

# Underdevelopment of financial markets and excess consumption volatility in developing countries

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## Abstract

A well documented pattern is that consumption volatility relative to output volatility is significantly higher in developing countries than in developed countries. This paper aims to explain this empirical regularity by linking it to one important difference between these two groups of economies, the development of financial markets. This paper proposes a model, in which the difference in development levels translates into the difference in development of financial markets. And shocks arise endogenously from incomplete financial markets in developing economies. While both consumption and output volatilities are higher in developing countries, consumption is more responsive, reflecting the permanent effects that shocks in financial markets induced. The quantitative results from the model are broadly consistent with empirical evidence. The model shows that even though both economies are subject to exactly the same exogenous shock process, the difference in productivity can translate into the excess consumption volatility pattern, through endogenous financial markets. The model has the ability to mimic a model which assumes exogenous permanent shock to productivity. In this sense, it provides at least a partial answer to why developing countries are subject to stonger permanent shocks.

## 1 Introduction

This paper aims to contribute to the understanding of excess consumption volatility in developing countries, relative to output volatility. A burgeoning literature has documented the existence of a negative relationship between macroeconomic volatility and income per capita. Particularly, data suggest that output growth is generally more volatile in developing countries. Naturally, one

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would expect that consumption growth exhibits higher volatility in developing countries as well. This premise is certainly confirmed in data. However, what is more interesting is that the negative relationship between volatility and development is even more pronounced for consumption volatility than for output. In other words, consumption volatility in developing countries is disproportionately higher than in developed countries, relative to output volatility. The purpose of this paper is to construct a theory that is consistent with these observations.

The focus on consumption volatility is well justified. The extent to which high volatility is a first-order problem for developing countries depends on the extent to which output volatility translates into consumption volatility. If, for instance, it was the case that poor countries can insure themselves through international risk-sharing and consumption growth is fairly stable, the welfare costs of output fluctuation would be less significant. However, that is not the case in reality. Evidence shows (e.g. Lewis 1996) that international consumption risk sharing is quite limited. As I document below, not only higher output volatility translates into higher consumption volatility in developing countries, but also consumption volatility is relatively larger, even controlling for output volatility. This implies that reducing volatility in developing countries would potentially bring substantial welfare gains. This implication is supported by De Ferranti et al. (2000)'s studies on various Latin American countries<sup>1</sup>. Another layer of welfare consequences is also important: fall in consumption during downturn or a crisis may have dramatic effect on countries that are already poor.

Figure 2 gives a comparison of consumption and GDP growth rate volatilities between Mexico and Canada. The left hand side graph presents demeaned GDP growth rates of these two countries. The blue line represents Canada while the red line is for Mexico. It is obvious that Mexico experiences more volatile output growth than Canada. It is even more obvious that consumption growth rate volatility in Mexico is higher than in Canada. Compared with GDP growth rate volatility, consumption growth rates seem to be more smoothed than GDP growth rates in Canada. It is interesting to see that it is not the case for Mexico where consumption growth is even more volatile, rather than more smoothed, than GDP growth. In other words, Mexico's consumption is more volatile than expected. I will show this sort of excess consumption growth volatility pattern is not unique in Mexico, and instead, as data suggest, it is what most of developing countries have experienced in the last 47 years.

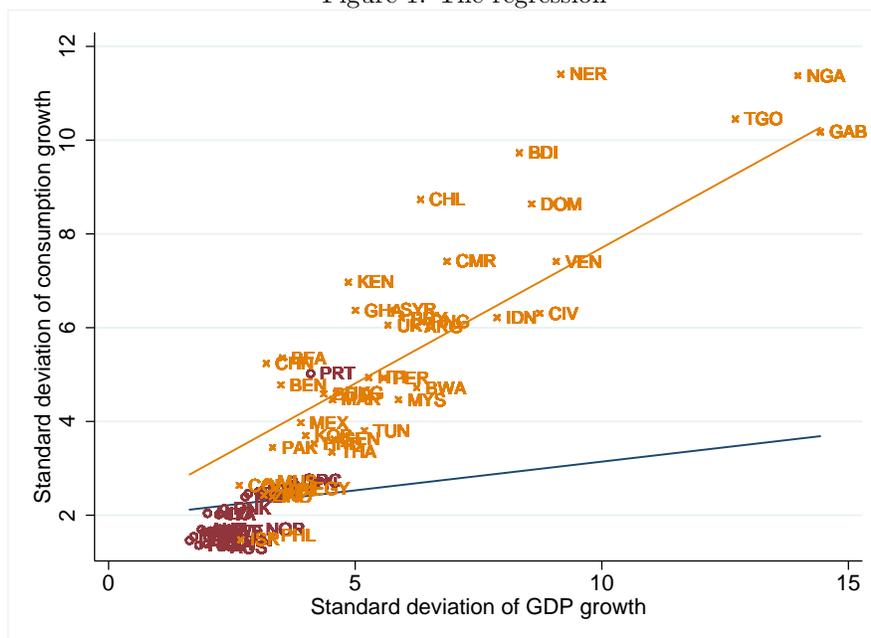
Using WDI data<sup>2</sup> from 1960 to 2007, I regress standard deviation of consumption growth on standard deviation of GDP growth. Figure 1 shows the regression lines for developed and developing countries, respectively. Developed countries cluster around the lower left corner, which means both consumption and GDP growth volatility are low. The picture for developing countries are quite dif-

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<sup>1</sup>"the World Bank (2000) estimates potential welfare gains of up to 5-10 percent of consumption in various Latin American countries, while these gains seldom reach 1 percent in developed economies."

<sup>2</sup>OECD countries are referred to as developed countries and the rest of the sample which have lower income level are labeled as developing countries.

Figure 1: The regression



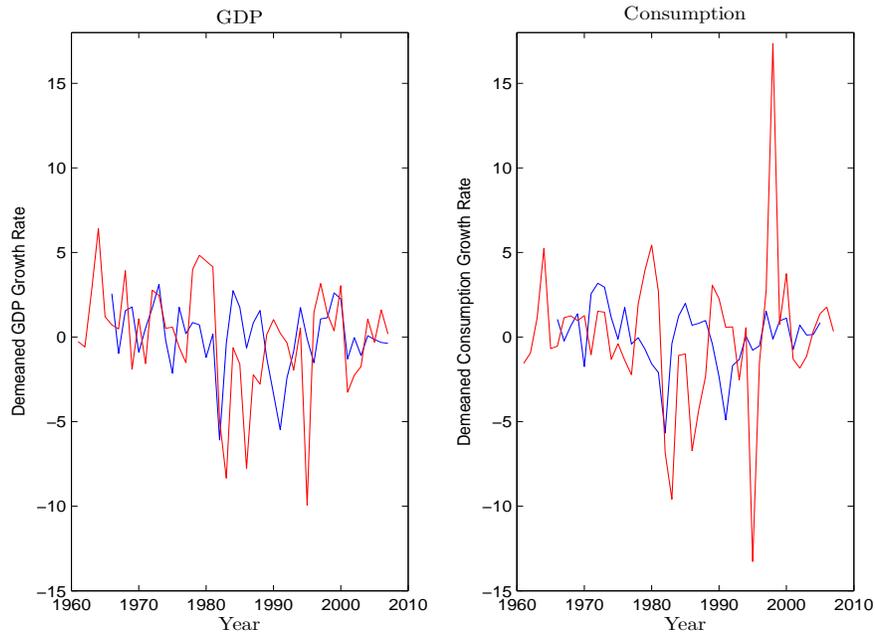
Source: WDI data, 1960-2007. Regression of standard deviation of consumption growth on standard deviation of GDP growth.

ferent: Most of them spread out towards the upper right corner, which means that both volatilities are higher in developing countries. Considering that GDP volatility may be interpreted as the underlying volatility of the economy, it is not a surprise that consumption also tends to be more volatile in developing countries. However, consumption volatility seems to follow a different pattern: consumption growth volatility increases much more, in response to GDP growth volatility. The positive slope of the regression line for developing countries is significantly higher.

To identify this pattern more clearly, I analyze the ratio of consumption volatility to GDP volatility. Table 1 gives the average standard deviations of consumption and output growth as well as their ratios in developing and developed countries, respectively. In the second column, the negative relationship between output growth volatility and income level is obvious, while the first column shows the same relationship holds for consumption. The third column gives the mean ratios in each group and shows that the ratio of consumption growth volatility to output growth volatility is disproportionately higher in developing countries. The gap between the two averages, roughly .3, is large and statistically significant.

Similar exercises have been conducted by researchers using different data in

Figure 2: Mexico and Canada



Source:WDI. Demeaned growth rates of GDP and consumption are presented on left hand side and right hand side, respectively. The blue curves are for Canada while the red curves are demeaned growth rates of Mexico.

terms of sample, time interval and frequency. Kose, Prasad, and Terrones (2003) documents a similar pattern. They also find that the ratio of consumption growth volatility to output growth volatility is significantly higher in developing countries, although the gap they find is relatively smaller than mine. Similarly, De Ferranti et al. (2000) investigate this issue by employing a different dataset.They show that the volatility of growth rate of real GDP in Latin American countries are twice as high as in industrial economies while consumption growth volatility is three times higher than industrial economies.

