Policy Implications of Central Bank Influence from Superior Forecasts

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Abstract

How might central bank communication of its internal forecasts assist the conduct of monetary policy? This paper analyzes through adaptive learning the policy implications of central bank influence of private forecasts when central bank forecasts are superior to private forecasts. The literature has shown that heterogeneous expectations may have destabilizing effects on aggregate dynamics and we show that influence from superior forecasts enables that the central bank must only respect the Taylor principle to ensure macroeconomic stability, in contrast to central bank influence from inferior forecasts. Influencing private expectations should therefore not be an objective *per se*; central banks should either increase the quality of their macroeconomic forecasts to guide private expectations, or respond in their interest rate rule to private forecasts - rather than to their internal forecasts - in the case of inferior forecasting performance.

Keywords: Endogenous influence; Adaptive Learning; Taylor Principle; Monetary Policy.

JEL Classification: E52, D83, D84

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1. Introduction

The expectations channel of monetary policy has become more and more important both in theory and practice and need the policymakers to be able to manage private expectations. However, policymakers' practices such as discussing private sector forecasts or publishing their internal forecasts may appear puzzling as the Rational Expectations (RE) hypothesis supposes all expectations are only based on the observable fundamentals. These practices can be explained when considering the hypotheses of imperfect information or imperfect knowledge of the model of the economy which modify the determination of aggregate variables and the policymakers' behaviour. Thus, Mankiw and Reis (2002, 2007) introduce information stickiness, Sims (1998, 2003) and Woodford (2003) focus on rational inattention, Guesnerie (2005, 2008) proposes an "eductive" learning approach and Evans and Honkapohja (2001) study the effects of adaptive learning. All these procedures intend to model a possible expectations formation process which departs from the full information RE hypothesis.

This paper proposes to analyze how central bank communication of internal forecasts may assist the conduct of monetary policy, when these central bank forecasts are superior to private forecasts. In order to structure the analysis, I define two concepts of management of private expectations. *Endogenous influence* is the central bank forecasting influence arising from superior central bank forecasts. *Exogenous influence* is the influence stemming only from the policy signals and the informational content of central bank forecasts and independent of its forecasting performance. In an environment of imperfect information, it is optimal for private agents to form their forecasts based on central bank forecasts, if they contain useful information and even if it is noisy information. Thus, as long as signals or information disclosed by the central bank do not deteriorate its relative forecasting performance, the key feature of this distinction is that endogenous influence drives private expectations towards the fundamental value of variables. The consequences of endogenous influence are then assessed in terms of their ability to stabilize expectations under adaptive learning according to the so-called E-stability criterion.

The theoretical framework used to analyse the interaction of forecasts' communication and policymaking through interest rate rules is the standard New-Keynesian model (Clarida, Gali, and Gertler (1999), McCallum (1999) and Woodford (2001)) which gives a central role for inflation and output expectations. Nevertheless, the precise role of both central bank and private forecasts in the decision making of central banks has not been put forward. Recent researches using adaptive learning have focused on the consequences of forecasting errors and the resulting correction devices. Bullard and Mitra (2002) and Evans and Honkapohja (2003b) show that some interest rate rules may lead to instability as private agents inefficiently seek to correct their forecast functions over time. A subsequent literature has then focused on the effects of the heterogeneity of forecasts between the central bank and private agents (Honkapohja and Mitra 2005, 2006, Preston 2008), on information asymmetry (Honkapohja and Mitra 2004) and influence of either the central bank of private agents on the other (Muto 2011). The contribution to the literature is therefore to analyze the effects of central bank influence when the central bank produces better forecasts. Based on forecasts' communication, endogenous influence is modelled through heterogeneity of forecasts between the central bank and private agents and superior forecasting performance of the central bank.

This paper does not support that all central banks have superior forecasts or influence private forecasts, but analyzes the theoretical implications of endogenous influence based an empirical literature showing that some central banks may have a superior forecasting performance compared to private agents (Romer and Romer 2000 and Sims 2002 among others) and that central banks may influence the dispersion of private expectations (see e.g. Fujiwara 2005, Ehrmann et al. 2009) or the level of private expectations (Hubert 2011).

This analysis is closely related but different from Muto (2011) and Eusepi and Preston (2010). The former analyzes the E-stability conditions when private agents condition their forecasts on central bank forecasts independently of the central bank forecasting performance and because the central bank is the leader and private agents the followers of the monetary process. Muto (2011) therefore assesses implicitly the effects of exogenous influence and this paper completes his seminal work. Eusepi and Preston (2010) analyzes through adaptive learning the effects of a set of different communication strategy whose the one consists in communicating the conditional forecast path for the nominal interest rate and the others constraints on the communication ability of the central bank. The framework of this analysis considers perfect communication of the central bank and focuses on macroeconomic forecasts of inflation and output; and more particularly on the distinction between central bank macroeconomic forecasts driving private expectations towards the fundamental value of variables and forecasts driving them away.

The present model assessing the policymaking effects of endogenous influence could also be connected to King, Lu and Pasten (2008) who define short-term and long-term credibility of the central bank associated with the commitment to low and stable inflation. Their analysis focuses on the central bank credibility (whether the central bank is of strong or weak type in the Barro and Gordon (1983)'s classic monetary game), while the focus is here on the influence ability due to the forecasting performance of the central bank. Moscarini (2007) reconciles both concepts and shows a reputation for competence implies credibility and transparency, while Geraats (2002, 2005) confirms that transparency enhances central bank credibility[1] and that central bank communication is very powerful tool to reach private expectations stabilization.

Another distinction of this work is related to the mechanism at work behind the expectations channel. King (1982), in an early paper, shows that monetary policy rules affect real outcomes in models with imperfect information through "prospective feedback actions" responding to shocks that are imperfectly known by private agents; the mechanism going through the effect of policy rules on the information content of prices. Amato, Morris and Shin (2002), Morris and Shin (2005) and Amato and Shin (2006) analyze the social value of public information and find that full transparency might deter the efficiency of monetary policy. Hellwig (2005) and Angeletos and Pavan (2004, 2007) challenge this conclusion[2] in the extent that more public information facilitates coordination. Walsh (2006) and Cornand and Heineman (2008) finally show that it may be advantageous for a central bank to make partial announcements to offset future shocks or if complete announcements intensify inflation variability. The main device at work in this paper considers perfect communication of policymakers' macroeconomic forecasts.

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[1] The focus of the paper is on communicating central banks and we assume, following Geraats (2002, 2005) that they are *strong* ones (in reference to the terms used in the framework of Kydland and Prescott (1977) and Barro and Gordon (1983)), that is to say, credible central banks without inflationary bias.

[2] Svensson (2006) also challenged it as the numerical condition under which transparency reduces welfare is unrealistic: it requires that the precision of public information is smaller than the one of private information.

The main result of this paper is that satisfying the Taylor principle is sufficient to reach E-stability and determinacy in a situation of endogenous influence. The intuition is straightforward: if the central bank has lower forecast errors than private agents and therefore that private agents follow the central bank forecasts, the central bank must only respond to its forecasts errors to reach macroeconomic stability. This result complements the one of Muto (2011): the central bank must respond more strongly to expected inflation than the Taylor principle suggests in the case of exogenous influence. Indeed, when central bank forecasts are relatively poor and still influences private forecasts, central bank errors and private forecasts errors add up to each other and the central bank need to respond more aggressively to inflation.

