Adjustment capacity in a monetary union: a DSGE evaluation of Poland and Slovakia

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Abstract

Once a country joins a monetary union, an efficient competitiveness channel is commonly considered to be the main substitute for the abandoned autonomous monetary and exchange rate policy. This paper attempts to make an empirical assessment of how the price competitiveness of domestic producers stabilizes the Polish and Slovak economies in comparison to the real interest rate, that could potentially be procyclical in EMU. To address this issue, we revisit the econometric studies of weighting the Monetary Conditions Index (MCI-ratio) to explore the relative importance of the two channels. We apply the IS-curve approach and – as opposed to previous MCI-ratio literature – we build a small open economy DSGE framework to be estimated with classical forward-looking techniques (FIML). We compare the estimates and impulse-response functions for Poland and Slovakia, concluding that the latter country seems to be more capable of handling asymmetric demand shocks under the common monetary policy. At the same time, it remains more vulnerable to asymmetric cost-push shocks. Also, we examine the consequences of a permanent fall in the interest rates in the home economy after the accession to the monetary union. If there was a natural interest rate disparity of 1 percentage point in favour of the catching-up economy and agents expected a 30-year long period of closing this gap, our model would predict a terms of trade appreciation for both countries in question, whereby the required appreciation would be more pronounced for Poland than for Slovakia (9.9% and 2.6% respectively). In the context of Slovak revaluations in ERM II, this fact could be taken into consideration when setting the final conversion rate, along with its further pros and cons.

Keywords: competitiveness channel, EMU, MCI-ratio, DSGE, forward-looking estimation.

JEL Classification: C22, C32, F15, F41.

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1 Introduction

Once a country joins a monetary union, its capacity of absorbing asymmetric shocks via policy tools is significantly reduced. Namely, it resigns from autonomous monetary policy and, to a large extent, nominal exchange rate volatility. The burden of adjustment shifts to market-based mechanisms, described in the literature as the competitiveness channel (de Grauwe, 2007; European Commission, 2008). After an asymmetric shock and in the absence of nominal exchange rate fluctuations, as well as fine-tuning possibilities with country-specific nominal interest rate, the speed of reversal to equilibrium is mainly conditional on the price dynamics, and thus on market flexibility, intrinsic inflation persistence and the process of forming expectations (see Torój, 2009).

In addition, this reversal could be hampered by procyclicality of country-specific real interest rates in a monetary union. When a positive demand shock raises the inflation rate and there is a close link between the inflation rate and inflation expectations on a country level, a real interest rate declines, which additionally fuels the boom (European Commission, 2006). A positive impulse for the economic activity can also stem from the fact that an economy with higher natural rate of interest joins a monetary union where lower interest rate level prevails.

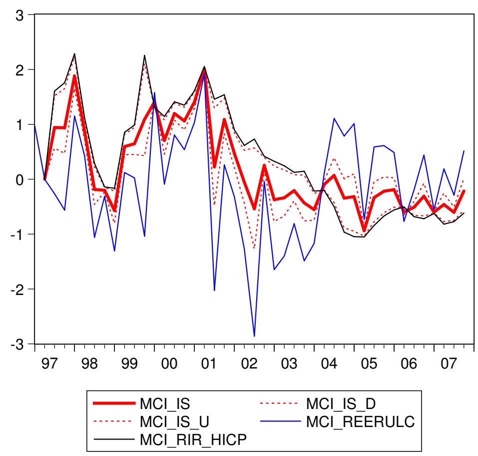
There are dynamic interactions between the competitiveness channel and the real interest rate effect. The former is commonly believed to be an equilibrating force in the long run, whereby the latter – to boost output and inflation volatility via short run effects (Arnold and Kool, 2004; Roubini et al., 2007; European Commission, 2008; Wójcik, 2008). However, a joint consideration of both mechanisms seems to be the appropriate approach to model the adjustment process in the aftermath of an asymmetric shock (see e.g. Hoeller et al., 2004; European Commission, 2006; Torój, 2009), as it provides better insight into the adjustment dynamics in general.

This paper aims to contribute to the literature by performing an *ex ante* econometric assessment of the adjustment capacity in Poland. Using historical data for the 1996-2009 period, it assesses the relative sensitivity of the Polish economic activity to the real interest rate and real exchange rate changes. For comparative reasons perform analogous estimations for Slovakia – a new EU member state that adopted the euro in 2009.

The Slovak case is of particular interest because of its ERM II experience. In the course of ERM II participation, the Slovak Koruna was twice revalued. Despite the fact that its appreciation was probably to a large extent in line with equilibrium, this may have partly affected the price competitiveness of Slovak goods on international markets. We attempt to establish a link between this permanent appreciation, followed by revaluations, and the permanent fall in Slovak nominal interest rates on their way to the euro area. In particular, we are interested whether these two shocks could be offseting each other as a positive and negative demand shock. We also attempt to perform a quantitative assessment of this offsetting power.

The issue of the relative impact of the real exchange and interest rates on economic activity in a small open economy has been heavily discussed at the turn of the centuries, when a concept of Monetary Conditions Index (MCI) was popular among policymakers (see Freedman, 1994, 1995, for the foundations of this concept). The main purpose of the MCI was to combine the interest rate

Figure 1: MCI for Poland (IS curve approach) with 90% confidence interval (due to weight uncertainty), 1997-2007



MCI_IS – the MCI, MCI_IS_D and MCI_IS_U – the MCI calculated with weights hitting upper and lower bound of the confidence interval for the estimated MCI ratio respectively, MCI_RIR_HICP – the real interest rate component of the MCI (calculated ex post with contemporaneous y/y HICP dynamics), MCI_REERULC – the real exchange rate component of the MCI (deflated with unit labour costs).
Source: Torój (2008).

with the exchange rate to provide a more adequate assessment of monetary policy stance for a small economy than the interest rate only (see Figure 1 for the estimates for Poland). Out of a number of econometric techniques developed for the purpose of MCI-ratio estimation (i.e. the impact of real interest rate divided by the impact of real exchange rate), we adopt here the IS-curve approach, i.e. we look at the relative importance of both variables in controlling the output gap dynamics. Our dynamic IS curve is part of a micro-founded New Keynesian DSGE model.

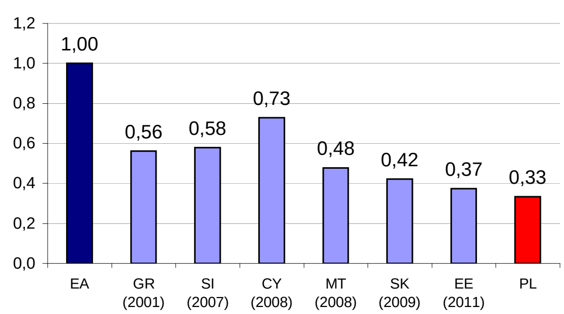
Having estimated the MCI-ratios for Poland and Slovakia, we perform simulations of the adjustment path after an asymmetric shock that hits both countries within the euro area. This can be seen as an econometric approach to an *ex ante* assessment of the competitiveness channel's efficiency in EMU, delivering some complementary information to institutional characteristics of both economies. Also, we argue that these ratios could provide some information on the required size of real appreciation that could potentially offset the positive demand pressure from the natural interest rate differential between the new member states and the euro area under common monetary policy with lower interest rates than in the NMS before the euro area entry.

The rest of the paper is organized as follows. Section 2 reviews some stylized facts about Slovakia's road to the euro, as well as the literature on estimating the relative impact of the real interest rate and the real exchange rate on the economic activity (MCI-ratio). In Section 3, a DSGE model is developed. Section 4 contains the estimation results for Poland and Slovakia. In Section 5, the influence of MCI-ratio on the adjustment capacity after an asymmetric shock is assessed on the basis of impulse-response functions. Section 6 investigates the mid-term consequences of natural interest rates misalignment and implications of central parity revaluation within the model developed in Section 3. Section 7 concludes.

2 Why does the MCI-ratio matter for the adjustment dynamics?

The history of euro area enlargements (Greece 2001, Slovenia 2007, Cyprus and Malta 2008, Slovakia 2009) exhibits a remarkable pattern (see Figure 2). When we look at the GDP *per capita* on the eve of euro adoption in individual countries, we can see a downward-sloping curve (with the only notable exception of Cyprus in 2008). On January 1st 2009 Slovakia joined the monetary union at a record low of 42% of euro area GDP *per capita*. If Poland had done the same in 2009, the Slovak record would have been beaten by 9 percentage points.

Figure 2: GDP *per capita* in the year preceding euro area entry [% EA]



Source: Eurostat data.

One can think of two ways in which this discrepancy matters for the country under ECB's common monetary policy. Following the approach adopted by Flaig and Wollmershäuser (2007) or Calmfors (2007), we can differentiate between two possible sorts of common interest rate misalignment: the **cyclical** and the **structural** one.

Firstly, economies with low levels of income could possibly be characterized by different structures of production and/or consumption than high-income economies. Empirical research confirms that this is to some extent the case for Poland (Adamowicz et al., 2009). This makes such countries exposed to asymmetric shocks after joining a monetary union. Once such a shock occurs, it would most probably lead to a cyclical divergence in inflation. If a central bank in an autonomous monetary regime follows a Taylor-type rule (Taylor, 1993), e.g. $i_t = \gamma_\pi \pi_t + \gamma_y y_t$ (i – nominal interest rate, π – inflation rate, y – output gap), it would bring the economy back to the unique equilibrium as long as it complies with the Taylor principle, which requires a more than one-to-one response to inflation ($\gamma_\pi > 1$).

However, with $\gamma_\pi < 1$, sunspot equilibria may occur. In a monetary union the central bank is concerned with the aggregate price stability. This means that in an example 2-country monetary union, it responds to a regional weighted average, $\pi_t = w\pi_{1t} + (1 - w)\,\pi_{2t}$. Under these circumstances, the response to the country-specific inflation rate, $\gamma_\pi w$, could be far below 1. This argument lied at the heart of "Walters critique" (Walters, 1994). However, the reasoning was incomplete because it left aside open economy considerations. When the domestic producers are involved in international trade and consumers could switch to imported products, an economy cannot afford a boom that deteriorates its competitiveness in the long run. It is the competitiveness channel that restores the long-term equilibrium.

Secondly, the New Keynesian literature (e.g. Clarida et al., 1999; Galí, 2008) establishes a widespread link between the natural interest rate (i.e. one that does not accelerate prices, in the spirit of Wicksell, 1907) and the level of an economy's technology. European Central Bank (2004) enumerates the level of an economy's development among the most essential factors behind the natural rate of interest. Countries with relatively lower income and productivity levels – like Poland and Slovakia – might therefore exhibit higher levels of natural interest rates.

Indeed, Bencik (2009) estimates the natural level of Slovak interest rates in the long term at around $2.5 - 3.0\%$. This seems to exceed the level estimated by the European Central Bank (2004) for the euro area in the interval of approximately 1% to 2%. The estimates by Brzoza-Brzezina (2003) suggest that the disparity can even be higher in the case of Poland, with Polish natural interest rate possibly even above 4%. A small economy that joins a monetary union whose level of interest rates is permanently lower could therefore be hit by a long-lasting postitive internal demand shock.

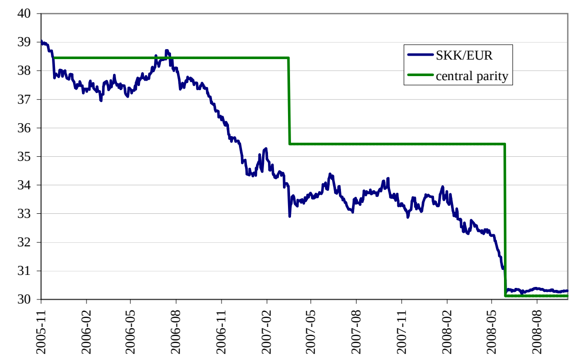
In both cases, the impact of cyclical or structural stress in monetary policy crucially depends on the sensitivity of economic activity to the discrepancy between the current real and the natural interest rate level. In the New Keynesian literature (see e.g. Clarida et al., 1999; Galí, 2008) it is usually modelled as an inverse function of intertemporal elasticity of substitution (σ).

Monetary policy stress was not the only demand disturbances on Slovakia's road to the euro. In the ERM II system, the Slovak koruna was twice revalued. The cumulative effect amounted to approximately 30% stronger currency over a of 2-year period (see Figure 3), and the final conversion rate was in line with the terminal value for the central parity.