Aguiar and Gopinath (2007) also confirm this finding with a relatively small sample of emerging and industrial economies. Their data suggest that emerging economies exhibit relatively volatile consumption at business-cycle frequencies, even though the already high income volatility is controlled for. The gap is roughly 0.5 in their finding.

Resende (2006) studies a sample of 41 small open economies. His findings are well consistent with the previous research: the consumption volatility to output volatility ratio is on average 30% lower in the developed economies subsample. Roughly speaking, emerging economies are, on average, two times higher in terms of output volatility and three times higher in terms of consumption

Table 1: The pattern

	$\sigma_c$	$\sigma_y$	$\sigma_c/\sigma_y$
Developed countries	2.155 (0.46)	2.403 (0.31)	0.896 (0.07)
Developing countries	5.385 (0.34)	4.503 (0.19)	1.197 (0.05)
Difference	3.23 (0.57)	2.101 (0.36)	0.302 (0.08)

Source: WDI (1960 - 2007). All the numbers are reported in percentage  $\sigma_c$  and  $\sigma_y$  are standard deviation for consumption growth and output growth, respectively.  $\sigma_c/\sigma_y$  is their ratio. Standard errors are in parenthesis.

volatility.

Although empirically well documented, the theoretical literature has devoted little attention to this interesting fact so far. It is well understood that consumption is less volatile than output in developed economies (Christiano 1987). The permanent income hypothesis suggests that people make their consumption decision based on the expected long run income, instead of the short run counterpart. Temporary productivity shocks are expected not to last and people would smooth consumption over time through financial market. Consumption, therefore, presents less volatility than output. Suppose that developing countries are subject to temporary shocks with bigger variance, both output and consumption growth volatilities will be higher. However, consumption is still expected to be less volatile, due to the same reason of consumption smoothing. The ratio of consumption growth volatility to output growth volatility would not be substantially different between these two groups. Therefore, the pattern described above can not explained away by simply assuming larger variance of temporary shocks. The permanent income hypothesis also implies that consumption would respond more than income, under the condition that the income shock is permanent. Assuming the shocks that developing countries are subject to are more persistent could help explain why the volatility of consumption relative to income is surprisingly higher for a broad sample of developing countries.

Instead of making these two specific assumptions (higher persistence and larger variance), this paper aims to explain this empirical regularity by linking it to one important difference between these two groups of economies, the development of financial markets. I will show the permanent shocks can endogenously arise from incomplete financial markets. In this sense, the model has ability to mimic a model which assumes exogenous permanent shocks. Or in other words, I provide at least a partial explanation why developing countries are subject to stronger permanent shocks.

Permanent shocks in developing countries seem to be critical to understand the issue. Resende (2006) hypothesizes that external borrowing constraint can be a candidate for explaining the relative consumption volatility differential. Poor countries are more likely to face borrowing constraints and as a consequence, consumption smoothing through international market is more limited. He uses a dynamic-general equilibrium model which features an endowment of two-goods, and small open economy, facing borrowing constraint. The author concludes that this mechanism alone has rather limited explanatory power, although the constrained economies do exhibit higher relative consumption volatility. He suggests that the reason why consumption volatility can't exceed income volatility is due to the lack of permanent shocks in his model.

Aguiar and Gopinath (2007) proposed a otherwise standard RBC model with exogenous permanent shock to growth rate. They show that shocks to trend growth are the primary source of fluctuations in emerging markets, while the transitory fluctuations around a stable trend are more important for developed economies. The difference between developed and developing countries lies in the stochastic process for Solow residuals. They also point out that the difference is probably a manifestation of deeper frictions in the financial market.

This paper shows that Aguiar and Gopinath (2007)'s conjecture is well justified. Underdevelopment financial markets could indeed induce permanent shocks endogenously. Comparing with Aguiar and Gopinath (2007), I don't need to assume exogenously any "trend shocks" in this model. Instead, I will show the model with endogenous incomplete financial markets presents "observational equivalence" to a model with exogenous permanent shocks.

This research tries to understand how well this mechanism can help explain the empirical regularity. Other factors are omitted, which of course can potentially contribute to understanding of this pattern, such as risk sharing with limited commitment (Levchenko 2005), external borrowing constraint (Resende 2006), different Solow residual process (Aguiar and Gopinath 2007) and interest rate volatility (Neumeayer and Perri 2005).

Towards this end, I study a closed economy with a simple enough structure, which only includes a financial market into an otherwise standard one sector stochastic growth model with infinite horizon. In this model, the only exogenous difference between developing and developed countries is the productivity level. Developed countries are richer in terms of higher productivity level and hence per capita income or capital. In contrast, developing countries are characterized by lower productivity and hence lower capital per capita.

I model the financial market by adopting Acemoglu and Zilibotti (1997)'s framework<sup>3</sup>. Following their work, I assume agents have access to a large number of imperfectly correlated projects in the intermediate sector, which transforms savings into capital. The critical assumption is the non-convexities in production at the micro level which is modeled as minimum size requirements: Projects

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<sup>3</sup>explain its influence

require certain amount of savings before being productive. If there are enough savings in the economy, then minimum size requirements will be irrelevant: all the projects will be open and all the risks will be diversified. On the contrary, if there are not enough savings in the economy, some projects will not be open and therefore not all the risks can be diversified. The capital stock, therefore, is subject to uncertainty from the financial market. Shocks arising from the financial market has permanent effects on the economy: if the uncertainty turns out to be favorable, other things equal, it results in higher savings in next period, which helps to better diversify the risks in the intermediate sector. This, in turn, increases the chance to receive favorable shocks in the following periods, hence increasing expected savings and capital. In other words, favorable shocks increase the expected permanent income in the economy, while less favorable ones reduce it.

I also assume there are exogenous temporary productivity shocks. The temporary shock not only increases volatility in the economy, but also brings in permanent effects by interacting with the shocks from financial markets. A good productivity shock, although temporary, increases savings in the economy and thus the chance to diversify the risks in the intermediate sector. Therefore, shocks to the intermediate sector have better odds to turn out to be favorable, which, as analyzed above, induces permanent effects on the economy.

Holding other things equal, a more productive economy has higher income and therefore savings which endogenously determine a more complete financial markets, which in turn, helps diversify most of the shocks. In the extreme case, all the risks in the intermediate sector can be diversified and only temporary shocks provide uncertainty. That is the "developed economy case", which actually replicates the behavior of standard stochastic growth model, in which consumption is more smoothed than output. On the other hand, lower savings in developing countries determine an incomplete financial market and not all the risks can be diversified. Obviously, comparing to the case with a complete financial market, uncertainty in financial market boosts volatility in both consumption and GDP growth. Moreover, consumption appears to be much more volatile relative to output, in response to the increase or decrease in permanent income. It implies their ratio should be higher in developing countries. In other words, the difference in productivity translates into the excess consumption volatility pattern, through the endogenous financial market. Note that I assume there is only one exogenous difference between these two groups, while many other aspects of the differences between these two groups of economies are assumed away.

I regard part of the value added of this paper to lie in its methodological contribution. The paper provides an interesting application and extension of the classical contribution of (Acemoglu and Zilibotti 1997). However, the differences are still not trivial in several respects. Actually, methodological contribution of this paper per se is important. First, I model an economy with infinite horizon which is better suited to studying high-frequency phenomena, in contrast to the two-period OLG framework in their paper, which is only appropriate for discus-

sion on development issues. The infinite horizon framework allows me to assess the model quantitatively. Second, my assumptions on preferences and depreciation are more general. And this yields important new insights and turns out to be critical to the research. The two-period OLG framework they adopt together with assumptions of Log utility and full capital depreciation allow them to derive analytical results. However, the simplicity comes at a cost: substitution effect, income effect and wealth effect cancel out exactly. Saving rate is fixed and therefore, the relationship between consumption volatility and GDP volatility can not be properly studied. To overcome these limitations, I assume CRRA preference and reasonable depreciation rate of capital. The existence of undepreciated capital provides reduces agents' exposure to risk. Therefore, the decision rules and equilibrium behavior change substantially. In particular, in their model, it is not possible to have a "steady state" with incomplete financial markets, while it is one of critical characteristics of developing countries in this model. Third, temporary shocks are included, in order to quantitatively compare the model economy with the data. Without these shocks, the economy would experience zero volatility, if the market is complete, as in the ... Acemoglu and Zilibotti (1997). Finally, more realistic setup of this general model provides technical difficulty. No analytical solution can be derived, except one special case. This paper provides a functional and successful algorithm for solving the general framework.

This paper finds its place in at least two lines of research on macroeconomic volatility. First, it is related to a growing literature on consumption volatility. Neumeyer and Perri (2005) address why emerging economies exhibit much more pronounced business cycle than developed economies. In their model, interest rate is affected by external shocks and also exacerbates or amplifies the effects of those shocks. If the temporary production shocks are dominant, consumption will be smoother than output. On the other hand, if, instead, the interest rate shocks dominate, the opposite happens. Levchenko (2005) adopts the Kocherlakota (1996) framework of risk sharing subject to limited commitment to address why increasing international financial integration is actually associated with higher consumption volatility in developing countries. In his model, financial opening has first order effect on domestic financial market. Agents turns to international market where they can insure themselves against both aggregate and idiosyncratic shocks and therefore the domestic risk pooling deteriorates, resulting in an increase in consumption volatility. The mechanism is consistent with the positive relationship between capital flows and consumption volatility found in the data.

