

# The determinants of euro area business investment over the cycle\*

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## Abstract

To analyse the contribution of financial factors to recent developments in euro area business investment, a small scale DSGE model of a closed economy with financial frictions and variable capacity utilisation rate is developed, mainly based on Smets and Wouter (2003), Christiano, Eichenbaum and Evans (2005), and Christiano Motto and Rostagno (2004).

The model is then brought to euro area data. Contrastingly with the literature, the observables refer mainly to the variables related more closely to the investment block: utilisation rate, external finance premium, price of capital, loans to non-financial corporations. Also, the stochastic parameters as well as some structural parameters, among them those related to the financial accelerator are then estimated on euro area data, while other parameters are calibrated with the estimates reported in Christoffel et al. (2008) or Christiano Motto and Rostagno (2004).

We found that shocks affecting the external finance premium can be estimated and their contribution to the dynamics of investments, loans and asset prices is not negligible.

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\*The views expressed in this paper are those of the authors and not necessarily those of the European Central Bank.

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# 1 Introduction

We develop and estimate a medium scale model of a closed economy to analyse recent developments in euro area non residential investment. Indeed, while over the most recent period, there is evidence (figures (1) and (2)) of a positive relation between profits and investment, a structural analysis is warranted to account for this relation. This motivates the set-up and the estimation of a medium scale DSGE model with both nominal and real rigidities, but also demand and productivity shocks and financial frictions. Introducing financial market frictions in the model and focussing on the investment block enable us to analyse both the effects on financial shocks on real variables but also the effects of non-financial shocks on financial variables.

The real rigidities present in the model are external habit formation in consumption decisions, adjustment cost in the production of capital and variable capital utilisation where on the nominal side we consider sticky prices and wages. Overall, 9 shocks affect the model dynamic. We consider preference and labour supply shocks in the household problem, a shock to the investment adjustment cost function, the usual government spending, TFP and monetary shocks plus wage and price markup shocks, thereby sharing most of the features of Smets and Wouters (2003).

The model incorporates financial frictions featured by a financial accelerator, a mechanism which has been extensively used in both theoretical and empirical macroeconomic literature since the seminal work of Bernanke and Gertler (1989). This represents a channel through which asymmetric information in credit markets makes the financing cost of an investment project dependent on the leverage ratio of the borrower. *Ceteris paribus*, an increase in investment pushes up the external finance premium requested by lender to finance the project. Since the financing cost become more procyclical, the mechanism implies an accelerating effect of financing conditions on the cycle. Our model differs from the benchmark Bernanke et al. (1999) DSGE model with financial frictions in two dimensions. First, we want to incorporate the extension brought to the literature by Christiano et al (2005) and Christiansen and Dib (2008). We assume that borrowers sign a debt contract that specifies a nominal interest rate, thereby allowing for debt deflation effect (Fisher effect). Second, we also introduce an exogenous shock to the external

finance premium in order to represent possible shifts in the market assessment of risk.

The model is then brought to euro area data using over the period 1985-2008. Contrastingly with most of the literature, we focus on the investment block and take as observables the variables related more closely to it: capacity utilisation rate, external finance premium, price of capital, loans to non-financial corporations and earnings are considered in addition to real GDP, GDP inflation, government consumption and the three months nominal interest rate. Differently from most of the literature where they are calibrated, the elasticity of the risk premium to the leverage ratio as well as the standard deviation of the shocks to it are estimated by Bayesian methods. Computing the shock contributions to the variance and to the growth rate of the observables, we show that shock to the risk premium account for a substantial part of the movements in non-residential investment, loans to non-financial corporation and stock prices.

The paper consists of four Sections and two appendix. The model is presented in Section 2 while the estimation and the results are discussed in Section 3. The steady state of the model is indicated in Appendix A, the linearised equations are listed in Appendix B and all figures and tables are presented in Appendix C.

## 2 The Model

Our baseline model is a closed economy DSGE model with most of the features of Smets and Wouters (2003) augmented with a financial accelerator in the spirit of Bernanke et al. (1999). As in Christiansen and Dib (2008), Quejo von Heideken (2008), or Christiano, Motto and Rostagno (2007), the financial accelerator is in nominal terms. To the endogenous dynamic of the risk premium modelled by those authors, we add a stochastic autoregressive exogenous shock. This accounts for the observation that the assessment of risks and therefore the premium requested by financial intermediaries to support it, sometimes shifts independently of the business cycle conditions, thereby becoming a driver to the business cycle.

The economy consists of 7 types of agents: Households, Retailers, Intermediate producers, Capital producers, Entrepreneurs, Financial Intermediaries and policy authorities (government and central bank). The model features nominal stickiness a la Calvo in wages and prices, with partial indexation on past inflation, and real rigidities, consumption habits and adjustment costs to capital. Moreover, we allow for variable capacity utilisation rate. Nine structural shocks are considered: consumption preference, labour supply, productivity, investment specific, risk premium, public consumption, wage and price markups and monetary policy. Except the three latter, all the shocks are persistent.

### 2.1 Households

While sharing the same utility, households provide differentiated labour services to the intermediate good sector. They consume the final good and save in the form of deposits. Each household  $j$  maximizes its expected lifetime utility discounted by  $\beta$ , the subjective rate of preference for the present, by choosing the consumption of final good  $c_t^j$ , the amount of the nominal deposits held at financial intermediaries,  $d_{t+1}^j$ , which pay a nominal gross free risk rate  $R_t$ , and supplying labor services to the intermediate sector,  $h_t^j$ . It is assumed that each household buy securities with payoffs contingent on the wage received so that, although households differ in their work intensity and wage received, they all consume at the same rate. The utility function, which allows for external consumption habits ( $\kappa$  being the habit parameter), and the budget constraints are the following:

$$U(c_t, h_t) = e_t \left[ \frac{1}{1-\sigma} (c_t^j - \kappa c_{t-1})^{1-\sigma} - \mu_t \frac{1}{1+\zeta} (h_t^j)^{1+\zeta} \right] \quad (1)$$

$$d_{t+1}^j + c_t^j p_t = w_t^j h_t^j + R_{t-1} d_t^j - t_t + D_t + X_t^j \quad (2)$$

Where  $t$  are lump sum taxes,  $w^j$  is the nominal wage of household  $j$ ,  $D$  are dividends received from ownership of firms and  $X^j$  are net cash flow from participating in state-contingent security markets.  $e_t$  and  $\mu_t$  are a stationary preference shocks to respectively intertemporal consumption and leisure. Standard maximization problem yields the following first order conditions:

$$E_t \left( \beta \frac{R_t \Gamma_{t+1}^c}{\Gamma_t^c \pi_{t+1}} \right) = 1 \quad (3)$$

$$\frac{\partial U}{\partial c_t^j} = 0 \iff e_t (c_t^j - \kappa c_{t-1})^{-\sigma} = \Gamma_t^c \quad (4)$$

$$\frac{\partial U}{\partial h_t^j} = 0 \iff -\Gamma_t^c w_t^j = e_t \mu_t (h_t^j)^\zeta \quad (5)$$

Where  $\Gamma_t^c$  is the lagrangian multiplier associated with the budget constraint and  $\pi_t$  is the gross inflation rate.

Households act as monopolistic suppliers of differentiated labor services to wholesale sector. Labor aggregator has the usual Dixit-Stiglitz form where the elasticity of substitution is affected by wage markup shocks ( $\tau w_t$  is the net wage markup shock). Total demand for household's labor by the intermediate sector is given by:

$$H_t = \left[ \int_0^1 (h_t^j)^{\frac{1}{1+\tau w_t}} dj \right]^{1+\tau w_t} \quad \text{and} \quad \frac{h_t^j}{H_t} = \left( \frac{W_t}{w_t^j} \right)^{\frac{\tau w_t + 1}{\tau w_t}} \quad (6)$$

So that,  $W_t$ , the aggregate wage, is given by:

$$W_t = \left[ \int_0^1 (w_t^j)^{-\frac{1}{\tau w_t}} dj \right]^{-\tau w_t} \quad (7)$$

Following Calvo (1983), we assume that, each period, households have a probability  $(1 - \theta_w)$  to reset their wage. In the case where they cannot, we assume that the

wage increase is partially indexed on gross past inflation. Formally, the wage of households that cannot re-optimize adjust according to:

$$w_t^j = \pi_{t-1}^{\iota_w} w_{t-1}^j \quad (8)$$

Where  $\iota_w < 1$  represents the degree of wage indexation.

