

# Banking and Optimal Capital Ratio in an Equilibrium Model\*

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## Abstract

I address the question of optimal capital ratio in banking, particularly the fact that banks' risk-weighted capital are substantially larger than the stipulated reserve requirements by Bank of International Settlements. My contribution is to show that when the underlying values of borrowers are correlated, banks should hold positive risk-weighted capital, regardless of the regulation. I use a derived distribution for debt portfolios to show that intermediation in a debt market will outperform direct lending, even if intermediaries are allowed to default. The model used is a generalization of Williamson (1986), with Costly State Verification as asymmetric information. By using a factor model for the value of entrepreneurs' projects, I introduce a positive probability for banks to default. It is shown that, in equilibrium, banks choose to hold capital reserves that are almost large enough to eliminate the expected auditing cost for their depositors. The reason is that auditing does not provide any utility and hence, the cake to be split between banks and depositors is enlarged by capital as an insurance against bad outcomes. It is also shown that the more correlation there is in the debt portfolio, the larger is the optimal share of risk-weighted capital. This could explain why small regional banks in Sweden often have more than twice the capital ratio of their nation-wide competitors.

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# 1 Introduction

The direct effect of the capital requirements on the banking system and their indirect effects on the real economy, are questions frequently investigated. The Bank of International Settlements' regulations, which are adopted by most developed countries, state that the Tier 1 capital ratio, i.e. basically shareholders' equity net of goodwill to risk weighted capital, should be 4 %. However, banks' capital is much larger. In Sweden, this Tier 1 ratio is often as high as 7-8 % among the market leading banks. Smaller banks typically have even larger capital ratios. Jackson et. al. (1999) find that G10 banks have capital ratios of 11.2% on average, including secondary capital. This means that the capital requirements by BIS rarely bind, thereby implying that banks hold capital for other reasons than the BIS capital requirements.

In this paper, I investigate why banks should have capital, and how large the stock should be. This is an important question due to the evidence that regulatory capital requirements rarely bind. My set-up is to assume that banks cannot diversify the entire market risk, notwithstanding how large they grow. In fact, data shows that on average, banks' credit losses are small, but occasionally there are sharp increases in these low levels, e.g. during the early nineties. My innovation of bringing in macroeconomic shocks that can severely hurt banks' profitability and thereby their survival, allows me to analyze banking within a more realistic framework. Using this set-up, it is shown that banks can exist in equilibrium, they carry substantial capital to reduce the probability of default and the destruction of value that occurs in defaults. This also explains why small regional banks have larger capital ratios than large banks: they are not able to diversify regional risk, which leaves them with more systematic risk in their debt portfolio. Moreover, banks are profitable in the sense that rich consumers have a higher return on capital when starting a bank than when investing in a private portfolio of loans to entrepreneurs. For this result to come through, it is essential that banks have sufficiently large bargaining power both against borrowers and depositors.

Since I am interested in debt markets and the concept of banking in particular, it is essential to have a model that does not rely on pure risk sharing with simple contingent claims of the Arrow and Debreu (1954) type. Introducing asymmetric information is one way of creating an environment that needs more complicated contracts than contingent claims. Classical ways of modeling asymmetric information in a market are through moral hazard or adverse selection; see e.g. Mas-Colell, Whinston and Green (1995). Both these approaches primarily deal with ex ante informational problems. Townsend (1979) explored another type of asymmetric information, costly state verification, hereafter CSV, which models ex post informational

problems. With CSV, the realization of a stochastic project is only known to the insider. Outsiders can, at a cost, monitor projects to reveal the true outcome. Several authors have shown that with CSV and ex post non-random audit, the efficient contract between an entrepreneur and an investor is a standard debt contract, e.g. Townsend (1979), Diamond (1984), Gale and Hellwig (1985), and Williamson (1986, -87a).

I follow Williamson (1986, 87a), where he derives a model with credit rationing and endogenous debt contracts in equilibrium; the 1986 paper also derives intermediation. With CSV, Williamson (1986, -87a) shows there to be one fundamental question for the lender: to audit and bear the associated cost or not to audit. Since there are two different “states” for auditing, there must be a threshold for possible signaled results for borrowers, bad results leading to audit or good results leading to no audit. Naturally, borrowers will never signal a result above the threshold since they would like to keep as much of the result as possible to themselves. The resulting efficient contract is then a standard debt contract, either the entrepreneur who received financing re-pays the loan with agreed on interest, or he is audited and the lender seizes his entire wealth. Further, private information is shown to give rise to intermediation if entrepreneurs’ projects require more capital than a single investor can supply. Every entrepreneur now needs to borrow from several investors, which leads to overlapping audits. Since each investor has an incentive to lie to the other investors in the same project, all investors need to audit themselves. To circumvent the problem with overlapping auditing, Williamson (1986) introduces delegated auditing through banks, which eliminates overlapping audit. To solve the monitoring problem of banks, the values of entrepreneurs’ projects are assumed to be independently distributed. All risk will then be pooled and the monitoring of banks is superfluous: therefore, repayments on bank deposits are larger than the expected return on lending directly to entrepreneurs, i.e. net of monitoring costs.

My model differs from that of Williamson (1986, -87a), in that I allow for heterogeneous consumers and correlation between entrepreneurs’ projects. A very small share of my consumers has substantially larger funds than the normal consumer. This means that these rich consumers can be as efficient in their auditing as banks, since they can completely fund projects. Hence, small and large investors will have different expected returns when lending. Further, I only consider rationed markets, i.e. there are more entrepreneurs wanting to borrow than consumers wanting to lend. Rationing together with the different efficiency in auditing costs, for my two types of agents, can lead to an equilibrium with two lending rates as in Williamson (1987b).

The most important innovation by far in my model is that I allow for systematic shocks. Introducing systematic risk might seem trivial. However, if direct lending is not prohibited, it is important that the

probability distribution for the value of the debt portfolio is derived from the probability distribution for entrepreneurs' values, rather than assumed. That is, the probability distribution for the value of the loan portfolio must be consistent with the probability distribution of entrepreneurs' projects. To compare bank lending with direct lending, these distributions of outcomes must also be computable. Vasicek (1987, -91, 2002) assumes entrepreneurs' projects to follow correlated Wiener processes. Thus, he derives a computable probability distribution for debt portfolio losses. His model also works well empirically and it is common within management of debt portfolios, e.g. JPMorgan Credit Metrics, CreditSuisse<sup>+</sup>, and Moodys' KMV products. Gordy (2003) use a similar setup to analyze the new BIS regulation on capital requirements which use asset classes assuming that individual loans are homogenous within the class. Using this probability distribution for the values of banks' loan portfolios, the intermediary is not just a delegated monitor as in Williamson (1986). Instead, banks supply the market with a new product, deposits with low verification costs.

In the spirit of Holmström and Tirole (1997), I incorporate the importance of banks' balance sheets into my model, and I only consider rationed markets, which have been thoroughly investigated by Stiglitz and Weiss (1981). I also view this work as a first step towards improving business cycle models with credit markets, such as Bernanke and Gertler (1989) or Carlstrom and Fuerst (1997), by including correlated investments.

Unlike the previously mentioned authors, and the vast majority of theoretical credit market work within economics, there is a strong link to the financial and statistical literature. I base my credit model on micro foundations from the financial literature. This way, I obtain a more realistic model environment, which then strengthens the relevance of my findings. There is a huge amount of research, both theoretical and empirical, performed within finance where the common model feature is the notion of both systematic and idiosyncratic risk, and the pricing of the two, e.g. Ross's Arbitrage Pricing (1976) or the famous CAPM by Sharpe (1964).

First, I thoroughly present the model in section 2, describing the optimal contract and the different ways for agents of interacting with each other. The solution to the problem derived by Williamson (1987a), consumers directly lending to entrepreneurs with my one factor distribution parametrized, is shown in the first subsection. The next subsection is devoted to the derivation of the entrepreneurial value and the resulting debt portfolio distribution, as this is a novelty in credit contracts research. An empirical example shows that the derived debt portfolio distribution fits real data quite well. In section 3, I start by setting

up the bank's problem of choosing what deposit rates to offer and how large the share of capital should be. Then, I solve the contract between the bank and the entrepreneurs to determine the distribution for banks' values. When the lending side of banking has been solved, I use the result to solve for optimal deposit rates and capital levels. Finally my concluding remarks are in section 4.