A permanent fall in nominal interest rates and a policy-induced deterioration of country's competitiveness might be considered as complementary shocks. Intuitively, a positive shock hitting the economy via internal demand might be offset in terms of economic activity by a negative shock hitting the external demand and the domestic price competitiveness in general. One might be interested in the net effect of both disturbances and, consequently, whether the choice of the final conversion rate of the domestic currency into the euro could be considered as an adjustment policy tool on the way to the euro. According to Bencik (2009), "...after Slovakia's accession to the monetary union, there will be a discrepancy between the neutral interest rate in the euro area and in Slovakia. It will be probably necessary to offset the expansive influence of interest rates by other economic policy instruments."

This paper attempts to formalize this argument and outline an empirical strategy to get the necessary quantitative information, which in turn would make this argument operational. In particular, in order to know how to offset such a set of shocks against each other, one needs to be aware of the relative

Figure 3: Slovak koruna appreciation and revaluations in ERM II



Source: Eurostat and ECB data.

impact of real interest rate and real exchange rate on the domestic economy. In mid- to late 1990s, this question was heavily discussed in the literature as the problem of estimating the "MCI-ratio".

The MCI (Monetary Conditions Index) is a weighted sum of real interest rate and real exchange rate. This index is supposed to reflect the joint impact of both factors on the domestic economic activity and evaluate the current monetary policy stance (Freedman, 1994, 1995). The econometric problem, associated with the construction of the index is related to the difficulties with the estimation of weights for both variables. Their ratio is the parameter in question. A value less than one suggested that the impact of the real exchange rate is stronger compared to the impact of real interest rate. The weights should be chosen as multipliers of some target variable with respect to both real interest and exchange rate. It was customary in the literature to select some measure of economic activity, e.g. the output gap (Freedman, 1994), although some authors (Kokoszczyński, 2004) argued that the inflation rate would be a preferable choice.

Specifying the index as a weighted average, we implicitly assume that both channels are perfectly substitutable and work independently (Bofinger, 2001). As a consequence, econometric studies in this field mostly involve an estimation of an IS curve, with a measure of an output gap linearly explained by real interest rate or real exchange rate (see e.g. Frochen, 1996; Mayes and Viren, 2000, 2002; Hyder and Khan, 2007). This assumption has been heavily criticized, e.g. by Stevens (1998). Batini and Turnbull (2002) attempted to amend this problem by implementing some additional interest rate and exchange rate dynamics in the model, so as to measure the short to mid-term impact of monetary conditions on the real economy.

Mayes and Viren (2002) argued that the MCI might be useful in the macroeconomic analysis of monetary integration in Europe. However, due to a number of policy errors associated with monetary policy conduct using MCI, especially in New Zealand (Drew, 2001; Mishkin and Schmidt-Hebbel, 2007; Svensson, 2001), the estimation of MCI-ratio was abandoned. It happened before the New Open Macro progress came into the central banks' modelling practice in the form of dynamic stochastic general equilibrium models (DSGE). However, using this framework could help to better address the deep econometric critique that Eika et al. (1996) formulated in response to models attempting to capture the MCI-ratio, i.e.:

- **dynamic specification**. A rigorous derivation allows applying empirically verified specification and the sources of model dynamics are explicit. Moreover, rational expectations additionally fine-tune this dynamics.
- **parameter stability**. DSGEs are commonly considered to be immune to the Lucas (1976) critique. Even if it was not fully the case for the economies here in question, they place a number of restrictions on a comparable VAR specification that allows us to pin down a relatively small number of parameters in an economically sensible way. This is especially valuable when the time series are short. Also, the presense of expectational terms allows us to apply the model for counterfactual analyses of scenarios after the euro adoption.
- **endogeneity**. In a log-linearized New Keynesian system, there are at least 2 more equations in addition to the IS curve: the Phillips curve and the nominal interest rate equation (Taylor rule). The absence of the latter was particularly stressed by many authors as a potential source of endogeneity bias (Eika et al., 1996; Gottschalk, 2001). The presence of the former allows us to specify the real interest rate correctly, i.e. in an *ex ante* manner.

In this paper, we consider the IS curve approach, treating the output gap as the target variable. We specify a New Keynesian small open economy DSGE model. It contains the features of interest: competitiveness channel, separate inflation processes in the tradable and nontradable sector, Euler equation establishing a dynamic link between present and future consumption via the *ex ante* real interest rate, as well as common monetary policy. It turns out that an IS curve specification as a linear equation with the real interest rate and the real exchange rate (in level; such as e.g. in Kot, 2003 or Torój, 2008) is dynamically inconsistent with the inter- and intratemporal optimizing setup. This is why we estimate the Polish and Slovak MCI-ratio, taking into account (i) the interest rate parameter in the Euler equation and (ii) the terms of trade parameter in the static equilibrium condition.

With our MCI-ratios, we simulate the dynamic responses of the Polish and Slovak economies to asymmetric disturbances within a monetary union, as well as to the permanent fall in interest rates. This brings us to the answer whether – and to what extent – procyclical real interest rates are dampened by the competitiveness channel, and how the fall in nominal interest rates changes the relative prices between the tradable sector and the rest of the monetary union.

3 DSGE model setup

The DSGE model developed here builds strongly upon the multi-region currency union models with possible heterogeneity, such as e.g. ones considered in the works by Benigno (2004); Lombardo (2006); Brissimis and Skotida (2008). The currency union consists of 2 regions. The whole economy of the monetary union, in line with a conventional treatment in the DSGE literature[1], is represented by the

[1] See Benigno (2004); Blessing (2008); Kolasa (2008).

interval $[0;1]$, whereby the first region (say, home economy) is indexed over $[0;w]$ (relative size of the region: w), and the second (foreign economy) is indexed over $[w;1]$.

As the behaviour of the nontradable sector is considered to be a crucial element of adjustment dynamics (see e.g. European Commission, 2008, 2009), both economies consist of two sectors. Each of them is characterized by price rigidities, modelled with Calvo (1983) mechanism. Conventionally, consumers in each region maximize their utility and producers in each sector – their present and discounted future profits. International exchange of goods incorporates the competitiveness channel of adjustment into the model and ensures that in the long run both economies return to their equilibrium after a shock. This is also true for a small economy that does not have an autonomous monetary policy, which is modelled for the entire currency union via a simple Taylor rule with smoothing.

While monetary policy is always symmetric (with a possibly asymmetric transmission mechanism though), there are three other shocks in the model that can be asymmetric (region-specific): demand shocks (to consumption) and cost-push shocks, both in the tradable and non-tradable sector.

Henceforth, parameters of the foreign economy are denoted analogously to home economy and marked with an asterisk, e.g. σ and σ^*. Lowercase letters denote the log-deviations of their uppercase counterparts from the steady-state values.

3.1 Household decisions

3.1.1 Intratemporal allocation of consumption

Households get utility from consumption and disutility from hours worked. In addition, utility from consumption depends on consumption habits formed in the previous period (see Smets and Wouters, 2003; Kolasa, 2008). The constant relative returns to scale utility function takes the form (as in Galí, 2008):

$$U_t\left(C_t, N_t, H_t\right) = \frac{\left(C_t - H_t\right)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \tag{1}$$

where C_t – consumption at t, H_t – stock of consumption habits at t, N_t – hours worked at t, $\sigma > 0$ and $\phi > 0$. Consumption habits are assumed to be proportional to consumption at $t-1$ (see Smets and Wouters, 2003):

$$H_t = hC_{t-1} \tag{2}$$

with $h \in [0;1)$ The overall consumption index aggregates the tradable and nontradable consumption bundles:

$$C_t \equiv \left[(1-\kappa)^{\frac{1}{\delta}} C_{T,t}^{\frac{\delta-1}{\delta}} + \kappa^{\frac{1}{\delta}} C_{N,t}^{\frac{\delta-1}{\delta}}\right]^{\frac{\delta}{\delta-1}} \tag{3}$$

where $\kappa \in (0;1)$ characterizes the share of nontradables in the home economy and $\delta > 0$ is elasticity of consumption substitution between the goods produced in both sectors.

The domestic consumption of tradables at t consists of goods produced at home, $C_{H,t}$, and abroad, $C_{F,t}$:

$$C_{T,t} \equiv \left[(1-\alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{4}$$

An analogous relationship holds for the foreign economy. Given this, α is an intuitive measure of degree of openness and $1-\alpha$ – home bias in consumption. $\eta > 0$ is the elasticity of substitution between home and foreign tradables.

A single type of good is indexed as k and belongs to good variety indexed over the interval $[0;1]$.

The consumption of domestic tradable goods in the home economy ($C_{H,t}$) and in the foreign one ($C^*_{H,t}$) is defined as:

$$C_{H,t} \equiv \left[\left(\frac{1}{w}\right)^{\frac{1}{\varepsilon_T}} \int_0^1 \left(\int_0^w C^j_{H,t,k} dj\right)^{\frac{\varepsilon_T-1}{\varepsilon_T}} dk\right]^{\frac{\varepsilon_T}{\varepsilon_T-1}} \quad C^*_{H,t} \equiv \left[\left(\frac{1}{w}\right)^{\frac{1}{\varepsilon_T}} \int_0^1 \left(\int_0^w C^{j*}_{H,t,k} dj\right)^{\frac{\varepsilon_T-1}{\varepsilon_T}} dk\right]^{\frac{\varepsilon_T}{\varepsilon_T-1}} \tag{5}$$

The parameter $\varepsilon_T > 1$ measures the elasticity of substitution between various types of goods in international trade, k indexes the variety of goods, and j – the households (integral over j reflects the difference in both economies' size).

We define in an analogous way the domestic and foreign consumption of goods produced abroad, $C_{F,t}$ and $C^*_{F,t}$:

$$C_{F,t} \equiv \left[\left(\frac{1}{1-w}\right)^{\frac{1}{\varepsilon_T}} \int_0^1 \left(\int_w^1 C^j_{F,t,k} dj\right)^{\frac{\varepsilon_T-1}{\varepsilon_T}} dk\right]^{\frac{\varepsilon_T}{\varepsilon_T-1}} \quad C^*_{F,t} \equiv \left[\left(\frac{1}{1-w}\right)^{\frac{1}{\varepsilon_T}} \int_0^1 \left(\int_w^1 C^{j*}_{F,t,k} dj\right)^{\frac{\varepsilon_T-1}{\varepsilon_T}} dk\right]^{\frac{\varepsilon_T}{\varepsilon_T-1}} \tag{6}$$

For both tradable consumption baskets (i.e. H and F), we define equal elasticity of substitution between various types of goods, ε^T, both at home and abroad.

The nontradable consumption bundles, domestic ($C_{N,t}$) and foreign ($C_{N*,t}$), are characterized in a similar fashion as:

$$C_{N,t} \equiv \left[\left(\frac{1}{w}\right)^{\frac{1}{\varepsilon_N}} \int_0^1 \left(\int_0^w C^j_{N,t,k} dj\right)^{\frac{\varepsilon_N-1}{\varepsilon_N}} dk\right]^{\frac{\varepsilon_N}{\varepsilon_N-1}} \qquad C_{N*,t} \equiv \left[\left(\frac{1}{1-w}\right)^{\frac{1}{\varepsilon_{N*}}} \int_0^1 \left(\int_w^1 C^{j*}_{N*,t,k} dj\right)^{\frac{\varepsilon_{N*}-1}{\varepsilon_{N*}}} dk\right]^{\frac{\varepsilon_{N*}}{\varepsilon_{N*}-1}}$$

Consequently, ε^N and ε^{N*} is defined as elasticity of substitution between various types of nontradable goods.

Households maximize at t the discounted flow of future utilities:

$$E_t \sum_t^{\infty} \beta^t U\left(C_t, N_t, H_t\right) \rightarrow \underset{C,N}{max} \tag{7}$$

where $\beta \in (0,1)$ is households' discount factor. Maximization of (7) is subject to a sequence of current and future budget constraints faced by a representative household:

$$\forall_t \quad \int_0^1 \int_0^w P_{H,t,k}^j C_{H,t,k}^j dj\, dk + \int_0^1 \int_w^1 P_{F,t,k}^j C_{F,t,k}^j dj\, dk + \int_0^1 \int_0^w P_{N,t,k}^j C_{N,t,k}^j dj\, dk + E_t\left\{Q_{t,t+1} D_{t+1}\right\} \leq D_t + W_t N_t \tag{8}$$

The right-hand side is a household's budget at t. Its income consists of payoffs of securities acquired in the previous periods (D_t), labour incomes (W_t – nominal wage for hours worked at t) and government transfers (T_t). The left-hand side of the inequality sums the consumption spendings of households (where P denotes a price of a particular consumption bundle, indexed in line with these bundles) and acquisition of securities. $Q_{t,t+1}$ is a stochastic discount factor for the payoffs at $t+1$, faced by the households.

