

Is Exchange rate - Customer order flow relationship linear? Evidence from the Hungarian FX market*

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Abstract

Over the last decade, the microstructure approach to exchange rates has become very popular. The underlying idea of this approach is that the order flows at different levels of aggregation contain valuable information to explain exchange rate movements. The bulk of empirical literature has focused on evaluating this hypothesis in a linear framework. This paper analyzes nonlinearities in the relation between exchange rates and customer order flows. We show that the relationship evolves over time and that it is different under different market conditions defined by exchange rate volatility. Further, we found that the nonlinearity can be captured successfully by the Threshold Regression and Markov Switching models, which provide substantial explanatory power beyond the constant coefficients approach. An important policy implication derived from our results is that market interventions would have bigger impact in periods of high volatility.

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1 Introduction

In standard macroeconomic models exchange rate is determined by fundamental factors, which are observed by all agents in the economy and constitute

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public knowledge. In these models, there is no private information and price determination is straightforward and immediate. Unfortunately, their empirical performance is very poor. Meese and Rogoff (1983a,b) show that the structural macro models almost do not have power to explain exchange rate movements and cannot out-perform a naïve random walk in out-of-sample fitting (see Frankel and Rose (1995), Isard (1995) or Taylor (1995) for a survey).

Inspired by Lyons (1995), the market microstructure approach has recently become popular. According to this approach, the information on the market is asymmetric, i.e., some agents have private information. When the market is not fully efficient, the informed agents can exploit this information to get profit. In the classical framework (Evans and Lyons (2002a)), (risk averse) market-makers get informed about local demand by observing orders from their own customers (1st-round trading), and receive information about global demand by trading with other dealers in a 2nd-round. Consequently, market-makers can infer the private information from the order flows, and adjust the quotes accordingly trying to close with zero net open positions. In this way, the information is embedded into the market through the order flows.

A growing amount of empirical literature focuses on examining the relation between exchange rates and order flows, mainly by estimating a linear regression of the price changes on the net order flows. The overall conclusion is that order flows contain relevant information for exchange rate determination (see e.g. Evans and Lyons (2002a) for inter-dealer order flow or Bjønnes et al. (2005) and Marsh and O'Rourke (2005) for customer order flows).

Although linearity has been a maintained assumption in the empirical literature, a time varying relation between exchange rates and order flows is, a priori, more realistic: why should the relation be the same under dramatically different market situations as crises or periods of growth? According to theoretical models (Admati and Pfleiderer (1988), Diamond and Verrecchia (1987), Foster and Viswanathan (1990), Subrahmanyam (1991) or Easley and O'Hara (1992)), exchange rate volatility is one of the key variable identifying the portion of information embedded to the market. Since the order flows are means of information transmission, the relationship between them and exchange rate is thought to be different in periods of high and low volatility.

In this paper, we first conduct an intensive analysis of the linear relation between exchange rate and customer order flow. Results of this analysis reveal that the relationship evolves over time and that it is different under distinct market conditions defined by volatility. We provide further evidence of this result through the estimation of two types of nonlinear models: Markov Switching (MS) and Threshold Models (TR). We use data on the Hungarian forint (HUF) - euro (EUR) spot exchange rate and different types of spot customer order flows: foreign participants and domestic non-banks. The data set was provided by the Magyar Nemzeti Bank (Central Bank of Hungary). In this sense, our work is related to Gereben et al. (2006), who use a similar data set to study the same relationship in a standard linear setting.

Although most of the empirical analysis has focused on the major currencies, the analysis of a CEEC currency is of major interest since the better understand-

ing of the information transmission mechanism has strong policy implications for countries that (as Hungary) plan to join to European Monetary Union, and that, eventually, have to make use of foreign exchange intervention to keep the currency into the prescribed bands.

Our main results include the following. First, we confirm that the customer order flows considered in this paper contain valuable information to explain contemporaneous HUF/EUR exchange rate movements. Second, the relationship between order flows and exchange rate is clearly nonlinear. Order flows have higher impact on exchange rates and are more informative in periods of high volatility. This result is in line with the information uncertainty model developed by Easley and O'Hara (1992), the conclusions obtained by Subrahmanyam (1991) and the results of Luo (2001). Third, the nonlinear relation between exchange rate and order flows can be successfully captured by nonlinear models. In particular, the nonlinear models, and specially the MS, provide substantial explanatory power beyond the constant coefficient (OLS) approach, and are crucial to understand the information transmission mechanism from specific groups of customers. To this respect, we find that although foreign participants are the major drivers of Hungarian exchange rate fluctuations, the domestic non-bank order flow also contain considerable information to explain exchange rate movements.

Despite the amount of literature devoted to nonlinear exchange rate modeling over the last decade, the question of the nonlinearity in the exchange rate – order flow relationship has still not received much attention. Up to our knowledge, only Lyons (1996), and Berger et al (2008) touched this issue slightly ¹, although none of them analyzes the nonlinearity intensively. A noticeable exception can be found in Luo (2001)². The main differences of this work with the existent literature and, in particular, with Luo (2001) are the following. First, this is the first work which studies nonlinearity in the exchange rate -customer order flow relation. The role of the customer order flows is central in the microstructure literature. They are the prime source of the private information in the market and a catalyst for the inter-dealer market activity in all canonical models (Kyle (1985), Glosten and Milgrom (1985) or Evans and Lyons (2006)), and therefore more important in the determination of exchange returns than inter-dealer order flow.

Second, there are differences in the set of models employed. Initially, we remain a priori agnostic about the sources driving the nonlinearities in the ex-

¹Lyons (1996) tests weather the information content of the order flow increases with market intensity. He concludes that the order flow is more informative when trading intensity is high. Berger et al. (2008) finds evidence of nonlinearities by comparing the estimates of the OLS regression of exchange rates on order flows with the ones obtained from a nonparametric regression.

²Luo (2001) tests the equally information hypothesis of the exchange rate inter-dealer order flow relationship under different market conditions. The author finds that the information contained in the order flow tends to increase with spread and volatility, and decrease with the volume of trade. He also finds that an interaction model and a Logistic Smoothe Threshold Regressive (LSTR), are able to capture this nonlinearities, although he does not evaluate their ability to explain exchange rate movements.

change rate - order flow relationship by estimating a Markov Switching (MS) model. Since the seminal work of Boothe and Glassman (1987), there is increasingly strong evidence that the conditional distribution of the nominal exchange rates is well described by a mixture of normal distributions, and that MS models fit exchange rate data very well. However, in spite of the sizable attention that MS models have currently received in the exchange rate literature, they still have never been employed for the exchange rate - order flow analysis. Given that we find very high correlation between the estimated regime changes and exchange rate volatility, we after consider several Threshold Regression (TR) specifications using volatility as a threshold variable. In this way, we evaluate if volatility by itself can direct regime changes. Since contemporaneous daily volatility is potentially endogenous, we evaluate the severity of this problem through the estimation of a Threshold Regression with Endogenous Threshold (THRET) model recently proposed by Kourtellos et al. (2009) that controls for endogeneity of the threshold variable.

Since statistical significance of the nonlinearities is a necessary but not sufficient condition for model adequacy, the third and last contribution is the evaluation of the nonlinear order flow specifications to explain in- and out-of-sample exchange rate movements which, up to our knowledge, has never been analyzed.

The remaining of the paper is organized as follows. Section 2 contains a description of the data set, preliminary estimations and nonlinearity testing. Section 3 makes a short description of the two nonlinear models that we use to fit the data: Threshold and Markov Switching models and presents the estimation results together with their performance, both in- and out-of-sample. Section 4 concludes.

2 Data description and Empirical Preliminaries

2.1 Data description

The data set consists from the HUF/EUR exchange rate and corresponding customer order flows data at daily frequency covering the period from the 2nd January 2003 to the 15th July 2009. The exchange rate contains quotes from the Reuters D2000-2 system. We use the midpoint of the best bid and ask quotes at 5:00 PM each day. We apply the logarithmic transformation to the exchange rate series to circumvent "Siegel paradox". After, we take first difference and multiply it by 100 to create a series of daily returns. The data on customer order flows is provided by the Central Bank of Hungary. The source of the data is the Daily Foreign Exchange Report of the Bank, which contains all foreign exchange transactions of significant size carried out by commercial banks residing in Hungary. In this sense, our order flows data set is not complete because it does not cover transactions produced by the financial institutions located offshore. According to the Central Bank of Hungary information, the offshore turnover is significant, although the most important market-makers are

said to be the locally based ones³. Additionally, we do not include the central bank order flow into analysis because of confidentiality. Central bank order flow was relatively stable in our data span with the only exception of the speculative attack on January 15 and 16, 2003, when the Bank was carrying out large-scale Hungarian forint sales to defend the exchange rate band⁴. Consequently, we exclude first 11 observations from the sample.

It is important to remark that compared with the data sets used so far, this data set has important advantages. First, our study focuses in customer (end-of-user) order flow data. Although the bulk of empirical literature studies the intra-dealer order flows, customer order flows are consistently more important in exchange rate determination than inter-dealer order flow according to the market microstructure literature (see e.g. Lyons (1995), Evans and Lyons (2005) or Sager and Taylor (2008)). Moreover, the customer order flow data is broken down according to the nature of customer: foreign participants and domestic non-banks, allowing us to test different information content of each of these order flows. Different types of customers can have different sources of private information and different aims, making the analysis particularly interesting. Concentration by the consumer allows us to use highly aggregated data with information content⁵. Second, the coverage of the data set is relatively high. Even taking into account that it does not contain information on transactions produced by the institutions located offshore, it provides a more complete picture of the market than the one offered by studies that use data from a single market-maker (Froot and Ramadoari (2002), Carpenter and Wang (2003), Mende and O'Rourke (2005) and others). Finally, as opposite to commercial data sets, the data from the MNB is not revealed to the market⁶, and is comparatively long for the standard microstructure literature⁷, so it is particularly well suited for the study of nonlinearities, since sample size is crucial to detect regime-change dynamics in the data.