## 2 Model

Consider a two-period economy where decisions are made in the first period and consumption takes place in the second. There are two basic types of agents, entrepreneurs and consumers, both of which are risk neutral. Each entrepreneur has an investment project that needs  $N$  units of capital to be started. The majority of consumers have an endowment of 1 unit of capital, but a limited fraction of consumers are very rich. An entrepreneur's investment project cannot be split or sold and needs capital to be initiated. If a project is initiated in period one, it results in a stochastic outcome  $\tilde{V} \sim LN(\theta, \sigma^2)$ , i.e. the final value of entrepreneurs' projects is log-normally distributed. I denote the probability distribution function by  $\psi(\cdot)$  and the cumulative distribution function by  $\Psi(\cdot)$ . The realizations of entrepreneurs' projects,  $V_i$ , are correlated with the linear correlation coefficient  $\rho$ , i.e. pooling of projects cannot remove all stochastic variation.

The market for capital arises since consumption takes place in the second period. Consumers need to "store" their capital, which can only be converted to consumption in period two. This is done by lending to entrepreneurs. There can be two market solutions, consumers lend directly to entrepreneurs and financial intermediation, that is banks. Banking requires capital in order to cushion possible losses, which means that banks' balance sheets are as in Figure 1. In order to simplify computations I assume that the capital is kept as reserves in the bank rather than lent to entrepreneurs.

In Figure 2, I present a scheme over my market and the possible contract solutions. There can be two possible market solutions, direct lending symbolized with dashed lines and a solution with banks symbolized by solid lines. It will be shown that banks are established if rich decide to act as intermediaries and investing their capital in a bank rather than lending directly. With a banking solution, either capital is needed and banks are started by the rich who use their capital as equity, or no capital is needed and any consumer can start banks. As I assume that all equity is stored as reserves I will use the term reserves from here on instead of capital or equity. If no reserves are needed, banking must be associated with zero profits, or there will be arbitrage opportunities. Returns on banks' portfolios fluctuate, but the deposit rate is fixed; hence banks

# Bank

Reserves	Equity
Loans	Deposits

Figure 1: The bank's balance sheet, where I have assumed that the capital rich invest in their bank as equity is kept as reserves to cover possible losses.

always need to be audited or there will be arbitrage.

In Williamson (1986), a small consumer can act as an intermediary due to *i.i.d.* project returns, which allows her to pool all risk by growing large and offer depositors a fix risk-free return. This solution is removed when systematic risk is introduced. For exactly the same reason that prevents delegated monitoring in Williamson (1986), a group of small investors cannot form a bank. They would have to rely on delegated monitoring and each of them would have an incentive to lie to the others and engage in side bets with entrepreneurs to reap private profits. The very rich, on the other hand, could reduce the risk in deposits by holding reserves, thereby removing some of the downside in banking. Hence, the solutions are as described in Figure 2.

An entrepreneur that contracts with rich consumers or banks borrows all his funds from the same source and the agreed repayment including interest is  $\bar{R}$ , resulting in an expected repayment of  $r$  to the lender. When entrepreneurs borrow from normal consumers with limited funds, they need to contract with  $N$  consumers with an agreed repayment including interest of  $\bar{R}_D$  yielding an expected repayment of  $r_D$  to the lender. In case consumers with limited funds decide to use the bank as an intermediary, they lend their money to the bank; that is, the bank offers the gross deposit rate  $D$ , which results in an expected repayment from the bank of  $d$ . The expected gross returns  $r$ ,  $r_D$  and  $d$  are lower than the agreed repayment, if there is a positive probability of default. If, for example, the value of project  $j$  is  $V_j < \bar{R}_D$  when a consumer is lending directly, the lender must audit and pay the cost  $c$ , implying that the expected return  $r_D$  is lower than  $\bar{R}_D$ . Note that higher repayment levels lead to a higher probability of default and therefore, higher expected auditing cost.

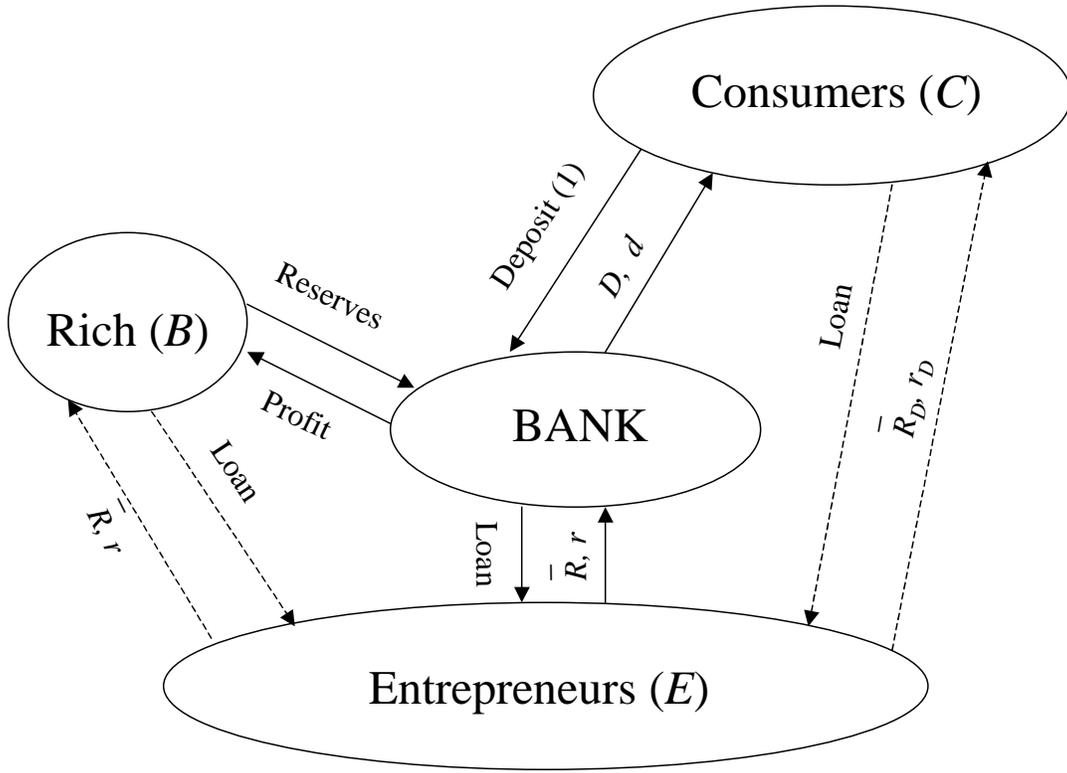


Figure 2: Graphical scheme over my credit model, I have the three different agents and a bank, which will be established if the rich choose to act as intermediaries. Dashed lines symbolize the case of no intermediation, though with my rationing assumption, it could be the case that both intermediation and loan contracts are written between consumers and entrepreneurs.

The entrepreneurs' problem is to offer lenders a repayment level,  $\bar{R}_D$ , that yields the largest possible expected value net of the repayment, equation (1)

$$\max_{\bar{R}_D} \int_{\bar{R}_D}^{\infty} (V - \bar{R}_D) \psi(V) dV. \quad (1)$$

But the entrepreneurs' offer must also satisfy lenders. Lenders' expected utility from lending is the total expected repayment net of auditing cost, i.e. when borrowers default, lenders seize all their assets and bear the auditing cost  $c$ , equation (2)

$$\int_0^{\bar{R}_D} (V - c) \psi(V) dV + \bar{R}_D (1 - \Psi(\bar{R}_D)) \geq r_D. \quad (2)$$

Williamson (1986, -87a) shows that the constraint must hold with equality, or the value of the objective function can be raised by reducing  $\bar{R}_D$  without violating the constraint. If the lender is also assumed to have all the bargaining power, the solution is found by maximizing lenders' expected utility (2).

Due to my use of a realistic probability distribution for project outcomes, I rely on numerical solutions

to a large extent. This means that the problem must be parameterized. Some insights as to the admissible range of parameters are gained from the second-order condition of equation (2)

$$-\psi(\bar{R}_D) - c\psi'(\bar{R}_D) < 0. \quad (3)$$

By re-writing the second-order condition (3) using my distribution function for values of entrepreneurs' projects  $\psi(\cdot)$ , I obtain expression (4) describing the relationship between the parameters and the optimal choice of repayment level  $\bar{R}_D$

$$c \frac{\ln(\bar{R}_D - \theta)}{\bar{R}_D - \theta - c} < \sigma^2. \quad (4)$$

It is immediately seen that in case the return on lending, i.e.  $\bar{R}_D - 1$ , is lower than  $\theta$ , which is the mean return parameter for the distribution of entrepreneurs' values, the condition for a maximum will be fulfilled for a wide range of plausible mean, variance and auditing cost parameters. Since the numerator is then negative and the denominator should be positive for reasonable parameters, the left-hand side will be smaller than any chosen variance. This is also the case in reality, where the return on lending tends to be lower than the expected return on equity. If instead the return on lending is larger than the mean parameter  $\theta$ , the left-hand side of (4) will be positive and it is harder to find parameters that will fulfill the condition, since the variance of return is in general a very small number.