The policy implications of this work are that, in a situation of imperfect information, influencing private expectations should not be an objective *per se* and central banks should increase the quality of their macroeconomic forecasts to guide private expectations. It would enable them to reach macroeconomic stability at a lower cost. Another solution to avoid the situation of exogenous influence would be that the central bank responds in its interest rate rule to private forecasts rather than to its internal forecasts in the case of inferior forecasting performance.

The rest of the paper is organized as follows. Section 2 presents the concepts of endogenous and exogenous influence and discusses the framework of analysis. Section 3 presents the assumptions and the conditions for stability and determinacy of endogenous influence. Results are compared to those of exogenous influence and other standard cases of the literature. Section 4 concludes this paper.

2. The Framework

The benchmark model of expectation formation in macroeconomics has been RE since the seminal papers of Muth (1961), Lucas (1972), and Sargent (1973). RE are based on the hypothesis that agents possess the correct equilibrium probabilities laws, in other words they know the correct form of the model, all parameters, and that other agents are rational. Households are considered as dynamic optimizers concerned with expected future incomes, employment and inflation while firms forecast future demand, costs and productivity levels. Monetary policymakers forecast future inflation and aggregate economic activity and consider both the direct impact of their policies and the indirect effect of policy rules on private expectations. Reconsidering the RE hypothesis, the learning approach proposes another model of expectation formation and describes agents (private ones and the central bank) forming their expectations by estimating and updating forecasting models in real time. They engage in a process of learning about the economy. They therefore can be considered as econometricians who estimate and re-estimate models as new data become available. In the case of this paper, the new data of interest is the central bank forecast.

2.1. Endogenous and Exogenous Influence

In an environment of imperfect information, the distinction between these two types of central bank influence of private expectations relies on the interaction of the central bank signals contained in central bank forecasts and the relative accuracy of these forecasts. Central bank forecasts are a function of y_{CB}, the central bank's private signal on the future realization of a given variable y, ε_{CB}, the noise associated to this signal, and s_{CB}, a stochastic term representing the policy signal the central bank may want to disclose to the public, and are conditional to I_{CB}, the information set of the central bank available when forming its forecasts. Private agents observe their own private signal y_{PA} - with a noise ε_{PA} - on the future realization of the same given variable y and the central bank forecast, and form their own forecasts as a linear combination of both. Central bank forecasts may therefore influence private agents and enter in the private forecasting function for two reasons: the good forecasting performance of future variables by the central bank and the policy content of central bank forecasts.

I thus define *endogenous influence* as the central bank forecasting influence arising from superior central bank forecasts. Private agents introduce the central bank forecast in their forecasting function because of the quality of central bank forecasts of future variables. The mechanism characterizing endogenous influence is simple and sound: relatively better forecasts of the central bank enable to influence directly private forecasts because they allow private agents to correctly forecast future outcomes. Thus, one would expect that rational private agents would follow the central bank due to its better forecasting record when forming their own expectations. In addition, one might suppose that relatively better central bank forecasts would enhance its credibility and therefore legitimate its influence.

Exogenous influence is defined as the central bank forecasting influence arising from the policy content of central bank forecasts and the disclosure of private information of the central bank. Central bank forecasting influence is labelled *exogenous* in the sense that it is independent of the central bank forecasting performance. Thus, central bank forecasts may be influential if the information sets used to construct the forecasts are not nested. Even if the central bank forecast errors are larger than those of private agents, it may be optimal for private agents to update their own forecasts following central bank forecasts as long as they may contain any marginal information outside the information set of private agents or policy signals. Exogenous influence is then characterized by the fact that it is rational that the weight on central bank forecasts is positive when information sets are not nested or the central bank sends signals.

The proposed distinction therefore consists in disentangling the nature of the influence and the reason for the weight put on central bank forecasts; in other words, whether influence arising from a better forecasting performance can be differentiated from the influence only due to policy signals. A fundamental difference between both types of influence is that exogenous influence is exclusive whereas endogenous influence is not, according to the criteria of the relative accuracy of central bank forecasts. Indeed, an endogenously influential central bank may also send policy signals. As long as the policy signal s_{CB} and/or the central bank information set I_{CB} do not deteriorate its forecast accuracy below the private forecast accuracy, then influence is still considered as endogenous as it moves private forecasts towards the fundamental value of future variables. And *in fine*, the general implication of this distinction is to differentiate influence moving private expectations towards the fundamental value of variables from the opposite: a self-fulfilling expectations-based value.

2.2 A standard version of the New Keynesian model

The aggregate demand or IS curve is obtained by log-linearizing the consumer's Euler equation and employing the goods market-clearing condition, so that the equation is expressed in terms of the output gap.

$$x_t = E_t^{PA} x_{t+1} - \varphi(i_t - E_t^{PA} \pi_{t+1}) + g_t \quad (1)$$

The aggregate supply or AS (or NK Phillips) curve is derived as a linearization of the firms' optimality condition under the price setting constraint.

$$\pi_t = \lambda x_t + \beta E_t^{PA}\pi_{t+1} + u_t \tag{2}$$

where x_t is the output gap, π_t is the inflation rate, i_t is the nominal interest rate, g_t is the demand shock and u_t is the cost push shock. E_t^{PA} denotes the private sector expectations (not necessarily rational as agents do not know the structural parameters) which influence the economy directly through aggregate demand and the Phillips Curve. Each variable is defined as the deviation from its steady state. φ is the elasticity of intertemporal substitution of the representative household, λ is the degree of price stickiness and β is the household's discount factor. These structural parameters satisfy $\lambda > 0$, $\varphi > 0$ and $0 < \beta < 1$.

The central bank uses a forward-looking rule[3] to set its interest rate according to its forecasts of future inflation and output gap.

$$i_t = \phi_0 + \phi_\pi E_t^{CB}\pi_{t+1} + \phi_x E_t^{CB} x_{t+1} \tag{3}$$

E_t^{CB} is the forecast of the central bank made at a date *t* for some variable, while ϕ_π and ϕ_x represents the response to future inflation and future output gap.
The shocks g_t and u_t are assumed to follow these processes:

$$w_t = \begin{pmatrix} g_t \\ u_t \end{pmatrix} = F \cdot \begin{pmatrix} g_{t-1} \\ u_{t-1} \end{pmatrix} + \begin{pmatrix} \tilde{g}_t \\ \tilde{u}_t \end{pmatrix} \text{ with } F = \begin{pmatrix} \mu & 0 \\ 0 & \rho \end{pmatrix}$$

Since our focus is on the learning behaviour, these expectations need not be rational. E_t^{CB} and E_t^{PA} are not rational as because of imperfect information and the uncertainty related to how shocks get mapped into outcomes for inflation and output. E_t^{CB} and E_t^{PA} denote subjective expectations that are instead formed as forecasts from an estimated model and observations.

