

# Productivity and Job Flows: Heterogeneity of New Hires and Continuing Jobs in the Business Cycle\*

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## Abstract

This paper focuses on the productivity dynamics of a firm-worker match as a potential explanation of "unemployment volatility puzzle". We assume that new matches and continuing jobs differ in their productivity levels and in their sensitivity to aggregate productivity shocks. Lower productivity in new hires implies that new matches have a higher destruction rate and lower wages than old matches, consistent with the empirical evidence, while the latter implies that new jobs have more volatile wages, as new hires receive productivity associated with the latest technology vintage. In our model, an aggregate productivity shock generates a persistent productivity difference between the two types of matches, creating an incentive to open new productive vacancies and to destroy old matches that are temporarily less productive. As a result, the model produces a well behaving Beveridge curve, despite endogenous job destruction, and more volatile vacancies and unemployment, without a need to rely on differing wage setting mechanisms of new and continuing jobs.

JEL classification: E24, E32, J64

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## Non-Technical Summary

New hires and continuing jobs exhibit substantially differing productivity and wage dynamics as well as job separation behavior. Whereas it is well known that productivity and wages increase with tenure and that the probability of a job ending declines with tenure, the differing wage cyclicality of new hires and continuing jobs has been the focus of attention in recent labour market matching literature. Haefke et al. (2008) and Carneiro et al. (2008) provide evidence on the strong responsiveness of wages in new hires to productivity fluctuations, while wages of continuing jobs exhibit substantial rigidity. Pissarides (2008) surveys and relates the empirical evidence to the 'unemployment volatility puzzle' in the Mortensen-Pissarides matching model. He argues that the observed wage cyclicality of new matches is consistent with the proportional relationship between wages and labour productivity in the standard model. Given the weak empirical support for wage stickiness, he argues that plausible explanations of the unemployment volatility puzzle should not rely on a sticky wage, but should rather be consistent with the observed proportional relation between labour productivity and wages of new matches.

This paper focuses on the productivity dynamics of a firm-worker match as a potential explanation of the "unemployment volatility puzzle". We study an economy in which new matches and continuing jobs differ in their productivity levels and in their sensitivity to aggregate productivity shocks. In order to capture the match heterogeneity, we use a vintage-type structure, where all matches are created as "new", but eventually transit exogenously from this state to "old". New and old matches differ in two ways: First, the productivity of matches is increasing with tenure, s.t. the average productivity of old matches is higher than that of new matches. In the literature, increasing productivity with tenure is attributed to e.g. learning by doing, learning of match quality or selection effects (e.g. Brown (1989), Topel (1991)). Lower productivity in new hires implies that new matches have a higher destruction rate and lower wages than old matches, consistent with the empirical evidence. Second, in the spirit of Campbell and Fisher (1998) new matches are more responsive to aggregate productivity fluctuations than old matches, suggesting that young firms respond more and possibly with different margins to business cycle shocks than do the old ones. In our model new hires obtain a temporary but persistent productivity advantage over old jobs, but in the long run the shock induced productivity differences even out. This captures the standard property in vintage models that new hires receive productivity associated with the latest technology vintage (Caballero and Hammour (1998) and Campbell (1997)), without producing a counterfactually higher productivity level of new hires relative to continuing jobs (Fos-

ter et al. 2006). Instead of pursuing the path of incorporating alternative wage setting mechanisms (wage rigidity) in the matching model, we analyze the factors that underlie the standard Nash wage. Our approach is thus in the spirit of Mortensen and Nagypál (2007a) and Pissarides (2008) who conclude that a flexible wage is not the principal problem with the model and that the need for wage rigidity has been overemphasized.

Allowing for heterogeneous productivity dynamics of new hires and continuing jobs tackles a number of problems in the standard matching model<sup>1</sup>. Our model produces a well behaving Beveridge curve despite endogenous job destruction and it narrows the gap between the volatility of the model's labour market variables and actual data. The well behaving Beveridge curve is due to aggregate productivity shocks creating a temporary productivity difference between the two match types. This creates an incentive to create new productive vacancies and destroy old matches that are temporarily less productive. Although employment adjustment does take place through the job destruction margin, it becomes less important relative to the standard model: employment adjustment shifts from the job destruction margin towards the job creation margin. The model thus produces a creative destruction or cleansing effect that Mortensen and Nagypál (2007a,b) suggest as a way to reconcile the Mortensen-Pissarides model with the data.