## 2.1 Parameterized solution without intermediaries

I use Swedish data to obtain reasonable numbers for the mean and standard deviation. Using a small sample from 1988-99 over twenty of the largest listed firms in Sweden, the estimated mean and standard deviation is .12 and .36, respectively. The average correlation is found to be .38. The mean and standard deviations are rounded to .1 and .3 to keep the calculation simple. Correlation is rounded down to .3, both to keep calculations simple and to account for non-traded firms in a bank's portfolio. The solutions' sensitivity to the choice of correlation will be analyzed later.

The expected utility for entrepreneurs and consumers is graphed in the left-hand panel of Figure 3. The auditing cost is set to .35, and the mean parameter and the variance are as estimated on the Swedish sample. With the estimated parameters for investment projects, the expected return on a contract is negative, and the expected payback from lending one unit of capital is just over .8, which does not seem to be very attractive. We also see the effect of having an interest rate on lending that is higher than the mean parameter for entrepreneurs' projects: the objective function (2) is almost flat around the optimal repayment level, which

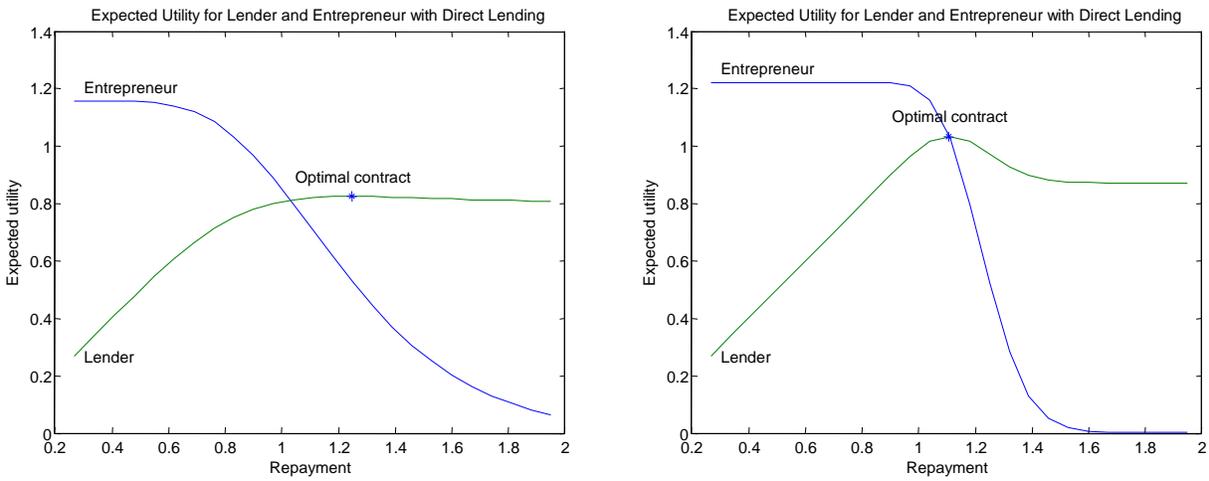


Figure 3: Graph of the expected utility of the lender and the entrepreneur, respectively, given direct lending. The left-hand panel shows the expected utility with parameters estimated on Swedish data, .1 and .3 for mean and standard deviation. In the right-hand panel, we replace the estimated risk and return parameters with a conjecture of these values. The mean parameter and the standard deviation on entrepreneurial projects are set to .2 and .1, respectively. The optimal repayment is 1.108, resulting in an expected return to the lender of 1.0357 and of 1.0346 to the entrepreneur.

was shown in equation (4). Raising or reducing the auditing cost will not be enough to obtain a sufficient contract with a positive return. Moreover, if auditing costs are reduced, the interest rate becomes very large as the asymmetric information disappears and we approach pure risk-sharing.

Does this mean that no debt financing should exist? The answer is that in this two-period economy there is neither reputation nor collateral/cosignatories for which I must compensate. A simple way of adjusting for the lack of these empirical features is to alter the parameters of my probability distribution. The use of collateral and cosignatories remove much of the downside of a project: this effectively raises the mean and reduces the variance. Adjusting the mean parameter to .2 and reducing the standard deviation to .1 results in the right-hand graph in Figure 3. The result is obvious; there is a much more pronounced hump in the lenders' expected utility function (2). The expected return on the efficient contract in the right-hand panel of Figure 3 is positive (.0357) at a repayment of (1.108), i.e. the expected auditing cost is (.0723).

To justify the high auditing costs chosen, the second-hand market for bankrupt firms' assets can simply be considered. The market value of firms' assets seldom amounts to the value of the firm just before bankruptcy; which is naturally why bankrupt firms are often reconstructed rather than defaulted. For example, debtors sometimes agree to convert debt into equity on the judgement that in time, more value can be extracted

from the equity than from an immediate bankruptcy. My theoretical probability distribution for the value of entrepreneurs' projects does not take this into account. Instead, the probability distribution is smooth over the default point; that is, I do not model the empirical feature that the value of firms' assets in a bankruptcy are worth less than just before the default. To compensate for this, we can instead have a large auditing cost, which will create a discrete jump of a firm's value at the defaulting point.

Williamson (1986) simply let banks' auditing costs be  $c/N$ , as there is no overlapping audit. Clark (1988) studied banks in the U.S. finding evidence of decreasing returns to scale in banking. Cerasi and Daltung (2000) model this feature as depending on auditing costs that increase with scale. Hence, it could be argued that some non-linear auditing structure should be used. For simplicity, I adopt the Williamson (1986) method and let banks' auditing costs be  $c/N$ , since a non-linear structure would only complicate the calculations. To be able to analyze markets with a positive probability for bank defaults, I need to decide on the auditing scheme for banks; how much more expensive should an audit of a very large bank be relative to the audit cost,  $c$ , of small entrepreneurs. A simple solution is to assume an auditing scheme with a fixed cost, equal to twice the overlapping audit cost for entrepreneurs, i.e.  $2c$ . The reasoning behind this assumption is as follows: essentially, a bank is just a large portfolio of loans, therefore it could be audited in two stages if it defaults. First, an "audit" is made where the loans in the bank are randomly distributed across the depositors. Second, the depositors audit the loan they just received in the first-stage division. As a result of the two-stage audit, the cost is doubled, i.e.  $2c$ .

For technical purposes, I will assume infinitely large credit markets, i.e. there is an infinite number of entrepreneurs as well as consumers. This simplifies the calculation of actual losses, which is shown in the below subsection, where the distribution for debt portfolios is derived.

## 2.2 Production distribution

Entrepreneurial outcome is driven by both a common factor, as well as an individual-specific factor. Systematic risk from the common factor will then remain, even if banks grow infinitely large. Since I analyze debt, the portfolio distribution will not be some simple transformation of the underlying distribution with smaller variance, as for equity portfolios. The reason is that extremely poor project outcomes will not be pooled by extremely good project outcomes for debt financing.

Denote the value on the  $i$ :th entrepreneurial project  $V_i$ , and assume it to follow a logarithmic Wiener process resulting in the following stochastic differential equation (5). The distribution model is based on a

continuous time Wiener process, though my model is a discrete two-period model. It only uses time periods  $T$  equal to 0 and 1

$$dV_i = \delta_i V_i dt + \sigma_i V_i dW_i, \quad (5)$$

where

$$dt \cdot dW_i = 0, \quad (dt)^2 = 0, \quad (dW_i)^2 = dt, \quad dW_i \cdot dW_j = \rho dt \quad \forall i \neq j. \quad (6)$$

$\rho$  is the ordinary linear coefficient of correlation between different firms' asset values. For a more intuitive interpretation, the Wiener process  $W_i$  can be represented as,

$$W_i = \sqrt{\rho} Y + \sqrt{1 - \rho} \varepsilon_i, \quad (7)$$

where  $Y$  is a common systematic factor, a macroeconomic shock, and  $\varepsilon_i$  an idiosyncratic noise component.