The model can be written under the following reduced-form:

$$y_t = \begin{pmatrix} x_t \\ \pi_t \end{pmatrix} = D + A^{PA} \cdot E_t^{PA} y_{t+1} + A^{CB} \cdot E_t^{CB} y_{t+1} + B \cdot w_t \tag{4}$$

where $D = \begin{pmatrix} -\varphi\phi_0 \\ -\lambda\varphi\phi_0 \end{pmatrix}$, $A^{PA} = \begin{pmatrix} 1 & \varphi \\ \lambda & \lambda\varphi + \beta \end{pmatrix}$, $A^{CB} = \begin{pmatrix} -\varphi\phi_x & -\varphi\phi_\pi \\ -\lambda\varphi\phi_x & -\lambda\varphi\phi_\pi \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ \lambda & 1 \end{pmatrix}$

2.3 The Expectations Formation Process: Adaptive Learning

In the set-up of adaptive learning, agents know the true values of the structural parameters and the uncertainty regards the reduced-form equilibrium dynamics of aggregate variables following stochastic shocks. Agents' expectations are described by a simple forecasting model. Agents are assumed to use a perceived law of motion (PLM) where true values of parameters are not known. They estimate this model to obtain estimates for the parameters in the PLM. It is postulated that agents use the most popular estimation method: least

[3] This form of monetary policy rules in terms of key macroeconomic variables is derived from the seminal Taylor (1993) rule. This interest rate rule can be derived explicitly to maximize a policy objective function of a quadratic form, which can alternatively be viewed as a quadratic approximation to the welfare function of a representative agent. The form of the policy rules affects the determinacy and learnability properties of the NK model. Bullard and Mitra (2002) assess various interest-rate rules under learning and find that rules responding to expectations of inflation and output deviations are the most desirable to reach stability and determinacy.

squares. Thus, agents estimate by recursive least squares (RLS) this equation with past and current data:

$$y_t = a + bw_t \quad with\ y_t = (\pi_t, x_t)'\ and\ w_t = (g_t, u_t)'$$

Given the parameters estimated, agents form forecasts for variables relevant to their decision problems, on the basis of the shock they observe. This forecast is therefore obtained by so-called adaptive learning.

$$E_t^i y_{t+1} = a + bFw_t \quad with\ y_t = (\pi_t, x_t)'\ and\ w_t = (g_t, u_t)'$$

2.4 The standard case of E-stability

We now describe the standard case of adaptive learning in which expectations are homogenous. We therefore have $E_t^{CB} y_{t+1} = E_t^{PA} y_{t+1}$. By replacing the forecast in the reduced form, we obtain the actual law of motion (ALM) that describes a temporary equilibrium of the economy, that is to say temporary equilibrium relations between the variables.

$$y_t = D + (A^{PA} + A^{CB})a + ((A^{PA} + A^{CB})bF + B)w_t \qquad (5)$$

The conditions of convergence of the model are given by the local stability conditions of the associated ordinary differential equations (ODE). Evans and Honkapohja (2001, chapter 2) shows that the local stability is determined by this ODE:

$$d\theta / d\tau = T(\theta) - \theta \qquad (6)$$

where τ is "virtual" or "notional" time and $T(\theta)$ is the mapping function (T-maps) from PLM to ALM. The expectational stability (E-stability) of this model depends on the local stability of this ODE under RLS learning. The E-stability condition defines the convergence of estimations under adaptive learning towards the fundamental values of these parameters under rational expectations, and therefore the convergence of the economy to the RE equilibrium. The equilibrium is E-stable if and only if all eigenvalues of the Jacobian matrix of the function mapping the Perceived Law of Motion estimated by agents to the Actual Law of Motion (describing the temporary equilibrium of the economy) have negative real parts.

The mapping functions from PLM to ALM are the following:

$$T(a) = D + (A^{PA} + A^{CB})a \qquad (7)$$

$$T(b) = (A^{PA} + A^{CB})bF + B \qquad (8)$$

Bullard and Mitra (2002) show in the present case with a simple forward-looking interest rule that the derivations of E-stability conditions yield to the following inequality:

$$\lambda(\phi_\pi - 1) > \phi_x(\beta - 1) \qquad (9)$$

which is exactly the Taylor principle (as put forward by Taylor (1993) and Woodford (2001) and consists in a response of the central bank to inflation higher than respectively one or a threshold value equal to $1 + \phi_x(\beta - 1)/\lambda$) and is the necessary and sufficient condition for the E-stability of this model.

3. Endogenous Central Bank Influence

We here develop the theoretical model that combines heterogeneous forecasts and superior forecasting performance so as to analyze the effects of endogenous central bank influence.

3.1 Assumptions and the timing of decisions

Assumption 1: the central bank and private agents have imperfect knowledge of the model of the economy and imperfect information about the future state of the economy. They forecast the future inflation rate and output gap.
This assumption makes persistent forecasting errors possible. This implication appears to match the results of a large literature which has questioned the empirical realism of full-information rational expectations. Pesaran and Weale (2006) survey this literature prior to this date, while Andolfatto, Hendry and Moran (2007), Capistrán and Timmermann (2009), Andrade and Le Bihan (2010) and Coibion and Gorodnichenko (2010) further explore the evidence against the assumption of full-information rational expectations.

Assumption 2: the central bank observes both shocks the economy may face, while the private agents (households and firms) only observe one of the shocks.

This assumption of asymmetric information in favour of the central bank may be subject to debate, but is not decisive *per se* in this framework. It responds to the need for modelling a relatively better forecasting performance of the central bank compared to private agents in order to set up the endogenous influence framework. In this paper, by making the assumption that the central bank benefits from information asymmetry, we do not mean that *all* central banks have larger information sets than private agents. We rather suppose that asymmetric information can exist: in some cases the private sector has superior information, and in some other cases the central bank has superior information. We focus on the second case in order to model the relatively better forecasting performance that characterizes endogenous influence.

The superior forecasting performance of the central bank is therefore the *crucial* assumption of this model. Romer and Romer (2000) and Sims (2002) among others provide empirical evidence of the superior forecasting performance of the Federal Reserve, while Hubert (2011) shows that the forecasts of the Riksbank, from Sweden, outperforms those of private agents.

In the end, we could imagine that a relatively better forecasting performance may arise from different sources than information asymmetry, as for instance a better model of the economy or a better use of the same information set. However, this does not change the underlying rationale for this assumption: endogenous influence characterizes the central bank influence which drives private expectations towards the fundamental value of variables, and therefore stems from a better precision of central bank forecasts.

Assumption 3: the central bank communicates its macroeconomic forecasts to the public.

Since the beginning of the nineties, central banks have shifted towards more and more transparency along with the emergence of the inflation targeting framework. Publication of macroeconomic projections has therefore become a widespread activity among the majority of central banks, including inflation targeting central banks and others like the Federal Reserve with FOMC forecasts, the ECB with the Eurosystem macroeconomic projections and the Bank of Japan with its Outlook for Economic Activity and Prices.

