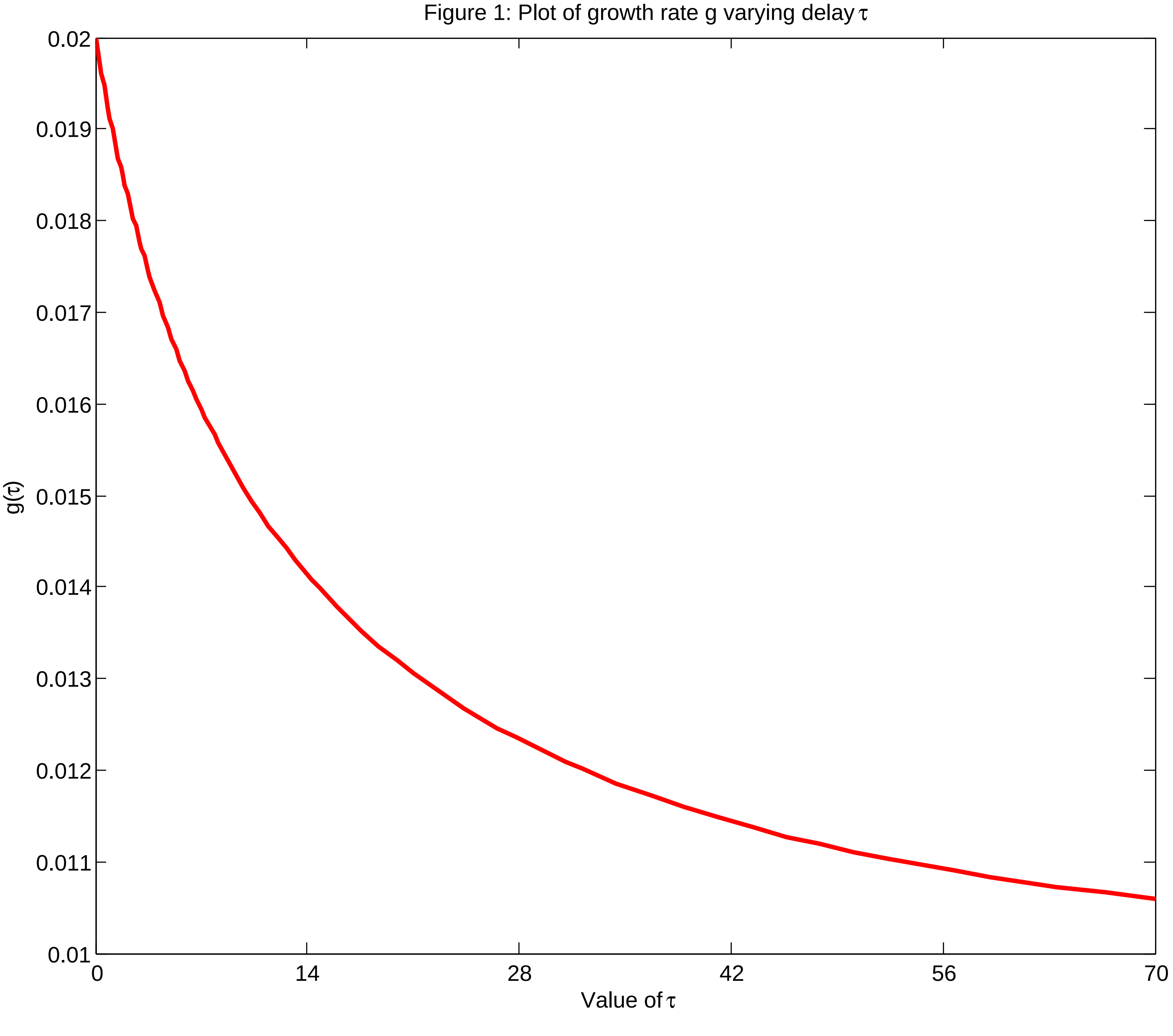


Figure 1: Plot of growth rate g varying delay τ
g(τ)
0.02
0.019
0.018
0.017
0.016
0.015
0.014
0.013
0.012
0.011
0.01
0
14
28
42
56
70
Value of τ