Maximizing (7) subject to (8) leads to the following first order conditions:

- demand equations (home and foreign) for individual goods k produced at home:

$$C_{H,t,k} = \frac{1}{w}\left(\frac{P_{H,t,k}}{P_{H,t}}\right)^{-\varepsilon_T} C_{H,t} \qquad C_{H,t,k}^{*} = \frac{1}{w}\left(\frac{P_{H,t,k}}{P_{H,t}}\right)^{-\varepsilon_T} C_{H,t}^{*} \tag{9}$$

- demand equations (home and foreign) for individual goods k produced abroad:

$$C_{F,t,k} = \frac{1}{1-w}\left(\frac{P_{F,t,k}}{P_{F,t}}\right)^{-\varepsilon_T} C_{F,t} \qquad C_{F,t,k}^{*} = \frac{1}{1-w}\left(\frac{P_{F,t,k}}{P_{F,t}}\right)^{-\varepsilon_T} C_{F,t}^{*} \tag{10}$$

- demand equations (home and foreign) for individual nontradable goods:

$$C_{N,t,k} = \frac{1}{w}\left(\frac{P_{N,k}}{P_N}\right)^{-\varepsilon_N} C_N \qquad C_{N,t,k}^{*} = \frac{1}{1-w}\left(\frac{P_{N,k}^{*}}{P_N^{*}}\right)^{-\varepsilon_{N*}} C_{N,t}^{*} \tag{11}$$

- demand equations (home and foreign) for domestic tradable goods:

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\eta} C_{T,t} \qquad C_{H,t}^{*} = \alpha^{*}\left(\frac{P_{H,t}}{P_{T,t}^{*}}\right)^{-\eta^{*}} C_{T,t}^{*} \tag{12}$$

- demand equations (home and foreign) for foreign tradable goods:

$$C_{F,t}=\alpha\left(\frac{P_{F,t}}{P_{T,t}}\right)^{-\eta}C_{T,t}\qquad C^*_{F,t}=(1-\alpha^*)\left(\frac{P_{F,t}}{P^*_{T,t}}\right)^{-\eta^*}C^*_{T,t}\tag{13}$$

- home and foreign demand equations for all tradable goods:

$$C_{T,t}=(1-\kappa)\left(\frac{P_{T,t}}{P_t}\right)^{-\delta}C_t\qquad C^*_{T,t}=(1-\kappa^*)\left(\frac{P^*_{T,t}}{P^*_t}\right)^{-\delta^*}C^*_t\tag{14}$$

- home and foreign demand equations for all nontradable goods:

$$C_{N,t}=\kappa\left(\frac{P_{N,t}}{P_t}\right)^{-\delta}C_t\qquad C_{N^*,t}=\kappa^*\left(\frac{P^*_{N,t}}{P^*_t}\right)^{-\delta^*}C^*_t\tag{15}$$

- home and foreign labour supply equations:

$$C_t^{\sigma}N_t^{\varphi}=\frac{W_t}{P_t}\qquad C_t^{*\sigma^*}N_t^{*\varphi^*}=\frac{W^*_t}{P^*_t}\tag{16}$$

The respective price indices are defined in the following way:

$$P_{H,t}\equiv\left[\frac{1}{w}\int_0^1\left(\int_0^w P^j_{H,t,k}dj\right)^{1-\varepsilon_T}dk\right]^{\frac{1}{1-\varepsilon_T}}\qquad P^*_{H,t}\equiv\left[\frac{1}{w}\int_0^1\left(\int_0^w P^{j*}_{H,t,k}dj\right)^{1-\varepsilon_T}dk\right]^{\frac{1}{1-\varepsilon_T}}\tag{17}$$

$$P_{F,t}\equiv\left[\frac{1}{1-w}\int_0^1\left(\int_w^1 P^j_{F,t,k}dj\right)^{1-\varepsilon_T}dk\right]^{\frac{1}{1-\varepsilon_T}}\qquad P^*_{F,t}\equiv\left[\frac{1}{1-w}\int_0^1\left(\int_w^1 P^{j*}_{F,t,k}dj\right)^{1-\varepsilon_T}dk\right]^{\frac{1}{1-\varepsilon_T}}\tag{18}$$

$$P_{T,t}\equiv\left[(1-\alpha)P_{H,t}^{1-\eta}+\alpha P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}\qquad P^*_{T,t}\equiv\left[(1-\alpha^*)P_{F,t}^{1-\eta}+\alpha^* P_{H,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}\tag{19}$$

$$P_{N,t}\equiv\left(\frac{1}{w}\int_0^1\left(\int_0^w P_{N,t,k}dj\right)^{1-\varepsilon_N}dk\right)^{\frac{1}{1-\varepsilon_N}}\qquad P^*_{N,t}\equiv\left(\frac{1}{1-w}\int_0^1\left(\int_w^1 P^*_{N,t,k}dj\right)^{1-\varepsilon_{N^*}}dk\right)^{\frac{1}{1-\varepsilon_{N^*}}}\tag{20}$$

$$P_t\equiv\left[(1-\kappa)P_{T,t}^{1-\delta}+\kappa P_{N,t}^{1-\delta}\right]^{\frac{1}{1-\delta}}\qquad P_t\equiv\left[(1-\kappa^*)P_{T,t}^{*1-\delta^*}+\kappa^* P_{N,t}^{*1-\delta^*}\right]^{\frac{1}{1-\delta^*}}\tag{21}$$

Log-linearization and differencing the formulas (19) and (21) lead to the following dependencies:

$$\pi_{T,t}=(1-\alpha)\pi_{H,t}+\alpha\pi_{F,t}\qquad \pi^*_{T,t}=(1-\alpha^*)\pi_{F,t}+\alpha^*\pi_{H,t}\tag{22}$$

$$\pi_t = (1-\kappa)\,\pi_{T,t} + \kappa\pi_{N,t} \qquad \pi_t^* = (1-\kappa^*)\,\pi_{T,t}^* + \alpha^*\pi_{N,t}^* \tag{23}$$

Using the above equations, we derive domestic demand functions for the domestic tradable, foreign tradable and nontradable goods:

$$C_{H,t,k} = \frac{1}{w}(1-\alpha)(1-\kappa)\left(\frac{P_{H,t,k}}{P_{H,t}}\right)^{-\varepsilon_T}\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\eta}\left(\frac{P_{T,t}}{P_t}\right)^{-\delta} C_t \tag{24}$$

$$C_{F,t,k} = \frac{1}{1-w}\alpha(1-\kappa)\left(\frac{P_{F,t,k}}{P_{F,t}}\right)^{-\varepsilon_T}\left(\frac{P_{F,t}}{P_{T,t}}\right)^{-\eta}\left(\frac{P_{T,t}}{P_t}\right)^{-\delta} C_t \tag{25}$$

$$C_{N,t,k} = \frac{1}{w}\kappa\left(\frac{P_{N,t,k}}{P_{N,t}}\right)^{-\varepsilon_N}\left(\frac{P_{N,t}}{P_t}\right)^{-\delta} C_t \tag{26}$$

Analogous equations hold for the foreign economy.

3.1.2 Intertemporal allocation of consumption

We define the stochastic discount factor as:

$$Q_{t,t+1} \equiv \frac{V_{t,t+1}}{\xi_{t,t+1}} \tag{27}$$

where $V_{t,t+1}$ is the price at t of an Arrow security, i.e. a one-period security paying 1 at $t+1$ when a specific state of nature occurs and 0 otherwise. $\xi_{t,t+1}$ is the probability that the state of nature in which 1 is paid materializes, conditional on the state of nature at t. Having the access to such a security market, households can transfer utility between periods, maximizing its discounted flow (see Galí and Monacelli, 2005).

The optimality of decisions requires that the marginal loss in utility due to buying the security at t instead of allocating this money to consumption must equal the discounted payoff at $t+1$, also expressed in terms of marginal growth of future utility:

$$\frac{V_{t,t+1}}{P_t}(C_t - H_t)^{-\sigma} = \xi_{t,t+1}\beta(C_{t+1} - H_{t+1})^{-\sigma}\frac{1}{P_{t+1}} \tag{28}$$

whereby C_{t+1} and P_{t+1} in the above equation should be interpreted as conditional expected values given the state of nature when the payoff is nonzero.

Applying the definition of $Q_{t,t+1}$ (27) and (2), the equation (28) can be written as:

$$\beta\left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right) = Q_{t,t+1} \tag{29}$$

We calculate the conditional expected value of both sides, which – along with $\Im_t \equiv E_t\left(Q_{t,t+1}\right)$ – leads to the Euler equation for consumption:

$$\Im_t = \beta E_t\left[\left(\frac{C_{t+1} - hC_t}{C_t - hC_{t-1}}\right)^{-\sigma}\left(\frac{P_t}{P_{t+1}}\right)\right] \tag{30}$$

Log-linearization of (30) around the steady state allows us to write the following dependence:

$$c_t - hc_{t-1} = E_t\left(c_{t+1} - hc_t\right) - \frac{1-h}{\sigma}\left[i_t - \left(E_t p_{t+1} - p_t\right) + ln\beta\right] \tag{31}$$

where lowercase variables are percentage deviations from the steady state for their uppercase counterparts. After basic simplifications, we obtain (see Smets and Wouters, 2003):

$$c_t = \frac{h}{1+h}c_{t-1} + \frac{1}{1+h}E_t c_{t+1} - \frac{1-h}{(1+h)\,\sigma}\left(i_t - E_t\pi_{t+1} - \rho\right) \tag{32}$$

where $i_t \equiv -ln\Im_t$ denotes short-term nominal interest rate at t, $E_t\pi_{t+1} = E_t p_{t+1} - p_t$ – expected domestic consumer price growth, $\rho = -ln\beta$ – natural interest rate corresponding to the households' discount factor β.

3.2 International prices

Define bilateral *terms of trade* between the home and foreign economy as:

$$S_t \equiv \frac{P_{H,t}}{P_{F,t}} \tag{33}$$

Log-linearizing (33) around a symmetric steady state $S_t = 1$ – the law of one price in the tradable sector – leads to the following relationship:

$$s_t = p_{H,t} - p_{F,t} \tag{34}$$

Also, define internal *terms of trade* as price ratio between tradables and nontradables:

$$X_t \equiv \frac{P_{T,t}}{P_{N,t}} \tag{35}$$

An analogous approximation allows us to write:

$$x_t = p_{T,t} - p_{N,t} \tag{36}$$

Although this symmetric steady state might seem to be controversial in the case of country pairs euro area vs Eastern Europe (Poland or Slovakia), it allows us to simplify the technical exposition substantially. When the steady state is asymmetric for some reason (e.g. persistent price level differentials due to heterogeneity in the level of economies' development), then $lnS_t \neq 0$ and equation (34) takes the form:

$$s_t = p_{H,t} - p_{F,t} - \bar{s} \tag{37}$$

An asymmetric steady stade has twofold implications. Firstly, equality (37) implies that a constant can appear in the equations containing s_t. The value $\bar{s}$ could probably be treated in an analogous manner to long-run equilibrium exchange rate in the sense of Williamson (1994), i.e. stabilizing the current account and production at their steady-state levels.

Secondly, generalizing the framework of Galí and Monacelli (2005) where a symmetric steady state was assumed, we obtain the following log-linearization of the tradable price index definition (19):

$$p_{T,t} = \frac{(1-\alpha)\,\bar{S}^{1-\eta}}{(1-\alpha)\,\bar{S}^{1-\eta}+\alpha} p_{H,t} + \frac{\alpha}{(1-\alpha)\,\bar{S}^{1-\eta}+\alpha} p_{F,t} \equiv (1-\tilde{\alpha})\, p_{H,t} + \tilde{\alpha} p_{F,t} \tag{38}$$

Note that (38) simplifies to $p_t = (1-\alpha)\,p_{H,t} + \alpha p_{F,t}$ when the steady state is symmetric. In addition, when the prices of domestic goods are lower than the foreign ones in the steady state ($\bar{S} < 1$), then $\alpha < \tilde{\alpha}$ if $\eta < 1$, $\alpha > \tilde{\alpha}$ if $\eta > 1$ and like in the steady state if $\eta = 1$. Analogous conclusions apply to the log-linearization of (21). Further derivations assume a symmetric steady state.