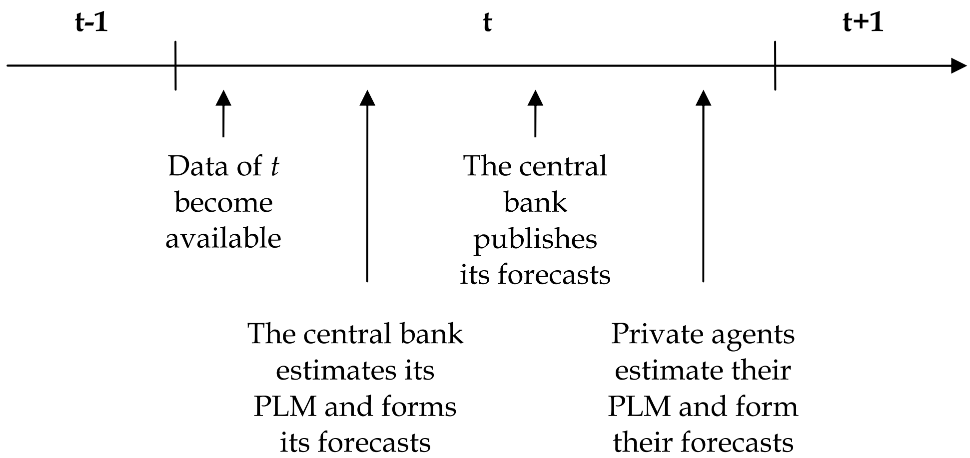
Assumption 4: private agents perfectly observe interest rates, but face uncertainty about the central bank procedure to set interest rate.

Private agents do not know the precise relationship between nominal interest rates, inflation and output – the policy rule – and preferences of the policymaker – the coefficients associated –. They therefore cannot infer central bank signals on future shocks from interest rate. The assumption that, despite increased transparency and communication, private agents do not know the exact procedure and weights used by monetary policymakers for interest rate setting seems plausible as the estimation of *ex post* monetary policy rules remains a subject of disagreement in the literature.

Assumption 5: in each period, private agents form their forecasts after the central bank ones.

Private agents are therefore able to incorporate central bank forecasts in their forecasting function. As soon as central bank forecasts, based on policy signals or private information, contain useful information for private agents, even if it is noisy information, private forecasts should be influenced by central bank forecasts. This assumption is supported by Fujiwara (2005) and Hubert (2011) that display strong evidence of the influential power of central banks on private agents, while there is no empirical evidence of the opposite.

Figure A: the timing of actions



The timing of actions is therefore essential to derive endogenous influence as it confers an advantage of second player to private agents. This advantage enables them to incorporate central bank forecasts in their forecasting function. The case of exogenous influence rests on the same timing assumption and makes central bank influence possible, but both types of influence differ in the extent that better central bank forecasts lead to smaller private forecast errors while central bank forecasts influential due to some policy signal content may increase private forecast errors. Thus, this second player advantage does not mean that private forecasts will necessarily be better.

3.2 The Model and E-Stability Conditions

In the following model, both private agents and the central bank have the same level of imperfect knowledge of the model of the economy and use a reduced-form equation to forecast future macroeconomic variables. On this point, they stand on an equal footing. However, the central bank and private agents have asymmetric information about the disturbances that affect the economy. Because of the central bank's superior forecasting performance, it is therefore rational for private agents to follow central bank forecasts.

The PLM of the central bank is

$$y_t = a^{CB} + b^{CB} \cdot w_t \tag{10}$$

and can be written

$$y_t = a^{CB} + b_g^{CB} \cdot g_t + b_u^{CB} \cdot u_t \qquad (11)$$

$$\text{with } a^{CB} = \begin{pmatrix} a_x^{CB} \\ a_\pi^{CB} \end{pmatrix}, \; b_u^{CB} = \begin{pmatrix} b_{xu}^{CB} \\ b_{\pi u}^{CB} \end{pmatrix} \text{ and } b_g^{CB} = \begin{pmatrix} b_{xg}^{CB} \\ b_{\pi g}^{CB} \end{pmatrix}$$

The functional form of the PLM (11) corresponds to the minimal state variable (MSV) solution of the reduced form (4), following McCallum (1983) who introduces this concept for linear rational expectations models. The MSV allows for a solution which depends linearly on a set of variables (here the coefficients associated to shocks and the intercept) and which is such that there is no solution which depends on a smaller set of variables.

The forecasting function of the central bank, when data from date *t* are available, is therefore:

$$E_t^{CB} y_{t+1} = a^{CB} + b_g^{CB} \cdot \mu \cdot g_t + b_u^{CB} \cdot \rho \cdot u_t \qquad (12)$$

The PLM of private agents is

$$y_t = a^{PA} + b^{PA} \cdot g_t + c^{PA} \cdot E_{t-1}^{CB} y_t \qquad (13)$$

and can be written

$$y_t = (a^{PA} + c^{PA} \cdot a^{CB}) + (b^{PA} + c^{PA} \cdot b_g^{CB}) \cdot g_t + c^{PA} \cdot b_u^{CB} \cdot u_t \qquad (14)$$

$$\text{with } a^{PA} = \begin{pmatrix} a_x^{PA} \\ a_\pi^{PA} \end{pmatrix} \text{ and } b^{PA} = \begin{pmatrix} b_x^{PA} \\ b_\pi^{PA} \end{pmatrix}$$

This form of the PLM of private agents is a simple forecasting function based on the only shock private agents observe, to which is added the central bank forecast they also observe. It is consistent with the linear combination of forecasts of Nelson (1972), Cooper and Nelson (1975), Fair and Shiller (1989, 1990) and Romer and Romer (2000). This approach consists in regressing the actual variable on forecasts made by both actors in order to know which one has the most accurate prediction of the actual variable. This linear combination of forecasts allows for comparing the relative forecasting performance of private agents and the central bank. If b^{PA} is equal to one, it means that private agents have better forecasts. At the opposite, if c^{PA} is equal to one, the central bank has more accurate forecasts. When private agents estimate (13), they determine whether the central bank forecasts contain information that could be useful to form their forecasts. In order to avoid explosive forecasts, the following condition should be met: $b^{PA} + c^{PA} = 1$.

Private agents thus compare their forecasting performance and the one of the central bank, using the criterion of the mean square forecast error. Private agents run the regression (13) and revise the weight given to each source of information over time according to the relative value of each source of information that they observe and their relation to output and inflation. The mechanism of endogenous influence can therefore be analysed as follows: if b^{CB} is equal to one, private agents put a weight of one on the central bank forecast – c^{PA} is equal to one – and they are influenced by the better forecasts of the central bank. At the opposite, the case of exogenous influence could be described as when b^{CB} is low, then c^{PA} will be equal to the value corresponding to what the marginal information or signal content of central bank forecasts would justify. In the end, the values taken by b^{CB} and b^{PA} reflect the forecasts' accuracy of both actors.

The forecasting function of private agents is then:

$$E_t^{PA} y_{t+1} = a^{PA} + b^{PA} \cdot \mu \cdot g_t + c^{PA} \cdot E_t^{CB} y_{t+1} \qquad (15)$$

and can be written

$$E_t^{PA} y_{t+1} = (a^{PA} + c^{PA} \cdot a^{CB}) + (b^{PA} + c^{PA} \cdot b_g^{CB})\mu \cdot g_t + c^{PA} \cdot b_u^{CB} \cdot \rho \cdot u_t \quad (16)$$

The analysis of central bank influence is therefore directly related to the analysis of the consequences of heterogeneous expectations between the central bank and private agents and how central bank influence may render both expectations homogeneous. The distinction between endogenous and exogenous influence therefore enables to differentiate central bank influence moving private expectations – the process of homogenisation of expectations – towards the fundamental value of variables from influence driving them away.