Using a standard textbook on stochastic mathematics, e.g. Björk, (1998), the solution for returns on wealth  $v_i$ , for a single firm is shown in equation (8), where the standard notation for continuous return is used,  $v_i = \ln V_i(1) - \ln V_i(0)$

$$v_i = \delta_i - \frac{1}{2} \sigma_i^2 + \sigma_i \sqrt{\rho} Y + \sigma_i \sqrt{1 - \rho} \varepsilon_i. \quad (8)$$

Here,  $Y$  and  $\varepsilon_i$  are *i.i.d.* standard normal, with cumulative distribution  $\Phi(0, 1)$  and probability distribution  $\phi(0, 1)$ . The dependence between firms is now captured by having the common process  $Y$  instead of correlated processes  $W_i$ . Hence, it is simple to find the default probability for a single firm, it is simply the probability that the firm's return  $v_i$  is lower than the predetermined interest rate  $\ln(\bar{R})$ , equation (9)

$$\underbrace{P(v_i < \ln \bar{R})}_{\equiv p} = \Phi\left(\frac{\ln \bar{R} - \delta_i + \frac{1}{2} \sigma_i^2}{\sigma_i}\right). \quad (9)$$

Note that from here on, I have defined the individual default probability in (9) as  $p$ . It should also be noted that the choice of borrowing rate for entrepreneurs directly affects the individual default probability. From equation (9), it is seen that the individual default probability is inversely related to the borrowing rate, equation (10)

$$\left(\ln \bar{R} - \delta + \frac{1}{2} \sigma_i^2\right) / \sigma_i = \Phi^{-1}(p). \quad (10)$$

### 2.2.1 Default Fraction for a Debt Portfolio

When risk is not idiosyncratic as in Williamson (1986, -87a), the distribution for banks' lending must be derived from the underlying distribution for entrepreneurs. This is because the defaulting projects cannot be fully pooled with successful projects, as entrepreneurs never repay more than the loan and the agreed

on interest. Vasicek (1987, -91, 2002) has derived the probability distribution for the unconditional default fraction for debt portfolios with a one factor setup. The unconditional default fraction is the share of defaulting loans in the portfolio. I follow Schönbucher (2000) and use the conditional independence to show how the unconditional default fraction can be found. Given the systematic shock, entrepreneurs are independent and normally distributed. Due to this independency of defaults, it is simple to find the conditional probability of default for an individual entrepreneur  $p(y) = P(v_i < \ln \bar{R} | Y = y)$ , shown in equation (11)

$$\begin{aligned}
p(y) &= P(v_i < \ln \bar{R} | Y = y) \\
&= P\left(\delta_i - \frac{1}{2}\sigma_i^2 + \sigma_i\sqrt{\rho}Y + \sigma_i\sqrt{1-\rho}\varepsilon_i < \ln \bar{R} | Y = y\right) \\
&= \Phi\left(\frac{\ln \bar{R} - \delta_i + \frac{1}{2}\sigma_i^2 - \sigma_i\sqrt{\rho}y}{\sigma_i\sqrt{1-\rho}}\right).
\end{aligned} \tag{11}$$

First, denote the share of defaulting firms, the default fraction, by  $X$ . Then, I can find the distribution for these default fractions,  $P(X \leq x)$ , of a bank by iterated expectations as follows

$$P(X \leq x) = E_Y [p(X \leq x | Y)] = \int_{-\infty}^{\infty} P(X \leq x | Y) \phi(y) dy. \tag{12}$$

To find the conditional probability for default fractions, I use the assumption that a bank's portfolio is infinite. Then, the fraction of actual defaults given  $y$  converges almost surely to the individual conditional default probability (11)

$$\lim_{I \rightarrow \infty} P(|X_i - p(y)| \leq \epsilon, \forall i \geq I) = 1, \forall \epsilon > 0. \tag{13}$$

Since the fraction of defaults is equal to the conditional default probability for an entrepreneur, expression (12) can be written as follows

$$\int_{-\infty}^{\infty} P(X = p(y) \leq x | Y) \phi(y) dy. \tag{14}$$

In (14), we see that the conditional default probability is either smaller or larger than default fraction  $x$ . This is used to split the support for the integral into two areas with an indicator function (15). It takes the value of one if the conditional default probability is lower than the default fraction  $x$ , and zero otherwise.

$$I_x = \begin{cases} 1 & \text{if } p(y) \leq x \\ 0 & \text{else} \end{cases} \tag{15}$$

We can now use the indicator function and substitute for the first term in the integral (14), which results in the cumulative distribution:

$$\int_{-\infty}^{\infty} I_x \phi(y) dy = \int_{-y^*}^{\infty} \phi(y) dy = \Phi(y^*). \tag{16}$$

The lower boundary  $y^*$  is found by inverting (11), the conditional individual default probability, which yields:

$$y^* = \frac{1}{\sigma_i \sqrt{\rho}} \left( \sigma_i \sqrt{1 - \rho} \Phi^{-1}(x) - \ln \bar{R} - \delta_i + \frac{1}{2} \sigma_i^2 \right). \quad (17)$$

That is, I find the boundary point where the business climate induces a default fraction of exactly  $x$ .

The intuition when going from (12) to (16) is that I know that the better the business climate is, large values for  $Y$  in my case, the smaller is the fraction of defaulting firms. It must be the case that there is some business climate  $-y^*$  in the support for  $Y$ , where smaller/larger realizations yield a larger/smaller default fraction than  $x$ . In terms of the common shock, I then obtain a truncation point; lower values on the common shock result in a functional value for  $p(X \leq x|Y)$  of zero, and do not affect the value of the integral.

To find the density function for the default fraction, I can just use Liebnitz's rule and differentiate the probability distribution (16) with respect to  $x$ , yielding (18)

$$f(x) = \frac{\sqrt{1-\rho}}{\sqrt{\rho}} \exp \left\{ \frac{1}{2} (\Phi^{-1}(x))^2 - \frac{1}{2\sigma_i^2 \rho} \left( \sigma_i \sqrt{1-\rho} \Phi^{-1}(x) - (\ln \bar{R} - \delta_i + \frac{1}{2} \sigma_i^2) \right)^2 \right\}. \quad (18)$$

The density for default fractions is extremely complicated, involving the inverse of the normal distribution, but it can be numerically computed.

To see why this approach is common in practical debt management, I show an example using the credit losses for one major Swedish bank, Svenska Handelsbanken (SHB), Figure 4. The probability of individual default for any project is set to 1 %, based on Carling et. al. (2004). I set the correlation to .3, based on the small sample estimate mentioned above. The left-hand panel shows credit losses as a time-series, and the right-hand panel is the resulting histogram for these losses, matched with the theoretical distribution.

Despite the small sample, the probability distribution does a good job matching the data, though the sample is too small to perform a formal test.<sup>1</sup>

I assume equal sized exposures; in reality a loan portfolio consists of varying sizes. Vasicek (2002) shows

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<sup>1</sup>A rule of thumb for a  $\chi^2$ -test for the goodness-of-fit of the distribution, is that the expected number of objects in any bin is at least five. To circumvent the problem with very low number of expected observations in the "large" loss bins, bins two and three could be merged into one and the whole area to the right of the third bin be considered as the third bin. This way, there are only three bins, but they all have expected observations larger than five. The resulting test statistic under the null that SHB credit-losses are distributed according to the derived distribution is 2.74, which is  $\chi^2$ -distributed with two degrees of freedom and a p-value of .25. Hence, the null cannot be rejected. This test procedure can be found in any standard literature for a basic statistics course, e.g. Aczel (1999).