The shifting of adjustment margin from job destruction to job creation also increases the volatility of vacancies and unemployment and the model captures the dynamic correlations between labour market variables and output (and unemployment) better than the standard matching model. As opposed to many earlier papers (e.g. Farmer (2004), Shimer (2005), Hall (2005), and Gertler and Trigari (2005)), we do not need to take the route of introducing rigid wages. Rather, due to the heterogeneity of productivity dynamics and the assumption that wages are negotiated separately in the new and continuing jobs, wages of newly created matches are more procyclical than wages of continuing matches, consistent with the empirical evidence.

Finally, following the recent literature which combines New Keynesian monetary policy models with a search labour market framework, we introduce price rigidities into the model. It turns out that, price rigidities do not alter the basic mechanism. Furthermore, the transmission of interest rate changes is very similar to the standard New Keynesian model with search frictions.

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<sup>1</sup>The standard matching model has difficulties to match key correlations and volatility of labour market variables and output (e.g. Shimer (2005), Hall (2005), Hornstein et al. (2005)). In particular, the standard matching model with endogenous job destruction fails to generate a strong positive (negative) correlation between output (unemployment) and vacancies: the Beveridge curve tends to be upward sloping. The reason is the sensitivity of the job destruction margin to exogeneous shocks. Moreover, the standard model fails to generate the high volatility of labour market variables observed in the data.

# 1 Introduction

New hires and continuing jobs exhibit substantially differing productivity and wage dynamics as well as job separation behavior. Whereas it is well known that productivity and wages increase with tenure (e.g. Brown (1989), Topel (1991)) and that the probability of a job ending declines with tenure (e.g. Farber 1999), the differing wage cyclicality of new hires and continuing jobs has been the focus of attention in recent labour market matching literature. Haefke et al. (2008) and Carneiro et al. (2008) provide evidence on the strong responsiveness of wages in new hires to productivity fluctuations, whereas wages of continuing jobs exhibit substantial rigidity. Pissarides (2008) surveys the empirical evidence about the cyclicality of wages in new and continuing jobs and relates the evidence to the discussion of the 'unemployment volatility puzzle' in the Mortensen-Pissarides matching model. He argues that the observed cyclicality of wages of new matches is consistent with the Nash wage equation, which gives a proportional relationship between wages and labour productivity, in the standard Mortensen-Pissarides model. Furthermore, he argues that given the weak empirical support for wage stickiness, plausible explanations of the unemployment volatility puzzle should not rely on a sticky wage, but should rather be consistent with the observed proportional relation between labor productivity and wages of new matches.

This paper focuses on the productivity dynamics of a firm-worker match as a potential explanation of the "unemployment volatility puzzle". We study an economy in which new matches and continuing jobs differ in their productivity levels and in their sensitivity to aggregate productivity shocks. The former feature, lower productivity in new hires, implies that new matches have a higher destruction rate and lower wages than old matches, consistent with the empirical evidence. The latter feature, in turn, captures the basic idea of Caballero and Hammour (1998) and Campbell (1997) where new hires receive productivity associated with the latest technology vintage, suggesting that young firms respond more and possibly with different margins to business cycle shocks than do the old ones (Campbell and Fisher, 1998). Thus, instead of pursuing the path of incorporating alternative wage setting mechanisms (wage rigidity) in the matching model, we analyze the factors that underlie the standard Nash wage equation. In this sense, our approach is in the spirit of Mortensen and Nagypál (2007a) and Pissarides (2008) who conclude that a flexible wage is not the principal problem with the model and that the need for wage rigidity is overemphasized in the literature.