In addition, another important aspect of heterogeneous expectations deals with the specific properties of each type of expectations. Looking at a standard learning case, it appears from computing the determinant of $(A^{PA} - I)$ that one eigenvalue has a positive real part and therefore that private agents through their expectations have a destabilizing effect on the economy, while central bank expectations have a stabilizing effect. The reason is simple: when private expectations deviate from the fundamental value, the actual inflation or output deviates in the same direction what leads the private expectations to continue to deviate. This is because all the values of A^P are positive. At the opposite, when the central bank forecasts deviate upward, the actual inflation or output decreases. This is due to the fact that all the values of A^{CB} are negative, which acts as a counter weight to deviations. This is why central bank and private forecasts may have different effects for stability of the economy in the case of heterogeneous expectations. Thus, monetary policy has an important role to play in this model: it must be designed to offset the tendency toward instability from private agents' learning.

The properties of heterogeneous expectations on stability conditions have first been studied by Evans and Honkapohja (1997) and Giannitsarou (2003). More recently, Honkapohja and Mitra (2006) find that the interaction of structural and expectational heterogeneity may make the conditions for stability more severe than those under homogeneous or transitional heterogeneous expectations. The benchmark E-stability condition continues nevertheless to have implications as it yields necessary conditions for convergence of persistently heterogeneous learning. In a companion paper, Honkapohja and Mitra (2005) show that when private agents and central banks have different expectations the Taylor principle is sufficient to reach E-stability if the learning algorithms are asymptotically similar. McCallum and Nelson (2004) moreover show that when the central bank uses its own forecasts and have different learning rules, we may have stability under the same conditions as in the homogeneous case if the central bank puts a relatively high weight on new information about the state of the economy and future shocks at each period (i.e. δ_{CB} is superior to 0.7 in their paper) what seems likely in theory and practice. In this paper, we consider heterogeneity of forecasts and use the Recursive Least Squares learning algorithms for both private agents and the central bank since it is the standard approach in the literature.

Putting central bank and private forecasts in the reduced-form describing aggregate dynamics and supposing without loss of generality that $\phi_0 = 0$ and so $D = 0$, the actual law of motion of the economy is then:

$$\begin{aligned} y_t &= A^{PA} \cdot (a^{PA} + c^{PA} \cdot a^{CB}) + A^{CB} \cdot a^{CB} \ldots \\ &\ldots + (A^{PA} \cdot (b^{PA} + c^{PA} \cdot b_g^{CB}) + A^{CB} \cdot b_g^{CB})\mu + B_g) \cdot g_t \\ &\ldots + ((A^{PA} \cdot c^{PA} \cdot b_u^{CB} + A^{CB} \cdot b_u^{CB})\rho + B_u) \cdot u_t \end{aligned} \quad (17)$$

Following Evans and Honkapohja (2003a), the T-maps defining E-stability and derived from the correspondence between PLM to ALM are then:

$$T(a^{CB}) = A^{PA} \cdot (a^{PA} + c^{PA} \cdot a^{CB}) + A^{CB} \cdot a^{CB} \quad (18)$$

$$T(a^{PA}) = A^{PA} \cdot (a^{PA} + c^{PA} \cdot a^{CB}) + A^{CB} \cdot a^{CB} - c^{PA} \cdot a^{CB} \quad (19)$$

$$T(b_g^{CB}) = (A^{PA} \cdot (b^{PA} + c^{PA} \cdot b_g^{CB}) + A^{CB} \cdot b_g^{CB})\mu + B_g \quad (20)$$

$$T(b^{PA}) = (A^{PA} \cdot (b^{PA} + c^{PA} \cdot b_g^{CB}) + A^{CB} \cdot b_g^{CB})\mu + B_g - c^{PA} \cdot b_g^{CB} \quad (21)$$

$$T(b_u^{CB}) = (A^{PA} \cdot c^{PA} \cdot b_u^{CB} + A^{CB} \cdot b_u^{CB})\rho + B_u \quad (22)$$

$$T(c^{PA}) = (A^{PA} \cdot c^{PA} + A^{CB})\rho + B_u (b_u^{CB})^{-1} \quad (23)$$

The equations for (a^{CB}, a^{PA}), (b_g^{CB}, b^{PA}) and (b_u^{CB}, c^{PA}) are independent of each other. Following the work of Honkapohja and Mitra (2006) on heterogeneous forecasts, the E-stability of the subsystems is satisfied if and only if all the eigenvalues of the Jacobian of M_1, M_2 and M_3 have negative real parts.

$$M_1 = \begin{pmatrix} T(a^{CB}) - a^{CB} \\ T(a^{PA}) - a^{PA} \end{pmatrix}$$

$$M_2 = \begin{pmatrix} T(b_g^{CB}) - b_g^{CB} \\ T(b^{PA}) - b^{PA} \end{pmatrix}$$

$$M_3 = \begin{pmatrix} T(b_u^{CB}) - b_u^{CB} \\ T(c^{PA}) - c^{PA} \end{pmatrix}$$

Proposition: The model is E-stable under learning if all eigenvalues of the following matrix $A^{PA} + A^{CB} - I$ have negative real parts. It corresponds to the following inequality:

$$\lambda(\phi_\pi - 1) > \phi_x(\beta - 1) \quad \textit{(24)}$$

The proof is in the Appendix. This condition is exactly the Taylor principle, the condition for stability in the RE case or with learning and homogenous forecasts.

Indeed, at the steady state of endogenous influence, expectations are homogeneous between private agents and the central bank. Expectations are the same as those of the MSV solution of the model. The Taylor principle is therefore a sufficient condition to reach E-stability when starting from heterogeneous expectations, superior central bank forecasts make them homogeneous. Compared to the existing literature on heterogeneous expectations, endogenous influence may be considered as another mechanism to reach E-stability only by satisfying the Taylor principle.

3.3 Determinacy

In the preceding subsection, we have obtained the E-stability condition but not the determinacy condition that defines the uniqueness of the equilibrium (if a unique stationary solution exists, the economy is said to be determinate; if multiple equilibria exist, the model is indeterminate). This issue is particularly important when the monetary policy rule of the central bank comprises forward-looking components as Bernanke and Woodford (1997) show. We have first focused on the MSV solution, unique in this model.

The standard determinacy condition is stated by Blanchard and Kahn (1980). In the case of RE, the equation (4) can be rewritten:

$$y_t = D + (A^{PA} + A^{CB}) \cdot E_t y_{t+1} + B \cdot w_t \quad (25)$$

and the determinacy condition is therefore $|A^{PA} + A^{CB}| < 1$ and leads to the resulting condition:

$$\phi_x(\beta + 1) > \lambda(\phi_\pi - 1) > \phi_x(\beta - 1) \quad (26)$$

Bernanke and Woodford (1997) conclude from this proposition that the central bank should not respond too aggressively to future inflation, since it may raise the possibility of sunspot equilibria. Bullard and Mitra (2002) shows that monetary rule based on expectations deliver both E-stability and determinacy compared to fundamentals-based rules.