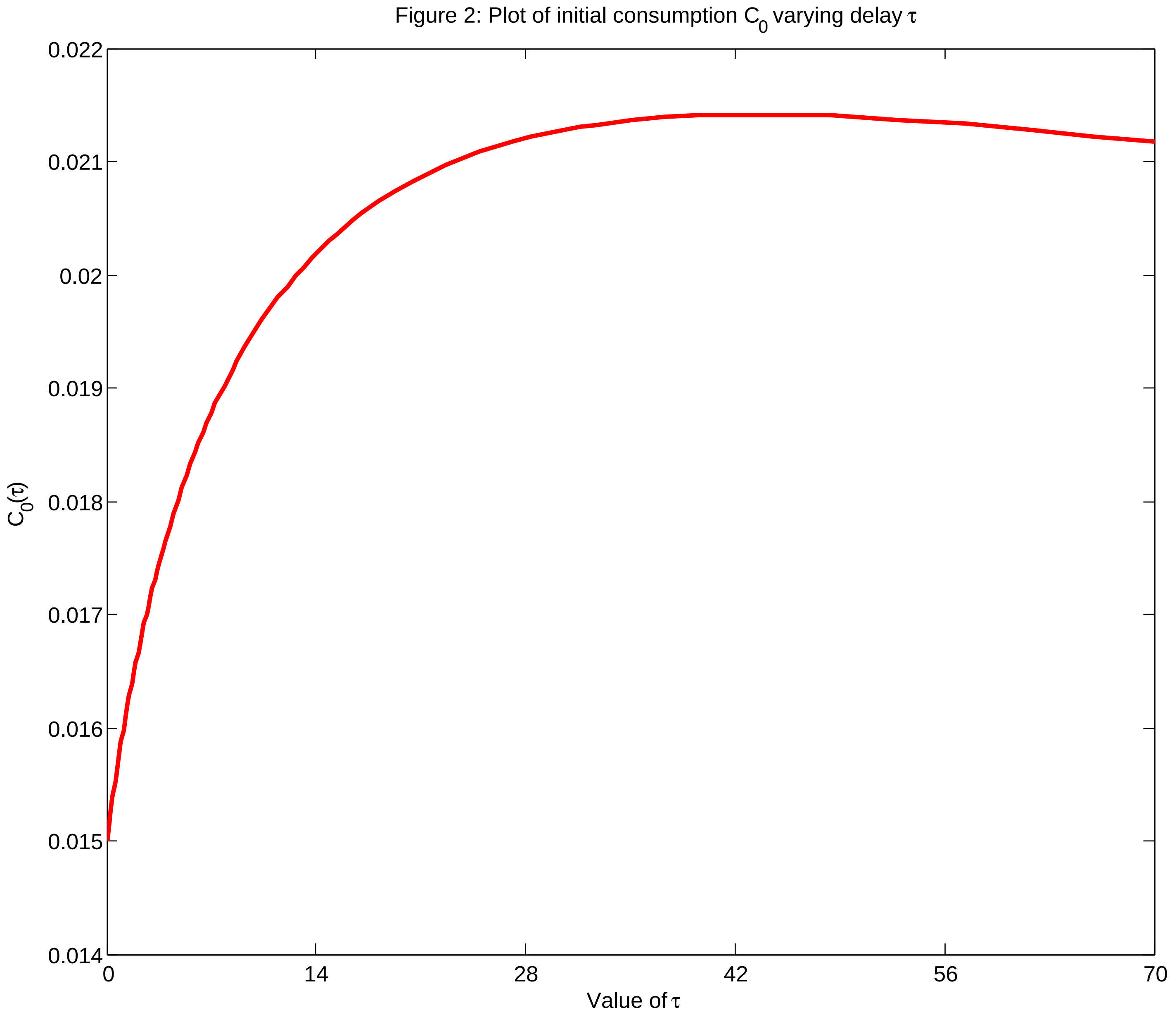


Figure 2: Plot of initial consumption C0 varying delay τ
0.022
0.021
0.02
0.019
0.018
0.017
0.016
0.015
0.014
C0(τ)
0
14
28
42
56
70
Value of τ