Figure 1 plots the log-HUF/EUR exchange rate (left axe) and two cumulated customer spot order flows: foreign participants and domestic non-banks (right axe). The descriptive statistics on the series is presented in the Table 1.

2.2 Preliminary Estimations and Linearity Analysis

In this section we analyze the linearity assumption in the relationship between exchange rate and consumer order flows. To do so, we start by computing the

³For more detailed data description look Gereben et al. (2006), Appendix 1

⁴The Hungarian forint's exchange rate is allowed to fluctuate within +/- 15 per cent band relative to euro.

⁵Completely aggregated over a trading day, total signed order flow of a bank is equal to zero, at least as an approximation. Therefore, aggregated over all banks total order flow is only randomly different from zero and uncorrelated with exchange rate changes.

⁶The power of commercial datasets to explain exchange returns has been recently questioned by Sager and Taylor (2008).

⁷Similar spans can only be found in Bjonnes et al. (2005) and Gereben et al. (2006) for the Swedish krona/euro and Hungarian forint/euro respectively. Comparatively with last, our data set is more aggregated and contains 4 years (approximately) more of observations.

sample cross-correlations between the HUF/EUR returns and the order flows. Results are presented in the Figure 2. The contemporaneous cross-correlations between returns and foreign participants and domestic non-banks order flow, are in both cases statistically different from zero, negative for the first series and positive for the second.

As a benchmark, we estimate a generic linear model for the exchange rate – order flow relationship. As in Bjønnes et al. (2005), we perform separate regressions for the different type of customers in our data (foreign participants and domestic non-banks):

$$y_t = \alpha_i + \beta_i X_{i,t} + \epsilon_{i,t} \quad (1)$$

where y_t are the exchange rate returns and $X_{i,t}$ is the respective net customer order flow. The results of this estimation are presented in the Table 2⁸. As expected by the cross-correlation analysis, the estimated slope coefficient is negative for the foreign participants order flow and positive for the domestic non-banks, and strongly significant in both regressions. In this respect, our estimations satisfy the notion of "push" and "pull" customers (see e.g. Bjønnes et al. (2005)) and can be interpreted accordingly.

Push customers are though to provide information to the market that is not yet common knowledge, initiating the orders and causing price changes (they are 1st-round traders according to the Evans and Lyons (2002a) setting). As a result, their trading will be positively correlated with price movements. Instead, pull customers are assumed to provide liquidity to the market, absorbing the open positions of market-makers generated when trading with push customers and their trading will be negatively correlated with price movement. Pull customers are attracted into the market by prices which suit them because they wish to trade on a certain side of the market and decide to act now rather than postpone the trade in the hope of achieving a better price. Motivated by the cross-correlation analysis above and the results of the estimation of the linear model (1)⁹, the foreign participants can be identified as push customers and domestic non-banks as pull. These was also the roll attributed to foreign participants and domestic non-banks by Gereben et al. (2006), which study the standar linear model using a very similar data set ¹⁰.

⁸We also performed the analysis plugging all the order flows in the same equation $y_t = \beta_0 + \sum \beta_i X_{it} + e_t$. However, as in Gereben et al. (2006), severe multicollinearity is detected when they are used together in the same equation. Although this problem does not affect the predictive power of the model, it affects the coefficient estimates which change erratically in response to small changes in the data. Multicollinearity is easily explained when one takes into account that push customers provides the liquidity to the market that makers need to close their positions, being consequently the mirror image of 1st round customers.

⁹Note that the usual quotation of the currency pair is HUF/EUR, therefore an increase in the exchange rate corresponds to a depreciation of the forint. Negative coefficients for the order flow indicate buying orders that cause Forint appreciations, while positive coefficients indicate a depreciating impact.

¹⁰However, as authors note, this definition is not completely in line with previous findings where the distinction that matters for determining the sign of the order flows' impact on

As can also be seen in the table, the foreign participants appear to be the most informative order flow in the linear setting, explaining a 37% of the variability in the returns, while domestic non-banks explains a 12%. We further test the estimated residuals for the presence of heteroskedasticity for all customers' order flows with the Breusch-Pagan and Cook-Weisberg tests. The null hypothesis of constant variance is rejected at 5% significance level in all estimations. We therefore impose a GARCH(2,1) structure for the conditional variance, but it does not increase the fitting significantly. In general, the results obtained with the linear model are in accordance with the ones obtained by Gereben et al. (2006) for the same currency.

Next, we proceed to question the linearity assumption in the relation given by (1). As a preliminary stage, we apply a standard BDS test to detect lack of independence in the estimated residuals of the linear model. The BDS is a fairly general linearity test that has power against most nonlinear dynamics, although the type of nonlinearity can not be exactly determined. The null of independent and identically distributed errors is rejected at 5% significance level for all estimations.

In order to inspect the linear relationship further, we implement a moving window regression to the equation (1). This allows us to check how the coefficients associated with the order flows evolve over time. We choose a window length equal to one hundred observations (five months approximate). The window regression slope coefficients (β_i^t) and their confidence intervals are plotted in Figure 3, together with the series of returns. As can be seen in the figure, the slope coefficient for all the order flows changes significantly with time, increasing (in magnitude) in periods when returns are more volatile.

To confirm this result, we proceed to test the stability of the estimated parameters under different volatility states. We start by constructing a simple daily exchange rate volatility series z_t as:

$$z_t = \frac{|\log(p_t) - \log(p_{t-1})|}{\sqrt{2/\pi}} \quad (2)$$

where p_t is the exchange rate at t . After, we divide the sample into two sub-samples according to the volatility regime: high volatility periods, when daily volatility at t is higher than its mean, and low volatility periods otherwise. After estimating equation (1) by separately in the two sub-samples, we test for equality of the estimates across sub-samples with Chow test. The results are presented in the Table 2. The equality hypothesis is strongly rejected for all order flows. It is interesting to note that the informativeness and the price impact (sensitivity) of all order flows are higher in periods of high volatility. For

the exchange rate is between financial and non-financial customers (Mende and Menkhoff (2003), Bjonnes, Rime and Solheim (2004) and Marsh and O'Rourke (2005)). A potential explanation that the authors give to this results "stems from the fact that Hungary is an emerging market economy, relying heavily on foreign capital flows. A large share of the economic fundamentals governing the forint's exchange rate are dependent on external factors. As a result, it is likely that foreign customers are more likely to convey non-public information about future fundamentals through their trades than domestic customers."

example, foreign participants explains a 47% of the variance of returns in periods of high volatility but only a 10% in periods of low volatility. Informativeness of the domestic non-banks order flow increases even more dramatically. Thus, in low volatility periods both order flows explain less than 1% of the variation of returns whereas in high volatility periods this number increases up to a 20 %. The increase in ‘sensitivity’ can be observed by looking to the slope coefficients under different volatility regimes. For all the order flows, the slope coefficients in high volatility periods are much larger in absolute value than in low volatility periods.

These results are consistent with implications of the theoretical models of Subrahmanyam (1991) and Easley and O’Hara (1992): in periods of high volatility customer order flows appear to be more informative and having bigger impact on the exchange rate.

As a resume, the inspection of the linear relationship given by (1) provides substantial evidence that there exists important nonlinearities in the data not captured by the generic model. Further, the volatility is strongly related to these nonlinearities, increasing the price impact of the order flows on returns.

3 Non-linear Estimation and Model Fitting

3.1 Modeling Non-linearity

In this subsection we briefly describe the two types of nonlinear models that are used in this paper: Markov switching and Threshold Regression.

3.1.1 Markov Switching Model

Markov Switching Models (MS) became popular for exchange rate modeling since seminal work of Engel and Hamilton (1990) and Engel (1994). Several studies relate exchange rates and macroeconomic fundamentals in a MS context (see e.g. Marsh (2000), Bessec (2003), Sarno et al. (2004) or Frömmel et al (2005)) or analyze spot and forward exchange rates comovements (see e.g. Clarida et al. (2002)). Up to our knowledge, this is the first attempt to model the relation between exchange rates and order flows in a MS framework.

Allowing for two regimes in the coefficients, the relation between y_t and X_t can be written in a MS as :

$$y_t = \alpha_{s_t} + \beta'_{s_t} X_t + e_t, \quad t = 1 \dots T \quad (3)$$

where y_t denote exchange rate returns, X_t is a vector of the net order flow and e_t is a vector of uncorrelated disturbances with zero mean and variance σ^2 .

The unobserved state variable $s_t = \{1, 2\}$ follows a two-state, first order Markov process with the following transition probability matrix:

$$P = \begin{pmatrix} \Pr(s_t = 1 | s_{t-1} = 1) & \Pr(s_t = 1 | s_{t-1} = 2) \\ \Pr(s_t = 2 | s_{t-1} = 1) & \Pr(s_t = 2 | s_{t-1} = 2) \end{pmatrix} = \begin{pmatrix} p_{11} & 1 - p_{22} \\ 1 - p_{11} & p_{22} \end{pmatrix}$$

where the transition probabilities p_{hh} give the probability that state h will be followed by another state $h = \{1, 2\}$. These transition probabilities are assumed to remain constant between successive periods. With the additional assumption that e_t in (3) are normally distributed (conditional to the information available at time t , Ω_t), the conditional density of y_t is normal.