Secondly, this research is also related to a large bulk of research which focus on relationship between financial markets and macroeconomic volatility. Among others, interest in the potential link between financial market and aggregate economic behavior has been growing since two decades ago (Gertler (1988) and Levine (1997)). A line of research has been devoted to the relationship between financial deepening and the severity of business cycles. The empirical link has been established that countries with more developed financial sectors experi-

ence less fluctuations in output and consumption (Denizer, Iyigun, and Owen (2000) and Easterly, Islam, and Stiglitz (2000)). Early works on the connection between financial system and volatility draw on information asymmetry theories (Bernanke, Gertler, and Gilchrist (1998) and Greenwald and Stiglitz (1993)). More recent works extend attention onto different types of market imperfection. Aghion, Banerjee, and Piketty (1999) argue that the combination of lower growth rate and higher volatility in developing countries results from less developed financial markets and a sharper physical separation between savers and investors. Similar endeavors, among others, include Rajan and Zingales (2001), Haan, Ramey and Watson (1999), and Fecht (2004). Notably, Acemoglu and Zilibotti (1997) stress that the underdevelopment of financial markets, and the induced lack of diversification, allow the "chance" to play a bigger role in the early stage of development. And it implies that the macroeconomic volatility decreases in income level.

The rest of the paper is organized as follows. The next section lays out the basic model and characterizes the equilibrium. An analytical special case is studied and it gives the motivation for studying the general version of the model. Section 3 conducts the quantitative analysis. The empirical pattern found in data is compared with the numerical results. Section 4 shows the model has the ability to mimic results from a model where exogenous permanent shocks are assumed. Section 5 concludes.

## 2 The Model

### 2.1 Environment

The model economy is populated by infinitely lived agents. A constant relative risk aversion utility function is assumed to parameterize their preferences. Agents maximize expected life time utility which is defined by

$$U = E_0 \sum_{t=1}^{\infty} \beta^{t-1} \frac{c_t^{1-\sigma}}{1-\sigma}$$

where  $c_t$  is consumption at period  $t$ ,  $\sigma$  is the coefficient of relative risk aversion and  $\beta$  is the discount factor. The population is constant and normalized to be unit. Therefore, labor supply is constant as well.

The production side consists of two sectors, the final good sector and intermediate sector. The final good sector uses capital and labor to produce a final output. The production function in the final good sector is assumed to be Cobb-Douglas with capital  $K_t$  and labor  $L_t$  as inputs

$$Y_t = A_t K_t^\eta L_t^{1-\eta}$$

where  $\eta \in (0, 1)$  is the elasticity of output to capital and  $A_t$  productivity at period  $t$ . Productivity is subject to an aggregate shock. Formally,  $A_t = e^{z_t}$  and  $z_t$  follows an  $AR(1)$  process

$$z_t = (1 - \rho)\mu + \rho z_{t-1} + \varepsilon_t$$

where  $|\rho| < 1$  and  $\varepsilon_t$  is a serially uncorrected normally distributed random variable with zero mean and constant variance, that is  $\varepsilon_t \sim \mathcal{N}(0, \sigma_z)$ .  $\mu$  is a non-negative constant.  $e^\mu$  therefore is the average productivity level in this economy.

Note that shock to the growth trend is an important source of volatility in output and consumption growth in developing countries, and it has been studied by Aguiar and Gopinath (2007). Since my goal is to explore and highlight the underdevelopment of financial market and its effects on consumption volatility, I assume away the growth trend of productivity, or in other words, assume the exogenous productivity growth is zero. This can be considered as a detrended version of a more general model. I provide a version of this model with deterministic trend in the appendix and show it is not essential to the results.

Agents work in the final sector and earn a competitive wage and also receive capital income through competitive renting market. The prices, precisely wage rate and return to capital, are determined competitively by the aggregate capital in the economy,  $K_t$ , and the productivity level,  $A_t$ . Agents decide how much to consume and save. They are also allowed to decide the allocation of their savings in financial market.

Following Acemoglu and Zilibotti (1997), I assume that the financial market, or the intermediate sector transforms savings into capital brought forward to the next period without using any labor. There is uncertainty which is represented by a continuum of equally likely states  $state \in [0, 1]$ . The transformation technology takes two forms: Safe and risky projects. The safe project gives non-stochastic return  $r$ . There is a continuum of risky projects, corresponding to the states of nature. Risky project  $j$  pays a positive return,  $R$ , only in state  $j \in [0, 1]$  and zero otherwise. It is assumed that  $R > r$ , which is consistent with the intuition that risky assets give higher return. All the projects are financed by issuing securities. Output from the intermediate sector is entirely distributed to the holders of securities. No profit is retained. Note that not all of the projects are necessarily active or available to be invested in. The number of active projects,  $n_t$ , is determined in equilibrium. In addition to deciding savings (and consumption) each period, agents are also allowed to decide how they allocate their savings in the financial market, that is the portfolio decision. They can invest in a set of active risky securities ( $i \in [0, n_t]$ ), which consists of state-contingent claims to the output of the risky projects, and safe asset, which consists of claims to the output of safe technology. It is obvious that agents invest in risky financial securities an equal amount of savings,  $F$ , due to the symmetry of risky assets: the expected return to each risky project is exactly the same. Formally,  $F_j = F_i = F, \forall i, j \in [0, 1]$ .

The key to this setup is that the number of active projects is endogenously determined in equilibrium. To appreciate its importance, consider the following case where all the projects are always active, or exogenously assuming  $n_t = 1$ . Given this assumption, agents would invest an equal amount in all of the risky assets and therefore all the idiosyncratic risks will be diversified. And the intermediate sector becomes deterministic and gives back constant return. In order to model the incompleteness of the financial market, the assumption is imposed that the  $j$ th project is productive only if it attracts a minimum amount of investment,  $M_j$ . Without loss of generality, it is assumed that project  $j < \gamma$  has no minimum size requirement and the minimum cost of the rest is increasing linearly<sup>4</sup>. Formally, the minimum size is specified by

$$M_j = \left\{ 0, \frac{D}{1-\gamma} (j - \gamma) \right\}$$

where  $D$  is the highest minimum size. At the beginning of period  $t$ , agents observe  $K_t$  and  $A_t$ , and decide  $n_t$  optimally. In equilibrium<sup>5</sup>,  $n_t(K_t, A_t)$  contains exactly the same information as state variables  $K_t$  and  $A_t$  combined.

## 2.2 Recursive Formulation

The agent's problem can be restated in the following recursive formulation.

$$V(K, k, A) = \max_{s \geq 0, 1 \geq \alpha \geq 0} \left\{ u(c) + \beta E_{A, n(K, A)} V(K', k', A') \right\}$$

The representative agent's value function is a function of aggregate capital,  $K$ , his own capital  $k$ , and aggregate productivity,  $A$ . The right hand side of the Bellman equation consists of utility derived from current consumption and the discounted expected continuation value. The expectation is conditional on  $A$  and  $n$ . The representative agent chooses saving and portfolio optimally.

The representative agent's choice is subject to the resource constraint

$$c + s = w(K, A) + \varphi(K, A) \cdot k$$

where  $w(K, A)$  is the wage rate,  $\varphi(K, A)$  is the return to capital and  $k$  is the capital of his own. The representative agent takes prices as given and makes saving decision,  $s$ , and therefore consumption decision,  $c$ .

Before describing the capital accumulation function, I define assets portfolio with  $\alpha$ , which is the percentage of savings invested in safe asset,

$$\phi = \alpha \cdot s$$

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<sup>4</sup>The results are not driven by the specification of the minimum size. In order to compare with Acemoglu and Zilibotti (1997), this specification is assumed here.

<sup>5</sup>Acemoglu and Zilibotti (1997) provide a micro foundation for this equilibrium condition. In order to avoid repeating their analysis which is not essential to this model, I provide an intuitive justification in next subsection. Note that, however, their micro structure could apply in this model as well.

where  $\phi$  is the total amount invested in safe asset. The total amount investment in risky assets is  $(1 - \alpha) \cdot s$ . Investment in each risky asset is  $F$ . And the number of projects which are active and available to invest in is  $n(K, A)$ . In equilibrium, it is determined by aggregate factors, precisely aggregate capital and productivity. Agents take it as given and make their choices. Therefore, the following relationship holds

$$n \cdot F = (1 - \alpha) \cdot s$$

Individual capital accumulation function takes two forms, depending on different realizations of state of nature. Suppose the state of nature  $j$  is realized at the end of the period. If project  $j$  happens to be active, agents collect returns from both safe and risky assets. In this case, capital next period,  $k'$ , consists of three components: return from safe asset,  $r \cdot \alpha \cdot s$ , return from risky asset,  $R \cdot \frac{(1-\alpha)}{n} \cdot s$  and undepreciated capital  $(1 - \delta) k$ , where  $\delta$  is depreciation rate in the economy. I denote  $k'$  in this case as  $k^g$ . Since all the states of nature have equal probability to be realized, the probability for  $k' = k^g$  is  $n$ . Conversely, if project  $j$  happens to be not active, agents' investment in risky asset gives no return and capital in next period only consists of return from safe asset and undepreciated capital. Similarly, I denote  $k'$  in this case as  $k^b$ . The probability for this case to happen is  $(1 - n)$ . Individual capital accumulation function is therefore as follows

$$k' = \begin{cases} r \cdot \alpha \cdot s + (1 - \delta) k & \text{if } j > n \text{ with prob } 1 - n \\ r \cdot \alpha \cdot s + R \cdot \frac{(1-\alpha)}{n} \cdot s + (1 - \delta) k & \text{if } j \leq n \text{ with prob } n \end{cases}$$