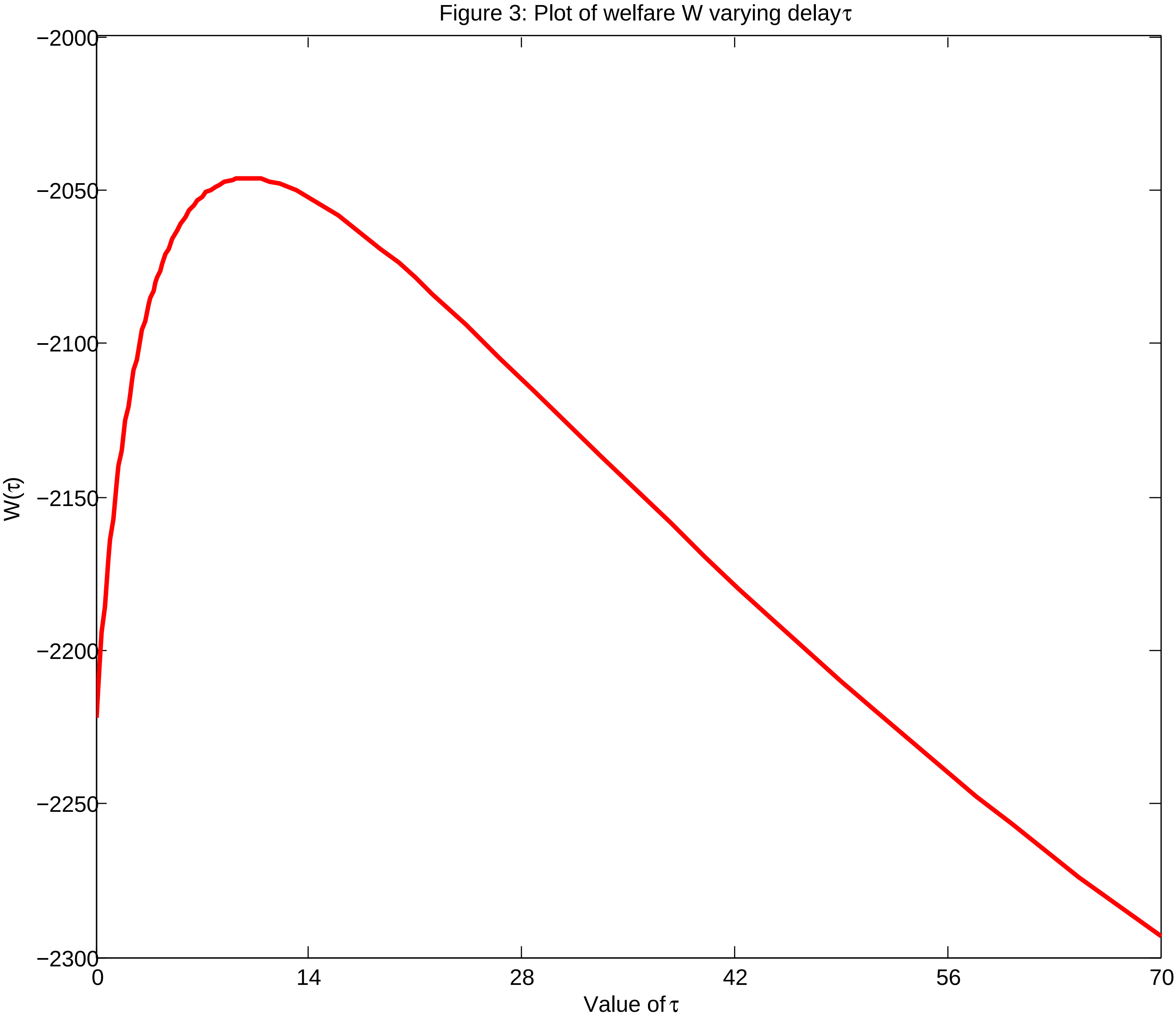


Figure 3: Plot of welfare W varying delay τ
−2000
−2050
−2100
−2150
−2200
−2250
−2300
W(τ)
0
14
28
42
56
70
Value of τ