The model can be estimated applying the Expectation Maximization algorithm (see e.g. Hamilton (1994) for further details). We will refer to the probability to be in state h based on information of the whole sample Ω_T as smoothed probability ($\Pr(s_t = h|\Omega_T)$).

3.1.2 Threshold Model

Threshold regressive (TR) models were first proposed by Tong (1978), Tong and Lim (1980) and Tong (1983), and a comprehensive statistical analysis was made by Hansen (2000). The main idea is that the evolution of the process, governing the dependent variable at any point of time, depends on the value of an observed *threshold variable* z_{t-d} relative to a *threshold value* c .

Formally, the Threshold Model can be written as:

$$y_t = b'_1 X_t + b'_2 X_t F(z_{t-d}, \gamma, c) + e_t \quad (4)$$

where X_t is a vector of explicative variables, which may contain lags of the endogenous variable, e_t is a martingale difference sequence with constant variance σ^2 and $F(z_{t-d}, \gamma, c)$ is known up to parameters vector γ and scalar c . In standard autoregressive specifications (Threshold Agressive, TAR), the threshold variable z_{t-d} is usually chosen to be a lagged value of the dependent variable (y_{t-d}), although it could be any other exogenous variable. The transaction function $F(z_t, \gamma, c)$ can be either continuous logistic

$$F(z_{t-d}, \gamma, c) = [1 + \exp\{-\gamma(z_{t-d} - c)\}]^{-1}$$

or exponential

$$F(z_{t-d}, \gamma, c) = 1 - \exp\left\{-\gamma(z_{t-d} - c)^2\right\}$$

or discontinuous with $\gamma \rightarrow \infty$, giving back 1 if $z_{t-d} \leq c$ and 0 otherwise. In the first case the model is logistic or exponential smooth TR (LSTR or ESTR), in the second, a standard TR.

Since we are interested in analyzing the contemporaneous relationship between exchange rate and customer order flow, as before we define y_t to be the exchange rate returns and X_t - a particular customer net order flow. According to theoretical models, regime changes depend on the contemporaneous ($d = 0$) market conditions defined by the threshold variable z_t .

To estimate the model, we maximize the likelihood function, assuming that the errors are normally distributed. To get the starting values for the estimation we apply the methods described in Franses and van Dijk (2000, p.91). The smoothness parameter γ is restricted to be positive ($\gamma > 0$) and the threshold value c is estimated internally.

One of the crucial assumptions of the TR model is that the threshold variable is exogenous. If correlation between the threshold variable with the contemporaneous error e_t is suspected, the exogeneity assumption cannot be insured. To overcome this problem a recent model developed by Kourtellos et al. (2009) can be employed: the Threshold Regression with Endogenous Threshold variable (THRET)¹¹. The model and the estimation method proposed by the authors (THRET-C2SLS) are detailed below.

The model is the following.

$$y_t = \beta_1' X_t + e_{1t}, \quad \text{if } z_t \leq c \quad (5)$$

$$y_t = \beta_2' X_t + e_{2t}, \quad \text{if } z_t > c \quad (6)$$

$$z_t = q_t \pi + v_t \quad (7)$$

Two first equations describe the relationship between the variables of interest in each of the two regimes, z_t is the threshold variable with c being the sample threshold value. The third equation is the selection equation that determines the regime that applies. The matrix q_t is $T \times l$, $q_t = [q_{1t}, q_{2t}]$, where q_{1t} is $T \times l - 1$ matrix of instruments for the threshold variable and the vector q_{2t} contains X_t . The variance covariance matrix of the errors (e_{1t}, e_{2t}, v_t) has the following properties: $E(e_{1t}, e_{2t}) = 0$, $E(e_{it}, v_t) = \sigma_{ve_i} > 0$, $E(e_{it}^2) = \sigma_i^2 > 0$, $i = 1, 2$, and $E(v_t^2) = \sigma_v^2 = 1$ due to a normalization. Notice that if $\sigma_{ve_i}^2 = 0$, $i = 1, 2$, the Threshold variable is exogenous.

We can also rewrite the THRET model (5), (6) and (7) in a single equation:

$$y_t = \beta_2' X_t + (\beta_1 - \beta_2)' X_{1t}(c) + \kappa_2 \lambda_t (c - q_t \pi) + (\kappa_1 - \kappa_2) \tilde{\lambda}_{1t}(c - q_t \pi) + \epsilon_t \quad (8)$$

where $X_{1t}(c) = X_t I(z_t \leq c)$, $\lambda_t(c - q_t \pi) = I(z_t \leq c) \lambda_{1t} + I(z_t > c) \lambda_{2t}$. The terms $\lambda_{1t} = -\frac{\phi(c - q_t \pi)}{\Phi(c - q_t \pi)}$ and $\lambda_{2t} = \frac{\phi(c - q_t \pi)}{1 - \Phi(c - q_t \pi)}$ are inverse Mills bias correction terms, where ϕ and Φ represent the pdf and cdf of a standard Normal. In the above equation we have also defined $\tilde{\lambda}_{1t}(c - q_t \pi) = I(z_t \leq c) \lambda_{1t}$, and renamed the covariance between error v_t and e_{it} as $\kappa_i = \sigma_{ve_i}$ for $i = 1, 2$. The error term in (8) is given by $\epsilon_t = I(z_t \leq c) (e_{1t} - \kappa_1 v_t) + I(z_t > c) (e_{2t} - \kappa_2 v_t)$. It can be easily seen that if the threshold variable is exogenous, or $\kappa_1 = \kappa_2 = 0$, the expression(4) is equivalent to (8) with $b_1 = \beta_2$, $b_2 = \beta_1 - \beta_2$, $F(z_t, \gamma, c) = I(z_t \leq c, \gamma \rightarrow \infty)$ and $e_t = \epsilon_t$.

The estimation procedure has three steps. First, we estimate the parameter vector π in the threshold equation (7) by Least Squares (LS). Second, we estimate the threshold parameter by minimizing a concentrated two stage least squares (THRET-C2SLS) criterion using the estimates of $\hat{\pi}$ from the first stage:

¹¹Carner and Hansen (2004) developed a generalized method of the model estimator for Threshold Regression models with endogenous regressors. Kourtellos et al. (2009) extended the model of Carner and Hansen (2004) to allow for endogeneity of the threshold variable.

$$\hat{c} = \arg \min_c S_n(c)$$

where

$$S_n(c) = \left(y_t - X_t \beta_2 + X_{1t}(c) (\beta_1 - \beta_2) + \kappa_2 \lambda_t (c - q_t \hat{\pi}) + (\kappa_1 - \kappa_2) \tilde{\lambda}_{1t} (c - q_t \hat{\pi}) \right)^2$$

Third, we estimate the parameters in (5) and (6) based on the split samples implied by \hat{c} by LS. For detailed description of the model, estimated procedure and its asymptotic properties see Kourtellis et al. (2009). The advantage of the THRET model is that it controls for the endogeneity of the threshold, but from another side this model restricts the transaction function to be zero or one. In this sense STR models are more flexible allowing the transaction function to be a function of the difference between threshold variable and the threshold parameter. Unfortunately, the IV estimation has still not been extended to STR models with endogenous threshold variable (but is a promising field of potential econometric research).

Given the results obtained in the previous section, we proceed to investigate further the presence of nonlinearities by the estimation of two standard nonlinear models, the Markov Switching (MS) and the Threshold Model (TR).

3.2 Estimation Results: MS and TR

The previous analysis of the linear regression (1) provided us evidence that, although the intercepts α appear to be non significant, the coefficients associated with the order flows evolve with time $\beta = \beta_t$. We remain a priori agnostic about the sources driving this nonlinearities by the estimation of a fairly standard MS model. Testing linearity against MS-type nonlinearity is not straightforward, since transition probabilities are not identified under the null, and conventional statistics does not follow asymptotic χ^2 distribution. Formal tests have been proposed by Hansen (1992), Hamilton (1996), Garcia (1998), and Di Sanzo (2009) and Carrasco et al. (2009). Last authors derive an optimal test against the alternative of Markov switching that requires only the estimation of the model under the null. Motivated by our previous findings, we follow here the approach of Carrasco et al. (2009) for the special case where the alternative is a model with MS in the coefficient associated to the net order flow (see e.g. Hamilton (2005) for more details about this test):

$$y_t = \alpha + \beta_{s_t} X_t + e_t, \quad t = 1 \dots T$$

Results of this test can be found in the Table 3. Empirical critical values are computed by parametric bootstrap from 1000 iterations for a sample size equal to the size of the original data. As can be seen in the table, the test rejects strongly the null of a linear model versus a Markov-switching alternative, suggesting that, at least, a model with two regimes in the coefficients associated to the order flows should be used to fit the data.

We then proceed to estimate a more general MS model allowing for additional regime shifts in the intercepts, as in (3). Results of the MS estimation are presented in the Table 4. In line with the results obtained in the previous section, the intercepts are small and usually non significant. Opposite, the coefficients associated with the order flows (β_{s_t}), are always significant and very different from one state to another. For all the order flows, the slope coefficients have always the same sign as in the OLS estimation (negative for foreign participants and positive for domestic non-banks) but the magnitudes are larger in the state 2 (high sensitivity state) and smaller in the state 1 (low sensitivity state): $|\beta_1| \leq |\beta^{OLS}| \leq |\beta_2|$.