The law of motion of aggregate capital is needed for agents to optimize.

$$K' = \Psi(K)$$

Finally, the exogenous shock process is AR(1),

$$\log A' = (1 - \rho) \mu + \rho \log A + \varepsilon$$

Given the model that I describe above, the definition of equilibrium is stated as follows:

1.  $V^*(K, k, A)$ ,  $\alpha^*(K, k, A)$  and  $s^*(K, k, A)$  solve the individual's maximization problem, taking  $n^* = n(K, A)$  as given.
2. Prices, namely wage rate,  $w(K, A)$  and capital return,  $\varphi(K, A)$ , are all competitively determined.
3. Consistency conditions: The law of motion of aggregate capital is consistent with the aggregation of individual capital,  $K' = \Psi(K) = \int k'$
4. Equilibrium conditions:  $n = n^*(K, A)$  such that the following hold:  $\alpha^*(K, k, A)$  and  $s^*(K, k, A)$  imply  $F^* = \frac{(1-\alpha^*)}{n^*} \cdot s^*$  and

$$F^* = \frac{D}{1-\gamma} (n^* - \gamma) \text{ if } 0 < n^* < 1$$

$$F^* \geq D \text{ if } n^* = 1$$

The consistency conditions need to be elaborated. Firstly, agents know the law of motion of aggregate shock. They also need to conjecture the law of motion of aggregate capital to make their decision. The conjecture,  $\Psi(K)$ , turns out to be correct and equal to the aggregation of individual capital. Secondly, agents take  $n^*(K, A)$  as given, and make their choice of  $\alpha^*(K, k, A)$  and  $s^*(K, k, A)$ .  $F^*$  is implied,  $F^* = \frac{(1-\alpha) \cdot s}{n^*(K, A)}$ . In equilibrium, the minimum size requirement  $M_n$  for the project  $j = n$  has to be the same as  $F$  (as long as  $n^* \in (0, 1)$ ). Otherwise,  $n$  can be either decreased or increased to achieve higher expected return or lower variance. To see the intuitions behind this equilibrium condition, suppose  $n$  is too low such that  $F > M_n$ . Agents will be better off if more projects are open. Although the expected return to the total investment will be the same, the variance will be lower since diversification opportunity improves<sup>6</sup>. On the other hand, it is not optimal either that  $n$  is too big such that  $F < M_n$ . Investment in projects which fall short of the minimum size requirement are not productive at all. Agents are better off and achieve higher expected return, if the number of projects reduces. If  $n = 1$ , the equilibrium condition becomes  $F \geq D$ . All the projects are active and investment in each risky project is no less than  $D$ . As will be seen soon, if the savings are not sufficient or equivalently  $F < D$ , the equilibrium active projects number  $n^*$  will be less than 1, which means not all the risks can be diversified away because of the endogenous incompleteness of the financial market. I will show that it plays an important role in generating the output and consumption volatilities.

### 2.3 Optimization

Taking  $n(K, A)$  as given and assuming interior solution where  $n < 1$ , the intertemporal Euler equations are derived as follows (refer to appendix for detailed solution),

$$U'(c) = \beta \cdot E \left[ U'(c_g) \cdot R \cdot \left( \eta \cdot A' \cdot K^{\eta(\eta-1)} + (1-\delta) \cdot \frac{1}{r} \right) \right] \quad (1)$$

$$U'(c) = \beta \cdot E \left[ U'(c_b) \cdot \frac{(1-n)}{\left(\frac{1}{r} - \frac{n}{R}\right)} \cdot \left( \eta \cdot A' \cdot K^{b(\eta-1)} + (1-\delta) \cdot \frac{1}{r} \right) \right] \quad (2)$$

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<sup>6</sup>The expected return is constant,  $r_E = F \cdot n \cdot R + (1-n) \cdot F \cdot 0 = (1-\alpha) \cdot s \cdot R$ .  
Variance is decreasing in  $n$ ,  $Var = n \cdot (F - (1-\alpha) \cdot s \cdot R)^2 + (1-n) \cdot (0 - (1-\alpha) \cdot s \cdot R)^2 = [(1-\alpha) \cdot s \cdot R]^2 \left[ \left(1 - \frac{2}{R}\right) + \frac{1}{R^2 n} \right]$

where  $c_g$  and  $c_b$  are the consumption choices, given next period's capital stock  $k^g, k^b$  and aggregate capital level  $K^g, K^b$ . Also note that the corner solution is obtained when  $n = 1$  and the risks are fully diversified. In that case backward inequality holds in the second equation.

The two equations are the Euler equations that relate current and future marginal utilities. The first one is similar to the standard Euler equation in a stochastic growth model, and the difference is that the return to capital in this model is more complicated, because of the structure of the financial market. The second equation is satisfied with equality in the interior solution. Backward inequality holds when the financial market is complete, in the sense that the idiosyncratic risks are fully diversified. The solution to this model is a decision rule for savings,  $s(K, k, A)$ , and a portfolio choice,  $\alpha(K, k, A)$ . As stated in the last subsection, I also need to solve for  $n(K, A)$  which ensures that the equilibrium condition is met. At this point, we can see the model is not too far away from the standard stochastic growth model. Note that in the corner solution case where the economy is fully developed (that's  $n = 1$ , and  $K$  is high enough), only the first equation is relevant. If I further assume  $R = 1$  and  $r = 1$ , the model simplifies to the standard stochastic growth model.

## 2.4 Special Case

To gain some intuition of the model, I study one special case, where  $\delta = 1$  and  $U(c) = \log(c)$ . This particular case can be solved by paper and pencil and it is the case where analytical solution is also obtained in standard stochastic growth models. I use the method of "guess and verify" to solve this case (refer to appendix for detailed solution). Decision rules are solved for:

$$\alpha = \frac{R \cdot (1 - n)}{R - r \cdot n}$$

$$s = \beta \cdot \eta$$

By using equilibrium condition, the number of active risk projects is also solved for:

$$n = \frac{(R + \gamma \cdot r) - \sqrt{(R + \gamma \cdot r)^2 - 4 \cdot r \left( \frac{(R-r) \cdot (1-\gamma)}{D} \cdot S + \gamma \cdot R \right)}}{2 \cdot r}$$

A few comments are in order. In this special case, the saving rate is constant, which means shock and capital levels do not affect the agent's saving rate. The result is the same as the standard stochastic growth model with these two special assumptions. The portfolio choice and equilibrium number of risky projects are the same as in Acemoglu and Zilibotti (1997), which means their two-period OLG model is a special case of this general setup. Interestingly,

exogenous shock and its persistence plays no role in determining the decision rules. One implication is that the consumption growth volatility will be identical to the output growth volatility. Actually, the full depreciation assumption severs one crucial channel of persistence. Moreover, substitution and income effect of inter-temporal prices cancel out under log preference. Study on the relationship between consumption and output volatility in this case will be trivial, due to the unrealistic assumptions. That's one of the reasons why I have to analyze the general model. On the other hand, this analytical solution provides a good starting point or initial guess for the numerical computation of the general model when rooting finding method has to be used (refer to appendix for detailed solution).

### 3 Numerical Analysis

Similar to most of stochastic dynamic general equilibrium models, the system of equations (equation (1) and (2)) does not have an analytical solution, except the special case. I solve the model numerically by exploring the recursive formulation. The combination of two-dimensional Spectral (Chebyshev polynomials) method (Judd (1998) and Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006)) and root-finding (Broyden) method is used to solve the model. See the appendix for detailed algorithm for solving decision rules and pinning down equilibrium conditions. I simulate the economy for 1100 periods and get rid of the first 100 periods in order to shake off the influence of the choice of starting point. I also repeat the same simulation for 100 times to compute the average statistics. Note that, in contrast to the analytical model with subgame perfect solution, I only solve for solution on the equilibrium path and ignore information outside the equilibrium path. It means I will switch to the case where  $K = k$ .

#### 3.1 One-Dimension example: decision rules and equilibrium condition

Policy functions, specifically saving rate and portfolio choice, are both two-dimension functions of capital and productivity levels. So is  $n(k, A)$ . Before calibration and simulation, I solve one special numerical example where temporary shock is not present. I plot the policy functions against capital for a given productivity level. In the general case, decision rules of different productivity levels are just parallel to the one shown in the figure 3.