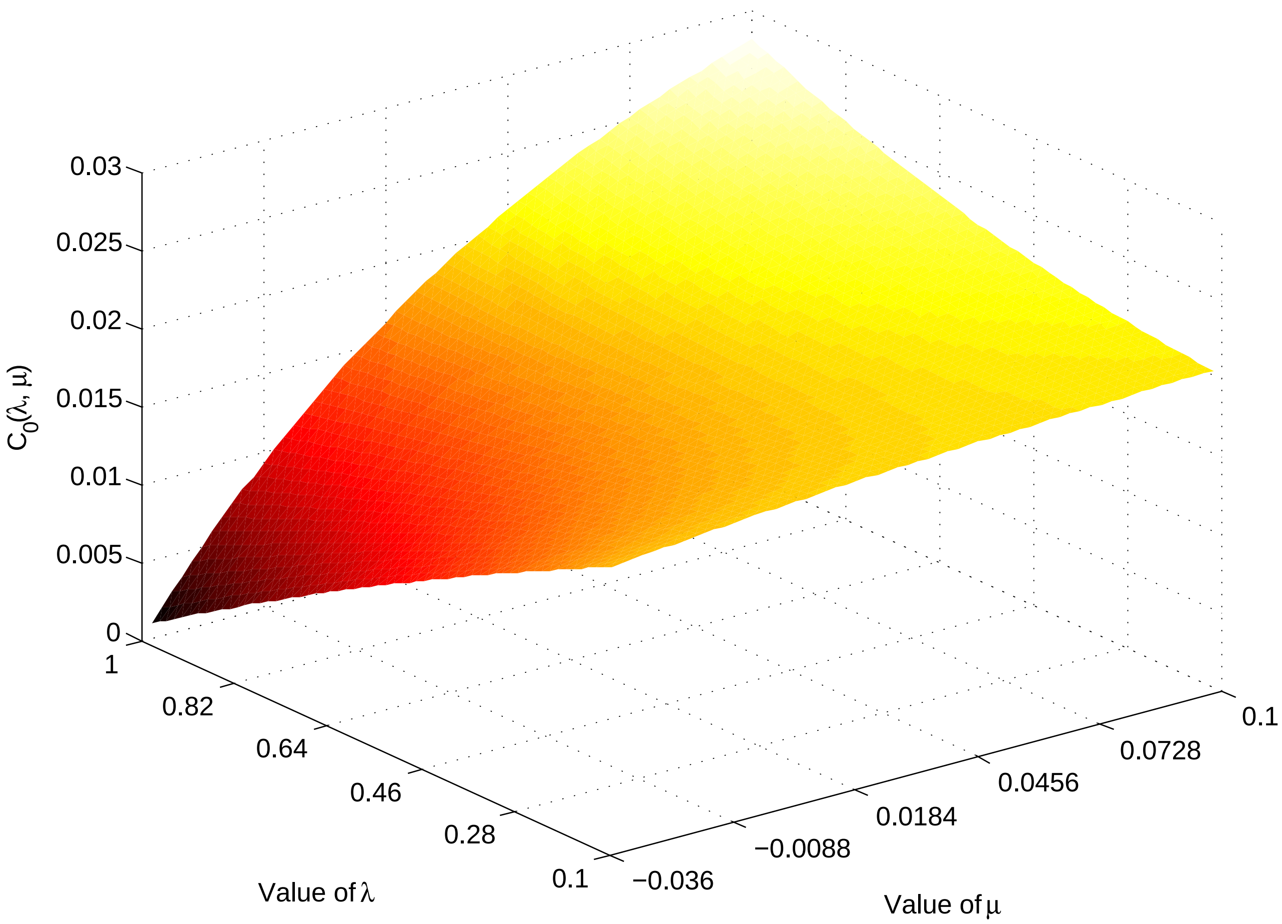


Figure 4: Plot of initial consumption C_0 varying λ and μ