Let  $\tilde{w}_t$  wage set by the households that can re-optimize.<sup>1</sup> If wage was perfectly flexible,  $\theta_w = 0$ , its optimum value would be obtained at each period by equating the marginal product of working with the opportunity cost of working in consumption terms ( $U'_L/U'_C = w/p$ ). The real wage would then be equal to a markup  $(1 + \tau w_t)$  over the current ratio of the marginal disutility of labour and the marginal utility of an additional unit of consumption. However, taking into account the probability that households will not be re-optimized in the near future ( $\theta_w > 0$ ), households set the wage so that the present value of the marginal return to working is a mark-up over the present value of marginal cost. This equality becomes:

$$\frac{\tilde{w}_t}{p_t} E_{t-1} \sum_{k=0}^{\infty} \left[ (\beta \theta_w)^k m_{t+k} e_{t+k} \left( (1 + \tau w_{t+k}) \Pi_{t+k} \tilde{w}_t U'_{t+k}{}^c + \Pi_{t+k} p_{t+k} \mu_{t+k} U'_{t+k}{}^h \right) \right] h_{t+k}^j = 0 \quad (9)$$

where  $\beta^s m_{t+j} = \beta^s \frac{u_c(t+j)}{u_c(t)}$  is the stochastic discount factor between periods  $t$  and  $t + 1$ .

From equation (7), we can obtain the law of motion of the aggregate wage index:

$$(W_t)^{-\frac{1}{\tau w_t}} = \theta_w ((\pi_{t-1})^{\iota_w} W_{t-1})^{-\frac{1}{\tau w_t}} + (1 - \theta_w) (\tilde{w}_t)^{-\frac{1}{\tau w_t}} \quad (10)$$

## 2.2 Capital Producers

Capital stock is produced by a continuum of identical competitive capital producers who combine final goods  $I_t$  purchased from retailers with installed capital  $K_t^i$  in order to produce new capital  $K_{t+1}^i$ . Capital producers use a linear technology,

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<sup>1</sup>Following Erceg, Henderson and Levin (2000) and Christiano et al.(2005) among others we have that all households that are able to reset their wages choose the same one. See Appendix in Christiano et al. (2005) for a proof.

and, we assume it takes one period for investment to be installed. Moreover, the capital producer faces a quadratic adjustment cost to capital,  $\Phi$ .<sup>2</sup> Noting  $\delta$  the depreciation rate, the law of motion for capital is given by:

$$K_{t+1}^i = (1 - \delta) K_t^i + (1 - \Phi(x_t I_t, I_{t-1})) I_t \quad \text{where} \quad (11)$$

$$\Phi(1) = \Phi'(1) = 0 \text{ and } \Phi''(1) > 0 \text{ for example, } \Phi = \frac{\chi}{2} \left( \frac{x_t I_t}{I_{t-1}} - \delta \right)^2 \quad (12)$$

The presence of lagged investment in the adjustment costs function means that there are costs to changing the flow of investment. We also introduce  $x_t$ , a shock to the investment cost function. Because of the adjustment cost, the capital producer's problem is dynamic. Calling  $q_t$  the real price of capital, the first order optimality condition for investment equates the marginal utility of consumption with the expected discounted shadow value of capital:<sup>3</sup>

$$q_t \Phi' \left( \frac{x_t i_t}{i_{t-1}} \right) \frac{x_t i_t}{i_{t-1}} - \beta E_t q_{t+1} \frac{\Gamma_{t+1}^c}{\Gamma_t^c} \Phi' \left( \frac{x_{t+1} i_{t+1}}{i_t} \right) \frac{x_{t+1} i_{t+1}}{i_t} \frac{i_{t+1}}{i_t} + 1 = q_t \left( 1 - \Phi \left( \frac{x_t i_t}{i_{t-1}} \right) \right) \quad (13)$$

## 2.3 Entrepreneurs

The entrepreneurs proceed in two steps. In the first step, they purchase the capital good on the basis of the expected price of capital, expected cost of financing and expected marginal productivity value. The production function is a standard constant return to scale Cobb-Douglas function:

$$Y_t = a_t K_t^\alpha L_t^{1-\alpha} \quad (14)$$

with  $a_t$  being the TFP term following a stationary process. In the first step, when deciding on the purchase of capital, the entrepreneurs also decide on the amount of money borrowed from the financial intermediaries. In the second step, after observing the productivity shock, they decide on the capacity utilisation rate, for

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<sup>2</sup>Adjustment cost to capital smooth the response of investment to shocks and affects directly the cost of capital, which would remain equal to 1 otherwise.

<sup>3</sup>for a detailed explanation of the investment problem see Smets and Wouters (2003).

which changed occur at a cost. In the third step, entrepreneurs will then sell output on a perfectly competitive market for a price that equals its nominal marginal cost. They do so by maximizing (14) which gives the following first order conditions:

$$Z_t = \alpha \Gamma_t^k \frac{Y_t}{K_t} \quad (15)$$

$$W_t = (1 - \alpha) \Gamma_t^k \frac{Y_t}{L_t} \quad (16)$$

where  $\Gamma_t^k$  is the lagrangian multiplier associated with the production function and denotes the real marginal cost (ie.  $\Gamma_t^k = MC_t$ ).

### 2.3.1 Borrowing and investing

The production process of intermediate goods combines the capital constructed by capital producers and labor supplied by both households and entrepreneurs, who are assumed to be risk neutral. Purchase of capital from capital producers is financed with internal finance, entrepreneurial net worth, and external finance, banking debt. Since the seminal work of Bernanke and Gertler (1989), it is known that financial frictions in the form of informational asymmetry between borrowers and lenders can make the external cost of finance depends on borrowers balance sheet conditions. We follow the same assumptions than Bernanke et al. (1999) or Gilchrist and Saito (2006) in order to introduce financial frictions, but add a stochastic component to the credit spread.<sup>4</sup>

At the end of period  $t$ , after the production of intermediate goods, the entrepreneur purchases  $K_{t+1}^i$ <sup>5</sup> from capital producers at the relative price  $q_t$ , and

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<sup>4</sup>To rule out self financing of capital needs, we assume finite expected lifetime horizon, the survival probability being  $\nu$ . New entrepreneurs enter to replace those who exit and in order to ensure that new entrepreneurs have some funds when starting, we make the assumption that each of them is endowed with  $h_t^e$  units of labor supplied inelastically to the wholesale production sector, and can be thought as managerial input, at nominal wage  $w_t^e$ .

In any case we will make calibration assumption that will make those quantities really small, as a usual feature in the literature, and not relevant for the calibrated and estimated version of the model. See BBG (1999) and Gilchrist and Saito (2006) among others for a more deep discussion of this point.