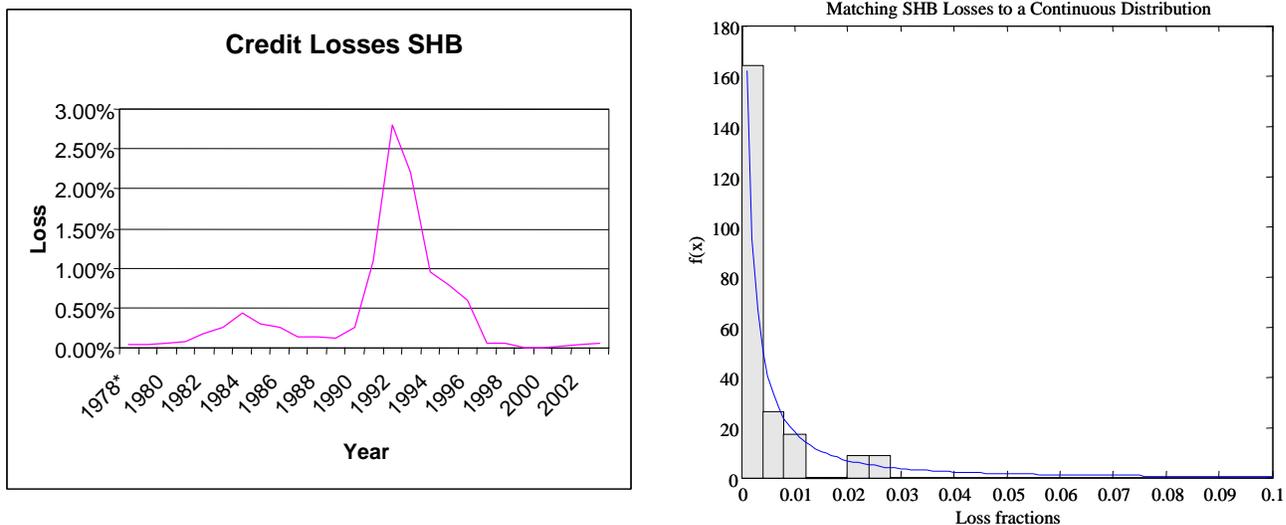


Figure 4: The left graph displays realized losses for SHB. In the right graph, the frequency of losses is plotted together with the theoretical loss-function as stated in equation (18). The histogram is standardized so that the “hole” between the third and fourth bar is filled when the total mass is calculated. The height of the bar is the mean of the height of bar three and four.

that the distribution is robust for varying exposure. A necessary and sufficient condition is that

$$\sum_{i=1}^n w_i^2 \rightarrow 0, \quad (19)$$

where  $w_i$  is the portfolio weight  $i$ . Condition (19) shows that as long as no exposure is too dominant in the portfolio, the result still holds. Schönbucher (2000) also shows that the exact distribution for a finite portfolio is almost identical to the limit distribution already for a portfolio of size 100. The infinite bank assumption is therefore not very restrictive when it comes to the derived distribution for default fractions.

### 2.2.2 Credit Losses

The standard way of solving for actual credit losses among practitioners and in the financial literature is to assume how large a share of a loan that can be recovered in a bankruptcy, henceforth called the recovery rate. I want to be slightly more general in this paper and have a varying recovery rate. This makes the portfolio loss fraction more consistent with the loss distribution for a single entrepreneur.

First, I study the bank’s actual gross cash-flow, the number of loans times the agreed repayment deducting losses made on defaulting borrowers, equation (20)

$$\pi_m = m\bar{R} - \sum_{i \in p^c} (\bar{R} - V_i). \quad (20)$$

$m$  is the size of the bank in terms of the number of firms to which it lends and  $p^c$  the group of defaulting loans. I divide (20) with  $m$  to obtain the average gross cash-flow per loan in the portfolio, equation (21)

$$\bar{\pi}_m = \bar{R} - \underbrace{\frac{p^c}{m}}_{\equiv X} \underbrace{(\bar{R} - \bar{V}^c)}_{\equiv l^c(\bar{R})}. \quad (21)$$

$\bar{V}^c$  denotes the average value on defaulting firms,  $X$  denotes the default fraction as before, and the last part of the second term on the right-hand side is the unconditional average loss per loan,  $l^c(\bar{R})$ .

$\bar{V}^c$  is easily calculated; I take its log to obtain the unconditional average rate of the loss, defined as  $\mu^c = \ln(\bar{V}^c)$ , which is calculated in (22)

$$\mu^c = \int_{-\infty}^{\infty} \left( \int_{-\infty}^{h(\bar{R}, Y)} \left( \delta - \frac{1}{2}\sigma_i^2 + \sigma_i\sqrt{\rho}Y + \sigma_i\sqrt{1-\rho}\varepsilon \right) \phi(\varepsilon) d\varepsilon \right) \phi(Y) dY. \quad (22)$$

The function  $h(\bar{R}, Y)$  is defined as (23) below,

$$h(\bar{R}, Y) = \frac{\ln \bar{R} - \delta + \frac{1}{2}\sigma_i^2 - \sigma_i\sqrt{\rho}Y}{\sigma_i\sqrt{1-\rho}}. \quad (23)$$

An explicit expression for the average mean recovery rate (22) cannot be obtained due to the normal kernel in the expectation. Using known central moments for the truncated normal, which can be found in e.g. Johnson et. al. (1994), (22) can be written as (24)

$$\mu^c = \int_{-\infty}^{\infty} \left\{ \left( \delta - \frac{1}{2}\sigma_i^2 + \sigma_i\sqrt{\rho}Y \right) \Phi(h(\bar{R}, Y)) - \sigma_i\sqrt{1-\rho}\phi(h(\bar{R}, Y)) \right\} \phi(Y) dY. \quad (24)$$

Through these calculations, I only obtain the unconditional average loss rate,  $l^c(\bar{R})$ . It is not possible to obtain a simple expression for the conditional average loss rate. Instead, I assume the recovery rate to be equal to the unconditional average loss rate for all loss fractions; that is, the average credit loss is a function of the lending rate  $\bar{R}$ , but not the fraction of defaulting firms  $X$ .

### 3 Banking

In earlier papers with no verification costs of banks, such as e.g. Williamson (1986), it is sufficient to solve the lending side of banking. When introducing my systematic component in the economy, this implies that banks need to be monitored in the same way as entrepreneurs. Therefore, banks must solve two problems: first, lending to entrepreneurs and also monitoring them and second, offering a satisfactory deposit rate to consumers. All agents in the economy know the probability distribution for entrepreneurs' values, but until

banks' lending rate,  $\bar{R}$ , is set, the probability distribution for banks' values is unknown. If treating  $\bar{R}$  as a parameter, I can still investigate the banks' problem of offering a deposit rate to consumers and the role for reserves without first solving the lending rate.

### 3.1 Stating the problem

Banks will need to set a contract with consumers in the same way that they settle the terms with entrepreneurs. This is done in the same way as entrepreneurs borrow from consumers described in (1)-(2). The bank offers the depositors the repayment  $D$ , and it holds reserves  $\mathfrak{R}$ , maximizing its expected return on available reserves (27). That is, banks maximize the return on invested capital. I let  $b$  denote the bank's value per unit lent,  $g(b)$  the probability distribution and  $G(b)$  the cumulative distribution for this value. Banks' returns were defined in (21). By deducting the auditing costs I obtain  $b$  as a function of the realized loss-fraction  $x$  of the stochastic variable  $X$  in (25)

$$b(x) = \bar{R} - \left( l^c(\bar{R}) + \frac{c}{N} \right) x. \quad (25)$$

Since the value of the bank is determined by the realized loss-fraction  $x$ , the probability distribution for banks' values will also be determined by the probability distribution of the loss-fraction,  $f(x)$ , defined in equation (18). I define the worst possible value of banks by  $\underline{b}$ , which takes a positive value if the recovery rate for banks is sufficiently large to cover the auditing costs. If credit losses are so large that the limited liability constraint binds,  $\underline{b}$  is zero. I then invert (25) so that  $x$  is expressed as a function of the parameters and  $b$ , this way I obtain my probability function for bank return  $g(b)$ .

$$g(b) = \begin{cases} a_1 f(x(b)) & \text{if } \underline{b} > 0 \\ \begin{cases} a_2 & \text{for } b = 0 \\ a_3 f(x(b)) & \text{for } b > 0 \end{cases} & \text{if } \underline{b} = 0 \end{cases} \quad (26)$$

The constants  $a_i$  are determined so as to make the probability mass integrate to 1.

Banks' objective function is then as follows in (27)

$$\max_{D, \mathfrak{R}} \frac{1}{\mathfrak{R}} \int_{D-\mathfrak{R}}^{\bar{R}} (b + \mathfrak{R} - D) g(b) db. \quad (27)$$

$D$  and  $\mathfrak{R}$  are as defined above, agreed repayment on deposits and reserves held. The bank's offer needs to be competitive to be accepted by consumers. The expected utility to consumers must be at least as high as if they were lending directly to entrepreneurs, i.e.  $r_D$ . Hence, the banks' maximization is constrained by

consumers' expected utility, which is shown in equation (28)

$$\int_{\underline{b}}^{D-\mathfrak{R}} (b + \mathfrak{R}) g(b) db + D(1 - G(D - \mathfrak{R})) - \gamma G(D - \mathfrak{R}) \geq r_D. \quad (28)$$

When banks' losses per lended unit of capital are greater than the promised gross deposit rate  $D$  minus reserves  $\mathfrak{R}$ , they cannot repay depositors. The first term in (28) is the expected value of a bank, including its reserves, for losses at its maximum to the point where it is no longer audited,  $D - \mathfrak{R}$ . The second term is the expected value for the depositor when banks' losses are small enough to permit it to repay depositors, i.e. the deposit rate  $D$  weighted with the probability of no default. The third term is the expected monitoring costs of banks, actual monitoring cost  $\gamma$  times the probability that an audit takes place. Monitoring costs  $\gamma$ , were set to  $2c$  in section 2.1.