Figure 5: Plot of welfare W varying λ and μ

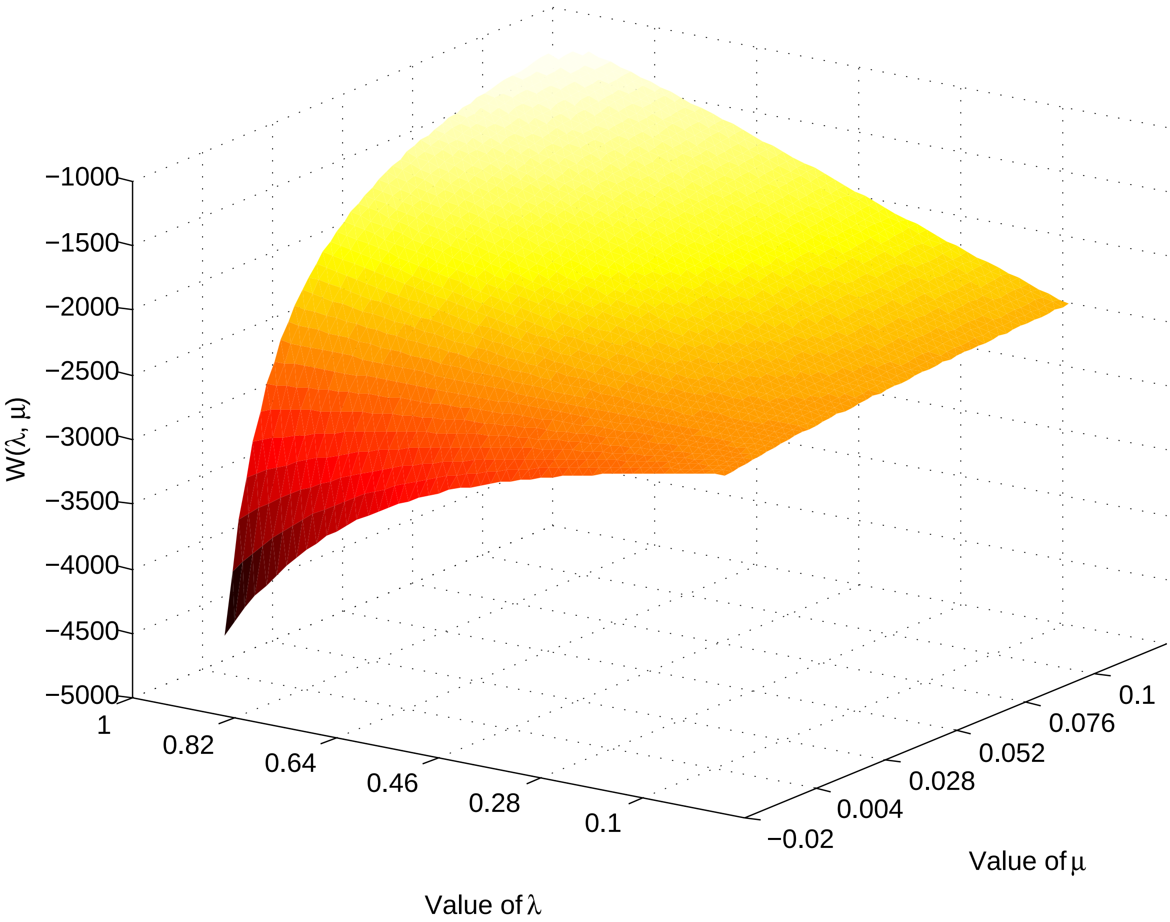


Figure 6: Plot of log of detrended