<sup>5</sup>Here the superscript  $i$  means capital installed which differs from the capital used,  $K^u$ . This distinction will became clear in the next section when we present the cpaital utilization decision of entrepreneurs.

this capital is used as an input in the production in period  $t + 1$ . The purchase is financed by net worth,  $N_{t+1}$ , and external borrowing of a nominal debt from a financial intermediary,  $B_{t+1}$ , so that  $q_t K_{t+1}^i = N_{t+1} + B_{t+1}/P_t$ . The financial intermediary raises funds from households deposits and faces an opportunity cost of funds equal to the economy's risk free nominal interest rate,  $r_t$ . The capital purchase decision depends on the expected gross real rate of return on capital, and the expected gross marginal cost of finance,  $F_{t+1}$ :

$$E_t F_{t+1} = E_t \left[ \frac{\bar{Z}_{t+1} + (1 - \delta)q_{t+1}}{q_t} \right], \quad \bar{Z}_{t+1} = \alpha \Gamma_t^k \frac{\bar{Y}_{t+1}}{K_{t+1}^i} \quad Y_{t+1} = \omega_{t+1} \bar{Y}_{t+1} \quad (17)$$

Where  $\omega_{t+1}$  is the idiosyncratic shock that entrepreneurs face and we have used the fact that  $E_t \omega_{t+1} = 1$ .  $\bar{Z}_{t+1}$  is the real marginal productivity of capital and  $(1 - \delta)q_{t+1}$  is the value of one unit of capital used in  $t + 1$ . The optimal contract between the borrower and the lender implies an external finance premium,  $s_t$ , that increases with the entrepreneur's leverage ratio. This premium can be defined as the ratio of borrower's cost of external funds to the cost of internal ones, where the latter is equal to the cost of funds in absence of financial market imperfections:

$$s_t = \frac{E_t (F_{t+1} \pi_{t+1})}{E_t (r_{t+1})} \quad (18)$$

Assume that the premium follows this form

$$s_t = \left( \frac{q_t K_{t+1}^i}{N_{t+1}} \right)^{\psi_t}, \quad (19)$$

Where  $\psi_t$ , the elasticity of the external finance premium with respect to the firm leverage depends on the parameters of the CSV problem. We introduce an exogenous shock to the elasticity of the external finance premium with respect to entrepreneurs leverage position, and assume that the shock has a persistent effect. This accounts for the varying appreciation of risk in the economy. From time to time, at a relatively low frequency, the risk assessment is allowed to move independently of the cyclical conditions. Aggregate entrepreneurial net worth at the end of period  $t$  is given by:<sup>6</sup>

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<sup>6</sup>Entrepreneurs going out of business in  $t$  consume the residual equity,  $c_t^e$ :

$$N_{t+1} = F_t q_t K_{t+1}^i - E_t F_t (q_{t-1} K_t^i - N_t) \quad (22)$$

From equation 17, we can see that expected changes in productivity (i.e.  $a_t$ ) and in asset prices (i.e.  $q_t$ ) determine the expected real return on capital, while equation 22 suggests that the unexpected movements in the real return on capital are the main force driving changes in the entrepreneurial net worth. Finally, equation (19) implies that changes in  $N_t$  together with the exogenous shock we introduce are the main source of movements in the external finance premium, confirming that movements in asset prices play a key role in the financial accelerator mechanism.

### 2.3.2 Deciding on the capacity utilisation rate

Entrepreneurs decide how intensively to use their capital stock after observing the shocks having occurred in  $t$ . Entrepreneurs own the physical stock of capital installed,  $K_t^i$ , and provide capital services,  $K_t^u$ , using capital at the rate  $u_t$  ( $K_t^i = u_t K_t^u$ ).

In any period they aim to maximize their profits, given by the production revenues minus the value of depreciated capital, by choosing  $u$ , and choosing households and entrepreneurial labor. Capital utilization is subject to a convex cost function in terms of the final good:

$$\max_{u_{t+1}} E_{t+1} [P_{t+1} \Gamma_{t+1}^c (u_{t+1} Z_{t+1} - \Upsilon(u_{t+1}))] \quad \text{with } \Upsilon, \Upsilon', \Upsilon'' > 0 \quad (23)$$

The optimal capacity utilisation equates the expected marginal benefits, the

$$c_t^e = (1 - \nu) \left( f_t q_{t-1} \tilde{k}_t - E_{t-1} f_t (q_{t-1} \tilde{k}_t - n_t) \right), \quad (20)$$

We follow Gilchrist and Saito (2006) by setting  $\Omega = 0$ , which make the effects of changes in the entrepreneurial net worth negligible.

Where the term in brackets represents the equity held by entrepreneurs that survive from  $t-1$  and  $w_t^e$  is the aggregate entrepreneurial wage which is given by the wage earned by the ones who survive in  $t-1$  and the wage of the new comers in  $t$ .

$$n_{t+1} = \nu [F_t q_{t-1} K_t^i - E_{t-1} F_t (q_{t-1} K_t^i - N_t)] + w_t^e \quad (21)$$

entrepreneurial wage is small.

marginal return to capital,  $F_{t+1}$  to the marginal expected costs,  $\Upsilon'$  :

$$u_{t+1}/E_t \left[ z_{t+1} - \Upsilon'(u_{t+1}) \right] = 0.$$

In the steady state,  $F = \Upsilon'$ .

$$E_t \left[ \frac{1}{\Upsilon''} \widehat{z}_t - \widehat{u}_t \right] = 0 \quad (24)$$

Where  $\Upsilon'' = \sigma_\Upsilon$  is the curvature of the capacity utilisation cost function around the steady state (since  $\Upsilon'(\bar{u}) = \bar{z}$ ) and  $1/\Upsilon''$  is the elasticity of the marginal capacity utilisation with respect to the rental rate of capital.

## 2.4 Retailers, final good producers

A continuum of monopolistically competitive retailers of unit measure buy wholesale goods from entrepreneurs and then differentiate the product slightly at zero cost.  $y_t^s$  is the retail goods sold by retail  $s$ ,  $s \in (0, 1)$ . Using the standard Dixit-Stiglitz aggregator where  $\tau p_t \geq 0$  is a (net) mark up *iid* shock with mean  $\lambda$  the first order conditions of this problem imply<sup>7</sup>:

$$\frac{y_t^s}{y_t} = \left( \frac{p_t}{p_t^s} \right)^{\frac{\tau p_t + 1}{\tau p_t}} \quad (25)$$

where  $p_t^s$  is the retail nominal price and the price index is given by:

$$p_t = \left[ \int_0^1 (p_t^s)^{-\frac{1}{\tau p_t}} ds \right]^{-\tau p_t} \quad (26)$$

As in Calvo (1983), we assume that retailers have market power and with probability  $(1 - \theta_p)$  they can choose prices in order to maximize expected profits. For the firms who cannot reset their prices, the price is indexed to the past inflation. Contrastingly with CEE (2001), we allow for partial price indexation,  $\iota_p < 1$ , to past inflation. Formally prices of firms who cannot reset them adjust according to:

$$p_t^s = (1 + \pi_{t-1})^{\iota_p} p_{t-1}^s, \quad (27)$$

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<sup>7</sup>Again see Smets and Wouters (2003) for a more detailed specification of the final good production problem.

where  $\iota_p$  is the degree of indexation to past inflation. Profit maximization by firms that receive the signal to rest their prices leads to the following problem under the constraint of equation 25.

$$Max_{\tilde{p}_t^s} E_{t-1} \sum_{k=0}^{\infty} (\beta\theta_p)^k m_{t+k} [\tilde{p}_t^s \Pi_{t+k} - mc_{t+k} p_{t+k}] y_{t+k}^s \quad (28)$$

$$\Pi_{t+k} = (\pi_t * \pi_{t+1} * \dots * \pi_{t+k-1})^{\iota_p} \quad (29)$$

$$mc_t = A(\alpha) \frac{(1+r_t)W_t^{1-\alpha}}{P_t} \quad \text{with} \quad A(\alpha) = \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^\alpha \quad (30)$$

results in the following first order condition:

$$0 = E_{t-1} \sum_{k=0}^{\infty} (\beta\theta_p)^k m_{t+k} y_{t+k}^s \left[ \tilde{P}_t \Pi_{t+k} - (1 + \tau p_{t+k}) P_{t+k} mc_{t+k} \right] = 0, \quad (31)$$

$$\frac{y_t^s}{y_t} = \left( \frac{p_t}{p_t^s} \right)^{\frac{\tau p_t + 1}{\tau p_t}} \quad (32)$$

where  $\beta^s m_{t+j} = \beta^s \frac{u_c(t+j)}{u_c(t)}$  is the stochastic discount factor between periods  $t$  and  $t+1$ . The price set by firm  $s$ , at time  $j$ , is a function of expected marginal costs. If prices are perfectly flexible ( $\theta_p = 0$ ), the prices are set as a mark-up  $(1 + \tau p_{t+k})$  over marginal costs. If not, prices are set as a mark-up over weighted expected marginal cost. Linearizing the equation

$$\tilde{p}_t = \frac{\tilde{P}_t}{P_t} \quad \text{and} \quad \hat{\tilde{p}}_t = \frac{\tilde{p}_t - \bar{p}}{\bar{p}} \quad (33)$$

$$\hat{\tilde{p}}_t = E_{t-1} \left[ \hat{m}c_t + \sum_{k=1}^{\infty} (\beta\theta_p)^k (\Delta \hat{m}c_{t+k} + \Delta \hat{\pi}_{t+k}) \right], \quad (34)$$