Using (37) and (38) we can write:

$$p_{T,t} = p_{H,t} - \alpha s_t \tag{39}$$

$$p_t = p_{T,t} - \kappa x_t \tag{40}$$

The real exchange rate Q_t (q_t for log-deviation from the steady state) versus the rest of the monetary union takes the form:

$$q_t = p_t - p_t^* = (1-\alpha-\alpha^*)\, s_t - \kappa x_t + \kappa^* x_t^* \tag{41}$$

Real exchange rate appreciation is then linked to the appreciation of external *terms of trade*, depreciation of domestic internal terms of trade (defined as in (36)) and appreciation of foreign internal terms of trade.

3.3 Producers

3.3.1 Real marginal costs

The producers of variety k in the tradable or nontradable bundle face the following production function (see Galí, 2008):

$$Y_{t,k}^{H} = A_t^H N_{t,k}^{H} \tag{42}$$

$$Y_{t,k}^{N} = A_t^N N_{t,k}^{N} \tag{43}$$

whereby $lnA_t^H \equiv a_t^H$ is an exogenous technological process (analogously for the nontradable sector N). Following Clarida et al. (1999), we assume away the price deviations of individual varieties within a sector as of second-order importance in the proximity of the steady state. This allows us to integrate the formulas (42) and (43) into sectoral production functions. It implies a real marginal cost, common for producers in a given sector, calculated as:

$$mc_t^H = w_t - p_{H,t} - a_t^H \tag{44}$$

$$mc_t^N = w_t - p_{N,t} - a_t^N \tag{45}$$

with w_t – wages in the home economy (expressed as log-deviations from the steady state).

Using log-linearized definitions of price aggregates (19) and (21) and the labour supply condition (16) we get:

$$mc_t^H = (w_t - p_t) + (p_t - p_{T,t}) + (p_{T,t} - p_{H,t}) - a_t^H = \sigma c_t + \phi n_t - \alpha s_t - \kappa x_t - a_t^H \tag{46}$$

$$mc_t^N = (w_t - p_t) + (p_t - p_{T,t}) + (p_{T,t} - p_{N,t}) - a_t^N = \sigma c_t + \phi n_t + (1-\kappa) x_t - a_t^N \tag{47}$$

Aggregating (42) and (43) over the good types k and assuming identical technologies across all producers, we obtain sectoral production functions, implying the following relationship:

$$n_t = \frac{N^N}{N} n_t^N + \frac{N^H}{N} n_t^H = \frac{\frac{Y^N}{A^N}}{\frac{Y^N}{A^N} + \frac{Y^T}{A^T}} n_t^N + \frac{\frac{Y^T}{A^T}}{\frac{Y^N}{A^N} + \frac{Y^T}{A^T}} n_t^H \approx \kappa n_t^N + (1-\kappa) n_t^H = \kappa y_t^N + (1-\kappa) y_t^H - \kappa a_t^N - (1-\kappa) a_t^H \tag{48}$$

The approximation makes use of steady-state technological symmetry between the sectors. Substituting this into (46) and (47) yields:

$$mc_t^H = \sigma c_t + \phi \left[\kappa y_t^N + (1-\kappa) y_t^H\right] - \alpha s_t - \kappa x_t - \left[1 + \phi (1-\kappa)\right] a_t^H - \phi \kappa a_t^N \tag{49}$$

$$mc_t^N = \sigma c_t + \phi \left[\kappa y_t^N + (1-\kappa) y_t^H\right] + (1-\kappa) x_t - \phi (1-\kappa) a_t^H - (1 + \phi\kappa) a_t^N \tag{50}$$

3.3.2 Pricing decisions

There are price rigidities in the economy. Following the usual approach in the New Keynesian literature, we model them by means of the Calvo (1983) scheme. In a given period, a fraction θ of producers are not allowed to reoptimize their prices in reaction to economic innovations and must sell at the price from the previous period. The probability of being allowed to reoptimize the price is equal across producers: $1-\theta$ in each period, independently of the amount of time elapsed since the last price change.

Some of the producers allowed to change their price do not really reoptimize. Following Galí and Gertler (1999) we assume that the change in price is partly implemented as an indexation to past inflation. This mechanism leads to a hybrid Phillips curve (see Galí and Gertler (1999); Galí et al. (2001)), empirically outperforming the purely forward-looking specifications in terms of goodness-of-fit. Following Kolasa (2008), inflation is modelled separately in the tradable and nontradable sector.

As Galí and Gertler (1999) we assume that a fraction $1-\theta$ of producers are able to change their price in t in each sector, which implies the following dependence between the price levels at $t-1$ and t:

$$p_t^H = \theta^H p_{t-1}^H + \left(1-\theta^H\right)\bar{p}_t^H \qquad p_t^N = \theta^N p_{t-1}^N + \left(1-\theta^N\right)\bar{p}_t^N \tag{51}$$

where $\bar{p}_t^H$ and $\bar{p}_t^N$ denote the prices set newly at t by the $1-\theta$ fraction of producers. Among the producers who reoptimize prices there is a fraction of $1-\omega$ producers reoptimizing in an anticipatory manner as in Calvo (1983). They maximize the discounted flow of future profits, using all information available at the time of decision and taking into account future constraints. The rest of producers (ω) reset their prices, according to past price dynamics:

$$\bar{p}_t^H = \omega^H p_{b,t}^H + \left(1-\omega^H\right) p_{f,t}^H \qquad \bar{p}_t^N = \omega^N p_{b,t}^N + \left(1-\omega^N\right) p_{f,t}^N \tag{52}$$

Prices set by the latter group of producers are modelled, as in Galí and Gertler (1999), as reoptimized prices from the previous period, indexed to past inflation:

$$p_{b,t}^H = \bar{p}_{t-1}^H + \pi_{t-1}^H \qquad p_{b,t}^N = \bar{p}_{t-1}^N + \pi_{t-1}^N \tag{53}$$

One can show (see Galí and Gertler, 1999; Galí et al., 2001; Galí, 2008 for details) that the reoptimized prices satisfy the following conditions:

$$p_{f,t}^H = \mu^H + \left(1-\beta\theta^H\right)\sum_{s=0}^{\infty}\left(\beta\theta^H\right)^s E_t\left(mc_{t+s}^H + p_{H,t+k}\right) \tag{54}$$

$$p_{f,t}^N = \mu^N + \left(1-\beta\theta^N\right)\sum_{s=0}^{\infty}\left(\beta\theta^N\right)^s E_t\left(mc_{t+s}^N + p_{N,t+k}\right) \tag{55}$$

where $\mu^T \equiv -ln\frac{\varepsilon^T}{\varepsilon^T-1}$ and $\mu^N \equiv -ln\frac{\varepsilon^N}{\varepsilon^N-1}$ are log-markups in the steady state (or markups in an economy without price rigidities), mc_t – real marginal cost at t.

Combined relationships (51)-(55) lead to the following hybrid Phillips curves in both domestic sectors:

$$\begin{aligned}\pi_t^H &= \tfrac{\omega^H}{\theta^H+\omega^H[1-\theta^H(1-\beta)]}\pi_{t-1}^H + \tfrac{\beta\theta^H}{\theta^H+\omega^H[1-\theta^H(1-\beta)]}E_t\pi_{t+1}^H + \\ &+\tfrac{(1-\omega^H)(1-\theta^H)(1-\beta\theta^H)}{\theta^H+\omega^H[1-\theta^H(1-\beta)]}mc_t^H\end{aligned} \tag{56}$$

$$\begin{aligned}\pi_t^N &= \tfrac{\omega^N}{\theta^N+\omega^N[1-\theta^N(1-\beta)]}\pi_{t-1}^N + \tfrac{\beta\theta^N}{\theta^N+\omega^N[1-\theta^N(1-\beta)]}E_t\pi_{t+1}^N + \\ &+\tfrac{(1-\omega^N)(1-\theta^N)(1-\beta\theta^N)}{\theta^N+\omega^N[1-\theta^N(1-\beta)]}mc_t^N\end{aligned} \tag{57}$$

where mc_t now denote the deviation of real marginal cost from its long-run value in the respective sector (analogously for the foreign economy).

3.4 Market clearing conditions

Equilibrium on the world markets of individual goods requires equality of overall production and consumption of every variety k:

$$\begin{aligned}Y_{t,k}^H &= \textstyle\int_0^w C_{H,t,k}^j dj + \int_w^1 C_{H,t,k}^{j*} dj = \\ &= wC_{H,t,k} + (1-w)\,C_{H,t,k}^* = \\ &= (1-\alpha)(1-\kappa)\left(\tfrac{P_{H,t,k}}{P_{H,t}}\right)^{-\varepsilon_T}\left(\tfrac{P_{H,t}}{P_{T,t}}\right)^{-\eta}\left(\tfrac{P_{T,t}}{P_t}\right)^{-\delta}C_t + \\ &+\tfrac{1-w}{w}\alpha^*(1-\kappa^*)\left(\tfrac{P_{H,t,k}}{P_{H,t}}\right)^{-\varepsilon_T}\left(\tfrac{P_{H,t}}{P_{T,t}^*}\right)^{-\eta^*}\left(\tfrac{P_{T,t}^*}{P_t}\right)^{-\delta^*}C_t^* = \\ &= \left(\tfrac{P_{H,t,k}}{P_{H,t}}\right)^{-\varepsilon_T}\left[(1-\alpha)(1-\kappa)\left(\tfrac{P_{H,t}}{P_{T,t}}\right)^{-\eta}\left(\tfrac{P_{T,t}}{P_t}\right)^{-\delta}C_t + \tfrac{1-w}{w}\alpha^*(1-\kappa^*)\left(\tfrac{P_{H,t}}{P_{T,t}^*}\right)^{-\eta^*}\left(\tfrac{P_{T,t}^*}{P_t}\right)^{-\delta^*}C_t^*\right]\end{aligned} \tag{58}$$

Plugging the above expression into the definition of aggregate domestic tradable product,

$$Y_t^H \equiv \left(\int_0^1 Y_{t,k}^{\frac{\varepsilon^T-1}{\varepsilon^T}}dk\right)^{\frac{\varepsilon^T}{\varepsilon^T-1}} \tag{59}$$

yields:

$$\begin{aligned}Y_t^H &= (1-\alpha)(1-\kappa)\left(\tfrac{P_{H,t}}{P_{T,t}}\right)^{-\eta}\left(\tfrac{P_{T,t}}{P_t}\right)^{-\delta}C_t + \tfrac{1-w}{w}\alpha^*(1-\kappa^*)\left(\tfrac{P_{H,t}}{P_{T,t}^*}\right)^{-\eta^*}\left(\tfrac{P_{T,t}^*}{P_t^*}\right)^{-\delta^*}C_t^* = \\ &= S_t^{-\alpha\eta}X_t^{-\kappa\delta}\left[(1-\alpha)(1-\kappa)C_t + \tfrac{1-w}{w}\alpha^*(1-\kappa^*)S_t^{\alpha\eta-\eta^*(1-\alpha^*)}X_t^{\kappa\delta}(X_t^*)^{-\kappa^*\delta^*}C_t^*\right]\end{aligned} \tag{60}$$

Log-linearizing around the steady state and using (41) leads to the following equilibrium condition:

$$y_t^H = (1-\alpha)(1-\kappa)c_t + \tfrac{1-w}{w}\alpha^*(1-\kappa^*)c_t^* - \tfrac{1-w}{w}\left\{\alpha^*\eta^*(1-\alpha^*)(1-\kappa^*) - \alpha\eta\left[\alpha^*(1-\kappa^*) - \tfrac{w}{1-w}\right]\right\}s_t + \\ +\kappa\delta\left[\tfrac{1-w}{w}\alpha^*(1-\kappa^*)-1\right]x_t - \tfrac{1-w}{w}\alpha^*\kappa^*\delta^*(1-\kappa^*)x_t^* \tag{61}$$

An analogous condition can be written for the foreign economy:

$$y_t^{F*} = (1-\alpha^*)(1-\kappa^*)c_t^* + \tfrac{w}{1-w}\alpha(1-\kappa)c_t - \tfrac{w}{1-w}\left\{\alpha^*\eta^*\left[\alpha(1-\kappa) - \tfrac{1-w}{w}\right] - \alpha\eta(1-\alpha)(1-\kappa)\right\}s_t + \\ +\kappa^*\delta^*\left[\tfrac{w}{1-w}\alpha(1-\kappa)-1\right]x_t^* - \tfrac{w}{1-w}\alpha\kappa\delta(1-\kappa)x_t \tag{62}$$

Market clearing conditions for the nontradable sector can be written using (11) as:

$$Y_{N,t} = C_{N,t} = \kappa\left(\frac{P_{N,t}}{P_t}\right)^{-\delta} C_t \qquad Y_{N,t}^* = C_{N,t}^* = \kappa^*\left(\frac{P_{N,t}^*}{P_t^*}\right)^{-\delta^*} C_t^* \tag{63}$$

Using the definition of internal terms of trade, (35), we get:

$$Y_{N,t} = \kappa X_t^{(1-\kappa)\delta} C_t \qquad Y_{N,t}^* = \kappa^*\left(X_t^*\right)^{(1-\kappa^*)\delta^*} C_t^* \tag{64}$$

Log-linearizing (64) around the steady state leads to the following equilibrium conditions:

$$y_t^N = \kappa(1-\kappa)\delta x_t + \kappa c_t \quad y_t^{N*} = \kappa^*(1-\kappa^*)\delta^* x_t^* + \kappa^* c_t^* \tag{65}$$

In further analyses, we treat all the log-linearized variables as deviations from a "natural" state of economy, driven by the exogenous technological processes a_t^T and a_t^N and undistorted by price relations. We therefore ignore a_t^T and a_t^N and treat y_t^T and y_t^N as output gaps in each sector.