We then test the hypothesis of equality of the intercepts in both states using a likelihood ratio (LR) in addition to constructing another LR for the null of equal slope coefficients (see Krolzig (1997)). As can be seen in Table 4, the null of no regime dependence in the intercept can not be rejected for the foreign participants order flow at usual significance levels. However, the test strongly rejects the null of equal slope coefficient across states for all the order flows. In general, the results of this testing procedure indicate that regime switches in the exchange rate - order flow relationship are characterized by different impact of the order flow on returns.

From the estimated transition probabilities p_{11} and p_{22} , we can calculate the duration of being in each regime¹². Since $p_{22} < p_{11}$ for all order flows, the high sensitivity periods have shorter duration than low sensitivity periods. For instance, in the case of the spot foreign participants order flow, the transition probabilities are estimated 94.2% and 75% (Table 4); this indicates that the average expected duration of being in the low sensitivity regime (state 1) is about 17 days compared to 4 days in the high sensitivity (regime 2).

Our analysis of the linear relationship pointed out that the impact of the order flows on the exchange returns was larger in high volatility periods. Since the results of the MS estimation indicate a higher impact of order flow during the regime 2 (high sensitivity), a natural question arises: how much is related the probability of being in the high sensitive state to volatility? Figure 4 (right column) depicts the smoothed probabilities of being in the high sensitive state (right axe) together with the exchange returns (left axe). Visual inspection of the two series shows that higher values of the smoothed probabilities correspond to periods of higher volatility. In fact, the correlation among the volatility and the smoothed probability series is positive and very high (0.64 and 0.69 for the foreign participants domestic non-banks order flows, respectively). Note that, this result is not a trivial implication of the MS model. The regime changes are not driven by observed volatility, but by an unobserved random variable S_t , which it is only assumed to follow an ergodic Markov Chain.

We then proceed to check if exchange rate volatility by itself can direct regime changes through the estimation of a threshold model (TR) using con-

¹²The average duration of each state can be calculated as (see e.g. Hamilton 1994): $\sum_{i=1}^{\infty} i p_{hh}^{i-1} (1 - p_{hh}) = (1 - p_{hh})^{-1}$

temporaneous daily volatility (2) as a threshold variable z_t ¹³.

When the threshold variable is lagged (as in standard autoregressive specifications), it is relatively easy to assume that it is uncorrelated with contemporaneous noises as long as the model is dynamically complete. This assumption is a priori more difficult to hold, if the relationships appear between contemporaneous variables. Note, however, that exchange rate returns are well-known to be unconditionally symmetric and highly leptokurtic. If the distribution of exchange returns y_t is symmetric, the dependent variable y_t is completely uncorrelated with contemporaneous volatility z_t ¹⁴. In fact, the empirical correlation coefficient is very small ($\simeq 0.1$) making us suspect that endogeneity cannot be strong. In order to evaluate the severity of endogeneity, we estimate a THRET model as in (8) using lagged values of volatility as instruments, together with a standard TR model (without instrument the threshold variable). Results of these estimations are reported in Table 4. For all the order flows, all the estimated parameters (including estimated threshold and variance) are statistically identical¹⁵ in both TR and THRET models. This result suggests that, if endogeneity exists, it is very small and does not cause parameters to be biased. As can be seen in the table, the estimated intercepts are again often not significant while the estimated slope parameters are statistically significant and different across regimes. The sign of the slope coefficients are the same in OLS and MS estimation and, as expected, their magnitudes are larger during the high volatility regime.

After optimal thresholds have been identified, a conventional Chow test can be conducted to test the null of linearity against the Threshold Regression specification. Since threshold parameters are not identified under the null, the test statistic has nonstandard distribution. Following Hansen (1997), we employ 200 parametric bootstrap replicas, and a modified grid search to find critical values. For all order flows the linearity hypothesis is strongly rejected (Table 3).

As commented before, the TR (and THRET) models impose strong restrictions to the shape of the transaction function that may reduce the ability of the model to track the data. Motivated first by the small correlation between volatility and exchange rate returns and later by the similar estimation results of the TR and THRET, we proceed to estimate a standard STR model, which allows for flexibility in the transaction function. The ability of this last model to explain in-and-out exchange rate movements is presented together with the results of the MS and THRET models in the next section. In this way, we will

¹³According to theoretical models (Admati and Pfleiderer (1988), Diamond and Verrecchia (1987), Foster and Viswanathan (1990), Subrahmanyam (1991) or Easley and O'Hara (1992)), the relation between exchange rates and order flow should varies with current volatility. Additionally, we also find that lagged values of volatility are not able to correctly detect regime changes. The simple volatility estimator also performs better than other measures of intraday volatility as the Parkinson's High-Low or the bid-ask spread.

¹⁴Let $y_t \sim F_t(0, \sigma_t^2)$ where F denote the cumulative distribution function. If the distribution is symmetric: $Cov(y_t, |y_t|) = E(y_t |y_t|) = -\int_{-\infty}^0 y_t^2 \frac{f(x)}{1-F(0)} dx + \int_{-\infty}^0 y_t^2 \frac{f(x)}{F(0)} dx = 0$ leading to the result.

¹⁵The results of the TR model belong to the 95% confidence of the estimated THRET

have a better picture of how regime changes directed by volatility can explain exchange rate returns, which is one of the main purposes of this work. The results of the estimation and testing of the standard STR models may be found in the Appendix II.

As an overall, the results of the estimation of nonlinear models confirm that the conclusions obtained in the last section are robust. The relationship between order flows and exchange rate is not linear, and the price impact of the order flows increase with volatility, which is consistent with the implications of the theoretical models of Subrahmanyam (1991) and Easley and O’Hara (1992).

3.3 In-Sample Fitting

To assess the ability of the nonlinear models to explain in-sample exchange rate movements, we compare their fitting performance with the usual linear (OLS) model. As a benchmark, we also include a Random Walk (RW). In order to evaluate fitting performance, we compute two standard measures: the Mean Absolute Errors (MAE) and the Root Mean Squared Errors (RMSE). Results are presented in the Table 5. The values in the first four columns are the MAE (up) and RMSE (down) of the competing order flow model relative to the ones of a RW. Last three columns present the performance of nonlinear specifications relative to the OLS. In both cases, a number smaller than one indicates better fitting of the competing model. Percentage gain can be obtained by subtracting those numbers from one.

In general, the OLS fits the data better than RW, especially for the spot foreign participants order flow. For the domestic non-banks spot order flow, however the gain is rather small. When considering the nonlinear specifications, the fitting increases substantially for both order flows. For the MS, which is the model that performs better, the fitting gain ranges between a 18 – 35% better than the RW and a 15 – 20% better than the OLS, depending on the order flow considered and the measure employed. The threshold models perform in general a bit worse than the MS, although the results are very similar once we allow the transaction function to have flexibility enough (STR).

In order to assess how the fitting performance changes with volatility, we compare the fitting of the models in two different parts of the sample, selected according to volatility in the exchange returns. First period (A) runs from 04/12/07 to 30/04/08 and second period (B) from 04/02/09 to 15/07/09 (5 months approx. each one). Period A is a pre-crisis period characterized by a relatively low volatility. Opposite, period B is situated in the middle of the crisis, corresponding to the last five months of our data. In particular, the average volatility in the period B is two times larger than in period A. Results are also reported in the Table 5. The fitting performance relative to the RW increases with volatility in all the order flow specifications, both linear and nonlinear. The performance of the nonlinear models is in general, considerably higher than OLS in the two periods, although the results are much better during high volatility. Figures 6, 7 and 8 plot the fitted HUF/EUR exchange rate by the MS and the two threshold models (THRET and STR) respectively (in green) together

with the OLS results (in red) for the two sub-periods considered (period A left column, period B right column). As can be observed in the figures, the nonlinear versions are able to track the data better than OLS in both periods (low and high volatility), but results are especially good during high volatility period, where the exchange rate changes are bigger and much more frequent.

When evaluating the information contained in the order flows from different type of customers to explain exchange rate movements, the spot foreign participants is the order flow that fits the data better. This result holds for all the models and all volatility periods. However, the explanation power of the domestic non-banks order flow increases substantially when considering the nonlinear specifications. In particular note that, according to the OLS, the domestic non-banks order flow has almost no additional power to explain exchange rate movements than the last observation of the exchange rate (actually, it is even outperformed by the RW according to the MAE during the low volatility period). However, using the nonlinear specifications, the same order flow outperforms the RW by a 15% according to the same measure.

3.4 Meese-Rogoff Test

In order to evaluate out-of-sample fitting, we employ a Meese-Rogoff (1985) type exercise ¹⁶. The Meese-Rogoff test has become the standard tool of model evaluation in the FX microstructure literature. In fact, microstructure oriented models are often able to beat the RW benchmark in out-of-sample fitting, even at short horizons. This result contrasts with the poor out-of-sample fitting of the structural macroeconomic models that are usually unable to outperform a RW at horizons shorter than a year.

Our evaluation period runs from 04/12/07 to 15/07/09 (last four hundreds observations). We estimate the models recursively by adding one observation each time, and we compute the predicted exchange returns. Table 6 gives detailed results for one day and one week ahead horizons using the MAE and RMSE as measures of accuracy. As before, the first four columns report the prediction errors of the competing order flow models relative to the RW and the last three columns the corresponding error measures of the nonlinear specifications with respect to OLS. Thus, again, numbers smaller than one indicate better performance of the competing specification. For model comparisons, we make use of the Diebold and Mariano (1995) test statistic. We employ a data dependent truncation lag to estimate the spectral density at zero frequency as described by Andrews (1991). The results are in line with the ones obtained in the in-sample fitting and indicate a clear superiority of the nonlinear specifications.