Normally firms maximize profits in economics. There are two reason to why I choose to let banks maximize return on reserves, i.e. on their invested capital. Since I assume infinitely large markets with rationed lending expected profits would always be infinite. Also banks themselves often measure their own success in terms of return on capital. What prevents banks from setting reserves to zero and earn infinite rate of return? In the model depositors have the possibility to lend directly to entrepreneurs earning an expected rate of return of 3.57 %. If expected return on deposits are lower than this banks cannot attract any deposits. When reserves are lowered expected auditing costs for the depositor increases. This prevents the bank from letting the reserves go to zero.

I let the bargaining power in the deposit market be with banks.<sup>2</sup> This assumption means that the solution method of Williamson (1986,-87a), optimizing depositors' expected utility, is no longer valid. Instead, there is a regular constrained maximization problem for which the standard solution method is to form a Lagrangian. In addition to the requirement that depositors are satisfied, there are also restrictions on the choice of controls,  $D$  and  $\mathfrak{R}$ . An offered deposit rate will be lower than the maximum return  $D \leq \overline{R}$ . Reserves are also constrained both from above and below; to prevent looting, reserves must be positive, and since banks have the bargaining power, no pure transfers to depositors should take place, i.e.  $0 \leq \mathfrak{R} \leq D - \underline{b}$ . Instead of explicitly including the lower bound, I follow Chow (1997) and let the first-order conditions be smaller than zero, if the constraints bind. The multiplier for the upper bound on deposits is  $\lambda_D$ , and for reserves  $\lambda_{\mathfrak{R}}$ . For

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<sup>2</sup>If consumers have the bargaining power, banking will not be profitable unless the monitoring costs of banks are extremely low. Naturally, there exists some solution where consumers and banks split the gains from banking somewhere in the middle, but I feel little to be gained from such an exercise, which is why I assume banks to have the bargaining power.

simplicity, I assume the net return on reserves to be zero. My Lagrangian is then defined by (29)

$$\begin{aligned} \mathcal{L} = & \frac{1}{\mathfrak{R}} \int_{D-\mathfrak{R}}^{\bar{R}} (b + \mathfrak{R} - D) g(b) db + \lambda_D (\bar{R} - D) + \lambda_{\mathfrak{R}} (D - \underline{b} - \mathfrak{R}) \\ & + \lambda_c \left( \int_{\underline{b}}^{D-\mathfrak{R}} (b + \mathfrak{R}) g(b) db + D(1 - G(D - \mathfrak{R})) - \gamma G(D - \mathfrak{R}) - r_D \right). \end{aligned} \quad (29)$$

Williamson (1986, -87a) observed that the constraint for the lenders' expected value must bind; otherwise the offered compensation could be reduced and the value of the objective function increased. This also holds true for my bank's optimization problem. There is no point in offering consumers a combination of reserves and a deposit rate resulting in a larger expected value than their alternative, i.e. directly lending to entrepreneurs earning  $r_D$ .

$$\lambda_c > 0 \Rightarrow \int_{\underline{b}}^{D-\mathfrak{R}} bg(b) db + D + (\mathfrak{R} - \gamma - D) G(D - \mathfrak{R}) - r_D = 0 \quad (30)$$

First-order conditions, w.r.t.  $D$  and  $\mathfrak{R}$ , are,

$$\text{w.r.t. } D \Rightarrow -\frac{1}{\mathfrak{R}} (1 - G(D - \mathfrak{R})) - \lambda_D + \lambda_{\mathfrak{R}} + \lambda_c (1 - G(D - \mathfrak{R}) - \gamma g(D - \mathfrak{R})) = 0 \quad (31)$$

$$\text{w.r.t. } \mathfrak{R} \Rightarrow -\frac{1}{\mathfrak{R}^2} \int_{D-\mathfrak{R}}^{\bar{R}} (b - D) g(b) db - \lambda_{\mathfrak{R}} + \lambda_c (G(D - \mathfrak{R}) + \gamma g(D - \mathfrak{R})) = 0. \quad (32)$$

I use the fact that the restriction on the minimum expected utility to depositors is binding, this yields equation (30).  $\mathfrak{R}$  set to zero and  $D$  to  $\underline{b}$  can be ruled out except for very special parameterizations since the first and third terms in (30) are zero, which yields  $\underline{b} - r_D = 0$ . This means that the scrap value of banks is so large per loan that the expected return on direct lending requires recovery rates that are extremely high. Further,  $D$  at the upper bound and  $\mathfrak{R}$  at the lower bound can also be ruled out since the first term in (30) is then banks' expected return on lending  $r$  and  $G(D - \mathfrak{R}) = 1$ , which yields  $r - \gamma = r_D$ . Once more, the calculated return on direct lending and the auditing cost of banks and parameters stipulated in sections 2 and 2.1 are not compatible with this condition. Due to my small variance in the production distribution, the expected return on banking would be at such a high level that the probability for any firm of having such a successful project is only .02 %. Hence,  $r = r_D + \gamma$  would not be a plausible average return on banks' portfolios, even if the cost for banks to audit entrepreneurs were zero.

Suppose banks set the deposit rate to  $D^* = r_D$  and that they insure depositors against expected monitoring costs by keeping reserves large enough to always meet its obligations  $\mathfrak{R}^* = r_D - \underline{b}$ , the first term in (30) is then zero and  $G(D - \mathfrak{R}) = 0$ , and (30) is satisfied. This will only be a solution if auditing cost of banks is sufficiently large to outweigh the increased return from bank leverage. To find the optimal controls

I first have to determine the banks' lending rate,  $\bar{R}$ , I then use hill-climbing in two dimensions.<sup>3</sup>

### 3.2 Finding the optimal lending rate $\bar{R}$

$\bar{R}$  is determined in the same manner as  $\bar{R}_D$  in section 2. Entrepreneurs offer banks the repayment  $\bar{R}$  that maximizes their own expected utility, equation (33).

$$\max_{\bar{R}} \int_{\bar{R}}^{\infty} (V - \bar{R}) \psi(V) dV \quad (33)$$

The maximization is constrained in that banks' expected utility should be at least as large as the endogenous current market rate,  $r$ . Since every loan granted by a bank is now part of a portfolio, it is the contribution of the loan's to the portfolio's expected return that is of importance. Hence, the constraint is that the average return on a loan should be at least as large as  $r$  in equation (34).

$$\bar{R} - \left( l^c(\bar{R}) + \frac{c}{N} \right) \int_0^1 x f(x, \bar{R}) dx \geq r \quad (34)$$

Just as in section 2, I use the observation by Williamson (1986, -87a) that if the bargaining power is with the lender, the solution is found by maximizing the constraint (34). To show how the Williamson (1986) result is affected by correlated project returns, I calculate two solutions; one where banks are able to diversify all systematic risk, and one with correlated project returns where the coefficient of correlation is set to .3. The auditing cost is  $c/N$  per project, where  $N$  is set to two. That is, we assume that every entrepreneur needs to borrow from two investors to have sufficient capital. Even with this choice of very small entrepreneurs, much of the hump around optimum is flattened. In Figure 5, we see that the bank's utility function over varying repayment levels has a much less pronounced hump than the graph of direct lending with twice the auditing cost; right-hand graph in Figure 3.

Without systematic risk, left-hand graph in Figure 5, we see that the optimal repayment level is much higher than when banks are exposed to risk, as in the right-hand graph in Figure 5. When the banks can diversify all risk, they set an optimal repayment level of 1.1925. This results in an expected net return of 8.31 % to banks, but entrepreneurs' utility is reduced to .7497. In my setting with systematic risk through correlated returns on entrepreneurs' projects, the expected net return per loan is reduced by 25 % to 6.26 %. The reason is that the systematic risk increases the expected auditing cost, which makes banks' preferred repayment level much lower, 1.1389, to compensate for the increased default probability among borrowers.

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<sup>3</sup>See Hendry (1995) for details.