Equation (26) implies the following law of motion of the price index:

$$P_t^{-\frac{1}{\tau p_t}} = (1 - \theta_p) \tilde{P}_t^{-\frac{1}{\tau p_t}} + \theta_p (\pi_{t-1}^{\iota_p} P_{t-1})^{-\frac{1}{\tau p_t}} \quad (35)$$

Using (34) and (35) we obtain an expression that relates the current inflation to past and expected inflation, to the current real marginal cost  $\xi_t$  and to the price

markup shock  $\lambda_t$ .<sup>8</sup>

## 2.5 Competitive Equilibrium and policy rule

In a competitive equilibrium, all the former presented optimality conditions are satisfied and all markets clears. The aggregate resource constraint is:

$$Y_t = C_t + I_t + G_t + \Upsilon(u_t) + csv \quad (36)$$

Final goods are allocated to consumption, investment, government expenditure, costs to change capital utilization and monitoring costs associated with defaulting entrepreneurs. Where  $g_t$  is the government expenditures that is taken as exogenous and we consider negligible in calibration the actual resource costs to bankruptcy. The monetary policy authority follows a standard (log-linear) taylor rule:

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1 - \rho_r)[\gamma^\pi \hat{p}_t + \gamma^y \hat{y}_t] + \epsilon_{rt}. \quad (37)$$

## 3 Simulations

In order to show the theoretical behaviour of the model we present in Section (8) the impulse responses functions of some of the variables to a positive 1% shock to the nominal interest rate, TFP, preference shock and to the external finance premium. The calibration used in this exercises follows standard numbers as in Christiano, Motto and Rostagno (2007) and/or Smets and Wouters (2007) and are presented in table (1).

Starting with a shock to the external risk premium (figure (3)), we see that, given that the capital to net worth ratio is above one in the steady state, an increase in the elasticity of risk premia to the leverage ratio pushes up the credit risk premia on shock. By pushing up the rental price of capital, this reduces investment. Output and inflation are reduced so that monetary policy rate declines thereby providing support for the economy to come back to the steady state. However, the decline in output having a large negative impact on the return to capital, has a negative impact on the price of capital which supplements the impact of higher risk premia

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<sup>8</sup>See appendix of Smets and Wouters (2003) for derivation.

and is not compensated by the lower monetary policy rate.

Starting with the monetary policy shock (figure (4)), an unexpected tightening results in a decline in the price of capital, partly through the lower return to capital and the higher real interest rate which increases financing costs. Overall, the net worth of entrepreneurs is reduced and therefore the external risk premium increased, adding to the increase in the real interest rates. The resulting increase in the rental price of capital is higher than in the case of no financial accelerator, thereby partly explaining why the decline in investment is stronger.

In Figure (5) responses to a positive TFP shock are presented. As usual a positive technological shock has a positive impact on output, consumption and investment. Although it has a relatively small impact on the latter. Nominal interest rate and inflation decrease, this increases the real cost of repaying existing debt creating a debt-deflation effect and pushing net worth down. This in turn causes an increase in the external finance premium which reduces the rise in the demand for capital and is the reason for the less pronounced impact of TFP shock on investment when the financial accelerator is present. Hours follows the usual behavior of sticky price models. Figure (6) regards the response to a positive preference shock, which raises the marginal utility of consumption and therefore the opportunity cost of holding deposits (savings). As households divert deposits towards consumption the real interest rate rises. The financial accelerator reduces the impact on investments while responses of output and consumption are quite similar under both scenarios.

These examples show that conceptually, in many cases, the financial accelerator can be an important amplifier of the shocks on the real economy. The estimation enables us to quantify these impacts.

## 4 Estimation

[WORK IN PROGRESS]

In contrast with most of the literature, we focus on the investment block and select as observables the variables related more to it. Non-residential investment is used as a proxy for business investment and the model is augmented by loans to non-financial corporations and profits in order to use these variables as observables in the estimation:

$$loan_{t+1}/P_t = q_t K_{t+1}^i - N_{t+1} \quad (38)$$

$$profit_t = P_t(Y_t - MC_t - F_t K_t^u - W_t H_t); \quad (39)$$

Overall, the model then includes 27 variables ( $y, c, i, h, \pi, w, a, r, f, s, mc, z, q, n, k^u, k^a, u, \Gamma^c, e, x, \mu, g, \tau p, \psi, loan, profit, \tau w$ ); as well as 9 shocks ( $\epsilon_e, \epsilon_\mu, \epsilon_x, \epsilon_g, \epsilon_a, \epsilon_{\tau w}, \epsilon_{\tau p}, \epsilon_\psi, \epsilon_r$ ).

## 4.1 Observables, data and Bayesian Procedures

On top of the three variables mentioned, we use series on capacity utilization rate, stock prices, real GDP, real government consumption, GDP inflation, the three months nominal interest rate, the spread between corporate bonds and 5 year maturity treasury bill, private consumption, and employment in heads <sup>9</sup>.

We mainly focus on the stochastic parameters and the financial accelerators ones. The parameters we estimate in our preliminary estimation (see Table 2) are all the standard deviations of the shocks, the autoregressive parameter of those shocks following an AR(1) process, the monetary policy rule weights, the financial accelerator elasticity, the curvature of the capacity utilization cost function, the parameter of the adjustment cost function, utility function parameters and the parameters of the calvo prices and wages. All the other parameters of the model are calibrated using exactly the same values as in the simulation exercise previously showed. We add measurement errors in the equation for the government consumption and capacity utilization rate observable equations.

The methodology we use is the standard one. First we log-linearize the equilibrium conditions around the non-stochastic steady state (see Appendix A and B). We assume a linear deterministic trend and detrend linearly the data, using a 2% yearly trend for real variables, inflation and interest rate and a 4% yearly trend for loans and earnings. Then, we calibrate some of the structural parameters

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<sup>9</sup>The series for stock prices and profits relate to euro area non-financial corporations, reported in DATASTREAM. When the data are not available over a long period for the euro area, they are backcast with data referring to Germany (in the case of spreads). National accounts are used for GDP components and employment and the deflators, backcast with the AWM database. Capacity utilisation is from the EC and loans non-financial corporations are from ECB.

that cannot be pinned down in the estimations and then we apply bayesian procedures to the rest of the parameters of the model. The sample covers the period 1980Q1-2008Q2, and we use the first five years of data to initialize the estimation. We apply bayesian procedures to estimate some parameters of the log-linearized version of the equilibrium conditions. We estimate the model by minimizing the posterior distribution of the model parameters based on the linearized state-space representation of the DSGE model.<sup>10</sup>

## 4.2 Estimation results

[ESTIMATION IS STILL WORK IN PROGRESS AND RESULTS ARE STILL PRELIMINARY]. In order to give an example of estimation results we present in Appendix C priors used and posteriors results from a preliminary estimation exercise. Although we do not comment them given the preliminary nature of the estimation and the lack of sensitivity analysis on priors used and number of replication for the Metropolis-Hastings algorithm.