¹⁶The Meese-Rogoff test relies on ex-ante data to estimate parameters but makes use of contemporaneous independent variables to produce the exchange rate 'forecasts'. Note that the Meese-Rogoff test cannot be considered as true out-of-sample forecasting exercise, since future information (except the estimated parameters) is used to produce forecasts. Rather, it should be considered as an out-of-sample fitting test, a test of stability of the estimated parameters or as a "weak" forecasting.

For one day ahead forecasting, all the order flows models (linear and nonlinear) perform much better than the RW according to both measures, although the gains of the OLS with the domestic non-bank order flow are rather modest. For one week ahead predictions, the relative performance of all the order flows models to the RW increases a lot. This last result is not surprising since, by construction, the Meese-Rogoff exercise penalizes the RW as the horizon increases. The nonlinear versions of the models substantially outperform the linear OLS at both time horizons. The improvements range between 20 – 25% for the MS, which again is the model that shows better overall performance. As before, the Threshold Specifications almost catch up the MS once enough flexibility in the transaction function (STR) is provided.

We also perform the Meese-Rogoff test in the two sub-samples described in the previous section: the period A (low volatility) and B (high volatility). The prediction errors for all order flow models are substantially smaller in periods of high volatility (Table 6). As before, the performance of the nonlinear versus the linear specification also increases with volatility.

Finally, when evaluating the informativeness of the order flows per group of customers, the foreign participants is again the order flow that performs better for all specifications. Notice that, during the low volatility sub-sample (A), the OLS with the domestic non-banks order flow is not able to make one day ahead predictions statistically different from the ones of a RW. Instead, both nonlinear models are able to beat the RW for all order flows in this sub-sample. In particular, for the domestic non-banks order flow, the predictions are statistically significant and considerably better than the ones derived from the OLS.

4 Conclusions

In this work we have questioned the linearity of the relationship between exchange rates and order flows that has been a maintained assumption in the empirical literature. We have employed a long database on customer order flows, which are at the cornerstone of microstructure literature. A first examination of the linearity hypothesis has revealed that the relationship evolves over time and that it is different under different market conditions defined by volatility. We have provided further evidence of this result through the estimation of a Markov Switching and two Threshold Models with volatility as a threshold variable (THRET and STR). The Markov Switching has received a lot of attention in the recent exchange rate literature, but has never been employed for the order flow analysis. Our main findings are, first, that the price impact and the information transmitted by the order flows increases with volatility, which is in line with the uncertainty model developed by Easley and O’Hara (1992) and the conclusions of Subrahmanyam (1991). An important policy implication of this first result, is that Central bank market interventions would have bigger impact in periods of high volatility. The second finding is that the non-linearity can be captured successfully by the Threshold Models and, specially,

Markov Switching, which provide substantial power to explain exchange rate movements beyond the constant coefficient approach. In particular, explicitly modeling nonlinearities is crucial to understand the information content of the order flow for determinate groups of customers.

In this research we have been primarily concerned with providing evidence that the relation between exchange rates and order flows is not linear, and that nonlinear models can explain exchange rate movements better than linear specifications. Future research might, consequently, analyze the source of these nonlinearities further and involve the nonlinear models into real exchange rate forecasting. In fact, order flows have already demonstrated their potential to explain future exchange rate movements (Evans and Lyons (2005)). In the light of our results, the increase in efficiency provided by the nonlinear specifications may lead to an improvement of forecast accuracy. However, one might be a priori cautious about the obtained results given that, although the models perform quite well conditional to be in a determined regime, the forecast of the regime changes may be wrought with difficulty. A reliable forecast of exchange rate volatility may help to this purpose. An appealing line of future research could include the use of cointegration between exchange rates and order flows (as in Bjønnes et al. (2005)) together with nonlinear specifications.

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APENDIX I: TABLES AND FIGURES

Table 1 Summary statistics

Variable	T	Mean	std	min	max
$\Delta \log(\text{Exchange rate})$ 100	1635	0.0091	0.6251	-3.3205	5.9372
Foreign participants (FP)	1635	-0.0024	0.0363	-0.2144	0.9023
Domestic non-banks (DNB)	1635	-0.0008	0.0141	-0.1020	0.1087
Volatility	1635	0.4120	0.47	0	5.9372

Notes: a) T – number of observations available, Mean and std – mean and standard deviation of a series, min and max – minimum and maximum values. b) Volatility is defines as absolute value of the first difference of natural logarithms of the HUF/EUR exchange rate. c) Order flows are NET order flows = Δ Cumulated order flows.

Table 2 Results of estimation of the generic model under different volatility conditions

		α	β	R ²	F stat
FP	OLS	-0.0330**	-13.3038***	0.3661	-
	OLS by Volatility:				
	Volatility(t)=L	-0.0163**	-3.5481***	0.1020	-
	Volatility(t)=H	-0.0511*	-17.31***	0.4694	157.31
DNB	OLS	0.0197	15.1743***	0.1184	-
	OLS by Volatility:				
	Volatility(t)=L	-0.0099	1.2252**	0.0045	-
	Volatility(t)=H	0.0517	24.93***	0.2024	65.15

Notes: a) FP and DNB indicate Foreign Participants and Domestic Non-bank spot customer order flow resp. b) Volatility is defined as the absolute value of the first difference of natural logarithms of the HUF/EUR exchange rate c) F stat is F statistics for the $H_0: \alpha(L)=\alpha(H)$ and $\beta(L)=\beta(H)$; d) *, **, *** indicate significance at 10%, 5%, and 1% respectively;

Table 3 Carrasco et al (2009) test statistics (MS) and Hansen (1997) Chow-type test (THRET) for the null of linearity against the respective nonlinear model.

Order Flow	(Carrasco et al. (2009)) MS	(Hansen (1997)) THRET
FP	16.410***	158.866***
DNB	9.993***	214.444***

Notes: a) FP and DNB indicate Foreign Participants and Domestic Non-bank spot customer order flow resp. b) The numbers in the columns are F-statistics. c) F_L and F_E are the values of the F-statistic for each of the hypothesis. d) *, **, *** indicate significance at 10%, 5%, and 1% respectively obtained with parametric bootstrap;

Table 4 Estimation Results, MS, TR and THRET:

Order Flow	MS													
	α_1		β_1		α_2		β_2		σ^2		ρ_{11}		ρ_{22}	
FP	-0.042*		-9.090***		0.019		-31.427***		0.184***		0.942***		0.750***	
DNB	-0.023		6.381***		0.307*		64.700***		0.249***		0.901***		0.301***	

Order Flow	TR / THRET											
	α_1		β_1		α_2		β_2		σ^{2A}		c^A	
FP	-0.024*	-0.014**	-6.752***	-5.234***	-0.028	-0.013	-20.932***	-19.618***	0.208***	0.212	0.768***	0.626
DNB	-0.005	-0.004	5.6316***	5.635***	0.024	0.011	52.925***	53.134***	0.257***	0.272	1.003***	0.998

Notes: a) FP and DNB indicate Foreign Participants and Domestic Non-bank spot customer order flow resp. b) *, **, *** indicate significance at 10%, 5%, and 1% respectively. A) The THRET-C2SLS procedure does not allow to test the significance of the estimated σ^2 and c.

Table 5 In-sample Fitting Results: MAE, RMSE

In Sample Fitting		Whole period								
		Model / RW				Model / OLS				
		OLS	MS	THRET	STR	MS	THRET	STR		
FP	mae	0.812	0.689	0.747	0.709	0.848	0.919	0.873		
	rmse	0.796	0.650	0.727	0.684	0.816	0.913	0.859		
DNB	mae	0.971	0.823	0.878	0.847	0.847	0.903	0.872		
	rmse	0.939	0.755	0.835	0.811	0.804	0.889	0.864		

In Sample Fitting		Period A						Period B							
		Model / RW				Model / OLS		Model / RW				Model / OLS			
		OLS	MS	THRET	STR	MS	THRET	STR	OLS	MS	THRET	STR	MS	THRET	STR
FP	mae	0.819	0.643	0.784	0.708	0.786	0.947	0.865	0.800	0.630	0.678	0.625	0.788	0.844	0.781
	rmse	0.814	0.646	0.810	0.773	0.794	0.987	0.950	0.763	0.602	0.684	0.636	0.789	0.896	0.834
DNB	mae	1.002	0.855	0.876	0.842	0.853	0.873	0.841	0.902	0.698	0.740	0.694	0.773	0.810	0.770
	rmse	0.960	0.799	0.829	0.798	0.832	0.864	0.832	0.884	0.666	0.740	0.698	0.753	0.828	0.790

Notes: a) The numbers in the cells indicate the MAE and RMSE of the competing model relative to RW (first 3 columns) or to OLS (last 2). A number smaller than 1 indicate better performance of the competing model. % Fitting gain can be obtained by subtracting the number in the cell from 1. b) FP and DNB indicate Foreign Participants and Domestic Non-bank spot customer order flow resp. c) Whole period: 01/02/2003 – 15/07/2009 Period ‘A’: 04/12/07-30/04/08, Period ‘B’: 20/02/0-17/07/09. d) STR is Logistic-STR