The left plot of figure 3 gives the saving decision which is concave and increases in capital. The decreasing curve in the right plot is the portfolio decision. For a given productivity level, the higher the capital, the less do agents invest in safe asset. After a threshold, agents invest nothing in the safe asset. If bad shocks realize in intermediate sector, while the investment in safe asset is zero,

Figure 3: Decision rules and equilibrium condition

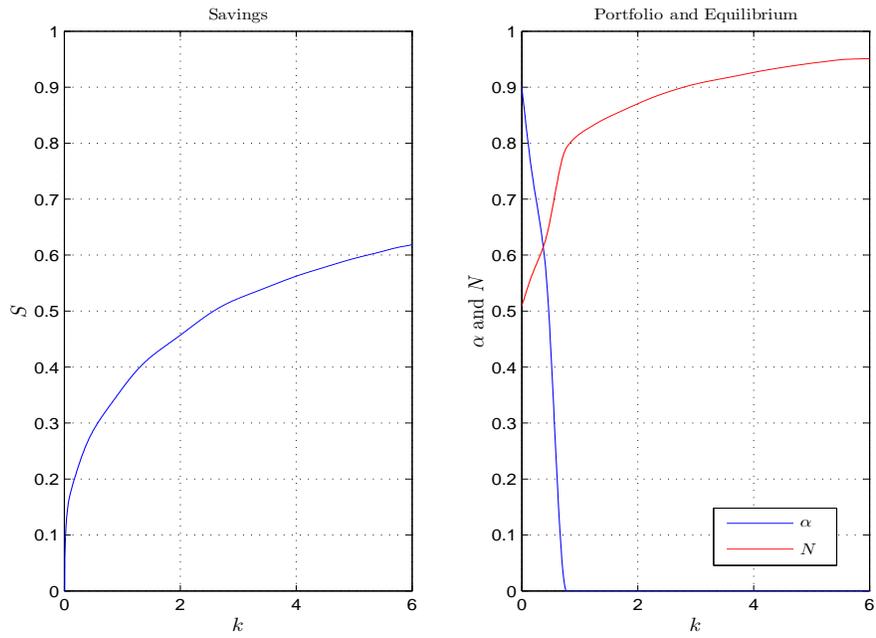
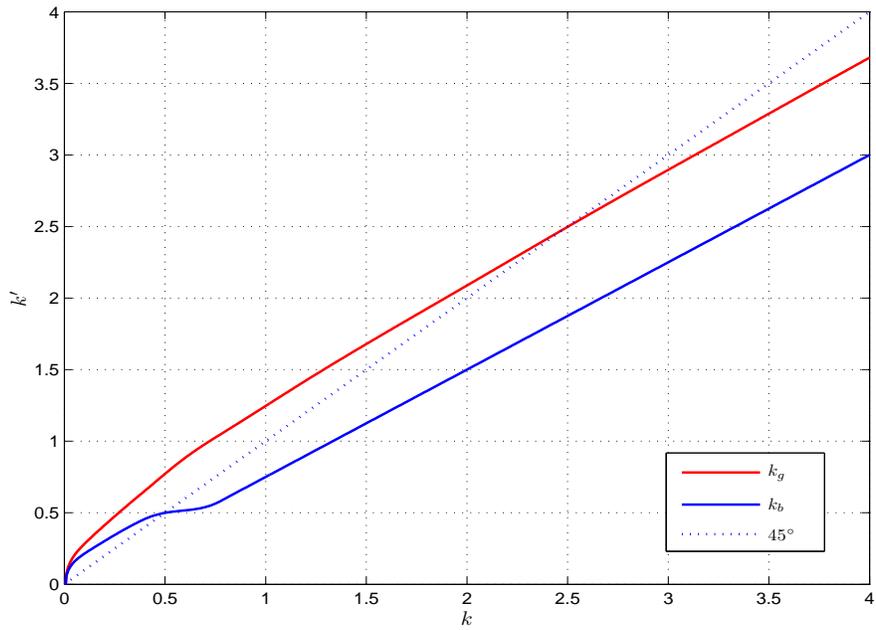


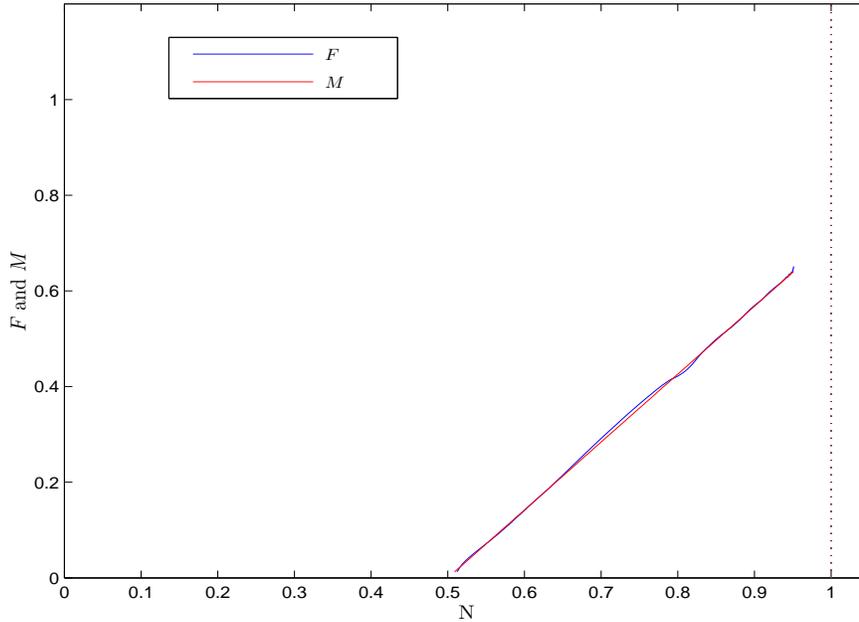
Figure 4: Capital dynamics



Note: Two possible realizations of capital in next period and 45 degree line

undepreciated capital works as a buffer and becomes capital in next period<sup>7</sup>. The Figure 4 gives two possible realizations of capital in next period. The plot for  $k_g$  is increasing in capital which describes the case where good shocks realize. It consists of not only undepreciated capital but also returns to the risky investment and safe asset.  $k_b$  looks more complicated. As analyzed before, if  $k$  is big enough, the chance to diversify increases and investment in safe asset decreases until zero. After a threshold of  $k$ , only undepreciated capital consists of  $k_b$ . It is expected that  $k_b$  follows the line of  $(1 - \delta) \cdot k$ . Below that threshold,  $k_b$  comprises both undepreciated capital and investment in safe asset. Note that both of the two realizations intercept with the 45 degree line. The two interceptions define the highest and lowest possible capital levels in the economy.

Figure 5: Minimum cost function  $M$  and investment curve  $F$



The equilibrium  $n^*$  is graphed against  $k$  in right subplot of figure 3. It is increasing and approaching 1 from below. It is intuitive that the more capital in the economy for a given productivity level, the more diversification opportunity available and the closer is the financial market to a complete one. The figure 5 is another way to present the equilibrium condition.  $F(n)$  is the amount of investment that agents decide to put into the risky projects, given the number

<sup>7</sup>That's a scenario which would not happen in Acemogolu and Zilibotti (1997). In their model, full depreciation implies that agents invest all their savings in risky assets only if the financial market is fully developed. Another implication is therefore that a reasonably rich economy could revert back to a really poor economy with a small probability. Introducing undepreciated capital helps provide the economy with persistence and avoid unrealistic implications.

of available projects,  $n$ .  $M$  is the minimum cost requirement curve. If the  $n(k)$  indeed satisfies equilibrium condition,  $F(n)$  will be the same as  $M$ . Figure 5 shows that the amount invested in each project, given a certain  $n$ , tracks the minimum cost curve very closely (almost indistinguishably), which means that equilibrium condition is met.

### 3.2 Choosing parameters

I parameterize the model using standard data in growth literature for most of the parameters. I use the standard *CRRA* utility function, where the risk aversion parameter is chosen to be 1.5. The discount rate  $\beta$  is standard from the literature, 0.96. The capital intensity is set to be 0.33, which is also common in growth models. Values of  $\rho = .95$  and  $\sigma_z = 0.015$  are widely used in the literature and close to the stochastic properties of the Solow residual of the U.S. economy. The AR(1) shock process is approximated by markov chain, using an extended version of Tauchen method.

Because the model aims to characterize a certain feature of developing countries, I choose the depreciation rate to be .25, which is unusually higher than the common setting 0.10. This choice is motivated by recent studies on depreciation rate in developing countries. Yisheng Bu (2006) employs World Bank data at the firm level and empirically estimates the depreciation rate in developing countries of different regions, including Africa, East Asia and Southeast Asia. The basic finding is that the depreciate rates in developing countries usually range from 20% to 40%. Sometimes, the rate can go up to 49% or even 84% in extreme cases. I choose 25% since depreciation rate in most of the countries is close to this number.

Since what matters for the decision in the model is the ratio of  $R/r$ , I normalize the  $r = 1$ , without affecting the results. I also normalize the average productivity to be 1, that is  $\mu = 0$ , for developing countries. There are the other four parameters left to be fixed, namely  $R, D, \gamma$  and  $\mu$  for developed economy. The idea is to match consumption rates ( $c/y$ ) and output growth volatilities ( $\sigma_y$ ) in both developing and developed countries by choosing these four need-to-be-calibrated parameters. I compute the average consumption rates for developing and developed countries and find there is a statistically significant difference between these two groups. It is important to let the model capture the first moment, namely the average consumption rates. It is also interesting to let the model deliver the correct output volatilities and see if consumption volatilities are close enough to the data.

So far, all these parameters, including persistence and variance of aggregate shocks and financial market setups, are shared between developing and developed cases. The only difference between developing and developed countries in this model economy is the average productivity level, where it is normalized to be 1 in developing countries and calibrated to be  $e^\mu$  in developed countries.