## 4.3 Shocks contribution

After the estimation of the model, we compute Bayesian impulse responses functions for each shock in the model and we use them to compute the shocks contribution to the dynamics of the observable variables using the following methodology.

Using the state space of the solved model:

$$Z_t = A(\theta)Z_{t-1} + B(\theta)s_t \quad (40)$$

$$s_t = P(\theta)s_{t-1} + \epsilon_t \quad (41)$$

Where  $Z$  is the vector of observed variables,  $\theta$  is the vector of unknown structural and stochastic parameters,  $s$  is the vector of state variables and  $e$  the vector of  $p$  shocks. After some straightforward algebra :

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<sup>10</sup>This procedure imply the evaluation of the likelihood function for arbitrary parameter values using the Kalman filter (Sargent 1989) following by the computation of the marginal likelihood using Monte-Carlo Markov-Chain algorithm (as the Metropolis-Hastings). Fernandez-Villaverde and Rubio-Ramirez (2004) proved that bayesian techniques works well even in the case of weakly misspecified models.

$$Z_t = \sum_{k=0}^{\infty} (I - A_{\theta}(L))^{-1} B_{\theta} P_{\theta}^k \epsilon_{t-k} \quad (42)$$

Partitioning by shocks  $j$ , where  $j = 1, \dots, p$  you obtain the shocks contribution to  $Z$  as :

$$Z_t = \sum_{s=1}^p \sum_{k=0}^{\infty} IRF_{s,k} \epsilon_{s,t-k} = \sum_{s=1}^p cont_{s,t} \quad (43)$$

Where  $IRF_{s,k}$  is the response of  $Z$  to a  $s$  shock that occurred  $k$  period before and  $cont$  is the contribution of current and past  $s$  shock to the variable  $Z$ .

Figures (10)-(11) shows those contribution for stock prices and non-residential investment. It is clear that the risk premium shock accounts for a substantial part of the movements in stock prices but also on investment and loans to non-financial corporations. [AS STRESSED IN PREVIOUS SECTION RESULTS ARE STILL VERY PRELIMINARY AND NEED TO BE CHECKED] [TO BE COMPLETED].

## 5 Concluding remarks

The preliminary results obtained suggest that, beyond the effect of the changes in the leverage ratio, shocks to the external finance premium contribute to the movements in euro area investment, and to overall economic activity. It is shown that shock to the risk premium account for a substantial part of the movements in profits but also in investment and loans to non-financial corporations.

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## 6 Appendix A - The steady state

We solve the model log-linearizing the equilibrium conditions around the non-stochastic steady state values. First we present the steady state calculations and then the log-linearized equations used in the estimation. Letters without time subscripts represents steady state values while letters with a hat represent log deviations from the steady state.

The adjustment cost function is specified in order to have a unit price of capital in the steady state ( $q = 1$ ). Given our calibration of  $\Omega = 0$  it follows from ?? that  $l = h$ . The rest of the relationships follows from usual calculations:

$$\frac{\tilde{k}}{y} = \alpha \frac{mc}{z}, \quad (44)$$

$$r = \pi/\beta, \quad (45)$$

$$f = SR/\pi, \quad (46)$$

$$z = f - (1 + \delta), \text{ also } z = \alpha mc \frac{y}{k} \quad (47)$$

$$\frac{i}{y} = \delta \frac{\tilde{k}}{y} \quad (48)$$

$$\frac{c}{y} = 1 - \frac{g}{y} - \frac{i}{y} - csv \quad (49)$$

$$\frac{c^e}{y} = 0 \quad (50)$$

## 7 Appendix B - Log linearized dynamic around the steady state

- Euler equation

$$\hat{\Gamma}_{t+1}^c = \hat{\Gamma}_t^c - \hat{r}_t + \hat{\pi}_{t+1} \quad (51)$$

$$\hat{e}_t - \frac{\sigma(\hat{c}_t - \kappa\hat{c}_{t-1})}{1 - \kappa} = \hat{\Gamma}_t^c \quad (52)$$

- The New-Keynesian Phillips curve

$$\hat{\pi}_t = \left( \iota_p \hat{\pi}_{t-1} + \beta E_t \hat{\pi}_{t+1} + \frac{(1 - \beta\theta_p)(1 - \theta_p)}{(1 - \theta_p)} [E_{t-1} \widehat{m}c_t + \hat{\epsilon}_{pm,t}] \right) \frac{1}{(1 + \beta\iota_p)} \quad (53)$$

- Firms FOC

$$\hat{w}_t = \hat{y}_t + \widehat{m}c_t - \hat{h}_t \quad (54)$$

$$\hat{z}_t = \hat{y}_t + \widehat{m}c_t - \hat{k}_t^u \quad (55)$$

- WagePhillipsCurve

$$\hat{w}_t = \left( \hat{w}_{t-1} + \beta(E_t \hat{w}_{t+1} + E_t \hat{\pi}_{t+1}) - (1 + \beta\iota_w) \hat{\pi}_t + \iota_w \hat{\pi}_{t-1} - \frac{(1 - \beta\theta_w)(1 - \theta_w)}{\theta_w} \dots \right) \quad (56)$$

$$\left( \dots \left( \hat{w}_t + -\sigma \frac{(\hat{c}_t - \kappa\hat{c}_{t-1})}{(1 - \kappa)} - \zeta \hat{h}_t - \widehat{m}u_t \right) + \hat{\epsilon}_{wm,t} \right) \frac{1}{(1 + \beta)} \quad (57)$$

- Production function

$$\hat{y}_t = \hat{a}_t + \alpha \hat{k}_t^u + (1 - \alpha) \hat{h}_t \quad (58)$$

- Variable Capital Utilization

$$\widehat{k}_t^u = \widehat{u}_t + \widehat{k}_{t-1}^i \quad (59)$$

- Capacity utilisation rate

$$\widehat{z}_t = \sigma_\Upsilon \widehat{u}_t^{11} \quad (60)$$

The higher  $\sigma_\Upsilon$ , the smoother the response of capacity utilisation.

- Expected real rate of return to capital. From (17)

$$\widehat{f}_{t+1} = \frac{\bar{z}}{\bar{z} + (1 - \delta)} E_t \widehat{z}_{t+1} + \frac{1 - \delta}{\bar{z} + (1 - \delta)} E_t \widehat{q}_{t+1} - \widehat{q}_t + \widehat{x}_{t+1} \quad (61)$$

- Investment

$$\widehat{i}_t = \frac{\left( \widehat{i}_{t-1} + \bar{\pi} \beta E_t \widehat{i}_{t+1} + \frac{1}{\Phi''} \widehat{q}_t \right)}{(1 + \bar{\pi} \beta)} \quad (62)$$

- Law of motion of capital

$$\widehat{k}_{t+1}^i = \delta \widehat{i}_t + \widehat{k}_{t-1}^i \quad (63)$$

- Resource constraint

$$\widehat{y}_t = \frac{c}{y} \widehat{c}_t + \frac{i}{y} \widehat{i}_t + \frac{g}{y} \widehat{g}_t + \bar{z} \widehat{u}_t + csv \quad (64)$$

- Monetary policy rule

$$\widehat{r}_t = \rho_r \widehat{r}_{t-1} + (1 - \rho_r) [\gamma^\pi E_t \widehat{\pi}_{t+1} + \gamma^y \widehat{y}_t] + \epsilon_{rt} \quad (65)$$

- Real Loans

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<sup>11</sup>This because  $z = \Upsilon'$  and  $u = 1$  in steady state and the optimality condition associated with the entrepreneur's capacity utilization decision is  $z_t - \Upsilon'(u_t)$  which linearized became  $\widehat{z}_t - \frac{\Upsilon''}{\Upsilon'} \widehat{u}_t$ .

$$\widehat{loan}_t = \hat{q}_{t-1} + \hat{k}_t^i - \hat{n}_t \quad (66)$$

- External Finance premium

$$\hat{s}_t = \bar{\psi} \widehat{loan}_{t+1} + \hat{\psi}_t \frac{\bar{k}}{\bar{n}} \quad (67)$$