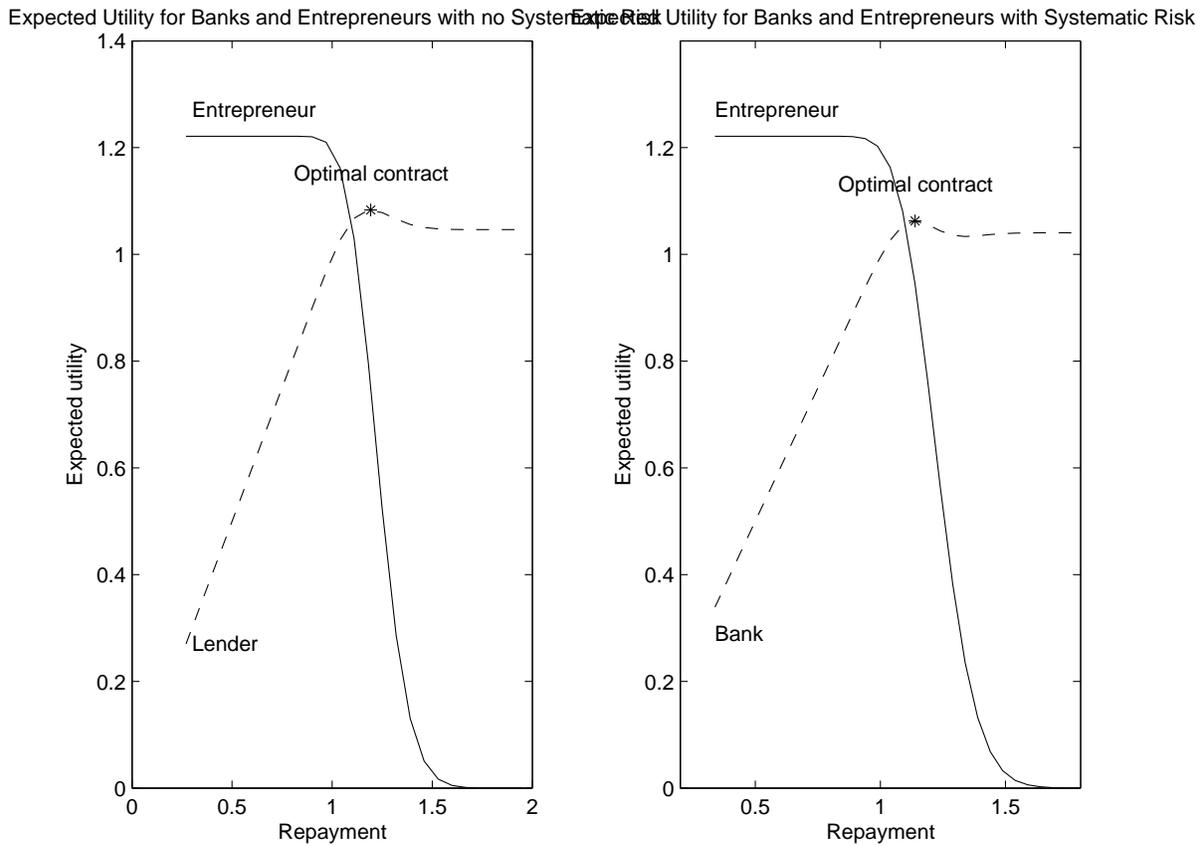


Figure 5: The left-hand graph shows bank lending when using the Williamson assumption of *i.i.d.*, i.e. no systematic risk. The optimal point is a repayment of 1.1925, yielding an expected utility for the bank of 1.0831, and for the entrepreneur of .7497. The right-hand graph shows bank lending with systematic risk, the coefficient of correlation is set to .3. Banks' expected utility falls by 25 % when systematic risk is introduced. The expected utility is now 1.0626 and the optimal repayment is 1.1389. With this borrowing contract, the expected utility for an entrepreneur is .9444.

If I compare the resulting optimum of bank lending to direct lending, there is a higher repayment level 1.1389, which naturally leads to more bankruptcies and lower expected utility for entrepreneurs. Banks' expected net return per granted loan is almost twice (6.26 %) that of the direct lending consumer (3.57 %), with double overlapping auditing. Entrepreneurs are roughly 9 % worse off with (.9444) borrowing from banks than directly borrowing from small consumers. Since entrepreneurs are worse off, I cannot say that the introduction of banks is Pareto improving.

To check if the choice of correlation,  $\rho = .3$ , is crucial for the optimal lending rate, some other coefficients of correlation are tried. Reducing the correlation to .1 only marginally raises the expected payback per loan, the sixth decimal increase marginally. The same is true when we increase the correlation to .5, but

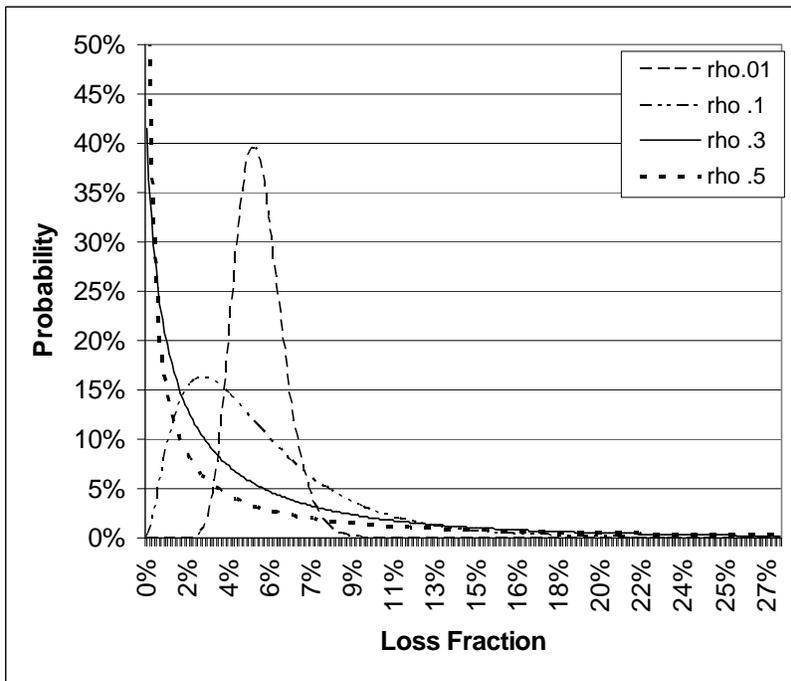


Figure 6: Plot of loss distributions when the correlation between entrepreneurs' projects outcome varies between .01 and .5. The individual probability of default is set to 5 %.

the expected payback is now naturally reduced. That is, the impact on the lending rate is obtained when going from idiosyncratic risk to just introducing marginal amounts of systematic risk. To some extent, this result is due to the fact that the recovery rate is the same across states for a given lending rate. But mainly because small amounts of systematic risk results in substantial increase in tail probabilities, in Figure 6 it is shown how the loss structure changes with different correlations. It is clear that the probability for large losses, the right tail, is not changed much when the correlation is altered.

### 3.3 Optimal controls $D$ and $\mathfrak{R}$

When I have solved for the optimal lending rate  $\bar{R}$ , and the resulting recovery rates, I can solve for optimal reserves and deposit rates. The solutions are graphed in Figure 7. On the x-axis, I have the probability of default for the bank. It can be seen that with the chosen parameters, the probability of default in optimum is rather low, .3 %. If monitoring costs are raised, the “optimal” default probability will be even lower. The optimal reserve ratio is 14.8 % and the expected utility on one unit of reserves is 1.168. Rich consumers greatly enhance their return on capital if forming a bank. By using their capital as reserves, they can earn more than 16 % return, which can be compared with directly investing in entrepreneurs' projects, earning a

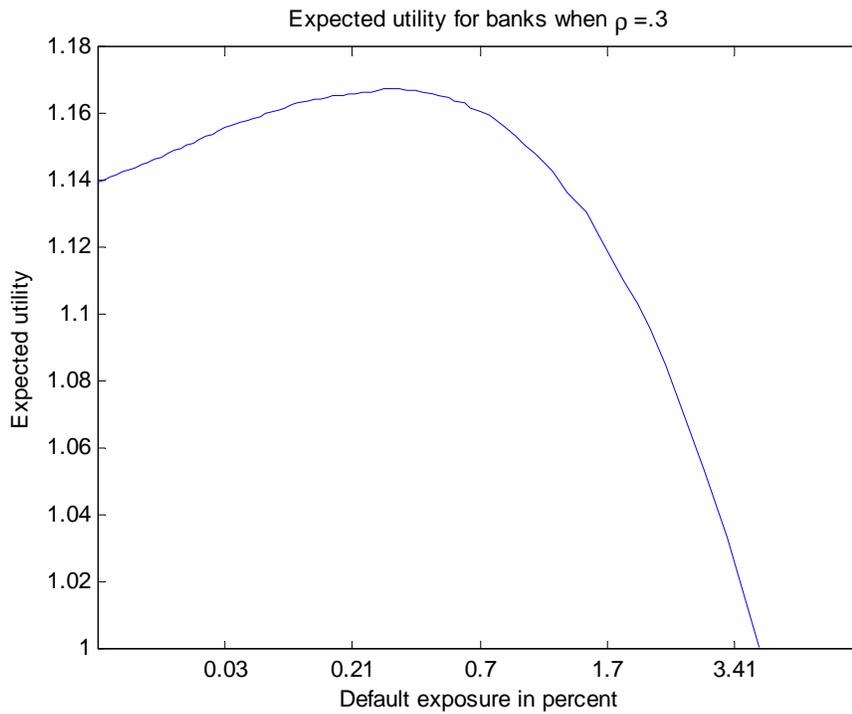


Figure 7: Graph of banks' expected utility in terms of percentage return on invested capital. The return is graphed over default exposure, i.e. how secure is the bank. When exposure is zero, the bank is fully insured against credit losses and depositors need not monitor. At the optimum point the probability for a bank to default is .3 %. The reserve ratio is 14.8 % and the expected utility per unit of reserves is 1.168 in optimum. 6 % return.