capital against time when τ=10

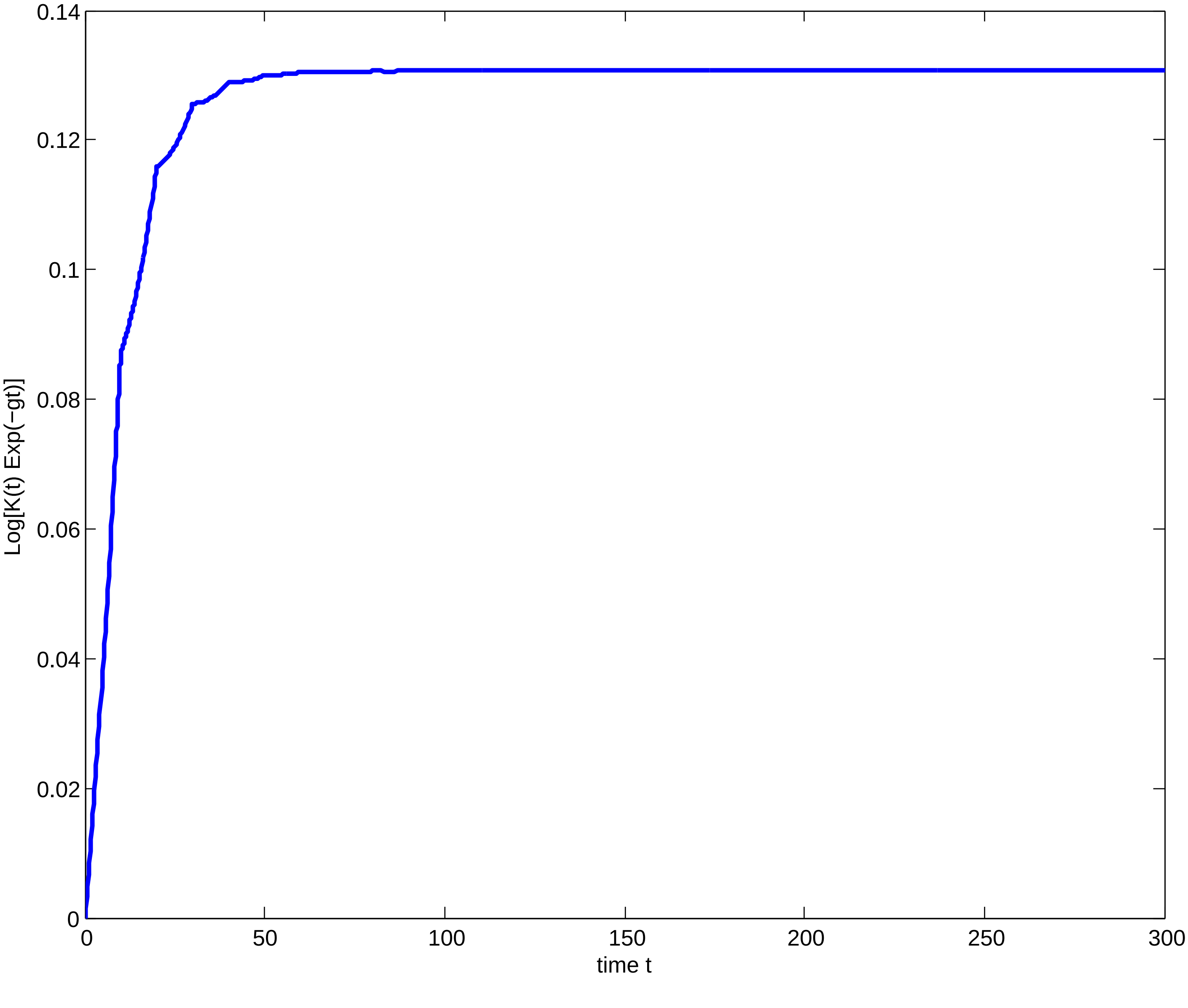
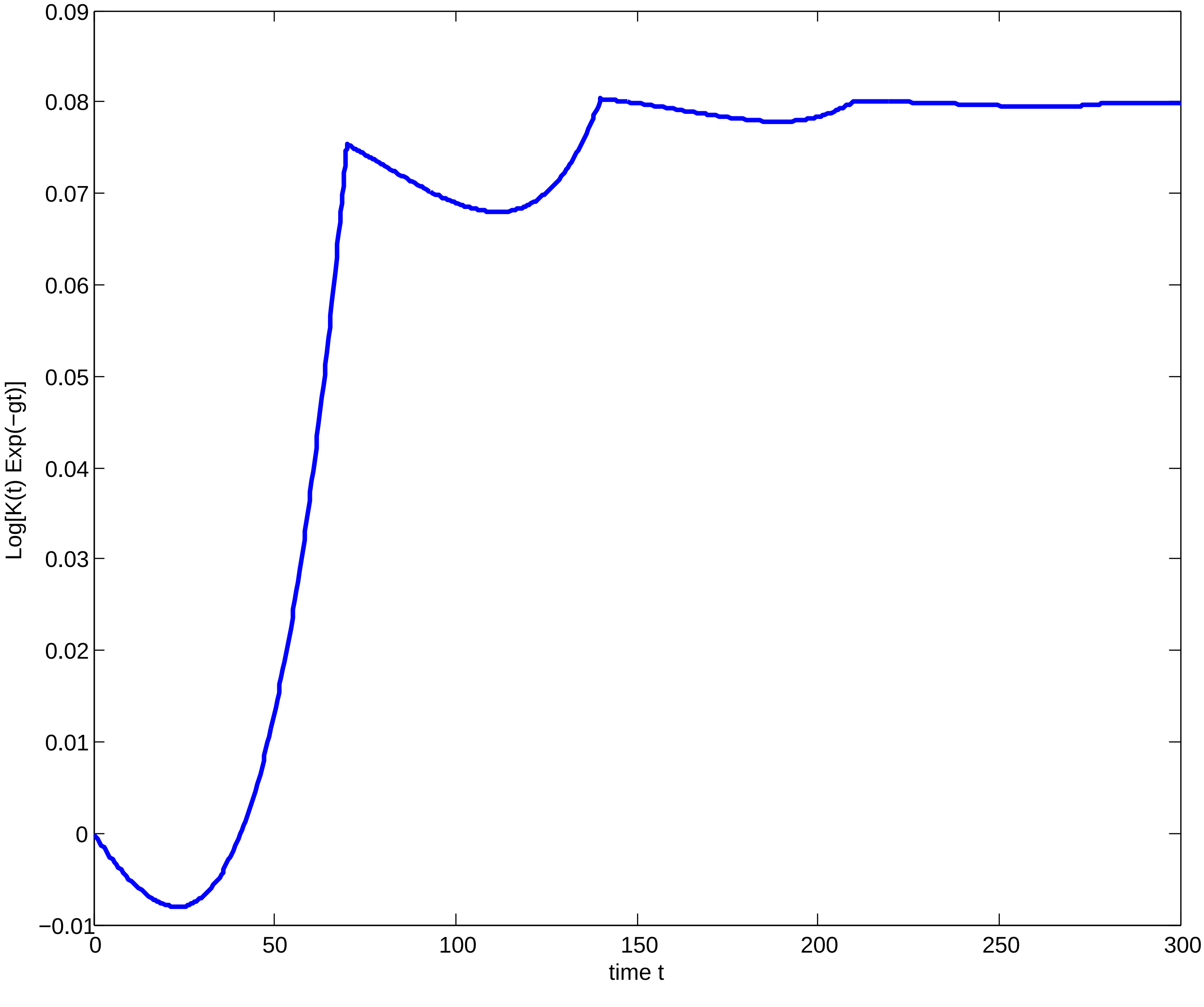