In the situation in which private agents are following the central bank forecasts because of a central bank's better forecasting record, the E-stability given by *(24)* and the determinacy condition is similar to (26):

$$\phi_x(\beta + 1) > \lambda(\phi_\pi - 1) > \phi_x(\beta - 1) \quad (27)$$

The determinacy condition is a sufficient condition for the E-stability of the equilibrium and this is consistent with McCallum (2007) that shows that in a forward-looking model where the current period information set is available to agents to form their forecasts through adaptive learning, the determinacy condition is sufficient for E-stability. Our result for a forward-looking monetary rule can be extended to different rules comprising a smoothing parameter or current-period data as Bullard and Mitra (2002) show that a monetary rule with current-period data becomes more robust for the determinacy because in the New Keynesian model, current inflation is determined by private agents' expected inflation and the central bank therefore respond to its inflation forecasts and private forecasts. Moreover, Bullard and Mitra (2007) show that a monetary rule with some interest smoothing is also more robust for the determinacy of the REE.

3.4 Reversing the shock observed by private agents

If we consider that private agents only observe the cost-push shock u_t (rather than the demand shock g_t) and the central bank still observes both shocks, the condition for stability related to the central bank preferences is the same as above and the second condition on the economy shown in the appendix is also similar with μ replacing ρ .

3.5 Comparison to exogenous influence

The distinction between endogenous and exogenous influence enables to differentiate central bank influence moving private expectations towards the fundamental value of variables from influence driving them away. Indeed, the case of exogenous influence corresponds to the situation where the central bank influences private agents because of policy signals or information sets are not nested and independently of its forecasting performance. Thus, private agents incorporate central bank forecasts in their forecasting function whereas those forecasts do not have lower forecast errors. This influential power may also stem from the central bank position of leader and private agents of followers in the monetary process.

Muto (2011) assesses one configuration of exogenous influence in which private agents only refer to the central bank forecast when forming their own forecasts. His main result is that if the central bank is the leader of expectations formation, the forward-looking monetary policy rule faces instability and therefore the central bank must be more restrictive than the Taylor principle suggests and satisfy this condition:

$$\lambda(\phi_{\pi} - 2) > \phi_x 2(\beta - 1) \tag{28}$$

The mechanism at work is the following. The central bank makes some forecast errors and because private agents can only refer to the central bank forecasts, the noise with which private agents follow the central bank forecasts is correlated to the central bank forecast errors. Then, private forecasts errors (or following errors) add to the central bank forecast errors and needs the central bank to respond more aggressively to inflation to reach E-stability. This result stems from the cumulative process of central bank and private forecasting errors, but is nevertheless common to all configurations of exogenous influence: if private and central bank errors are in average similar - defining exogenous influence -, the central bank does not know at each period whether the forecasting performance of private agents is better than its own and must therefore respond at each period more aggressively to inflation to avoid potential self-fulfilling expectations. The higher response to inflation actually stems from the uncertainty faced by policymakers at each period about the possibility that private and central bank forecasts remains heterogeneous. Even in a framework where private forecasts are a combination of the private signal and the central bank forecasts – as described in equation (15) – and where the given weight associated with the central bank forecast represents the marginal value of the policy signal or private information disclosed by the central bank, the central bank must respond more aggressively to inflation than the Taylor principle suggests because the central bank must offset the potential destabilizing effect of private forecasts.

In comparison, in the situation of endogenous influence private agents reduce their forecast errors and both expectations become homogeneous. The central bank therefore needs to respond only to its forecast errors. The Taylor principle is thus a sufficient condition to reach E-stability.

A complementary question would therefore be whether the central bank should follow private forecasts rather than influence them when central bank forecasts are not clearly superior. Indeed, homogeneous expectations would only require the central bank to satisfy the Taylor principle as shown by Bullard and Mitra (2002) and confirmed in the case of private agents influencing central bank forecasts by Muto (2011). The objective of central banks should therefore be to avoid the situation of exogenous influence to reach macroeconomic stability at a lower cost. In the case of inferior forecasting performance, the central bank should respond in its interest rate rule to private forecasts to ensure macroeconomic stability more easily, by contrast to the case of superior forecasting performance in which it should respond to its internal forecasts.

3.6 Comparison to other heterogeneous cases without influence

Honkapohja and Mitra (2006) analyse the situations in which a part of the agents (private or the central bank) is learning, while the other part (the central bank or private agents) have rational expectations (RE). In a similar New-Keynesian framework, they show that when the central bank has RE and private agents are learning, the condition for stability of the economy is precisely the Taylor principle as for homogenous rules under learning. The intuition is that because the central bank has more information than the private agents, it is able to stabilize the economy whatever the potential destabilizing effect of private expectations (which stems from the positive eigenvalue of $A^{PA} - I$). At the opposite, when private agents have RE and the central bank is learning, the necessary condition for the economy to be stable needs the Taylor principle to be violated. However, violation of the Taylor principle is not a sufficient condition as the conditions are related to μ and ρ. According to benchmark numerical parameters used in the literature, the economy may be stable or unstable even if the Taylor principle is not met.

Honkapohja and Mitra (2004) focus on the case where both the central bank and private agents are learning, but there is some information asymmetry between the two types of agents. They show that the conditions are more restrictive when the central bank has less information compared to when it has more information then private agents. Asymmetric information in favour of the central bank makes stability easier to reach. This paper therefore suggests that central bank should allocate enough resources for information acquisition about the state of the economy and future shocks.

4. Conclusion

We study central bank policymaking in a situation of endogenous influence in which the central bank has a better forecasting performance. In contrast to the situation in which the central bank influences private agents without better forecasts (i.e. the case of exogenous influence) and must respond more strongly to future inflation than the Taylor principle suggests; this paper shows that the central bank must only respect the Taylor principle and need not be more restrictive to ensure macroeconomic stability. The policy implications of this work are that, in a situation of imperfect information, influencing private expectations should not be an objective *per se* and central banks should increase the quality of their macroeconomic forecasts to guide private expectations. It would enable them to reach macroeconomic stability at a lower cost. Another solution to avoid the situation of exogenous influence would be that the central bank responds in its interest rate rule to private forecasts rather than to its internal forecasts in the case of inferior forecasting performance.

References

Amato, J., S. Morris and H.S. Shin (2002), "Communication and Monetary Policy", *Oxford Review of Economic Policy,* 18, 495–503.

Amato, J. and H.S. Shin (2006), "Imperfect common knowledge and the information value of prices", *Economic Theory*, vol. 27(symposium issue on monetary policy), 213–41.

Andolfatto, D., S. Hendry and K. Moran (2007), "Are Inflation Expectations Rational?", *Journal of Monetary Economics*, 55(2), 406-422.

Andrade, P., and H. Le Bihan (2010), "Inattentive Professional Forecasters", mimeo.

Angeletos, G.-M. and A. Pavan (2004), "Transparency of information and coordination in economies with investment complementarities", *American Economic Review*, 94(2), 91–8.

Angeletos, G.-M. and A. Pavan (2007), "Efficient Use of Information and Social Value of Information", *Econometrica,* 75 (4), 1103–1142.