Even if the default probability is low, .3 %, this still means that depositors would occasionally need to audit banks, which reduces their expected return. The reason why it is not optimal to fully insure depositors and totally eliminate the non-productive auditing cost is that banks benefit from leverage. When banks' reserves are reduced, they can lend to more entrepreneurs, yielding a lower return per loan but a greater base of loans generating a positive return.

I perform a sensitivity analysis by changing the correlation in the same way as was done in section 3.2. Unlike the sensitivity analysis for the lending rate, which only gave marginal effects of altered correlation, the change of systematic risk has a large impact on the determination of reserves and deposit rates. Naturally, this is due to the increased concavity imposed on the deposit problem through the larger monitoring cost, when double auditing is needed. When the probability of large default fractions in a bank's portfolio is large, the cost for depositors in terms of non-productive auditing costs is increased. To compensate the depositors for this expected cost, banks' optimal choice is to raise reserves neutralizing some of the expected audit

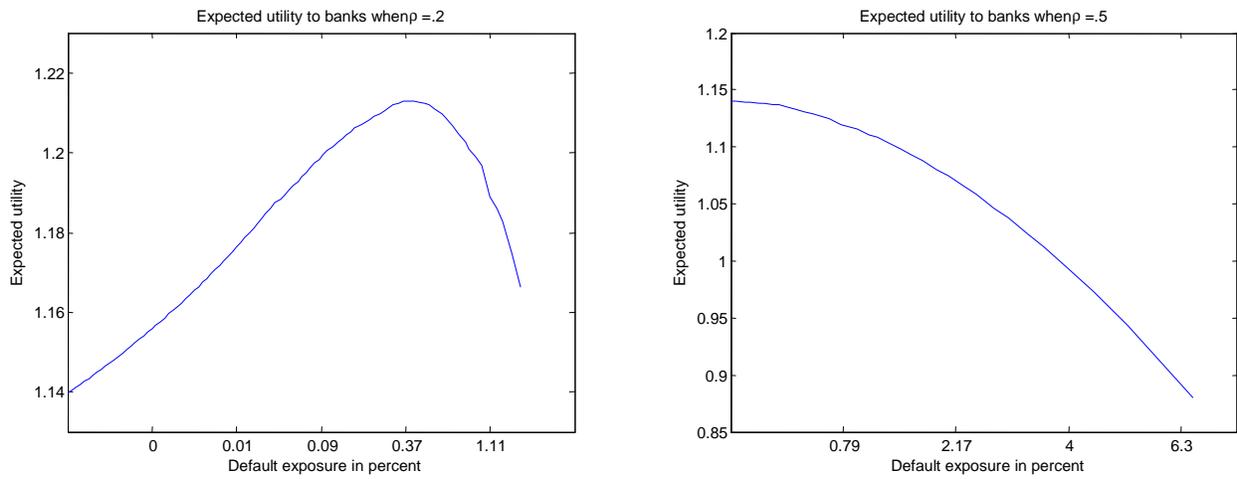


Figure 8: Sensitivity check of the choice of value for  $\rho$ . The left-hand graph shows how the expected gross return to the bank is changed when  $\rho$  is reduced to  $.2$ . The optimal reserve level is then  $11\%$ , the resulting probability of default is  $.37\%$ , and net return on reserves is  $21\%$ . In the right-hand graph, the correlation is raised to  $.5$ , which results in a solution where banks fully insure depositors by holding reserves sufficiently large to cover the maximum possible credit loss. Reserves are then  $19\%$  and the rate of return on them is  $14\%$ .

states. The right-hand panel of Figure 8 shows the expected utility for a bank when the systematic risk is increased by setting the correlation to  $.5$ . With this large amount of systematic risk it is, in fact, optimal for banks to fully insure depositors. Banks carry reserves equal to the difference between consumers' outside option, lending directly, and the minimum payback on the bank's loan portfolio.

When the correlation is instead reduced to  $.2$ , I obtain the opposite result showed in the left-hand panel of Figure 8, i.e. banks reduce their reserves to increase their leverage. This can be done due to the lower default risk for the bank, which reduces the expected auditing costs for depositors. The optimal choice for reserves with a correlation of  $.2$  is  $11\%$  and results in a default probability of  $.37\%$ . With the lower reserves, the return on investment to the bank is  $21\%$ .

The sensitivity analysis captures the actual behavior among banks in Sweden. Small provincial banks have much larger capital ratios than the large banks servicing all of Sweden. It can be argued that by mainly granting loans in one province, it is not possible to obtain geographical diversification which, in turn, implies a larger correlation in the debt portfolio. That is, my model actually captures some of the behavior among banks in the Swedish market.

Another implication of my model is that in the described economy, available financing through banks is

directly related to the wealth of bankers. If banks' total reserves are large, many loans are granted. This could point at a possible link to the real economy through the credit market in a dynamic model. A negative real shock to the economy erodes banks' reserves, which contracts bank lending, leading to fewer firms that can generate real output in the next period.

## 4 Conclusion

I use mathematical methods to derive a distribution for a bank's debt portfolio in a two-period model to study under what conditions financial intermediation will be established. Assuming the value of borrowers' projects to be log-normal and correlated, the direct lending problem is derived. I show that for an efficient debt contract to arise, with log-normality, it is important that the mean-variance ratio is large, i.e. the project needs to be rather safe. When empirically estimated parameters are used, the resulting debt contract has a negative interest rate. The auditing cost must be set so low that the contract is almost pure risk-sharing, i.e. lenders always audit. This could be the reason why collateral and cosignatories are so frequently used in banking. An implication of this is that policies reducing the value of banks' collateral could make credit markets work less efficiently, driving out debt financing which is mostly used by poorly capitalized firms.

With the derived distribution for debt portfolios, expected utility for direct lending and debt portfolios can be compared. I analyze under what conditions financial intermediation yields higher expected utility for investors/consumers than no intermediation. A main finding is that banks can only be started by rich investors, since it requires substantial amounts of capital which is used as reserves; due to the correlated borrowers, a bank's portfolio will not generate a non-stochastic outcome. Since the return on banks' debt portfolios are stochastic the expected auditing costs of banks are positive. Therefore banks are inefficient unless they have capital, which is used as reserves in my model to reduce the expected auditing cost for depositors. That is, banks need to carry reserves that are sufficiently large to almost eliminate the probability of a default. Moreover, banks must have bargaining power both against borrowers and depositors. If consumers have too much bargaining power, deposits will be too expensive for banking to be lucrative.

The requirement that banking must be backed by reserves means that less wealthy banks results in a tightening of bank supplied loans. This is similar to the result of Holmström and Tirole (1997), where they use moral hazard and perfect correlation between entrepreneurs. An implication is that banks could enhance and prolong business cycle downturns, since a macroeconomic shock that reduces banks' reserve levels leads

to less granted loans, thereby slowing down the recovery of the economy.

I feel that my findings are well in line with stylized facts from credit markets, in spite of the two-period model. Banks' capital ratios are large to keep them afloat through a rather deep recession, given that they have been involved in sound banking practice which only involves lending to low risk projects. Capital requirements only come into play when banks have been involved in more or less reckless lending.

I can also explain why small provincial banks carry much larger capital ratios than the large market leading banks. Since they cannot diversify geographical risk, their portfolio will be more risky and it is optimal to fully insure depositors as the expected auditing costs grow quickly.

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