Figure 7: Plot of log of detrended capital against time when τ=70



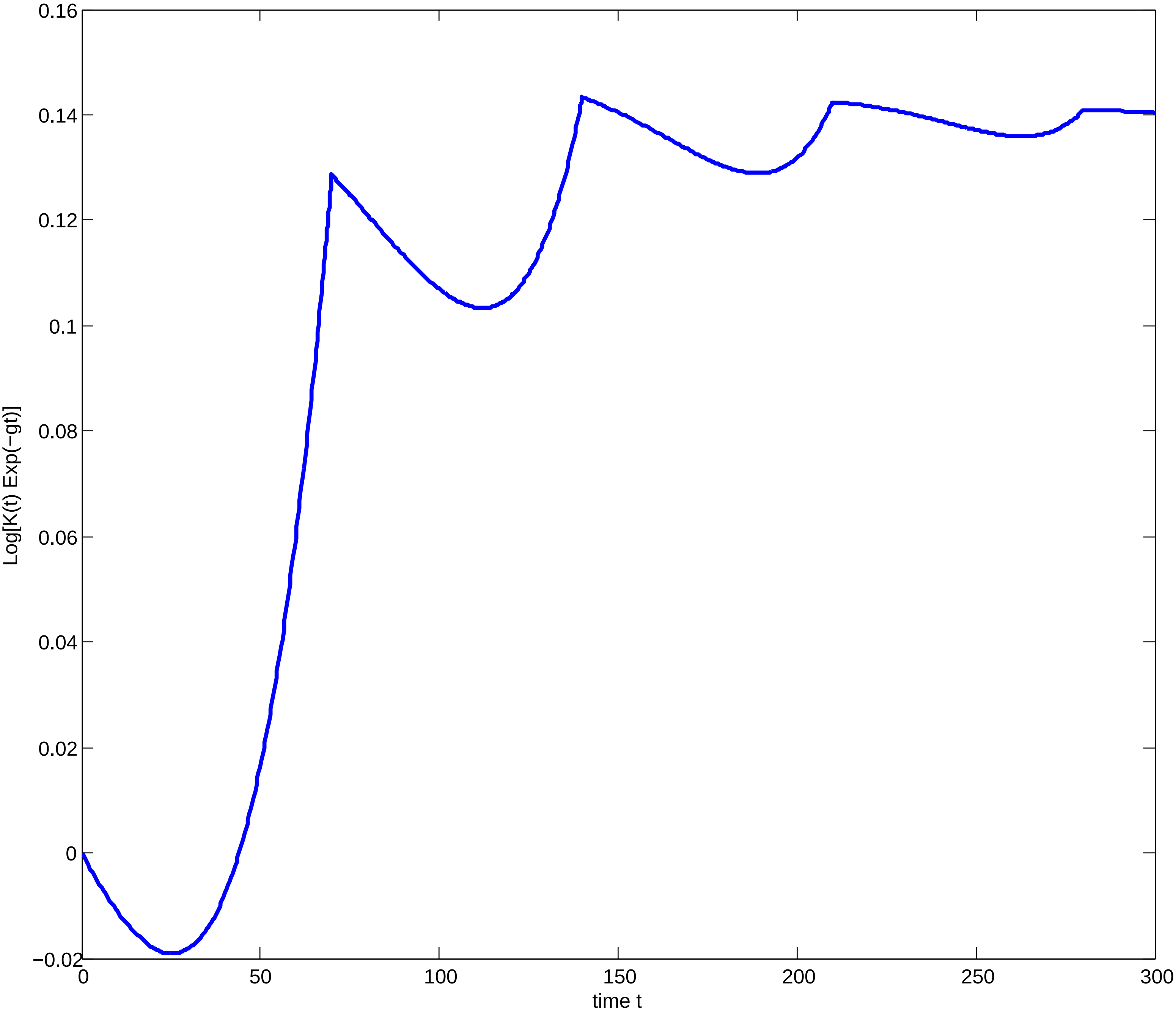


Figure 8: Plot of log of detrended capital against time when λ=0.94 and τ=70