- Rental price of capital

$$\hat{f}_{t+1} = \hat{s}_t + \hat{r}_t - \hat{\pi}_{t+1} \quad (68)$$

- Dynamic of net worth

$$\hat{n}_{t+1} = \nu \bar{f} \left( \frac{\bar{k}}{\bar{n}} \hat{f}_t - \left( \frac{\bar{k}}{\bar{n}} - 1 \right) (\hat{s}_{t-1} + \hat{r}_{t-1} - \hat{\pi}_t) + \hat{n}_t \right) \quad (69)$$

- Real Profits

$$\widehat{profit}_t = \hat{y}_t - \widehat{mc}_t - \hat{f}_t - \hat{k}_t^u - \hat{w}_t - \hat{h}_t \quad (70)$$

## 8 Appendix C - Figures and Tables

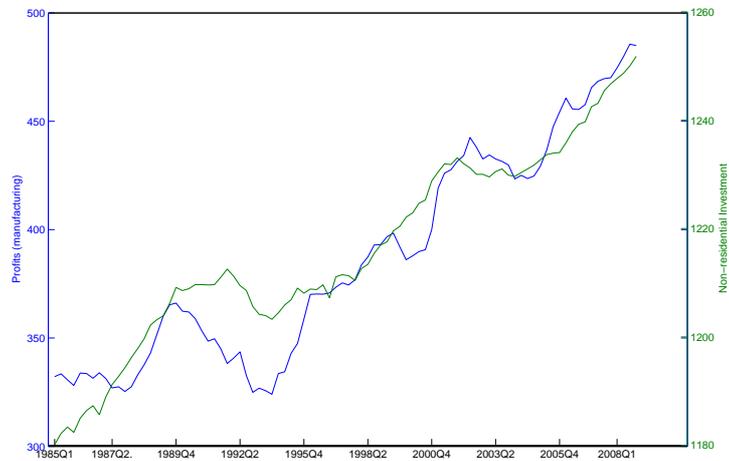


Figure 1: Profits and Investments

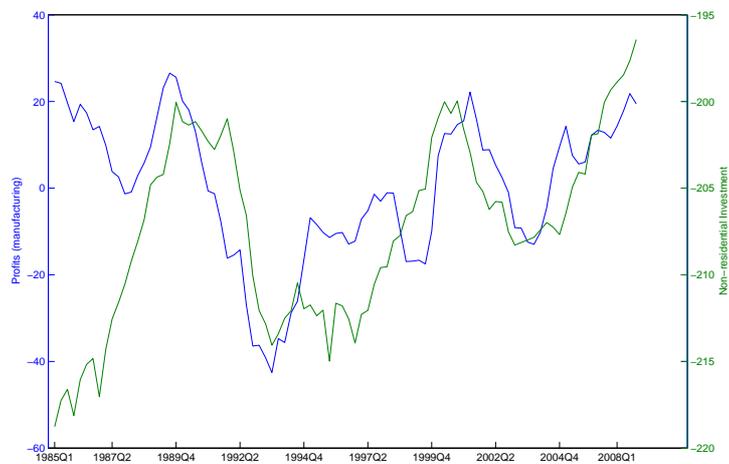


Figure 2: Profits and Investments (linearly detrended data)

$\beta = 0.99$	$\kappa = 0.6$	$\iota p = 0.3$	$\rho_g = 0.8$
$\zeta = 1$	$\bar{r} = 1.0152$	$\theta_w = 0.7$	$\chi = 7.69$
$\sigma = 2$	$f = 1.0202$	$\tau \bar{w} = 1.05$	$\rho_\psi = 0.5$
$\delta = 0.02$	$\bar{z} = 0.0402$	$\iota w = 0.7$	$\sigma_\Upsilon = 0.3691$
$\bar{\pi} = 1.005$	$\frac{k}{\bar{y}} = 10.7457$	$\gamma_\pi = 1.684$	
$v = 0.9728$	$\frac{g}{\bar{y}} = 0.17$	$\gamma_y = 0.01$	
$S = 1.01$	$\frac{c}{\bar{y}} = 0.5649$	$\rho_r = 0.87$	
$\frac{k}{\bar{n}} = 2$	$\frac{i}{\bar{y}} = 0.2149$	$\rho_a = 0.7625$	
$\alpha = 0.36$	$\psi = 0.042$	$\rho_e = 0.7$	
$A = 1$	$\theta_p = 0.9$	$\rho_\mu = 0.7$	
$csv = 0.01$	$\bar{m}c = 1.2$	$\rho_x = 0.6562$	

Table 1: Parameter values Calibrations

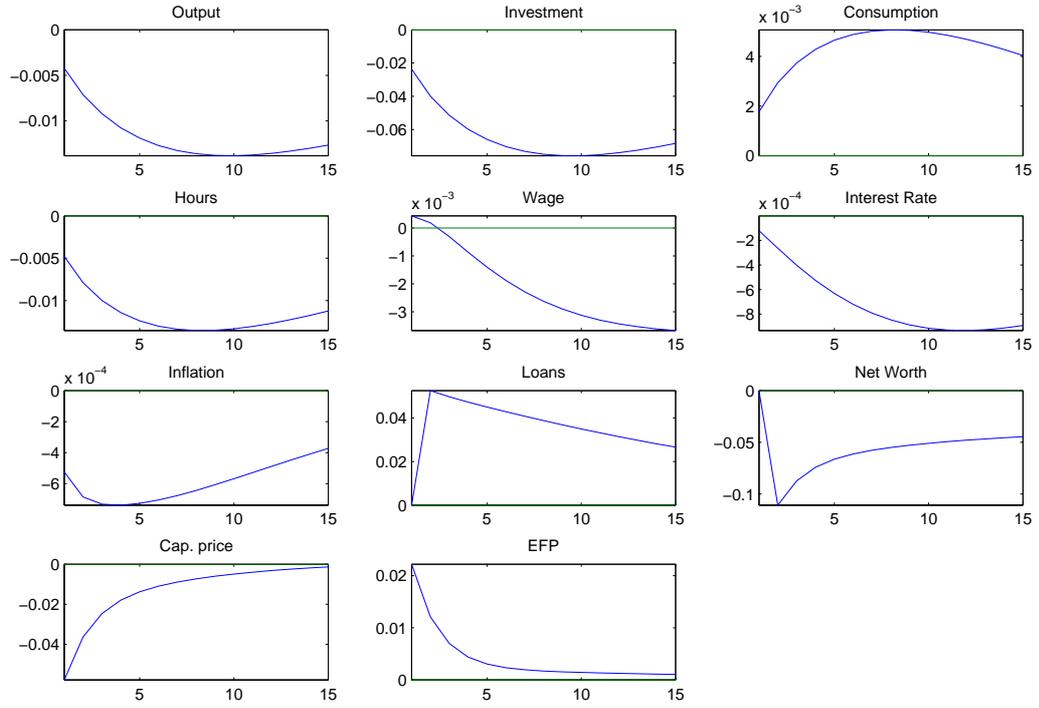


Figure 3: The response to a positive EFP shock (1%)