Table 2: Moments to match

Type	Consumption rate	Output volatility
Developed Countries	$c/y = .57$	$\sigma_y = 2.3$
Developing Countries	$c/y = .66$	$\sigma_y = 4.5$

Source: WDI 1960-2007

Table 3: Baseline Parameters

Parameter	Economic interpretation	Value
$\sigma$	CRRA risk aversion	1.5
$\beta$	Annual discount rate	0.96
$\eta$	Capital intensity	.33
$\delta$	Depreciation rate	.25
$\rho$	Shock persistence	0.95
$\sigma_z$	Shock standard deviation	0.015
$R$	Return to risky projects	1.12
$D$	Largest minimum cost	0.71
$\gamma$	Financial market parameter	0.5
$\mu$	Log of average of productivity	0.215

Source: Standard and calibrated parameters

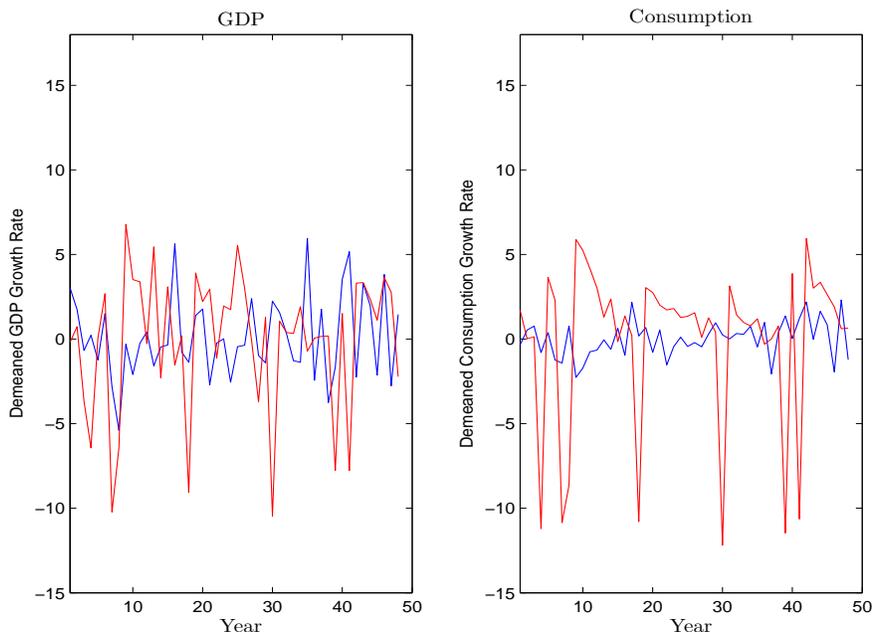
### 3.3 Baseline results

Before showing the statistics from the simulation, I present representative simulation series for developing and developed countries' experiments in Figure 6. In both of the experiments, exogenous shock processes are set to be exactly the same. It is interesting to see Figure 6 mimics the Figure 2 pretty well. Excess consumption growth volatility is quite clear and even more striking than what's presented in Figure 2. One pitfall is worth noting that consumption growth volatility in developed country experiment is much more smoothed than GDP growth.

The main results are summarized in table 4. Firstly, the first column shows that the negative relationship between development and consumption volatility exists in the model and it is even more pronounced than output growth volatility.

Secondly, a quick look at the third column reveals the fact that, in this model, the ratio of consumption volatility to output volatility is substantially higher in developing economy cases. Ratios are approximately 1 in the developing economy case, while the ratio in developed economy case is only slightly above .47. The pattern found in the model, is well consistent with the empirical findings in data and in Kose, Prasad and Terrones (2003), Loayza, Ranciere, Serven and Ventura (2007) and Aguiar and Gopinath (2007).

Figure 6: Representative Simulation



Source: Simulation data. Demeaned growth rates of GDP and consumption are presented on left hand side and right hand side, respectively. The blue curves are for the experiment of the developed country case while the red curves are demeaned growth rates from the experiment of the developing countries. In both of the experiment, the exogenous shock processes are the same.

Thirdly, although it is obvious that the model replicates the empirical pattern pretty well, there are still notable differences between the model and data. Volatilities, in terms of consumption, are lower in the model than their counterparts in data. I compute the ratio of consumption volatility ( $\sigma_c$ ) in the model to consumption volatility ( $\sigma_c^d$ ) in data. The ratio is quite high for the developing economy case, 83%, which means the model does a good job replicating the volatility level in developing countries. In contrast, the ratio is quite low in the developed economy case, 50%, which means the model doesn't generate enough volatility in consumption for developed countries. It is relatively less surprising. In the model, developed economies spend most of their time in a world with perfect diversification or complete financial market. All the shocks with permanent effects are diversified away and only temporary aggregate shocks provide exogenous uncertainty. The model behaves similarly to a standard one sector stochastic growth model which consumption smoothing mechanism helps the economy to achieve lower consumption volatility. Consumption, therefore, presents much less volatility than output.

Finally, in addition, the fact that there are only two different sources of uncer-

Table 4: Results

Type	$\sigma_c$	$\sigma_y$	$\sigma_c/\sigma_y$	$\sigma_c/\sigma_c^d$
Developed countries	1.0894	2.305	0.472	50%
Developing countries	4.4715	4.532	0.986	83%

Source: Simulation

tainty in the model, contributes to the low volatility results. I didn't assume exogenous permanent shocks or "growth trend shocks". However, it is not hard to imagine that it would increase the volatility level of consumption growth. Empirical evidence (e.g. Easterly, Islam, and Stiglitz (2000) ) also shows that fiscal policy, public consumption and nominal shocks all help increase the volatility in both developing and developed countries. Another source of uncertainty comes from international sector which exposes the economy to external shocks. It is expected that volatility level will be higher, if more sources of uncertainty are included in the model.

### 3.4 Analysis

In this model, lower productivity level is associated with both higher output and consumption growth volatilities, keeping the variance and persistence of exogenous shocks unchanged. It is not expected in the standard stochastic growth models. Even more interestingly, consumption volatility responds more strongly, relative to output volatility.

At first sight, it seems striking that simply decreasing (increasing) the productivity level leads to asymmetrical increase (decrease) in consumption and output volatilities. The intuition behind the results lies in the permanent income theory. A developed economy is more productive and has higher income and capital per capita around steady state. Hence more savings. The corner solution ( $n = 1$ ) in the model corresponds to the case where saving is even higher than the highest minimum size requirement. Agents invest all the savings in the risky projects and all the idiosyncratic risks are fully diversified. A developed economy spends most of its time in a world with full diversification. Then it behaves more like a standard stochastic growth model which is subject only to temporary aggregate shocks. Consumption smoothing motive leads to substantially low consumption volatility, relative to output volatility.

On the contrary, developing countries are less productive and the income and capital is therefore lower around the "expected steady state"<sup>8</sup>. Savings are not enough to satisfy all the minimum size requirements and some of the projects

<sup>8</sup>Unlike standard stochastic growth model, there is no deterministic steady state in the model. I define an expected steady state,  $k^e$ , where capital in next period is expected to be  $k^e$ , given the current level of capital is  $k^e$  without any exogenous disturbance. Formally,  $k^e = k^g \cdot n + k^b \cdot (1 - n)$

are left unopen. Part of the securities are unavailable to invest in. Therefore, risk arises endogenously from the financial market. Firstly, lack of diversification provides a significant source of uncertainty. On top of temporary aggregate shock, capital accumulation is also subject to uncertainty. The bottom right plot in figure 3 shows that the equilibrium number of active sectors or the probability to receive good shocks is less than 1 around the "expected steady state". Secondly, good (bad) shocks increases (decreases) the expected permanent income in the economy. Given capital  $k$ , the capital next period  $k'$  can be  $k^g$  and  $k^b$ , determined purely by chance. If  $k'$  turns out to be  $k^g$ , instead of  $k^b$ , then capital to be used in production next period is relatively higher and savings next period are higher as well, increasing the number of active projects, and hence better chance to receive better shocks again. Other things equal, higher expected discounted permanent income results from better realization,  $k^g$ , in the financial market. Intuitively, consumption responds to shocks from the financial market the way it responds to permanent exogenous shocks. Interestingly, the interaction between the exogenous shock and the endogenous uncertainty further increases the variance of the permanent shocks. Given capital  $k$ , a good temporary shock (higher  $A$ ) enhances the productivity and therefore the savings and diversification opportunity in the economy. Similar to the analysis above, higher savings increase the chance to receive favorable shock in the financial market. Conversely, a bad temporary shock (lower  $A$ ) decreases the expected permanent income.

As analyzed above, the model can actually translate the difference in productivity or different levels of development, into the relative consumption volatility differential pattern identified in empirical research.

## 4 Observational equivalence

This section evaluates if the model has the ability to mimic the a model in which permanent shocks are exogenously assumed. I conduct this exercise by following procedure.