Barro, R.J., and D.B. Gordon (1983), "A Positive Theory of Monetary Policy in a Natural Rate Model", *Journal of Political Economy,* 91(4), 589–610.

Bernanke, B. and M. Woodford (1997), "Inflation forecasts and monetary policy", *Journal of Money, Credit and Banking*, 24, 653-684.

Blanchard, O., and C. Kahn (1980), "The Solution of Linear Difference Models under Rational Expectations", *Econometrica*, 48 (5), 1305-1313.

Bullard, J. and K. Mitra (2002), "Learning about monetary policy rules", *Journal of Monetary Economics*, 49, 1105-1129.

Bullard, J. and K. Mitra (2007), "Determinacy, Learnability, and Monetary Policy Inertia", *Journal of Money, Credit, and Banking*, 39 (5), 1177-1212.

Capistrán, C. and A. Timmermann (2009), "Disagreement and Biases in Inflation Expectations", *Journal of Money, Credit and Banking*, 41(2-3), 365-396.

Clarida, R., J. Gali and M. Gertler (1999), "The science of monetary policy: a new Keynesian perspective", *Journal of Economic Literature*, 37, 1661-1707.

Coibion, O. and Y. Gorodnichenko (2010), "Information Rigidity and the Expectations Formation Process: A Simple Framework and New Facts", *NBER Working Paper* 16537.

Cooper, J.P. and C.R. Nelson (1975), "The Ex Ante Prediction Performance of the St. Louis and FRB-MIT-PENN Econometric Models and Some Results on Composite Predictors", *Journal of Money, Credit and Banking*, 7(1), 1-32.

Cornand, C. and F. Heinemann (2008), "Optimal Degree of Public Information Dissemination", *Economic Journal*, 118, 718-742.

Ehrmann, M., S. Eijffinger and M. Fratzscher (2009), "The Role of Central Bank Transparency for Guiding Private Sector Forecasts", CEPR Discussion Paper No 7585.

Eusepi, S. and B. Preston (2010), "Central Bank Communication and Expectations Stabilization", *American Economic Journal: Macroeconomics*, 2.

Evans, G. and S. Honkapohja (1997), "Least squares learning with heterogeneous expectations", *Economic Letters* 52, 197–201.

Evans, G. and S. Honkapohja (2001), Learning and Expectations in Macroeconomics. Princeton Univ. Press, Princeton, NJ.

Evans, G. and S. Honkapohja (2003a),"Adaptive learning and monetary policy design", *Journal of Money, Credit and Banking*, 35, 1045–1072.

Evans, G. and S. Honkapohja (2003b), "Expectations and the Stability Problem for Optimal Monetary Policies", *Review of Economic Studies*, 70, 807-824.

Fair, R.C. and R. Shiller (1989), "The Informational Content of Ex Ante Forecasts", *Review of Economics and Statistics*, 71, pp. 325-331.

Fair, R.C. and R. Shiller (1990), "Comparing Information in Forecasts from Econometric Models", *American Economic Review*, 80, pp. 375-389.

Fujiwara, I. (2005), "Is the Central Bank's Publication of Economic Forecasts Influential?", *Economics Letters*, 89, 255-261.

Geraats, P. (2002), "Central Bank Transparency", *Economic Journal*, 112(483), 532–65.

Geraats, P. (2005), "Transparency and reputation: The publication of central bank forecasts", *Topics in Macroeconomics,* 5(1.1), 1–26.

Giannitsarou, C. (2003), "Heterogeneous learning", *Review of Economic Dynamics*, 6, 885–906.

Guesnerie, R. (2005), Assessing Rational Expectations 2: Eductive Stability in economics. MIT Press.

Guesnerie, R. (2008), "*Macroeconomic and monetary policies from the "eductive" viewpoint", in* Monetary Policy under Uncertainty and Learning, edited by Klaus Schmidt-Hebbel and Carl Walsh, 2008, Central Bank of Chile.

Honkapohja, S. and K. Mitra (2004), "Monetary Policy with Internal Central Bank Forecasting. A Case of Heterogenous Information" in From National Gain to Global Markets, Essays in Honour of Paavo Okko on His 60th Birthday. Edited by Mika Widgren.

Honkapohja, S. and K. Mitra (2005), "Performance of Monetary Policy with Internal Central Bank Forecasting", *Journal of Economic Dynamics and Control*, 29, 627-658.

Honkapohja, S. and K. Mitra (2006), "Learning stability in economies with heterogeneous agents", *Review of Economic Dynamics*, 9, 284–309.

Hellwig, C. (2005), "Heterogeneous Information and the Welfare Effects of Public Information Disclosures", Working Paper UCLA.

Hubert, P. (2011), "Do Central Bank Forecasts Influence Private Agents? Forecasting Performance vs. Policy Signals", mimeo.
King, R. (1982), "Monetary Policy and the Information Content of Prices", *Journal of Political Economy,* 90(2), 247–279.
King, R.G., Y. Lu and E. Pasten (2008), "Managing Expectations", *Journal of Money, Credit and Banking,* 40(8), 1625 – 1666.
Kydland, F.E., and E.C. Prescott. (1977), "Rules Rather Than Discretion: The Inconsistency of Optimal Plans." *Journal of Political Economy,* 85:3, 473–91.
Lucas, R. E. (1972), "Expectations and the Neutrality of Money," *Journal of Economic Theory*, 4, 103—124.
Mankiw, N. G. et R. Reis (2002), "Sticky Information versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve", *Quarterly Journal of Economics*, 110.
Mankiw, N. G. et R. Reis (2007), "Sticky Information in General Equilibrium", *Journal of the European Economic Association*, 5(2–3), 603–613.
McCallum, B. T. (1983), "On Non-Uniqueness in Linear Rational Expectations Models: An Attempt at Perspective," *Journal of Monetary Economics*, 11,134–168.
McCallum, B.T. (1999), "Issues in the Design of Monetary Policy Rules", Chapter 23, in Taylor and Woodford (eds.) 1483-1530.
McCallum, B.T. and E. Nelson (2004), "Timeless perspective vs. discretionary monetary policy in forward-looking models", *Federal Reserve Bank of St. Louis Review* 86, 43–56.
McCallum, B.T. (2007), "E-stability vis-à-vis Determinacy Results for a Broad Class of Linear Rational Expectations Models", *Journal of Economic Dynamics and Control*, 31, 1376-1391.
Morris, S. and H.S. Shin (2005), "Central bank transparency and the signal value of prices", *Brookings Papers on Economic Activity*, 2, 1–66.
Moscarini, G. (2007), "Competence Implies Credibility", *American Economic Review*, 97(1), 37-63.
Muth, J. F. (1961), "Rational Expectations and the Theory of Price Movements," *Econometrica*, 29, 315-335.
Muto, I. (2011), "Monetary Policy and Learning from the Central Bank's Forecast", *Journal of Economic Dynamics and Control*, 35(1), 52-66.
Nelson, C.R. (1972), "The Prediction Performance of the FRB-MIT-PENN Model of the U.S. Economy", *American Economic Review*, 62(5), 902-917.
Pesaran, M. and M. Weale (2006), "Survey Expectations", in *Handbook of Economic Forecasting*, Elliott, Granger and Timmermann (eds.), North-Holland Press.
Preston, B. (2008), "Adaptive Learning and the Use of Forecasts in Monetary Policy", *Journal of Economic Dynamics and Control*, 32.
Romer, C. and D. Romer (2000), "Federal Reserve Information and the Behavior of Interest Rates", *American Economic Review,* 90, 429–457.
Sargent, T. J. (1973), "Rational Expectations, the Real Rate of Interest and the Natural Rate of Unemployment", *Brookings Papers on Economic Activity*, 2, 429—472.
Sims, C. (1998), "Stickiness", *Carnegie-Rochester Series on Public Policy*, 49(1), 317-356.
Sims, C. (2002), "The Role of Models and Probabilities in the Monetary Policy Process,"*Brookings Papers on Economic Activity*, 2, pp. 1–62.
Sims, C. (2003), "Implications of Rational Inattention", *Journal of Monetary Economics*, 50(3), 665-690.
Taylor, J. (1993), "Discretion versus Policy Rules in Practice", Carnegie-Rochester Conference Series in Public Policy, 39, 195-214.
Svensson, L. (2006), "Social value of public information: Morris and Shin (2002) is actually pro transparency, not con", *American Economic Review*, 96(1), 448–51.