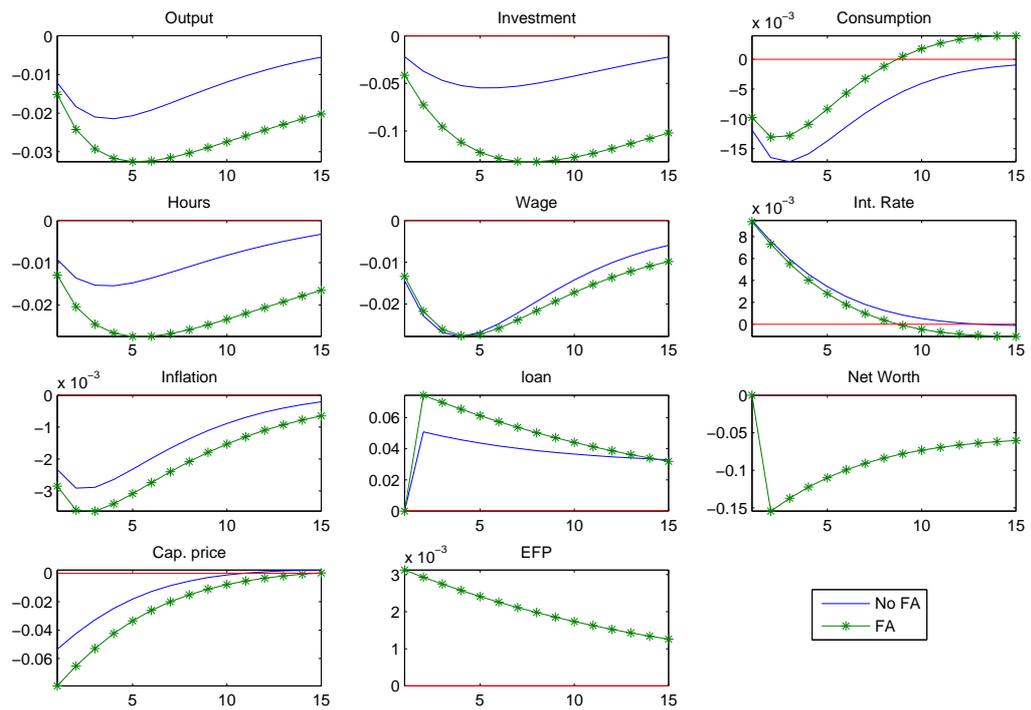


Figure 4: The response to a tightening monetary policy shock (1%)

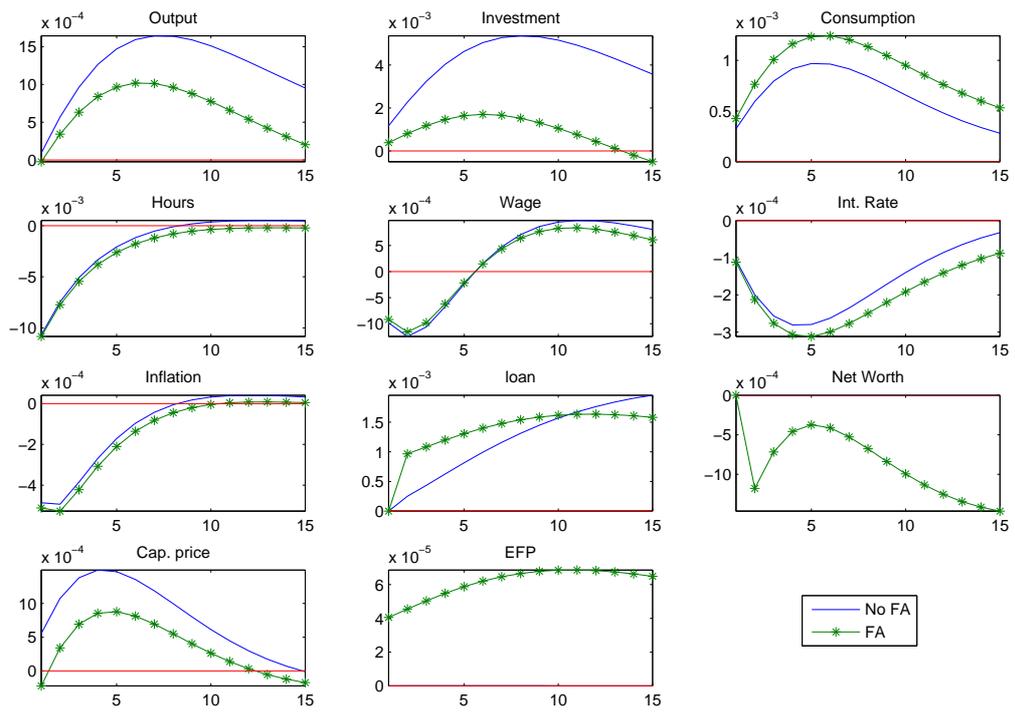


Figure 5: The response to a positive technology shock (1%)

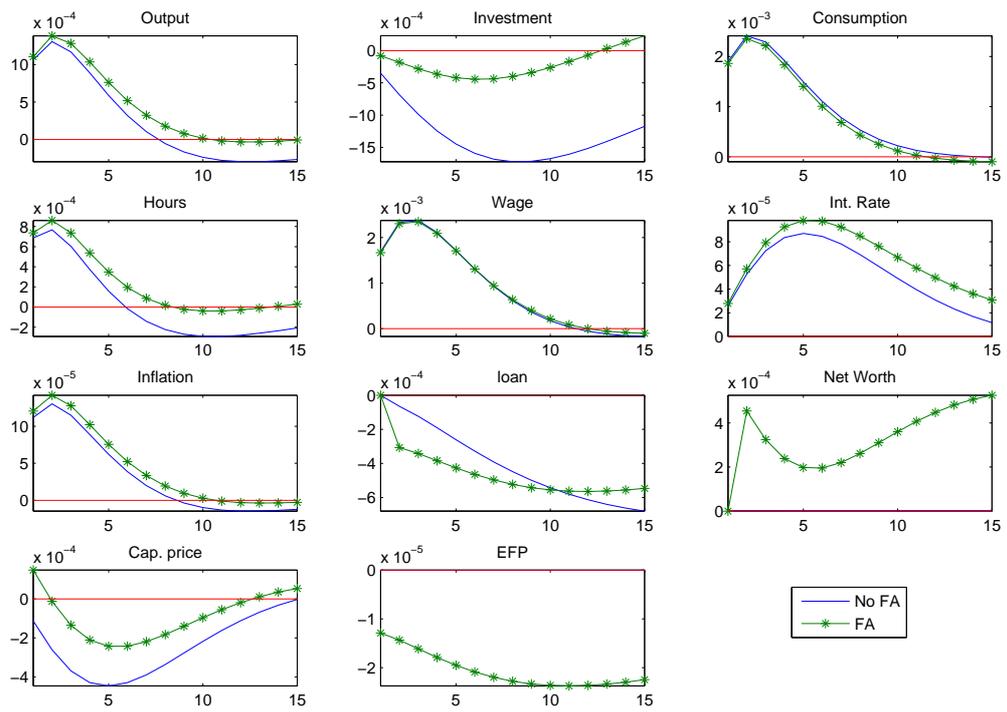


Figure 6: The response to a positive preference policy shock (1%)

	Prior distribution			Posterior Distribution	
parameter	type	mean	std	mode	std
$\rho_r$	beta	0.5	0.2	0.4999	0.1588
$\rho_a$	beta	0.7	0.2	0.9999	0.0002
$\rho_g$	beta	0.5	0.2	0.9962	0.0030
$\rho_{mu}$	beta	0.5	0.2	0.9607	0.0120
$\rho_x$	beta	0.5	0.2	0.9165	0.0195
$\rho_e$	beta	0.5	0.2	0.9247	0.0145
$\gamma_\pi$	normal	1.7	0.1	1.3700	0.0335
$\gamma_y$	normal	0.05	0.05	0.0103	0.0059
$\kappa$	beta	0.6	0.1	0.4973	0.0465
$\theta_p$	beta	0.5	0.2	0.7238	0.0088
$\theta_w$	beta	0.5	0.2	0.7123	0.0089
$\iota_p$	beta	0.5	0.2	0.3724	0.0279
$\iota_w$	beta	0.5	0.2	0.3999	0.4628
$\sigma$	normal	1.5	0.375	0.9669	0.0191
$\zeta$	normal	1.5	0.375	0.9915	0.0157
$\sigma_\Gamma$	beta	0.5	0.2	0.3979	0.0645
$psi$	beta	0.1	0.2	0.0652	0.0078
$\Phi''$	normal	5	1	4.0000	0.5310
$\epsilon_e$	inv.gamma	0.1	2*	0.2933	0.144
$\epsilon_\mu$	inv.gamma	0.1	2*	0.2861	0.1517
$\epsilon_{pm}$	inv.gamma	0.1	2*	0.2986	0.0683
$\epsilon_x$	inv.gamma	0.1	2*	0.2025	0.0424
$\epsilon_g$	inv.gamma	0.1	2*	0.2570	0.0071
$\epsilon_a$	inv.gamma	0.1	2*	0.3009	0.0096
$\epsilon_{wm}$	inv.gamma	0.1	2*	0.4616	0.0311
$\epsilon_r$	inv.gamma	0.1	2*	0.1911	0.0177
$\epsilon_\psi$	inv.gamma	0.1	2*	0.2023	0.1879