Suppose the model proposed in the paper captures the reality exactly, but one tries to understand the data from simulations with a "misspecified" model which is otherwise standard RBC model with exogenous permanent shocks.

$$Y_t = e^{z_t} K_t^\eta (\Gamma_t L_t)^{1-\eta}$$

The production function is assumed to be Cobb Douglas with two types of shocks.  $z_t$  is a standard  $AR(1)$  process.

$$z_t = \rho_z z_{t-1} + \varepsilon_z$$

$\Gamma_t$  is used to represents the cumulative product of permanent shock to the growth rate. Specifically,

$$\Gamma_t = e^{g_t} \Gamma_{t-1} = \prod_{s=0}^t e^{g_s}$$

$$g_t = (1 - \rho_g) \mu_g + \rho_g g_{t-1} + \varepsilon_g$$

where  $|\rho_g| < 1$  and  $\varepsilon_t^g$  represents a innovation from a normal distribution with zero mean and standard deviation  $\sigma_g$ . The parameter  $\mu_g$  is the productivity's long-run mean growth rate.

In order to directly estimate the underlying shock process, one wants to make use of information from Solow residuals. Simulation data provides a sequence of savings which are assumed to be translated into investment one for one in a close economy. Given the initial capital  $k_0$  from the simulation, a sequence of pseudo capital can be constructed. With information on output sequence, therefore, the sequence of pseudo Solow residual can be backed out from the production function.

To extract information on permanent shocks from the pseudo Solow residual, I make use of the standard method proposed by Beveridge and Nelson(1981), which implies that, log of Solow residual, if following and  $I(1)$  process, can be decomposed into random walk component and a transitory component.

$$sr_t = \tau_t + s_t$$

where  $sr_t$  is the log of Solow residual. And  $\tau_t$  represents random walk and follows the process below,

$$\tau_t = (1 - \eta) \mu_g + \tau_{t-1} + \left( \frac{1 - \eta}{1 - \rho_g} \right) \varepsilon_g$$

Following the measure advocated by Cochrane (1988), I estimate variance of the permanent component with the method below,

$$\lim_{K \rightarrow \infty} K^{-1} Var (sr_t - sr_{t-K}) = \sigma_{\Delta\tau}^2$$

where  $sr_{t-K}$  is the  $K$  period lag of Solow residual. The method is meaningful only conditional on  $K$  is sufficiently big so that the estimation is accurate enough. In practise, it is difficult to choose  $K$  to be big enough, since the limitation of the data length. However, that's not a problem for this exercise since I can simulate the model for long enough period of time, so that I can choose  $K$  to be unusually bigger than what would be chosen in practise. Therefore, I choose  $K$  to be 200, 500, 1000 and 2000, respectively, in different experiments. The critical point of this excursive is to find out the difference of permanent shocks between experiments of developing and developed economies. Towards

this end, I look at the ratio of standard deviation of permanent shocks in developing economy to that in developed economy. The essential finding is that the ratio is quite stable and roughly 2.

$$\frac{\sigma_{\Delta\tau}^{developing}}{\sigma_{\Delta\tau}^{developed}} \approx 2$$

The message is clear that the model proposed in this model without any permanent shocks could essentially replicate a model with exogenous permanent shocks. And it, therefore, delivers endogenously that or partially explain why developing countries experience stronger permanent shocks.

## 5 Conclusion

Discussion on the output growth volatility has been growing recently. In contrast, studies on consumption growth volatility are scarce. This paper aims to fill the gap and improve the understanding of consumption volatility in developing countries. Mounting evidence shows that consumption exhibits excess volatility in developing countries. This pattern has been documented by several recent research. Permanent income theory suggests that, in order to rationalize this pattern, developing countries should be subject to shocks with higher persistence and larger variance. Deeper thinking reveals that financial underdevelopment in developing countries could endogenously increase the persistence and variance of shocks, even though they receive exactly the same exogenous shocks as developed countries. This paper models this connection based on Acemoglu and Zilibotti's classic contribution (1997) in this field. Numerical results obtained from the model show that the model is well in line with the empirical regularity documented in different dataset, although volatility of consumption in both group is obviously lower in the model than in data.

A way to extend this model is to include one international sector into the model so that the model will be useful to address the relationship between openness and volatility. Both theoretical and empirical research conclude that openness have ambiguous effects on volatility. On one hand, openness provides another channel to smooth consumption through the risk sharing opportunities between countries and reduce the volume of volatility; on the other hand, the international sector exposes the economy to more external uncertainties. This current research has potential to provide a guideline for studying this ambiguous issue and determine which kind of openness helps increase or decrease consumption volatility. For example, suppose that openness can help diversify the risks in local financial market by inducing more capital inflow, which therefore brings down the variance of shocks from financial market, consumption smoothing motives could dominate uncertainty which comes along with openness. And openness helps reduce the consumption volatility in developing countries. On

the other hand, if international market provides agents with better diversification opportunity, the capital outflow will happen. Therefore, the risk sharing channel would turn out to be less relevant and consumption volatility increasing in openness will be observed.

## 6 Appendix

### 6.1 Solution

Inter-temporal Euler equations are derived by following the procedure below:

1. Rearrange the resource constraint and replace consumption in Bellman equation.
2. Rearrange the expected continuation value.  $V(K^{g(b)}, k^{g(b)}, A')$  is the continuation value if the good (bad) shock realizes and temporary shock turns out to be  $A'$ .  $[n \cdot V(K^g, k^g, A') + (1 - n)V(K^b, k^b, A')]$  is the expected continuation value, given  $A$ . Since  $A'$  is also stochastic, the expected continuation value, conditional only on  $A$ , is

$$E_A [n \cdot V(K^g, k^g, A') + (1 - n)V(K^b, k^b, A')]$$

Therefore, the Bellman equation becomes the following,

$$V(K, k, A) = \max_{s \geq 0, 1 \geq \alpha \geq 0} \left\{ \begin{array}{l} U(w + \varphi k - s) + \\ \beta \cdot E_A [n \cdot V(K^g, k^g, A') + (1 - n)V(K^b, k^b, A')] \end{array} \right\}$$

3. Assume interior solution (or  $n < 1$ ) and derive first order conditions (from now on,  $E_A$  is replaced by  $E$  for simplicity)

first order condition with respect to  $s$ :

$$U'(c) = \beta \cdot E \left[ \begin{array}{l} n \cdot \left( r \cdot \alpha + R \cdot \frac{(1-\alpha)}{n} \right) \cdot V_k(K^g, k^g, A') \\ + (1 - n) \cdot r \cdot \alpha \cdot V_k(K^b, k^b, A') \end{array} \right]$$

first order condition with respect to  $\alpha$ :

$$0 = E \left[ \begin{array}{l} n \cdot \left( r \cdot s - R \cdot \frac{1}{n} \cdot s \right) \cdot V_k(K^g, k^g, A') \\ + (1 - n) \cdot r \cdot s \cdot V_k(K^b, k^b, A') \end{array} \right]$$

4. Rearrange the two first order conditions, the system becomes

$$U'(c) = \beta \cdot R \cdot E[V_k(K^g, k^g, A')]$$

$$U'(c) \left( \frac{1}{r} - \frac{n}{R} \right) = \beta \cdot (1 - n) \cdot E[V_k(K^b, k^b, A')]$$

5. Derive the Envelope condition

$$V_k(K, k, A) = U'(c) \cdot \left( \eta \cdot A \cdot K^{\eta-1} + (1 - \delta) \cdot \frac{1}{r} \right)$$

6. Use the Envelope condition and move it one period forward. Replace  $E[V_k(K^g, k^g, A')]$  and  $E[V_k(K^b, k^b, A')]$  on the right hand side of the equation system. The equation system ends up with two inter-temporal equations,

$$U'(c) = \beta \cdot R \cdot E \left[ U'(c_g) \cdot \left( \eta \cdot A' \cdot K^{g(\eta-1)} + (1 - \delta) \cdot \frac{1}{r} \right) \right]$$

$$U'(c) = \beta \cdot \frac{(1 - n)}{\left( \frac{1}{r} - \frac{n}{R} \right)} \cdot E \left[ U'(c_b) \cdot \left( \eta \cdot A' \cdot K^{b(\eta-1)} + (1 - \delta) \cdot \frac{1}{r} \right) \right]$$

where

$$c_g = c(K^g, k^g, A') \text{ and } c_b = c(K^b, k^b, A')$$

Note that corner solution is obtained when  $n = 1$  and it corresponds to the case where the risks are fully diversified. In that case backward inequality holds in the second equation.

## 6.2 Special case

Analytical solution can be derived from a special case where  $\delta = 1$  and  $U(c) = \log(c)$ . Using the guess that consumption is a function of aggregate capital and not dependent on individual capital, more precisely, consumption is a constant fraction of the aggregate output. That is,  $c = t \cdot A \cdot K^\eta$ . Also the conjecture of law of motion of aggregate capital is the same as individual capital.