Walsh, C.E. (2006), “Transparency, flexibility, and inflation targeting”, in (F. Mishkin and K. Schmidt-Hebbel, eds.), Monetary Policy under Inflation Targeting, 227–263, Central Bank of Chile, Santiago, Chile.

Woodford, M. (2001), “The Taylor Rule and Optimal Monetary Policy”, *American Economic Review,*91(2), 232-237.

Woodford, M. (2003), “Imperfect Common Knowledge and the Effects of Monetary Policy.” in *Knowledge, Information and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, P.Aghion, R.Frydman, J.Stiglitz and M.Woodford (eds), Princeton University Press.

APPENDIX

The T-maps defining E-stability and derived from PLM to ALM are then:

$$T(a^{CB}) = A^{PA} \cdot (a^{PA} + c^{PA} \cdot a^{CB}) + A^{CB} \cdot a^{CB} \quad (18)$$

$$T(a^{PA}) = A^{PA} \cdot (a^{PA} + c^{PA} \cdot a^{CB}) + A^{CB} \cdot a^{CB} - c^{PA} \cdot a^{CB} \quad (19)$$

$$T(b_g^{CB}) = (A^{PA} \cdot (b^{PA} + c^{PA} \cdot b_g^{CB}) + A^{CB} \cdot b_g^{CB})\mu + B_g \quad (20)$$

$$T(b^{PA}) = (A^{PA} \cdot (b^{PA} + c^{PA} \cdot b_g^{CB}) + A^{CB} \cdot b_g^{CB})\mu + B_g - c^{PA} \cdot b_g^{CB} \quad (21)$$

$$T(b_u^{CB}) = (A^{PA} \cdot c^{PA} \cdot b_u^{CB} + A^{CB} \cdot b_u^{CB})\rho + B_u \quad (22)$$

$$T(c^{PA}) = (A^{PA} \cdot c^{PA} + A^{CB})\rho + B_u (b_u^{CB})^{-1} \quad (23)$$

The expectational stability (E-stability) of the Actual Law of Motion (ALM) is satisfied if these T-maps are locally stable, what is satisfied if and only if all eigenvalues of the Jacobian of M_1, M_2 and M_3 have negative real parts. Those Jacobian matrices are computed at the equilibrium values of endogenous influence, i.e. $b_u^{CB} = b_g^{CB} = 1$ according to the *Assumption 2*. It derives from this assumption that a^{PA}=0, b^{PA}=0, and c^{PA}=1.

$$J_1 = \begin{pmatrix} A^{PA} + A^{CB} - I & A^{PA} \\ A^{PA} + A^{CB} - I & A^{PA} - I \end{pmatrix}$$

$$J_2 = \begin{pmatrix} \mu(A^{PA} + A^{CB}) - I & \mu A^{PA} \\ \mu(A^{PA} + A^{CB}) - I & \mu A^{PA} - I \end{pmatrix}$$

$$J_3 = \begin{pmatrix} \rho(A^{PA} + A^{CB}) - I & \rho A^{PA} \\ -B_u & \rho A^{PA} - I \end{pmatrix}$$

Following Honkapohja and Mitra (2006), the determinant for computing the eigenvalues of J_1, J_2 and J_3 may be simplified as follows

$$\det(J_1) = \begin{vmatrix} A^{PA} + A^{CB} - I & A^{PA} \\ A^{PA} + A^{CB} - I & A^{PA} - I \end{vmatrix}$$

$$\det(J_1) = \begin{vmatrix} 0 & I \\ A^{PA} + A^{CB} - I & A^{PA} - I \end{vmatrix}$$

After subtracting the second row from the first, the computation shows that J_1 has eigenvalues with negative real parts if and only if $A^{PA} + A^{CB} - I$ has the same property. Similarly, we obtain:

$$\det(J_2) = \begin{vmatrix} \mu(A^{PA} + A^{CB}) - I & \mu A^{PA} \\ \mu(A^{PA} + A^{CB}) - I & \mu A^{PA} - I \end{vmatrix}$$

$$\det(J_2) = \begin{vmatrix} 0 & I \\ \mu(A^{PA} + A^{CB}) - I & \mu A^{PA} - I \end{vmatrix}$$

After subtracting the second row from the first, the computation shows that J_2 has eigenvalues with negative real parts if and only if $\mu(A^{PA} + A^{CB}) - I$ has the same property. Because $0 < \mu < 1$, it suffices to have only the eigenvalues of $A^{PA} + A^{CB} - I$ for E-stability. The necessary and sufficient condition of J_2 is therefore similar to the one for J_1.

As the system of M_3 is not linear, the Jacobian J_3 is analyzed differently. For b_u^{CB}, the standard E-stability arguments apply and yield to the same property than for J_1 and J_2, because $0<\rho<1$. For c^{PA}, the E-stability condition is $\rho A^{PA}-I$.

For the special case of a 2× 2 matrix *A*, it can be shown that the condition that both roots of *A* have negative real parts is equivalent to the condition that the trace of *A* is negative and the determinant of *A* is positive. Thus, all the eigenvalues of $A^{PA}+A^{CB}-I$ have negative real parts if and only if the two conditions apply. It corresponds to the following inequalities:

$$\varphi\phi_x-\lambda\varphi+\lambda\varphi\phi_\pi-\beta\varphi\phi_x>0 \tag{29}$$

$$\varphi\phi_x-\lambda\varphi-\beta+\lambda\varphi\phi_\pi+1>0 \tag{30}$$

If (29) holds then (30) holds. The E-stability condition therefore corresponds to (29) that can be rewritten:

$$\lambda(\phi_\pi-1)>\phi_x(\beta-1) \tag{24}$$

The second condition needs ρ the autocorrelation of the cost push shock to be sufficiently small as A^{PA} has an eigenvalue higher than one (see Honkapohja and Mitra (2004) for more details), but has no effect on the optimal responses to inflation or output of the central bank to reach E-stability.