Figure 9: Plot of ratio D(t)/Y(t) against time when λ=0.5 and τ=10

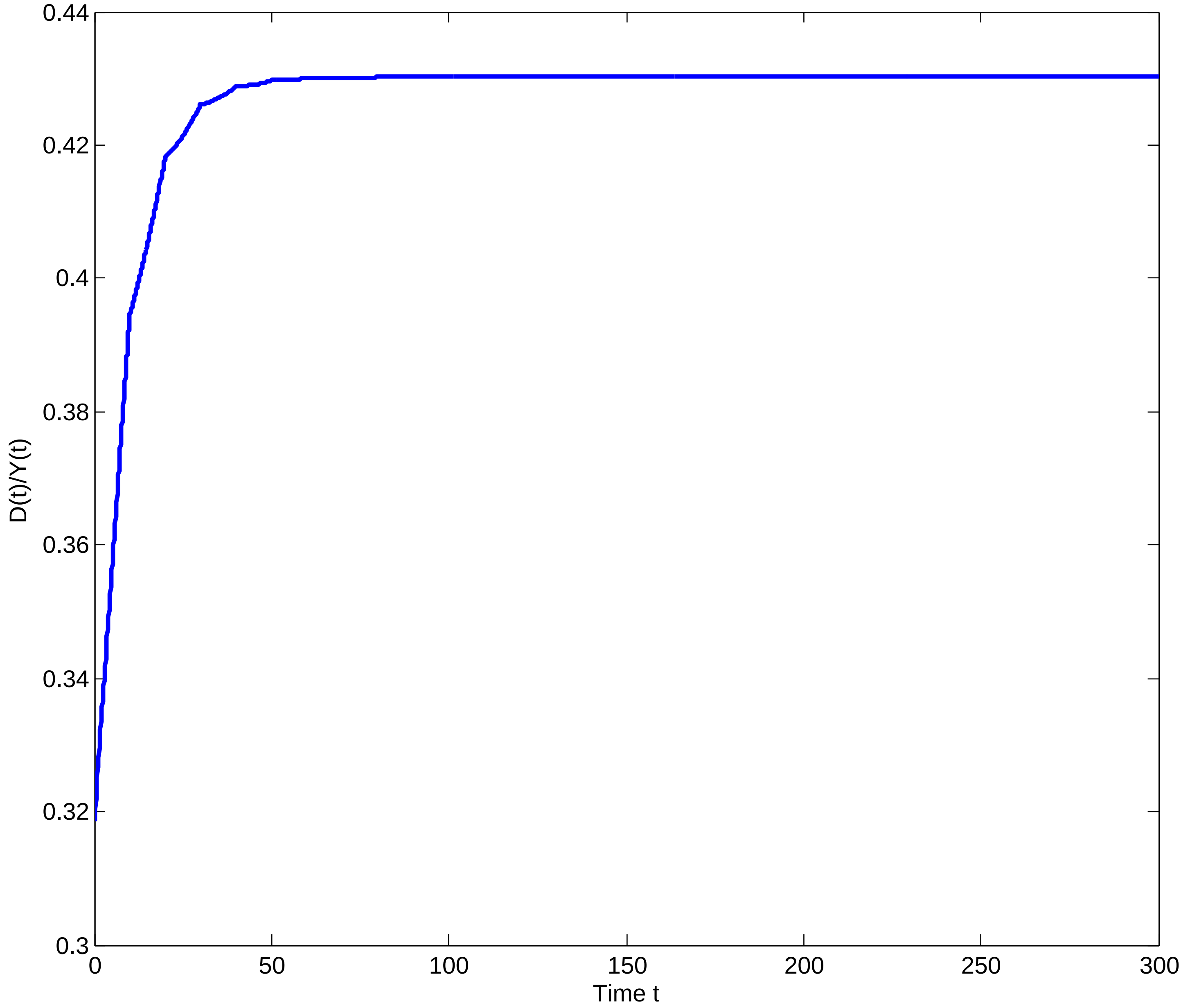


Figure 10: Plot of ratio D(t)/Y(t) against time when λ=0.94 and τ=70