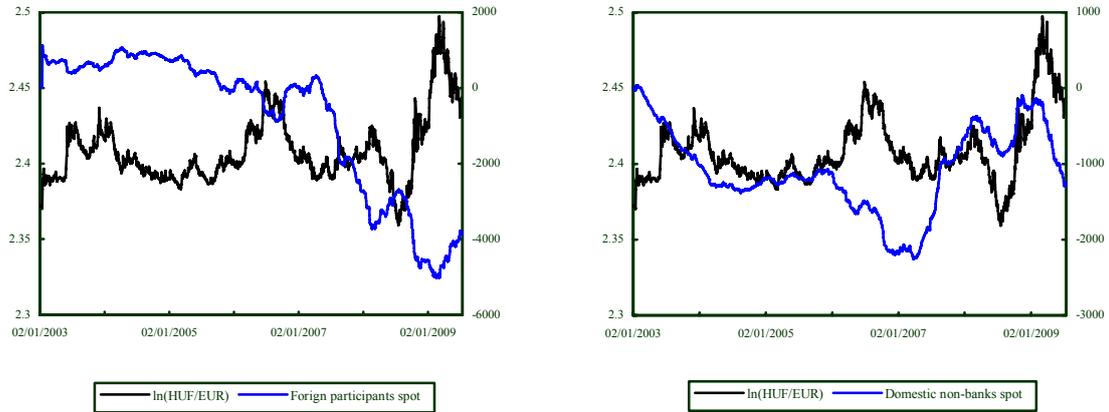
Table 6 Out-Sample Fitting Results (Meese-Rogoff): MAE, RMSE

Out of Sample Fitting (Meese-Rogoff)		Sample						
		Model / RW				Model / OLS		
		OLS	MS	THRET	STR	MS	THRET	STR
1 DAY								
FP	mae	0.767 ^{***}	0.625 ^{***}	0.6961 ^{***}	0.633 ^{***}	0.815 ^{***}	0.904 ^{***}	0.825 ^{***}
	rmse	0.766 ^{***}	0.617 ^{***}	0.6908 ^{***}	0.631 ^{***}	0.806 ^{***}	0.900 ^{***}	0.824 ^{***}
DNB	mae	0.938 ^{***}	0.720 ^{***}	0.7981 ^{***}	0.735 ^{***}	0.767 ^{***}	0.848 ^{***}	0.784 ^{***}
	rmse	0.920 ^{***}	0.745 ^{***}	0.7819 ^{***}	0.771 ^{***}	0.810 ^{***}	0.848 ^{***}	0.838 ^{***}
1 WEEK								
FP	mae	0.322 ^{***}	0.267 ^{***}	0.2923 ^{***}	0.271 ^{***}	0.828 ^{***}	0.904 ^{***}	0.842 ^{***}
	rmse	0.340 ^{***}	0.288 ^{***}	0.3075 ^{***}	0.300 ^{***}	0.848 ^{***}	0.900 ^{***}	0.882 ^{***}
DNB	mae	0.394 ^{***}	0.307 ^{***}	0.3355 ^{***}	0.314 ^{***}	0.778 ^{***}	0.848 ^{***}	0.797 ^{***}
	rmse	0.408 ^{***}	0.340 ^{***}	0.3482 ^{***}	0.348 ^{***}	0.832 ^{***}	0.849 ^{***}	0.853 ^{***}

Out of Sample Fitting (Meese-Rogoff)		Sub-Sample A							Sub-Sample B						
		Model / RW				Model / OLS			Model / RW				Model / OLS		
		OLS	MS	THRET	STR	MS	THRET	STR	OLS	MS	THRET	STR	MS	THRET	STR
1 DAY															
FP	mae	0.750 ^{***}	0.705 ^{***}	0.790 ^{***}	0.721 ^{***}	0.940 ^{**}	1.023	0.961 [*]	0.810 ^{**}	0.631 ^{***}	0.7046 ^{***}	0.631 ^{***}	0.779 ^{***}	0.846 ^{***}	0.779 ^{***}
	rmse	0.770 ^{***}	0.702 ^{***}	0.763 ^{***}	0.729 ^{***}	0.912 [*]	0.984	0.947 [*]	0.771 ^{***}	0.604 ^{***}	0.6962 ^{***}	0.644 ^{***}	0.783 ^{***}	0.898 ^{***}	0.835 ^{**}
DNB	mae	0.993	0.918 ^{**}	0.903 ^{***}	0.822 ^{***}	0.925 ^{**}	0.887 ^{***}	0.828 ^{***}	0.915 ^{***}	0.696 ^{***}	0.7548 ^{***}	0.706 ^{***}	0.761 ^{***}	0.806 ^{***}	0.772 ^{***}
	rmse	0.956	0.854 ^{**}	0.820 ^{***}	0.768 ^{***}	0.893 ^{**}	0.853 ^{***}	0.803 ^{***}	0.894 ^{***}	0.666 ^{***}	0.7359 ^{***}	0.711 ^{***}	0.745 ^{***}	0.820 ^{***}	0.796 ^{**}
1 WEEK															
FP	mae	0.357 ^{***}	0.338 ^{***}	0.292 ^{***}	0.341 ^{***}	0.947 ^{**}	1.029	0.955 [*]	0.317 ^{***}	0.248 ^{***}	0.2674 ^{***}	0.249 ^{***}	0.781 ^{***}	0.842 ^{***}	0.786 ^{***}
	rmse	0.371 ^{***}	0.340 ^{***}	0.307 ^{***}	0.353 ^{***}	0.917 ^{**}	0.985	0.951 [*]	0.327 ^{***}	0.256 ^{***}	0.2950 ^{***}	0.277 ^{***}	0.784 ^{***}	0.902 ^{***}	0.847 ^{**}
DNB	mae	0.476 ^{***}	0.440 ^{***}	0.335 ^{***}	0.391 ^{***}	0.924 ^{**}	0.880 ^{***}	0.821 ^{***}	0.356 ^{***}	0.265 ^{***}	0.2875 ^{***}	0.275 ^{***}	0.746 ^{***}	0.808 ^{***}	0.774 ^{***}
	rmse	0.463 ^{***}	0.414 ^{***}	0.348 ^{***}	0.371 ^{***}	0.893 ^{**}	0.848 ^{***}	0.801 ^{***}	0.375 ^{***}	0.277 ^{***}	0.3122 ^{***}	0.303 ^{***}	0.738 ^{***}	0.831 ^{***}	0.806 ^{**}

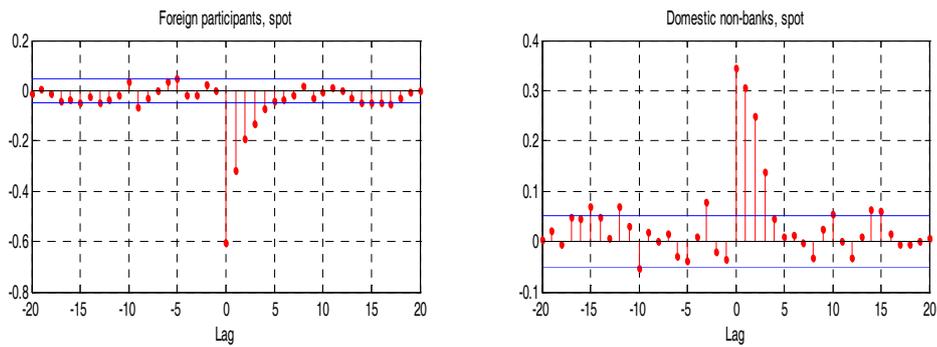
Notes: a) The numbers in the cells indicate the MAE and RMSE of the competing model relative to RW (first 3 columns) or to OLS (last 2). A number smaller than 1 indicate better performance of the competing model. % Fitting gain can be obtained by subtracting the number in the cell from 1. b) FP and DNB indicate Foreign Participants and Domestic Non-bank spot customer order flow resp. c) Sample: 04/12/07-15/07/09 Sub-sample 'A': 04/12/07-30/04/08; Sub-sample 'B': 20/02/0-17/07/09. d) STR is Logistic-STR. e) *, **, *** indicate statistical significance at 10%, 5%, 1% resp. according to DM statistic.