3.5 Monetary policy

The central bank's monetary policy is described by a Taylor (1993) rule with smoothing, which is a commonly applied description in the literature and empirically tested as an adequate tool for both the euro area (Sauer and Sturm, 2003) and Poland (see i.a. Kolasa, 2008; Gradzewicz and Makarski, 2009). The common nominal interest rate is set according to the equation:

$$i_t = \rho + (1-\gamma_\rho)(\gamma_\pi\tilde{\pi}_t + \gamma_y\tilde{y}_t) + \gamma_\rho i_{t-1} \tag{66}$$

where i_t – central bank policy rate at t, $\tilde{y}_t$ – the output gap in a currency union, $\tilde{\pi}_t$ – inflation rate in a currency union, $\gamma_\rho \in (0;1)$ – smoothing parameter, $\gamma_\pi > 1$, $\gamma_y > 0$ – parameters of central bank's response to deviations of inflation and output from the equilibrium levels. The condition $\gamma_\pi > 1$ is necessary to satisfy the Taylor rule (Taylor, 1993), leading to a unique equilibrium.

Inflation and output gap in the currency union are aggregated over member countries:

$$\tilde{\pi}_t = \int_0^1 \pi_t^j dj \tag{67}$$

$$\tilde{y}_t = \int_0^1 y_t^j dj \tag{68}$$

In practice, for a finite number of economies, country weights (vector $\mathbf{w}_{n\times 1}$) reflect relative sizes of economies (in a 2-country case: w and $1-w$).

3.6 Model equations

The log-linearized dynamic model is composed of the Euler equation for consumption (32), sectoral Phillips curves (56) and (57), real marginal cost (49) and (50) along with their foreign counterparts, equilibrium conditions (61), (62) and (65), equation of common monetary policy (66) and a set of identities defining the aggregate values for the monetary union (67) and (68), aggregate price dynamics and deflators.

The list of the shocks includes:

- $\epsilon_t^D = \rho_c \epsilon_{t-1}^D + \xi_t^D$ and $\epsilon_t^{D*} = \rho_c^* \epsilon_{t-1}^{D*} + \xi_t^{D*}$, interpreted as demand shocks respectively in home and foreign economy and applied to the Euler equation (32) (and its foreign analog);
- $\epsilon_t^T = \rho_{\pi_T} \epsilon_{t-1}^T + \xi_t^T$ and $\epsilon_t^{T*} = \rho_{\pi_{T*}} \epsilon_{t-1}^{T*} + \xi_t^{T*}$, interpreted as cost-push shocks in the tradable sector of home and foreign economy and applied to the Phillips curve for tradables (56) (and its foreign analog);
- $\epsilon_t^N = \rho_{\pi_{NT}} \epsilon_{t-1}^N + \xi_t^N$ and $\epsilon_t^{N*} = \rho_{\pi_{NT*}} \epsilon_{t-1}^{N*} + \xi_t^{N*}$, interpreted as cost-push shocks in the nontradable sector of home and foreign economy and applied to the Phillips curve for nontradables (57) (and its foreign analog);
- $\epsilon_t^i = \rho_i \epsilon_{t-1}^i + \xi_t^i$, interpreted as the monetary policy shock.
-

4 Estimation

The presence of expectational components in the model requires using specific estimation techniques. As shown in Torój (2009), careful estimation of forward- and backward-looking parameters in the Euler and Phillips curves is critical for accurate modelling of adjustment dynamics after asymmetric shocks.

In the literature, there are two standard manners of handling this problem. Firstly, following a seminal paper of Galí and Gertler (1999), the system can be estimated equation-by-equation using the generalized method of moments (GMM, see Hansen, 1982). Secondly, one can specify a closed system and solve out the forward-looking components using standard algorithms (Blanchard and Kahn, 1980; Klein, 2000; Sims, 2001) and estimate the log-linearized system as a structural VAR, either with classical or Bayesian methods. The classical approach is based on full information maximum likelihood (FIML) estimation.

It is commonly argued that the latter approach outperforms the former in a number of aspects:

- the choice of instruments is usually arbitrary; they can be weak (Staiger and Stock, 1997; Shea, 1997; Stock and Yogo, 2003), non-orthogonal or too many (in the last case, it can lead to spuriously significant estimates - see e.g. Baum et al., 2003);
- in a number of New Keynesian studies, problems with identification of structural parameters have been risen (Mavroeidis, 2005; Nason and Smith, 2005);
- single equation GMM is a limited information method, inferior to system estimation in fully specified multi-equation rational expectations framework (Fuhrer and Rudebusch, 2004; Lindé, 2005; Mavroeidis, 2005);
- in the case of an Euler equation, multiple expectational terms must be instrumentalized in a single equation (Fuhrer and Rudebusch, 2004).

Although Monte Carlo studies of Fuhrer and Rudebusch (2004) and Lindé (2005) confirmed the superiority of FIML over GMM in relatively basic New Keynesian systems such as a closed-economy, 3-equation trinity model, the FIML method is not free of drawbacks. It is based on the assumption of normally distributed disturbances. Also, it requires a careful selection of starting values for parameters in the maximization procedure, possibly close to the global maximum. Nonetheless, due to severity of potential problems with GMM estimation – especially with remarkably short data samples available for Poland and Slovakia – we apply the FIML approach in this study.

The log-linearized model can be summarized in a matrix form as:

$$\mathbf{A}E_t\mathbf{x}_{t+1} = \mathbf{B}\mathbf{x}_t + \mathbf{C}\varepsilon_t \tag{69}$$

The model is solved under the assumption of expectations rationality into the following form (Blanchard and Kahn, 1980; Klein, 2000):

$$\mathbf{x}_t = \mathbf{M}\mathbf{x}_{t-1} + \mathbf{E}z_t + \mathbf{N}\varepsilon_t \tag{70}$$

where $\mathbf{M}_{n_1\times n_1}(\mathbf{A},\mathbf{B},\mathbf{C})$ and $\mathbf{N}_{n_1\times n_1}(\mathbf{A},\mathbf{B},\mathbf{C})$, n_1 – number of endogenous variables in the reduced system. Solving out identities (e.g. price deflator equations) in order to obtain a square matrix N requires to push some lagged variables (price relations) into an exogenous vector z_t of length k, accompanied by a matrix of coefficients $\mathbf{E}_{n_1\times k}(\mathbf{A},\mathbf{B},\mathbf{C})$, also being a function of deep model parameters.

In rational expectations models, it is common to assume AR(1) residuals by construction (see e.g. Mavroeidis, 2005, for motivation):

$$\boldsymbol{\varepsilon}_t = \boldsymbol{\Phi}\boldsymbol{\varepsilon}_{t-1} + \boldsymbol{\xi}_t \tag{71}$$

where $\boldsymbol{\Phi}$ has nonexplosive eigenvalues and $\boldsymbol{\varepsilon} \sim N(0, \mathbf{D})$.

FIML estimation requires to maximize the following log-likelihood function:

$$\mathcal{L}\left[\boldsymbol{\xi}\left(\mathbf{M},\mathbf{N},\mathbf{E},\boldsymbol{\Phi}\right),\mathbf{N},\mathbf{D}\right] = -\frac{Tn}{2}ln\left(2\pi\right) - \frac{T}{2}ln\left|\mathbf{D}\right| + \frac{T}{2}ln\left|\mathbf{N}\right|^2 - \frac{1}{2}\sum_{t=1}^{T}\boldsymbol{\xi}_t\mathbf{D}^{-1}\boldsymbol{\xi}_t \quad \rightarrow \underset{\mathbf{A},\mathbf{B}}{max} \tag{72}$$

Function (72) is maximized iteratively with respect to structural parameters of the model, contained in matrices $\mathbf{A}$ and $\mathbf{B}$, as well as to parameters describing the stochastic properties of the shocks, $\boldsymbol{\Phi}$ and $\mathbf{D}$. The solution of the model is a unique mapping from these parameters to matrices $\mathbf{M}$, $\mathbf{N}$ and $\mathbf{E}$, that are associated with the estimated SVAR model being fit to observable data. Note that the employed estimation procedure places enough restrictions to parametrise this model relatively parsimoniously and to identify the structural disturbances.

The vector of starting values for the iterative procedure has been calibrated in accordance with data and the previous literature (see Table (1)).

The model is estimated in two region pairs:

- euro area and Poland ($w = 0.03$);
- euro area and Slovakia ($w = 0.005$),

whereby the calibration of weights reflects relative real GDP as 2000-2008 average.

In order to fit it to the data over the period 1996q2-2009q3, when Poland and (most of the time) Slovakia were not the euro area members, some necessary adjustments in the model structure need to be done:

- Every region leads an autonomous monetary policy. The smaller region (Poland or Slovakia) has therefore its own Taylor rule.
- As a consequence, the driving variable in country-specific Euler equations is the country-specific nominal interest rate.
- There are nominal exchange rate fluctuations between the two regions. The nominal exchange rate is an endogenous variable, given by the following difference equation reflecting the simple uncovered interest parity condition:

$$e_t = E_t e_{t+1} + i_t - i_t^* + \varepsilon_t^e$$

Table 1: Calibration of the starting values for FIML

Parameter \ Region	**Calibration and source**
consumption habits (h)	0.59 (Smets and Wouters, 2003, estimates for EA)
impatience (β)	0.99 (Kolasa, 2008, calibrated for PL/EA)
elasticity of substitution H/F (η)	1.5 (Lipińska, 2008, calibrated for CZ)
elasticity of substitution T/NT (δ)	0.5 (Lipińska, 2008, calibrated for CZ)
price indexation in T (ω^T)	0.21 (EA), 0.28 (PL/SK) (Kolasa, 2008, estimates for PL/EA)
price indexation in NT (ω^N)	0.162 (Kolasa, 2008, estimates for PL/EA)
price rigidities in T (θ^T)	0.49 (EA), 0.55 (PL/SK) (Kolasa, 2008, estimates for PL/EA)
price rigidities in NT (θ^N)	0.75 (EA), 0.79 (PL/SK) (Kolasa, 2008, estimates for PL/EA)
elasticity of intertemporal substitution (σ)	1.91 (EA), 1.94 (PL/SK) (Kolasa, 2008, estimates for PL/EA)
labour supply elasticity (ϕ)	4 (Lipińska, 2008, calibrated for CZ)
CB response to inflation (γ_π)	1.5 (EA), 1.31 (PL) (Gradzewicz and Makarski, 2009, estimates for PL/EA)
CB response to output (γ_y)	0.46 (EA), 0.21 (PL) (Gradzewicz and Makarski, 2009, estimates for PL/EA)
CB smoothing (γ_ρ)	0.71 (EA), 0.76 (PL/SK)(Gradzewicz and Makarski, 2009, estimates for PL/EA)
serial correlation of demand shock (ρ_c)	0.5 (based on Lindé, 2005)
serial correlation of T inflation shock (ρ_{π_T})	0.1 (based on Lindé, 2005)
serial correlation of NT inflation shock ($\rho_{\pi_{NT}}$)	0.1 (based on Lindé, 2005)
serial correlation of monetary policy shock (ρ_i)	0.8 (based on Lindé, 2005)

Source: author.