\* degree of freedom

Table 2: Priors and Posteriors distributions

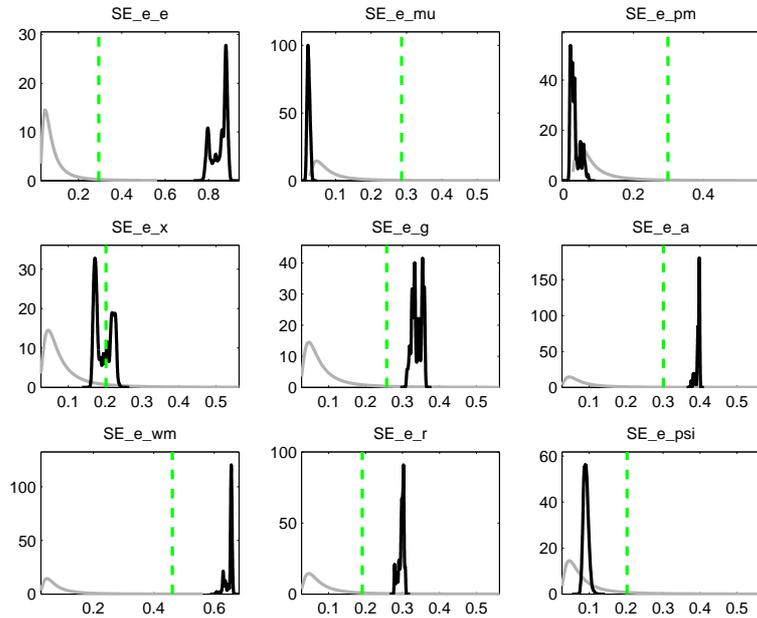


Figure 7: Priors and Posteriors distributions

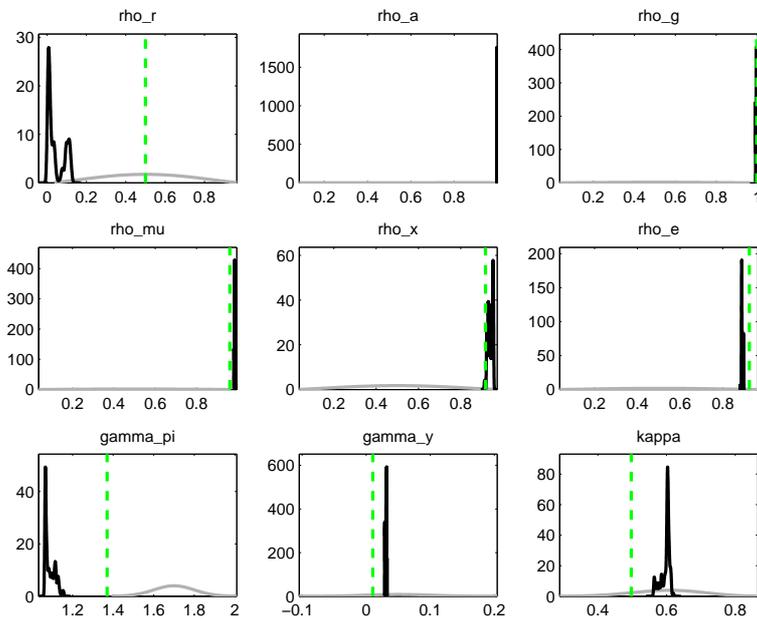


Figure 8: Priors and Posteriors distributions

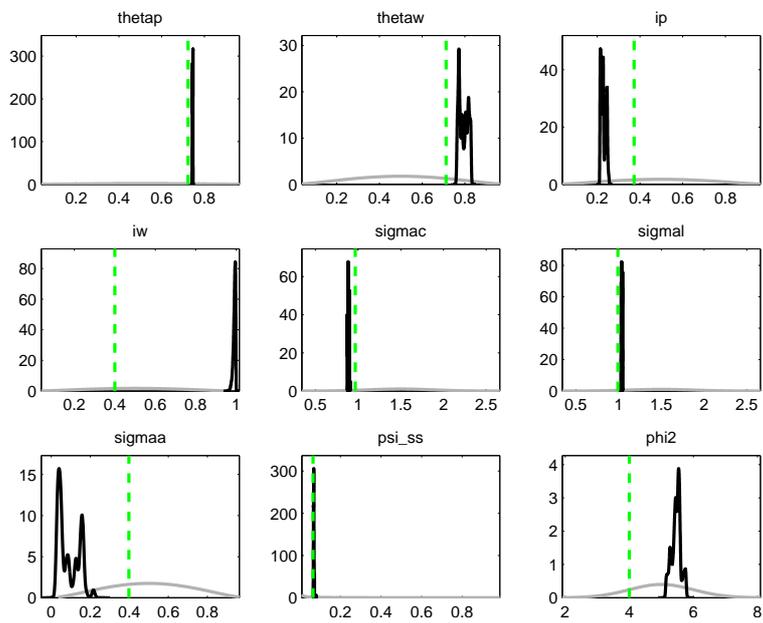


Figure 9: Priors and Posteriors distributions

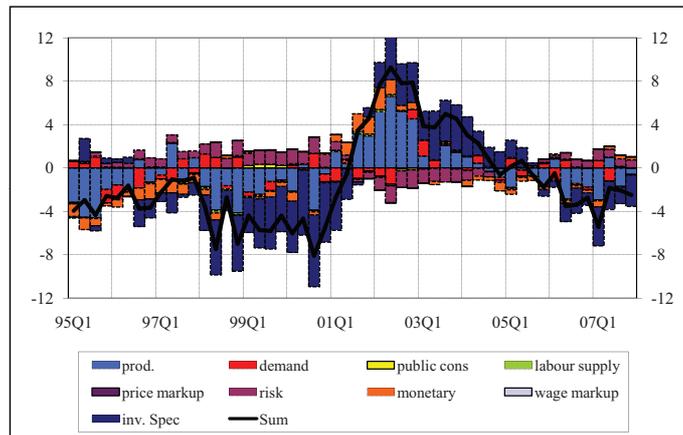


Figure 10: Shocks contribution to stock prices

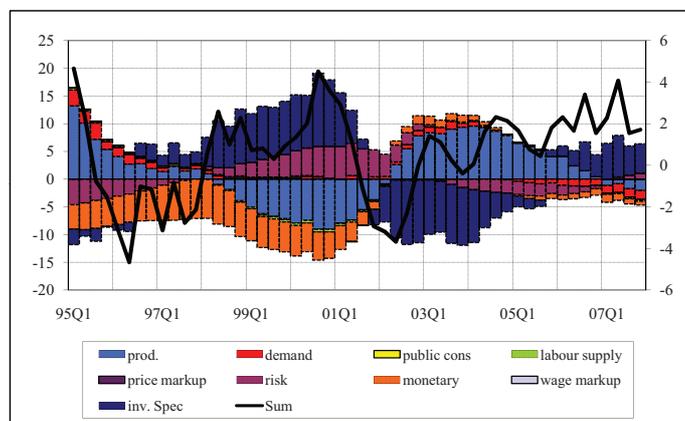


Figure 11: Shocks contribution to non-residential investment