Fig. 1 The data: log-exchange rate and cumulated order flow



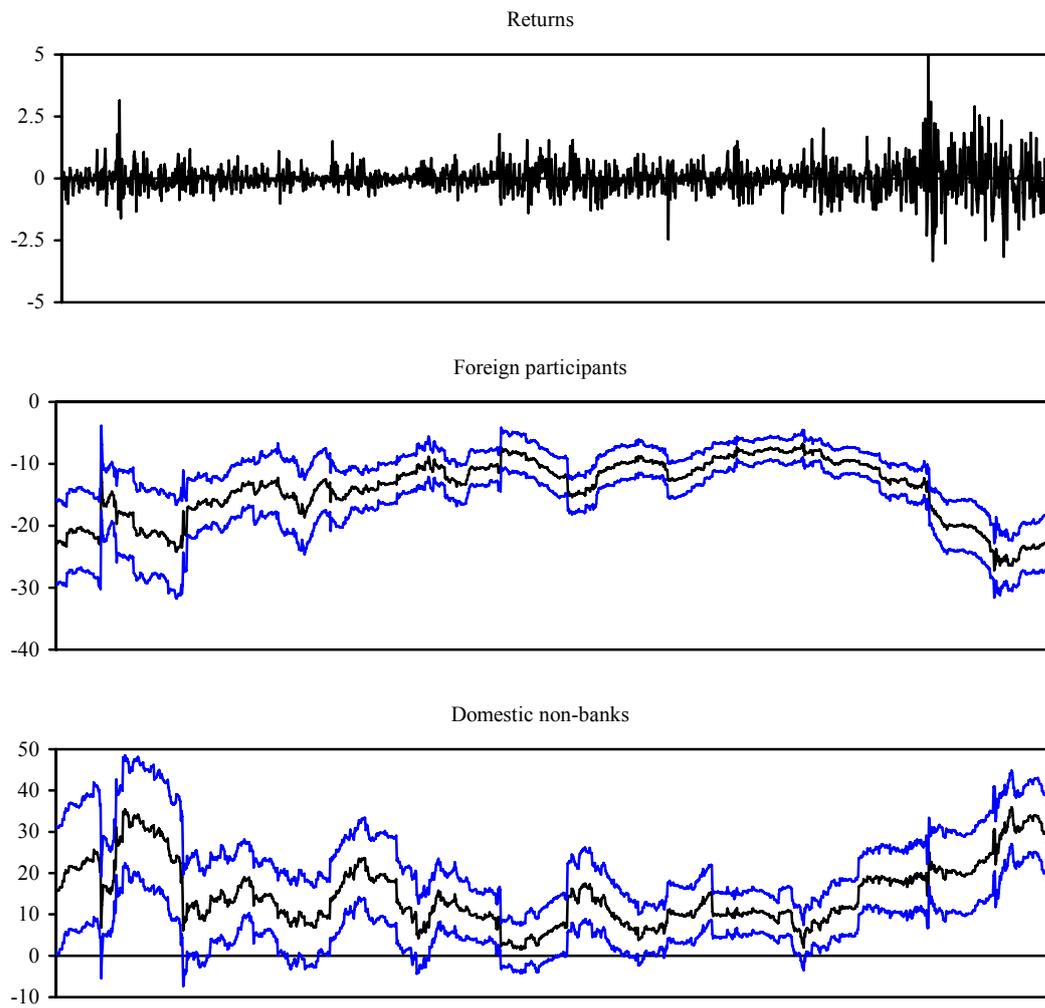
Notes: $\log(\text{HUF}/\text{EUR})$ – left axe. (Cumulated) Order flows – right axe.

Fig. 2 Sample Cross Correlation: Returns, net Order flows



Notes: a) Returns: $\Delta(\log(\text{HUF}/\text{EUR})) \times 100$ (% change in exchange rates). b) Confidence intervals in the graph correspond to 5% significance level.

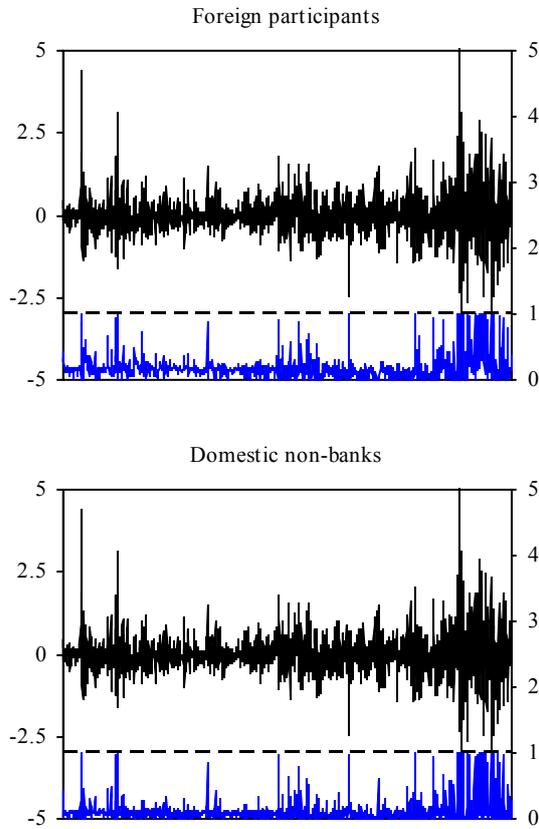
Fig. 3 Results of the Moving Window regression



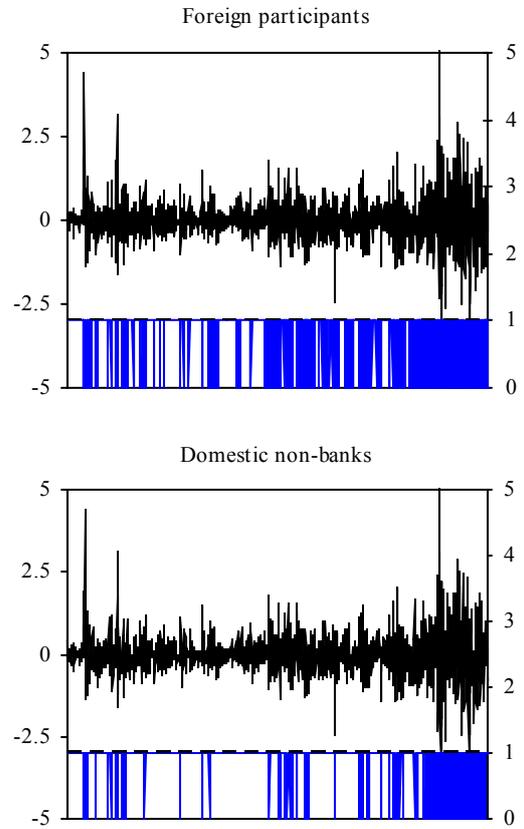
Notes: Upper and Lower bounds are 5% confidence intervals

Fig. 4 Returns, Values of transaction function (THRET) and Smoothed probabilities (MS)