Table 2: Data sample (1996q2-2009q2)

Variable	Data	Transformations	Source
C	consumption (national accounts)	dlog, demeaned	Eurostat
inflation in T	PPI in manufacturing	dlog, demeaned	Eurostat (EA-PL), OECD (EA-SK) – due to longer data availability for SK
inflation in NT	HICP in services	dlog, demeaned	Eurostat
interest rate	money market 3M rate	net of disinflation and convergence effects (see the text)	Eurostat
nominal exchange rate	EUR/PLN, EUR/SKK	log, demeaned	Eurostat

Source: author.

- Therefore, the logarithmic terms of trade definition is reformulated with the nominal exchange rate e_t as another component.

The model was fitted using data on consumption, interest rates, nominal exchange rate and sectoral inflation rates (see Table (2)). Consumption and price indices were log-differenced and demeaned. The nominal exchange rate series in logarithms was demeaned. Due to strong seasonal effects in HICP in the first quarter of every year (at least in 1990s) in Slovakia, this series was seasonally adjusted using TRAMO/SEATS.

The data needed some additional adjustment due to disinflationary effects in Poland and Slovakia over the sample period that make the series nonstationary.

- In the case of Poland, the interest rate and nontradable inflation series were detrended using the National Bank of Poland's data on inflation target. This data is not continuous in quarterly terms, and it was smoothed using the Hodrick-Prescott filter.
- No such data is available for Slovakia, as there was no explicit inflation targeting strategy until early 2005 (NBS, 2004). Instead, the main monetary policy objective was defined as a low inflation rate that would allow the fulfilment of the Maastricht criterion. This is why we interpret the Slovak disinflation as an element of euro adoption strategy. In consequence, we disentangle the nominal interest rate on the Slovak money market into an element due to convergence to the euro area and an the residual component of regular monetary policy and policy shocks. We do this by estimating the following equation of Slovak interest rate convergence to the euro area:

$$\triangle i_t = \rho^{\hat{SK}} \left(i_t - i_t^* \right) + \hat{\triangle i_t}$$

Table 3: FIML estimates of model parameters

Parameter \ Region	**PL**	**SK**	**EA**
calibrated values			
openness (α)	0.480	0.899	0.010
share of NT sector (κ)	0.263	0.310	0.401
estimated values			
consumption habits (h)	0.465	0.564	0.522
impatience (β)	0.978	0.999	0.989
elasticity of substitution H/F (η)	0.294	1.136	0.608 (vs PL) / 1.100 (vs SK)
elasticity of substitution T/NT (δ)	0.505	0.558	0.611
price indexation in T (ω^T)	0.304	0.534	0.288
price indexation in NT (ω^N)	0.441	0.249	0.458
price rigidities in T (θ^T)	0.579	0.585	0.665
price rigidities in NT (θ^N)	0.803	0.990	0.768
elasticity of intertemporal substitution (σ)	1.592	2.157	1.958
disutility from work (ϕ)	3.621	2.308	3.378
CB response to inflation (γ_π)	1.170	1.086	1.784
CB response to output (γ_y)	0.484	0.350	0.498
CB smoothing (γ_ρ)	0.614	0.471	0.487
serial correlation of demand shock (ρ_c)	0.499	0.435	0.392
serial correlation of T inflation shock (ρ_{π_T})	0.096	0.084	0.108
serial correlation of NT inflation shock ($\rho_{\pi_{NT}}$)	0.100	0.050	0.114
serial correlation of monetary policy shock (ρ_i)	0.402	0.613	0.607
variance of demand shock (σ_c^2)	0.094	0.875	0.780
variance of T inflation shock ($\sigma_{\pi_T}^2$)	1.294	0.921	0.579
variance of NT inflation shock ($\sigma_{\pi_{NT}}^2$)	0.800	0.782	0.755
variance of monetary policy shock (σ_i^2)	1.257	1.025	0.774
variance of UIP shock (σ_e^2)	1.152	0.909	-

All standard errors of the structural parameters, computed via the Hessian method (see e.g. Calzolari and Panattoni, 1988), are lower than 0.01.
Source: author.

We obtain an estimate of $\rho^{\hat{S}K} = -0.031$ (with a standard error 0.02). Using the values of $\hat{\triangle i_t}$ and the terminal value of $i_{2009Q1} = i^*_{2009Q1}$, we construct the "net of convergence" component of the nominal interest rate. We also use the other, "convergence" component to detrend the data on inflation in nontradables.

Note that we do not detrend the data on tradable inflation. Both ADF and KPSS tests confirm that these series are stationary for both Poland and Slovakia. Furthermore, such detrending would distort the inference regarding the competitiveness channel. As noted by Stevens (1998) in his MCI-ratio considerations, inflation rates of tradable and nontradable goods seem to be completely different processes and the data used confirm this view.

Finally, to ensure a more efficient estimation with relatively short time series, we calibrate 2 model parameters for each region. Following Kolasa (2008), we set α as a corresponding measure of economies' openness and κ as the share of services in the HICP basket (both averaged over 2000-2008).

Table 3 contains the estimation results. The dynamic properties of the adjustment process hinge mainly on the strength of consumption habits (h) and Phillips curve parametrisation (price indexation ω and Calvo probabilites θ). In the former case, the estimates for both countries lie within a narrow range aroud the estimate for the euro area (0.52): 0.47 for Poland and 0.56 for Slovakia.

However, both countries differ in terms of market rigidities. Product market rigidity in the nontradable sector (as measured by the estimated Calvo probability) has turned out to be higher in both regions than in the euro area. In the case of Slovakia, the Calvo probability in the nontradable sector seems to be extremely high (0.99). In Poland it is remarkably lower (0.80), but still exceeds the one for the euro area (0.77). As we do not explicitly model here the labour market mechanisms, one might in fact presume that these parameter values capture the effects of price and wage stickiness jointly. On the other hand, the Calvo parameter in the tradable sector in Poland (0.58) is almost equal to the Slovak one (0.59). Both estimates lie somewhat below the one for the euro area (0.66). Unsurprisingly, three regions in question exhibit more price rigidity in the nontradable sector than in the tradable sector.

In terms of estimated intrinsic inflation persistence, Poland seems to be more similar to the euro area than Slovakia. In the tradable sector, the fraction of backward-looking price setters amounts to 0.304 (against 0.288 in the euro area). In line with expectations and empirical evidence from other studies, the respective estimates for the Phillips curves of the nontradable goods producers are remarkably higher: 0.441 (Poland) and 0.458 (euro area). Against this background, the outcomes for Slovakia are relatively surprising. They suggest a higher persistence in the tradable sector (at 0.534) and lower in the nontradable one (at 0.249). This might result from a high share of administered prices over the sample period.

These results, however, need to be treated with much prudence in the inference about future adjustment capacity of both economies in the case of asymmetric shocks. Firstly, the sample covers almost 14 years in which both economies in transition were deeply reformed, including the field of market regulations. Secondly, the parameter of intrinsic inflation persistence is argued to be regime-dependent and not immune to Lucas (1976) critique. As shown by Benati (2008), this is particularly the case when it comes to the monetary integration in Europe.

In terms of the main factors that affect the competitiveness channel, Slovakia seems to be better equipped with adjustment capacity than Poland. This is above all due to high elasticity of substitution between foreign and domestic tradable goods (1.136 in Slovakia and only 0.294 in Poland). This makes the economic activity in Poland less sensitive to real appreciations or depreciations required to restore the equilibrium in the monetary union. This result is accompanied by a lower degree of openness to foreign consumption. Also, euro area's elasticities of substitution between domestic and Polish or Slovak goods show that Slovak goods are closer subsitutes for euro area consumers (at 1.100 elasticity) than Polish goods (0.608). As a result, the Polish economy is less sensitive to real appreciation or depreciation, which implies a longer period of adjustment via competitiveness channel than in Slovakia. On the other hand, the elasticity of substitution between tradable and nontradable goods in Poland and Slovakia are comparable (0.505 and 0.558 respectively, against 0.611 in the euro area).

The elasticity of intertemporal substitution in Poland ($\hat{\sigma} = 1.592$) is lower than in Slovakia ($\hat{\sigma} = 2.157$). This result suggests that the economy can be more sensitive to the real interest rates in Poland than

Table 4: Estimated "MCI-ratio" for Poland and Slovakia

	PL		SK	
interest rate parameter in Euler curve	**0.229**	*[0.000]*	**0.136**	*[0.000]*
terms of trade parameter in equilibrium condition	**0.177**	*[0.000]*	**1.107**	*[0.000]*
MCI-ratio	**1.293**		**0.123**	

Test for significance: p-values in brackets.
Source: author.

in Slovakia. As a result, it can be more exposed to the procyclical effects of the real interest rate mechanism, especially given the fact that the efficiency of the competitiveness channel would probably be lower.

The estimated, micro-founded model does not correspond to the MCI-ratio analyses based on reduced-form IS curves, incorporating measures of both real interest rate and real exchange rate on the right-hand side and a measure of output gap on the left-hand side of the equation (see e.g. Kot, 2003; Torój, 2008). However, we attempt to construct an analog of this relation by picking:

- the interest rate parameter in the Euler equation (31), $\frac{1-h}{(1+h)\sigma}$;
- the terms of trade parameter in the equilibrium condition on the market of domestic tradable goods (61), $\frac{1-w}{w}\alpha^*\eta^*(1-\alpha^*)(1-\kappa^*)-\alpha\eta\left[\alpha^*(1-\kappa^*)-\frac{w}{1-w}\right]$.

This MCI-ratio (defined as the ratio of interest rate parameter to terms of trade parameter), based on our estimates, has been calculated in Table 4. It follows that the relative sensitivity of Polish economic activity to the real interest rate in comparison to the terms of trade (1.293) remarkably exceeds the analogous relation in Slovakia (0.123). These estimates suggest that the mid-term dominance of the competitiveness channel over the real interest rate effect could emerge more effectively in Slovakia than in Poland.

5 Adjustment dynamics under EMU: simulation analysis

The estimated model parameters (Table 3) are used to simulate impulse-response functions to asymmetric demand and cost-push shocks in the home (Polish or Slovak) economy. In order to keep the comparative perspective, the magnitude of the shocks for both countries are set to 1. In the case of demand shocks (Figure 4), this corresponds to a sudden 1% increase in consumption (a positive shock). For cost push shocks in the tradable and nontradable sector (Figures 5 and 6 respectively) this means a surprise 1 percentage point increase in sector-specific inflation (a negative shock).

In every case, the response of common monetary policy to country-specific domestic shocks is negligible. As the nominal interest rate remains virtually unchanged, we concentrate on alternative adjustment channels.

Impulse-response functions after an asymmetric demand shock confirm the general insight from the estimation results: the competitiveness channel in Slovakia seems to be more effective, as the response of most categories in consideration is flatter and they return more quickly to the steady state than in Poland. In particular, Poland needs a deeper response in terms of trade in order to restore the equilibrium. There is also more volatility in internal terms of trade, as well as tradable and nontradable inflation rates. The realignment after a period of overheating is visible in the nontradable production, as well as in consumption, where some mid-term drop can be observed as a consequence. This is also the case for both tradable and nontradable inflation.

Note the most remarkable difference in impulse-response functions in tradable output of both economies. In Poland, the positive demand shock is accomodated more slowly and the deviation of output from the steady state remains positive over the first few quarters. On the contrary, Slovak output already drops one period after the shock occured. The response of the competitiveness channel is immediate and deep, confirming that in a highly open and exchange rate sensitive economy it can act very effectively as a replacement for the autonomous monetary policy. The Slovak tradable output starts growing after the hump in terms of trade erodes.

A different picture emerges when we analyze the cost-push shocks in the tradable sector (Figure 5). Here, Slovakia turns out to be relatively worse equipped with its (i) more open and exports-dependent economy and (ii) relatively high inflation persistence (compare our previous results in Torój, 2009, for the EA12 countries). This is why we can observe a deep drop in tradable output in Slovakia, while its Polish counterpart suffers relatively moderately from an adverse supply shock. This shock is a direct hit to the country's competitiveness, provided that it is asymmetric. This would not be the case for oil shocks, but it might matter after sectoral shocks (as in the car industry for Slovakia) or disturbances stemming from the labour market.