1. Replace consumption in the inter-temporal equations with this guess, I have

$$\frac{1}{c} = \beta \cdot R \cdot E \left[ \frac{1}{t \cdot A' \cdot K^{g\eta}} \cdot \left( \eta \cdot A' \cdot K^{g(\eta-1)} \right) \right]$$

$$\frac{1}{c} = \beta \cdot \frac{(1-n)}{\left(\frac{1}{r} - \frac{n}{R}\right)} \cdot E \left[ \frac{1}{t \cdot A' \cdot K^{b\eta}} \cdot \left( \eta \cdot A' \cdot K^{b(\eta-1)} \right) \right]$$

2. Rearrange both of the equations. Then,  $A'$  drops out, and so does expectation operator,

$$\frac{1}{c} = \beta \cdot R \cdot \frac{\eta}{t \cdot K^g}$$

$$\frac{1}{c} = \beta \cdot \frac{(1-n)}{\left(\frac{1}{r} - \frac{n}{R}\right)} \cdot \frac{\eta}{t \cdot K^b}$$

3. Replace  $K^g$  and  $K^b$  with their law of motion,

$$\frac{1}{c} = \beta \cdot R \cdot \frac{\eta}{t \cdot \left( r \cdot \alpha \cdot s + R \cdot \frac{(1-\alpha)}{n} \cdot s \right)}$$

$$\frac{1}{c} = \beta \cdot \frac{(1-n)}{\left(\frac{1}{r} - \frac{n}{R}\right)} \cdot \frac{\eta}{t \cdot r \cdot \alpha \cdot s}$$

4. Savings are replaced by  $(1-t) \cdot A \cdot K^\eta$ , since savings are also a constant fraction of output,

$$\frac{1-t}{t} = \beta \cdot R \cdot \frac{\eta}{t \cdot \left( r \cdot \alpha + R \cdot \frac{(1-\alpha)}{n} \right)}$$

$$\frac{1-t}{t} = \beta \cdot \frac{(1-n)}{\left(\frac{1}{r} - \frac{n}{R}\right)} \cdot \frac{\eta}{t \cdot r \cdot \alpha}$$

5. Solving for  $t$  and  $\alpha$  from this equation system, leads to the solution in the text. And it is obvious that  $t$  is indeed a constant. The implied demand for risky assets is

$$F^*(n) = \frac{R-r}{R-r \cdot n} s$$

6. Impose the equilibrium condition,  $F^*(n^*) = \frac{D}{1-\gamma} (n^* - \gamma)$ , or

$$\frac{R-r}{R-r \cdot n^*} s = \frac{D}{1-\gamma} (n^* - \gamma)$$

the equilibrium  $n^*(k)$  is obtained.

### 6.3 The model with deterministic trend

In this section, a similar model with deterministic trend is provided. One of the critical assumptions I make here is that the minimum cost parameter  $D$  is growing with the economy at the same growth rate. This assumption captures the idea that the incompleteness of the financial market can not be eliminated by growth alone. Examples are not rare that many countries enjoy high growth rate for a long time but are still plagued by less sound financial market in the mean while.

I assume the labor productivity grows constantly at the rate of  $g$ , that is  $\Gamma_t = \Gamma_{t-1}e^g$ , (where  $\Gamma_t$  is the labor productivity level in period  $t$ ) therefore the production function at period  $t$  is

$$Y_t = A_t K_t^\eta \cdot (\Gamma_{t-1} L_t)^{(1-\eta)}$$

The system can be normalized by the growth factor  $\Gamma_{t-1}$ . I use hat version to denote the normalized variables.

$$\hat{x}_t = \frac{x_t}{\Gamma_{t-1}}$$

I start with the definition of value function in sequential form

$$V_t = E_t \sum_{s=t+1}^{\infty} \beta^s u(c_s)$$

1. Rearrange it with hat variables, yielding

$$V_t = (\Gamma_{t-1})^{(1-\sigma)} \frac{(\hat{c}_t)^{1-\sigma}}{1-\sigma} + \beta \cdot \left( E_t \sum_{s=t+1}^{\infty} \beta^s (\Gamma_{s-1})^{(1-\sigma)} \frac{(\hat{c}_s)^{1-\sigma}}{1-\sigma} \right)$$

2. Divide both sides by  $(\Gamma_{t-1})^{(1-\sigma)}$

$$\frac{V_t}{(\Gamma_{t-1})^{(1-\sigma)}} = \frac{(\hat{c}_t)^{1-\sigma}}{1-\sigma} + \frac{1}{(\Gamma_{t-1})^{(1-\sigma)}} \beta \cdot \left( E_t \sum_{s=t+1}^{\infty} \beta^s (\Gamma_{s-1})^{(1-\sigma)} \frac{(\hat{c}_s)^{1-\sigma}}{1-\sigma} \right)$$

3. Denote  $\widehat{V}_t = \frac{V_t}{(\Gamma_{t-1})^{(1-\sigma)}}$ , so that

$$\widehat{V}_t = \frac{(\widehat{c}_t)^{1-\sigma}}{1-\sigma} + \frac{1}{(\Gamma_{t-1})^{(1-\sigma)}}\beta \cdot \left( E_t \sum_{s=t+1} \beta^s (\Gamma_{s-1})^{(1-\sigma)} \frac{(\widehat{c}_s)^{1-\sigma}}{1-\sigma} \right)$$

4. Use  $\widehat{V}_{t+1} = \frac{V_{t+1}}{(\Gamma_t)^{(1-\sigma)}}$

$$\widehat{V}_t = \frac{(\widehat{c}_t)^{1-\sigma}}{1-\sigma} + \frac{1}{(\Gamma_{t-1})^{(1-\sigma)}}\beta \cdot \left( (\Gamma_t)^{(1-\sigma)} \cdot E_t \widehat{V}_{t+1} \right)$$

5. Rewrite the resource constraint, capital accumulation and portfolio definition with hat version

$$\widehat{V} = \frac{(\widehat{c})^{1-\sigma}}{1-\sigma} + \widehat{\beta} \cdot E \left( \widehat{V}' \right)$$

$$\widehat{c} + \widehat{s} = w(\widehat{K}, A) + \varphi \left( \widehat{K}, A \right) \cdot \widehat{k}$$

$$\widehat{\phi} = \alpha \cdot \widehat{s} \text{ and } n \cdot \widehat{F} = (1 - \alpha) \cdot \widehat{s}$$

$$\widehat{k}^j = \begin{cases} r \cdot \alpha \cdot \widehat{s} + (1 - \delta) \widehat{k} & \text{if } j > n \text{ with } 1 - n \\ r \cdot \alpha \cdot \widehat{s} + R \cdot \frac{(1-\alpha)}{n} \cdot \widehat{s} + (1 - \delta) \widehat{k} & \text{if } j \leq n \text{ with } n \end{cases}$$

where  $\widehat{\beta} = e^{g(1-\sigma)}\beta$  and it is easy to show that  $w(\widehat{K}, A) = w(K, A)$  and  $\varphi(\widehat{K}, A) = \varphi(K, A)$ , respectively. The solution to the model will be same as the one I solved for in the text.

## 6.4 Algorithm and Simulation

In this section, I will outline the solution algorithm and the simulation procedure. Since I assume the economy always stays on the equilibrium path, I need to solve for  $n(k, A)$ ,  $s(k, A)$  and  $\alpha(k, A)$ , which make the two inter-temporal equations and the equilibrium condition hold. This numerical exercise poses interesting challenges. Different from the standard stochastic growth models, which focus on a support around steady state, the endogenous uncertainty in this model can shift capital in the economy within a broad ergodic set, namely

between "bad" and "good" quasi steady states<sup>9</sup>. I have to provide decision rules over a large range, in which the curvature of the decision rules I want to solve for change dramatically. Another difficulty comes from the kinky shape of the portfolio decision, therefore I have to choose the approximation method wisely to provide a good approximation to the true policy functions. Since it is a general equilibrium model, I need to solve for  $n(k, A)$  or in other words, update the parameter matrix in the outer loop. Unfortunately, no existing algorithm guarantees that it will converge. I design a generalized version of the bisection method, which helps it to converge successfully.

I take two steps to solve the general equilibrium problem. Firstly, I take  $n(k, A)$  as given and solve the two equations with a root-finding method. I discretize  $A$  and  $k$  in a two dimensional space with Chebyshev nodes and then interpolate the consumption rule  $c(k, A)$  with two dimensional *Chebyshev approximation* (with  $nk \times nA$  coefficients, where  $nk$  and  $nA$  are the numbers of collocation points of capital and productivity level, respectively). Chebyshev approximation is used to take advantage of the high accuracy of the solution, when using a large number of basis functions. However, if the policy function displays kinks, the scheme may deliver a poor approximation. To avoid it, I have to interpolate portfolio rule  $\alpha(k, A)$  with *shape-preserving piecewise cubic approximation* (with  $nk \times nA$  coefficients). The following steps are taken sequentially.

Step 1: Compute the left hand side value at each collocation points of the  $k - A$  mesh;

Step 2: Compute  $k^b$  and  $k^g$ , according to the law of motion of capital;

Step 3: Compute the right hand side value at each collocation point of the  $k - A$  mesh, which requires computation of the expectation;

Step 4: Compute expectation, by using numerical integration with a *modified Tauchen algorithm*;

Step 5: Solve the  $2 \times nk \times nA$  nonlinear equation system with Broyden's method and obtain the two policy functions.

Secondly, I set up an outer loop to solve for  $n(k, A)$

Step 1: Guess a pseudo function for  $n(k, A)$ ;

Step 2: Take the function as given and solve for  $s(k, A)$  and  $\alpha(k, A)$  (see above);

Step 3: Compute  $F(k, A)$  and  $\frac{D}{1-\gamma}(n(k, A) - \gamma)$ ;

Step 4: If the difference is under the tolerance level,  $n^*(k, A)$  is found;

Step 5: If not, update  $n(k, A)$  with a *generalized bisection method*;

Step 6: Go back to Step 2 until the equilibrium condition is satisfied in Step 4.

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<sup>9</sup>The good and bad quasi steady states are the intersections between  $k^g$ ,  $k^b$  and 45 degree line, respectively.

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