Returns and Smoothed Probability. MS

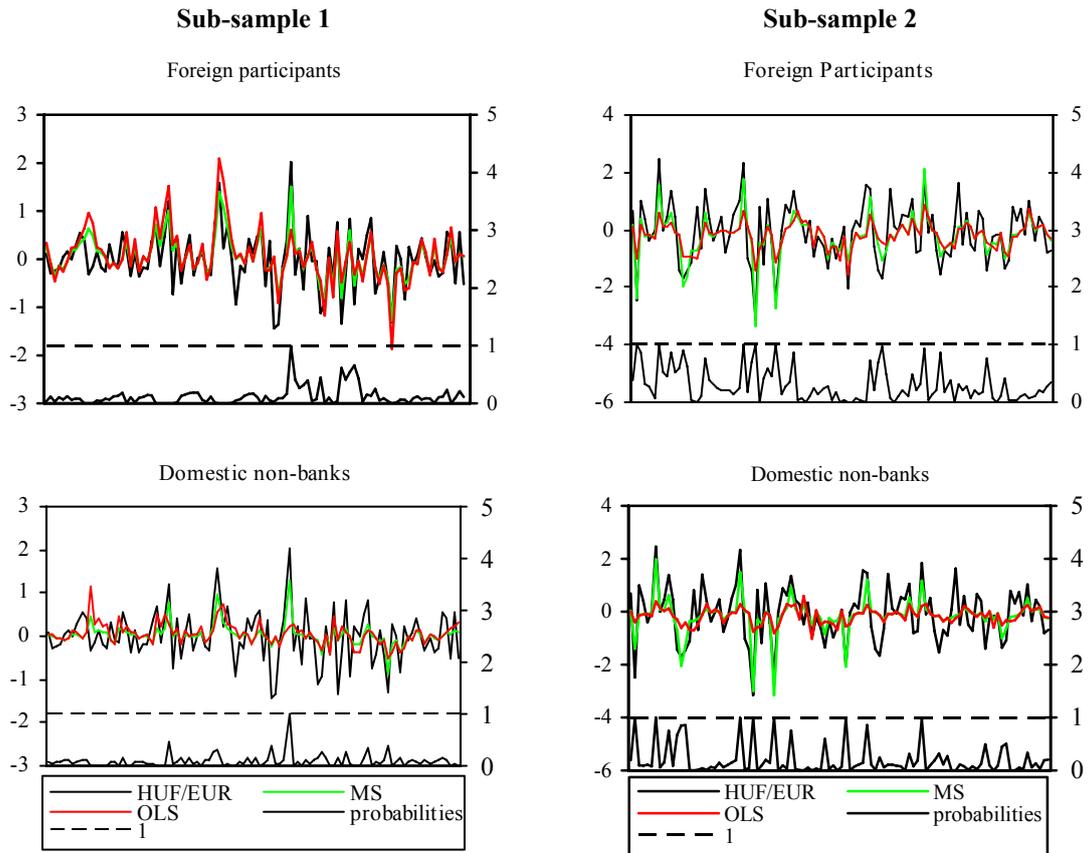


Returns and Transaction function. THRET



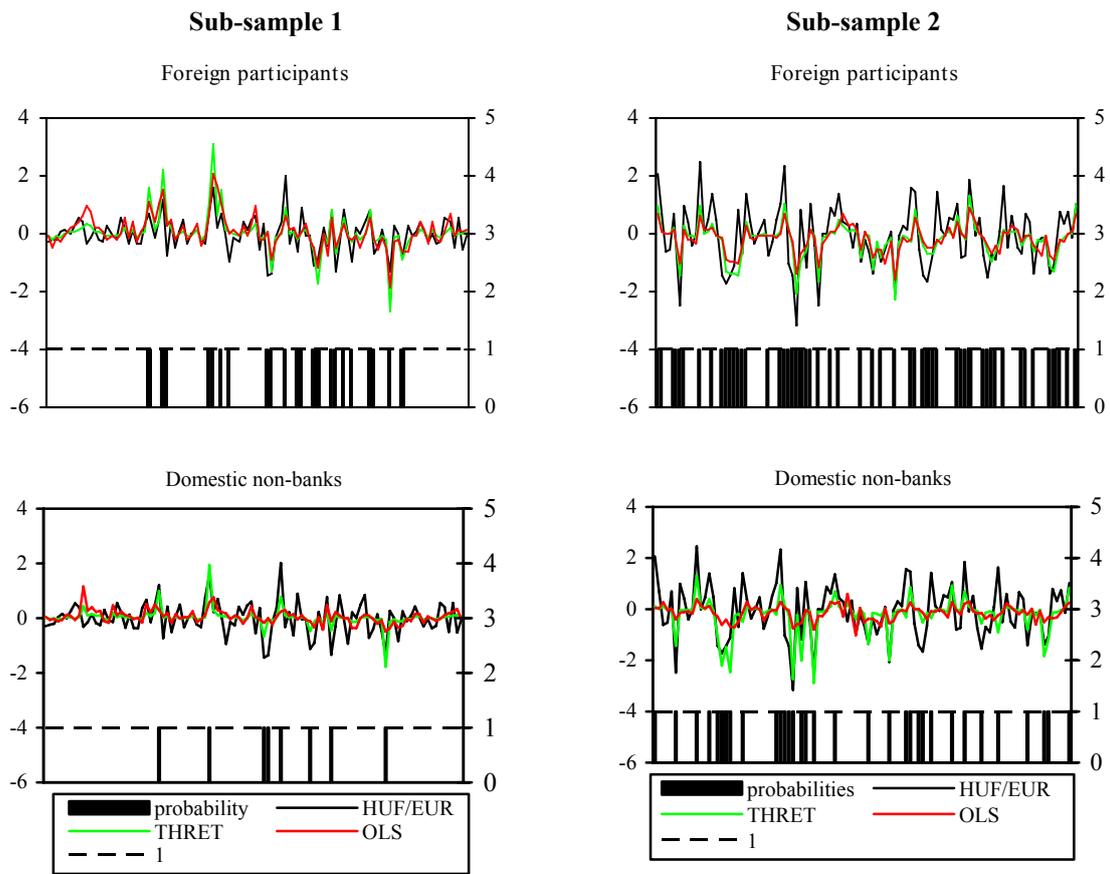
Notes: a) Left Scale: Returns = $\Delta (\log (\text{HUF}/\text{EUR})) \times 100$ (% change in exchange rates). b) Left Column - Smoothed probability of high sensitivity state (MS) Right Scale: Right column - Values of transaction function (THRET).

Fig. 6 In-sample Fitting. MS and OLS



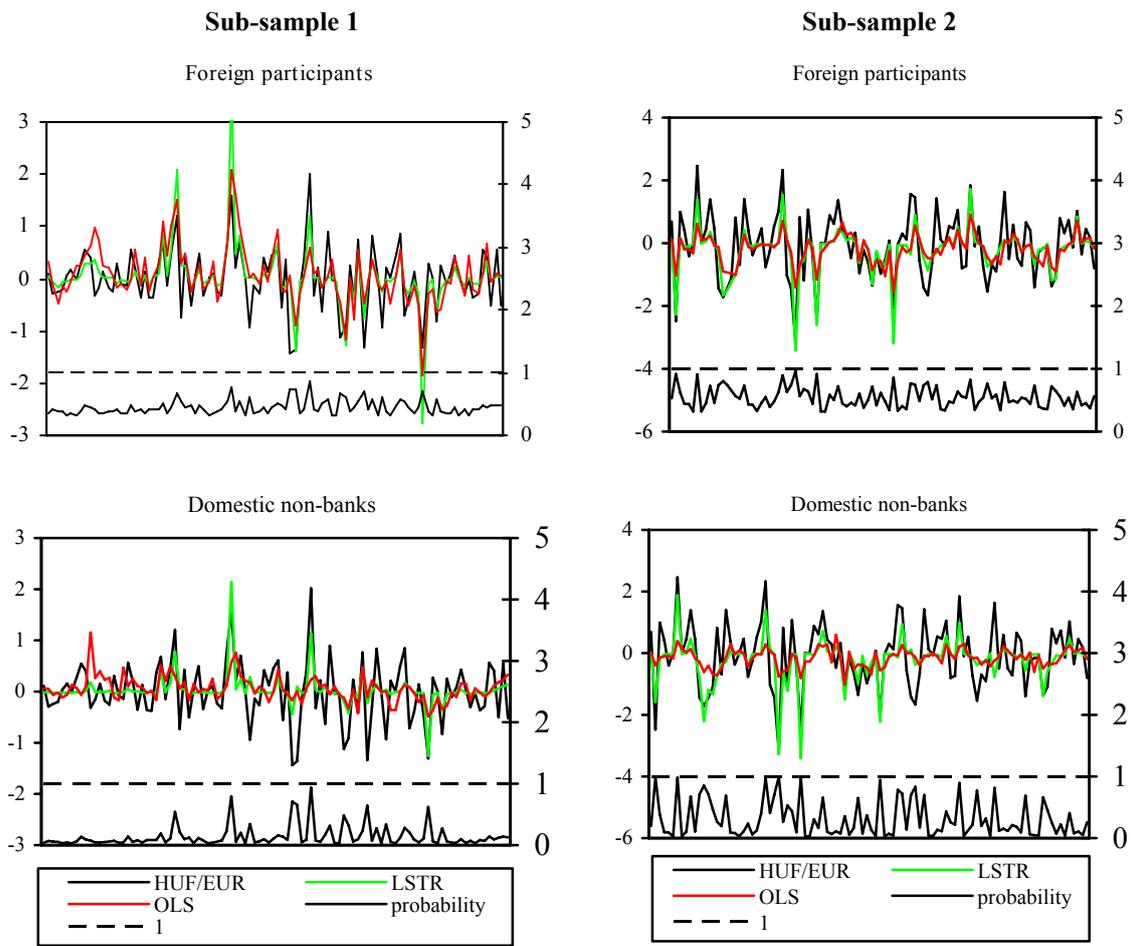
Notes: a) Sub-sample 1: 04/12/07-30/04/08 b) Sub-sample 2: 20/02/09 -15/07/09

Fig. 7 In-sample Fitting. THRET and OLS



Notes: a) Sub-sample 1: 04/12/07-30/04/08 b) Sub-sample 2: 20/02/09 -15/07/09

Fig. 8 In-sample Fitting. LSTR and OLS



Notes: a) Sub-sample 1: 04/12/07-30/04/08 b) Sub-sample 2: 20/02/09 -15/07/09

APPENDIX II: STR Estimation

Testing linearity against STR-type nonlinearity implies testing the joint null hypothesis

$H_0: \alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$ in (2). However, under the null, parameters γ and c are not identified. To overcome the difficulties with testing, we use the method based on an auxiliary regression proposed by Escribano and Jorda (2001) developed on the basis of the testing procedure proposed by Saikkonen and Luukkonen (1988), Teräsvirta (1994), Teräsvirta et al (1994). Additionally, this procedure allows us to test model specification for the STR: logistic (LSTR) against exponential (ESTR). The method involves the following steps:

First, we run an auxiliary regression of the type:

$$y_t = \alpha + \beta X_t + \delta_1 X_t z_t + \delta_2 X_t z_t^2 + \delta_3 X_t z_t^3 + \delta_4 X_t z_t^4 + e_t \quad (8)$$

Where as before, y_t is the returns, X_t is the consumer order flow, and z_t is the observed threshold variable (volatility). The linearity hypothesis has as null: $H_0: \delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$. Second, if linearity is rejected, we can proceed to select the specification of the model by computing usual F-statistics for the following null hypothesis: $H_0: \delta_2 = \delta_4 = 0$ and $H_0: \delta_1 = \delta_3 = 0$ in (8). If the F-statistic associated to the first hypothesis is higher than the one associated to the second the resulting specification is Exponential (ESTR), and if opposite, the resulting specification is logarithmic (LSTR).

Escribano-Jorda Procedure	P-values (F-statistics):			Resulting specification
	$\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$	$\delta_2 = \delta_4 = 0$	$\delta_1 = \delta_3 = 0$	
Ho:				
FP	157.37***	10.40	33.22	LSTR
DNB	126.95***	2.11	19.20	LSTR

Notes: a) FP and DNB indicate Foreign Participants and Domestic Non-bank spot customer order flow resp. b) The numbers in the columns are F-statistics. c) F_L and F_E are the values of the F-statistic for each of the hypothesis. d) *, **, *** indicate significance at 10%, 5%, and 1% respectively

For all order flows the hypothesis of linearity is rejected at all reasonable significance levels. The testing procedure suggests the Logistic STR for all the specification. The results of the estimation of the standard TR and LSTR models are presented below.

Order Flow	LSTR						
	α_1	β_1	α_2	β_2	σ^2	γ	c
FP	0.0132	14.7672*	-0.0943	-48.958***	0.1832**	1.2489***	0.6081**
DNB	-0.0320*	-2.5431*	0.2662***	70.8212***	0.2575***	3.0752***	1.1485***

Notes: a) FP and DNB indicate Foreign Participants and Domestic Non-bank spot customer order flow resp. b) *, **, *** indicate significance at 10%, 5%, and 1% respectively.

The estimated threshold coefficients are positive and statistically significant. Estimates of smoothness parameters are significant and quite high. It means that the coefficients change fast when the latent variable is around the threshold. The estimated intercepts are often not statistically different from zero. Opposite, the slope coefficients β are always strongly significant and statistically different in each regime. The signs of the resulting estimated slope coefficients for period t , $\beta_t = \beta_1 + \beta_2 F(z_t, \gamma, c)$ are always as in the OLS, MS and THRET/TR estimation (negative for spot foreign participants, and positive for domestic non banks), and increasing, in magnitude, with volatility.

The ability of the Logistic-STR model to explain in and out of sample exchange rate movements is presented together with the one of MS and THRET models in Tables 5, 6.