In both countries, the appreciated terms of trade gradually fall towards the steady-state value (the adjustment seems to be slightly slower in Slovakia due to higher estimated inflation persistence). However, there are two remarkable differences in the realignment after an adverse cost-push shock in the tradable sector. Firstly, Polish consumers switch more eagerly to nontradable goods. The effect of nontradable output growth is additionally boosted by procyclical the real interest rate effect induced by a transitory inflation growth (note the hump-shaped responses of consumption). As a result, Polish nontradable sector plays a more significant role in rebalancing the economy (note the deep reaction in internal terms of trade).

Secondly, note a deeper drop in consumption in Poland than in Slovakia. Higher elasticity of substitution between foreign and domestic tradable goods allows the consumers to swich to goods imported from the euro area and thereby smooth their consumption more efficiently. The Polish economy is less open and the elasticity of substitution against the euro area is lower, which leads to a deeper drop. These responses obviously imply that Slovakia could run higher current account deficits after cost-push shocks in the euro area.

Figure 4: Impulse-response functions for PL/SK after an asymmetric demand shock

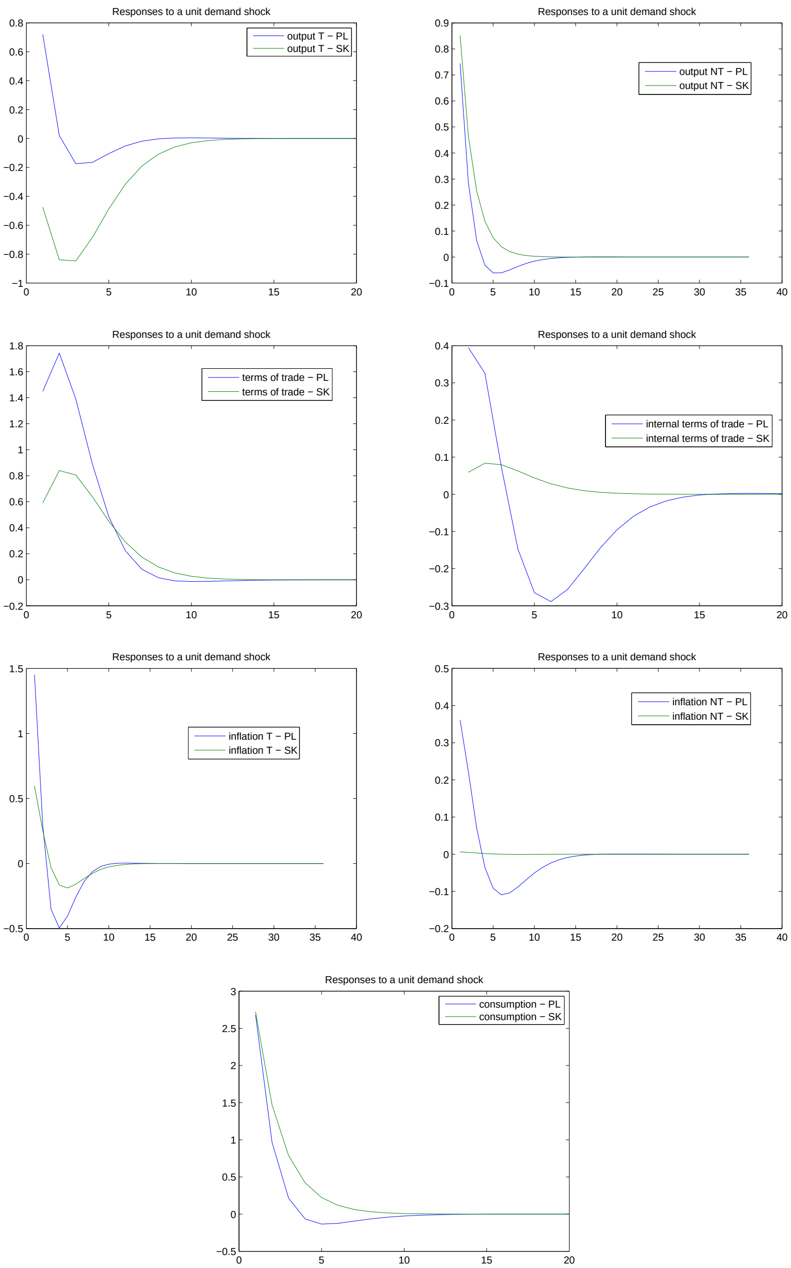


Figure 5: Impulse-response functions for PL/SK after an asymmetric cost-push shock in T

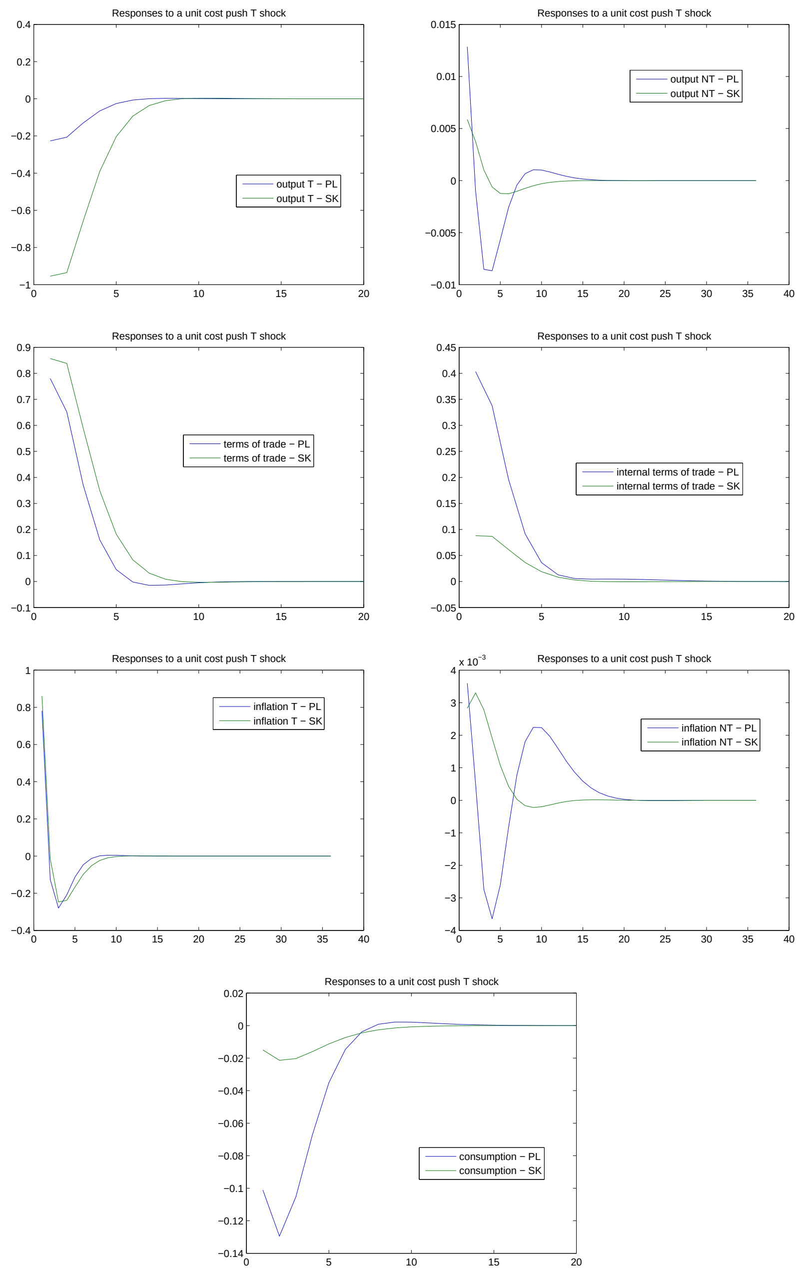


Figure 6 completes the picture with dynamic responses to a cost-push shocks in the nontradable sector. These shocks do not matter directly for the adjustment process. However, they are not directly addressed by either common monetary policy (focused on the foreign big economy) or the competitiveness channel (focused on the tradable sector), so they might have particularly persistent effects.

Directly after the shock, both economies face a similar drop in nontradable output and a hump-shaped internal terms of trade depreciation. In Slovakia, due to higher rigidities in the nontradable sector, the volatilities in responses seem to slightly exceed their Polish counterparts.

Interestingly, there is a qualitative difference in the response of the tradable sector in our setup. In Slovakia, consumers substitute more expensive nontradable goods with tradables, both domestic and imported from the euro area – in comparable amounts. In Poland, they focus on domestic tradables. As a result, domestic tradable output grows (contrary to what is happening in Slovakia). Higher demand for domestic tradables induces inflation and a consequent external terms of trade appreciation. Note that the inflation growth in both sectors causes a bust of a high magnitude, which is illustrated by the dynamic response of consumption. In Slovakia, the size of nontradable sector is lower (as compared to Poland), so that the effects do not spread to the tradable sector to such an extent. The real interest rate effect is weaker, economic slowdown begins immediately and external terms of trade depreciate, restoring the equilibrium more quickly.

6 MCI-ratio and adjustment to permanent interest rate shock

We use the DSGE model derived in Section 3 and estimated in Section 4 to address the issue of structural misalignment in monetary policy for both Central European economies in question. In particular, we are interested in the consequences of a permanent interest rate fall after the accession to the euro area. According to the estimates of Flaig and Wollmershäuser (2007) and Calmfors (2007), a number of lower-income economies have probably suffered from a structural stress in the initial years after the euro adoption.

Reconsider the region-specific Euler curve (32), containing i_t as an autonomous policy rate in the home economy. Assuming that the home economy is in its zero-inflation steady state, i_t equals ρ, i.e. the country-specific natural interest rate.

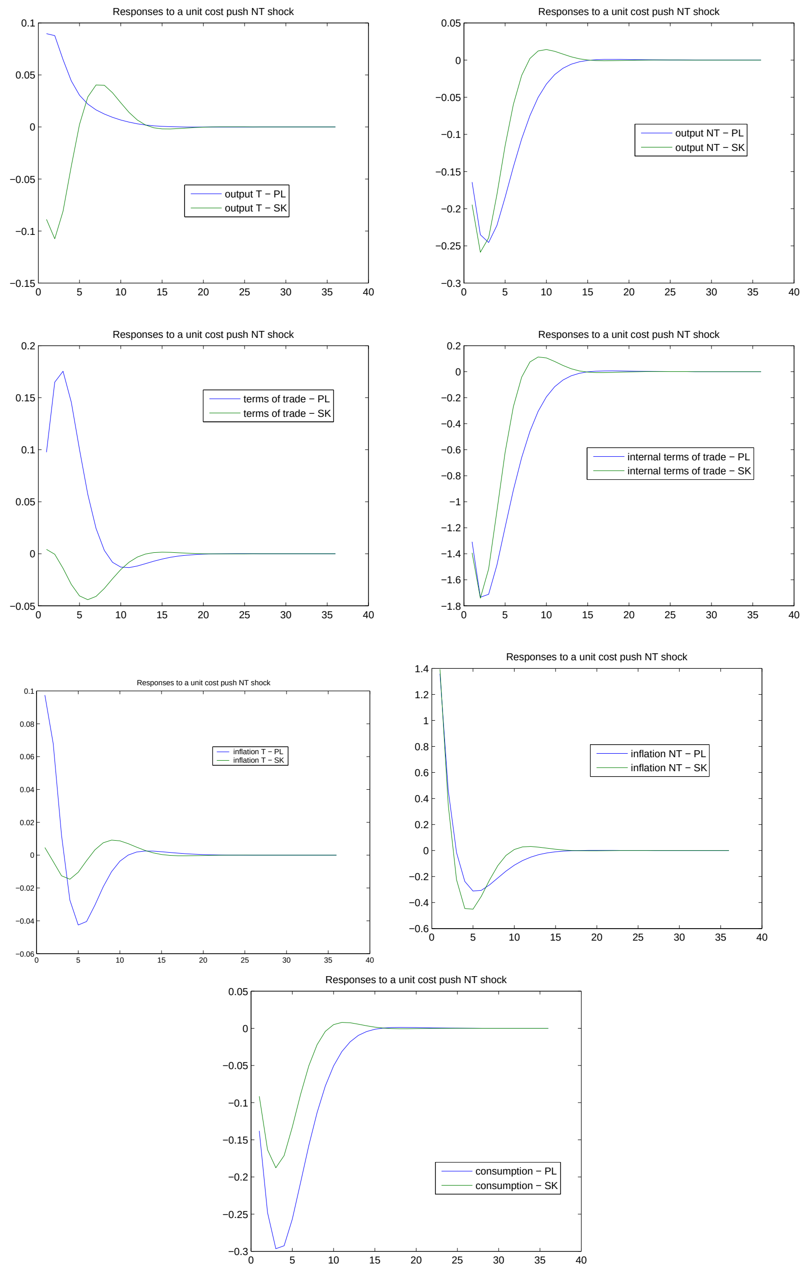
$$c_t = \frac{h}{1+h} c_{t-1} + \frac{1}{1+h} E_t c_{t+1} - \frac{1-h}{(1+h)\sigma} \left(i_t - E_t \pi_{t+1} - \rho \right) + \varepsilon_t^c$$

Let $\Delta i \equiv \rho - \rho^*$ denote an exogenous shift associated with the euro adoption and switch to the common monetary policy regime. This component can be modelled as a permanent shock:

$$c_t = \frac{h}{1+h} c_{t-1} + \frac{1}{1+h} E_t c_{t+1} - \frac{1-h}{(1+h)\sigma} \left(i_t - E_t \pi_{t+1} - \rho \right) + \left[\varepsilon_t^c - \Delta i \frac{1-h}{(1+h)\sigma} \right]$$

In the simulations, we set the fall in nominal interest rates to 1 p.p. in annual terms, which corresponds to $\Delta i = -0.25$ [p. p.] in our quarterly model. To keep the results comparable, we apply the same

Figure 6: Impulse-response functions for PL/SK after an asymmetric cost-push shock in NT



shock for the Polish and the Slovak economy. However, the magnitude of the results differs according to what natural interest rate disparity one might assume for each of these countries.

A critical point in this simulation is calibrating the persistence of this shock. It is highly implausible (and numerically more challenging) to assume that the shock would last forever. The catching-up process will probably tend to close the natural interest rate disparity, at least when we do not take into account other factors possibly involved. This implies the shock's serial correlation below unity. An exact value strongly depends on an arbitrary assumption of how long the process of technological convergence would last. In what follows, we assume the serial correlation in the demand shock $\rho_c = 0.962$, which corresponds to a scenario where 99% real convergence would materialize itself within 30 years. Sensitivity of the main result to this assumption has been tested in Table 5.

It needs perhaps to be stressed that this shock persistence affects the results of the simulations mainly via the channel of agents' expectations. As the adjustment to the new mid-term "steady state" is relatively quick (approximately 2-3 years), the effects of this shock dying out and reversal are not yet observable.

The results of the simulation are presented in Figure 7. In the upper panels, the response of main macroeconomic characteristics of the region are presented for Poland (left-hand) and Slovakia (right-hand). We can disentangle the realignment into several aspects:

1. **Permanent terms of trade appreciation.** When the economic activity is constantly boosted by the low nominal interest rate, the competitiveness channel must counteract this effect by reducing the external demand. This can only be achieved when the terms of trade appreciate. The size of this permanent appreciation is a function of many model parameters, but the estimated MCI-ratio plays here the most prominent role.

   An estimated appreciation of 9.94% in Poland and 2.6% in Slovakia in not surprising. On the one hand, the impact of the shock depends on the intertemporal elasticity of substitution, the main determinant of MCI-ratio's numerator. On the other hand, the required terms of trade appreciation that counterbalances this impulse is all the more pronounced, the less sensitive an economy is to external real appreciation.

   These numbers rest on the assumption that the expected period of closing the natural interest rate gap is 30 years. Taking into account an interval of 25 to 50 years, it might differ from 8.4% to 16.7% for Poland and from 2.2% to 4.3% for Slovakia.

2. **Opening (or deepening) a current account deficit.** The impact on consumption clearly exceeds the effect on output in both sectors. This leads to a persistent current account deficit. The access to the savings of the foreign big economy is necessary to keep the interest rate low. This effect is perfectly in line with the experience of catching-up economies within the euro area (such as Greece or Portugal; see e.g. Fagan and Gaspar, 2007). Importantly, our model does not contain any corrective mechanism for the current account or net foreign asset stock in the home economy. According to Blanchard and Giavazzi (2002), "up to a first order" this should not be seen as inherently problematic. Indeed, the questions about thresholds and mechanisms of such correction in a monetary union remain, to the best of our knowledge, without any consensus

answer in the literature today. One possible implementation would be to introduce collateral constraints for households into the model.

3. **Vague impact on the nontradable sector.** Simulation results for both economies differ qualitatively. In Poland, the internal terms of trade slightly depreciated, while in Slovakia – slightly appreciated. This can be seen as an indirect effect associated with sectoral rebalancing in response to external terms of trade appreciation, required by the competitiveness channel. The impact is a function of a set of structural parameters, mainly the elasticity of substitution between the tradable and nontradable goods of domestic households.

4. **Possible drop in the tradable output.** There is a qualitative difference between Poland and Slovakia in terms of the new tradable output level after the permanent shock under consideration. While the Polish output of tradable goods grew slightly, the Slovak one dropped. The reason for this is similar to the motivation of the differing patterns of adjustment of this variable after an asymmetric demand shock. The effect in Poland is mainly driven by rising domestic demand. Consumers are less discouraged to buy domestic products due to lower elasticity of substitution between Polish goods and goods imported from the euro area (the same applies to the consumers in the euro area). Lower openness of the economy makes this effect additionally limited. On the other hand, permanently appreciated terms of trade hit the Slovak exporting sector, leading to its contraction in the mid-term.

In the lower panels of Figure 7, an alternative adjustment scenario is considered. Instead of a fully market-based transition to the new mid-term equilibrium (dashed lines), it is assumed that policymakers know how much the terms of trade will need to appreciate on a permanent basis. This shift is precisely applied in conjunction with the interest rate shock. It would be equivalent to setting the conversion rate of the national currency against the euro (or the central parity) at a stronger level than the pre-euro economic fundamentals (including the nominal interest rates that would prevail if euro adoption was not expected) would justify. The revaluations of the Slovak koruna in the ERM II might possibly be seen as a part of this process.

For Slovakia, in the alternative scenario, all the variables on the adjustment path seem to be less volatile. It is then highly probable that a welfare analysis run with this model would indicate it as a preferable scenario. However, to confirm this in general, a much more sophisticated investigation would have to be repeated in a large-scale model with a more detailed sectoral breakdown, financial constraints and frictions, investment and multiple stages of production. The case seems to be even more dfficult to assess for Poland, as consumption and tradable output exhibit some overshooting pattern under the alternative scenario. A detailed analysis of this issue would also need a more elaborate application of the permanent shock (i.e. a gradual fall in interest rates, as agents start to discount euro area entry in their expectations).

In general, both scenarios might have their advantages and disadvantages whose thorough consideration exceeds the depth of this analysis. The market-based scenario (dashed lines) would probably induce a prolongued period of higher inflation, which seems to be particularly dangerous for expectations in countries with a short history of low inflation. Here, on the contrary, rationality of expectations was

Figure 7: Permanent 1 p.p. shock in nominal interest rates

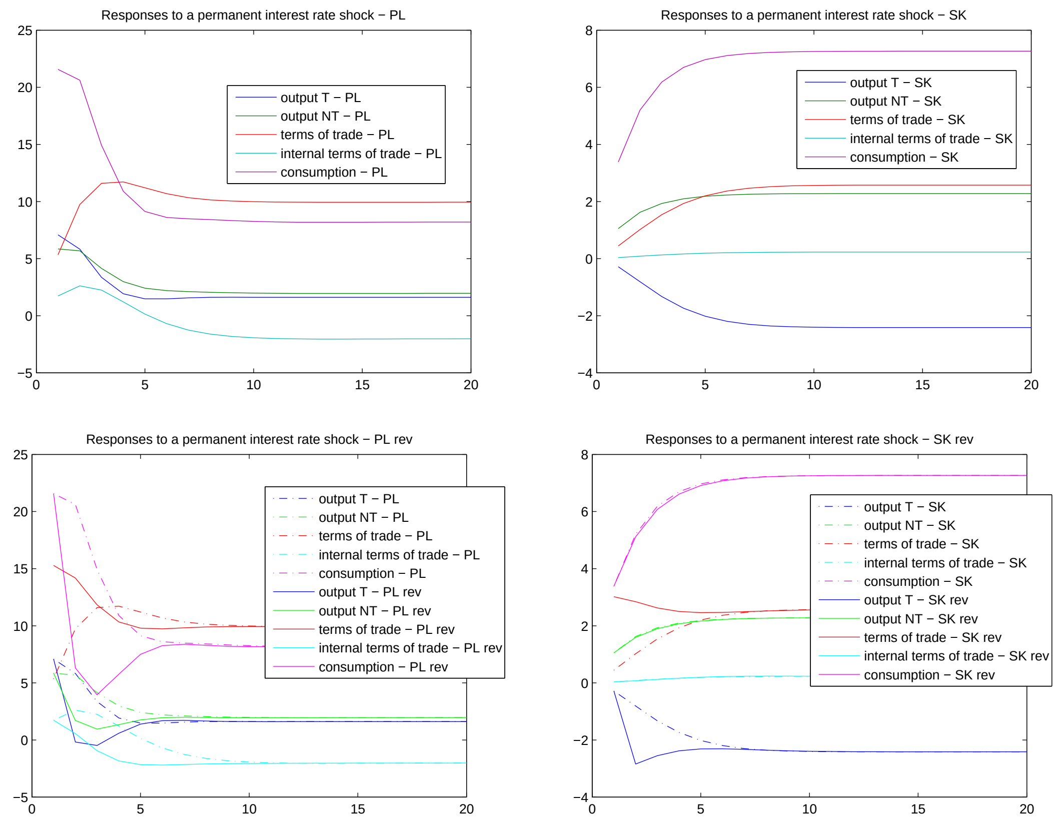


Table 5: Persistence of structural stress in monetary policy vs required terms of trade appreciation

time of natural rate disparity [years] expected by agents	25	**30**	35	40	45	50
corresponding ρ_c	0.955	**0.962**	0.968	0.972	0.975	0.977
permanent terms of trade appreciation – PL	8.42	**9.94**	11.79	13.45	15.05	16.35
permanent terms of trade appreciation – SK	2.17	**2.57**	3.07	3.51	3.94	4.28

assumed. This scenario might also be undesirable when the inflation persistence or price rigidities are high. On the other hand, the revaluation-based scenario (solid lines) could potentially hit the tradable sector severely, including advanced and innovative branches. In Slovakia (see Figure 7), a sharp drop of the tradable output was observed. This would put the producers of tradable goods under substantial time pressure. On the contrary, under gradual appreciation, they would have more time for the necessary restructuring processes.

7 Conclusion

In this paper, we attempted to compare Poland's and Slovakia's capacity to absorb macroeconomic asymmetric shocks in the euro area. We also considered a permanent fall in the nominal interest rates as a stimulus, shifting the economy to a new, "mid-term" steady state. For this purpose, we applied a New Keynesian DSGE model of a 2-region, 2-sector currency union, including various sources of cross-regional heterogeneity.

The estimates suggest that, when it comes to the competitiveness channel, Slovakia is better equipped to adjust after asymmetric demand shocks than Poland. This difference is moderate, and it mainly stems from the fact that Slovakia is a smaller, more open economy with higher elasticity of substitution between the foreign and domestic tradable consumption. This should make the competitiveness channel – a mechanism of adjustment via external and internal demand for domestic tradables – more efficient.

The picture after cost-push shocks in both sectors is somewhat more nuanced, but does not substantially affect the general conclusion. After an adverse shock, an economic slowdown in the tradable sector in Slovakia – given the estimated parametrisation – could be deeper than in Poland, as adverse supply-side developments hit more severely its external competitiveness. On the contrary, the adjustment in Polish production is flatter but slower than in the Slovak case.

When the fall in nominal interest rates is persistent, the economy shifts to a different position. The main feature of this permanent adjustment is an appreciation of external terms of trade. Its quantitative assessment strongly depends on the assumption how long the natural interest rate differential will last. With a 30-year long period of real convergence, the estimated terms of trade appreciation for Poland is 9.9%, whereby for Slovakia 2.6%. This difference is intuitive, given the fact that the Polish economy is less sensitive to external competitiveness developments than the Slovak one.

The latter finding suggests that the ultimate conversion rate could to some extent be considered as a tool faciliating the transition to the new equilibrium point. However, the impact of such measure on the tradable sector requires a more profound investigation.

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