In order to capture the match heterogeneity, we use a vintage-type structure, where all matches are created as "new", but eventually transit exogenously from this state

to "old". New and old matches differ in two ways: First, following a long tradition of works on productivity and job duration (or tenure), the productivity of matches is increasing with tenure, s.t. the average productivity of old matches is higher than that of new matches. In the literature, increasing productivity with tenure is attributed to e.g. learning by doing, learning of match quality or selection effects (e.g. Brown (1989), Topel (1991)). Second, in the spirit of Campbell and Fisher (1998) new matches are more responsive to aggregate productivity fluctuations than old matches.<sup>2</sup> In our model new hires obtain a temporary but persistent productivity advantage over old jobs, but in the long run the shock induced productivity differences even out. This captures the standard property in vintage models that new hires receive productivity associated with the latest technology vintage, without producing a counterfactually higher productivity level of new hires relative to continuing jobs (Foster et al. 2006).

Our setup emphasizes the distinction between new and continuing matches, as opposed to e.g. Reiter (2006) who focuses on the timing of job creation in the business cycle. Reiter (2006) studies the role of embodied technological change on fluctuations in a matching model, where the productivity of a match depends partially on the aggregate productivity prevailing at the time of creating the match. We study an economy where matches are not locked into the prevailing technology at the job creation date as they eventually transit from new to old. We also model job destruction as arising endogenously from the optimizing decisions of agents in response to productivity shocks. This allows us to address the role of tenure related productivity differences and the non-trivial role of endogenous job destruction and creative destruction effects for the 'unemployment volatility puzzle' in the Mortensen-Pissarides matching model, as recently discussed by Mortensen and Nagypál (2007a, 2007b).

Allowing for heterogeneous productivity dynamics of new hires and continuing jobs tackles a number of problems in the standard matching model<sup>3</sup>. Our model produces a well behaving Beveridge curve despite endogenous job destruction and it narrows the gap

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<sup>2</sup>This feature is consistent with the idea that young plants (or firms) adopt more flexible organisations to cope with the greater risk and to exploit new opportunities. This suggests that young firms respond more and possibly with different margins to business cycle shocks than do the old ones (Campbell and Fisher, 1998).

<sup>3</sup>The standard matching model has difficulties to match key correlations and volatility of labour market variables and output (e.g. Shimer (2005), Hall (2005), Hornstein et al. (2005), Mortensen and Nagypál (2007a)). In particular, the standard matching model with endogenous job destruction fails to generate a strong positive (negative) correlation between output (unemployment) and vacancies: the Beveridge curve tends to be upward sloping. The reason is the sensitivity of the job destruction margin to exogenous shocks. Moreover, the standard model fails to generate the high volatility of labour market variables observed in the data.

between the volatility of the model's labour market variables and actual data. The well behaving Beveridge curve is due to the fact that an aggregate productivity shock creates a temporary productivity difference between the two types of matches. This creates an incentive to create new productive vacancies and destroy the old matches that are temporarily less productive.<sup>4</sup> Although employment adjustment does take place through the job destruction margin, it becomes less important relative to the standard model: there is a shift of employment adjustment from the job destruction margin towards the job creation margin. The model thus produces a creative destruction or cleansing effect that Mortensen and Nagypál (2007a,b) suggest as a way to reconcile the Mortensen-Pissarides model with the data.

The shifting of adjustment margin from job destruction to job creation also increases the volatility of vacancies and unemployment and the model captures the dynamic correlations between labour market variables and output (and unemployment) better than the standard matching model. As opposed to many earlier papers (e.g. Farmer (2004), Shimer (2005), Hall (2005), and Gertler and Trigari (2005)), we do not need to take the route of introducing rigid wages. Rather, due to the heterogeneity of productivity dynamics and the assumption that wages are negotiated separately in the new and continuing jobs, wages of newly created matches are more procyclical than wages of continuing matches, consistent with the empirical evidence.

Finally, following the recent literature which combines New Keynesian monetary policy models with a search labour market framework, we introduce price rigidities into the model following Walsh (2005). It turns out that, price rigidities do not alter the basic mechanism. Furthermore, the transmission of interest rate changes is very similar to the standard New Keynesian model with search frictions.

The rest of the paper proceeds as follows. In section 2 we construct a Mortensen-Pissarides type matching model with endogenous job destruction and heterogeneity of matches. Section 3 describes the calibration of the model and in section 4 we analyze the behavior of the model in response to productivity shocks. Section 5 introduces nominal rigidities into the model and discusses the transmission of monetary policy in the model. Section 6 concludes.

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<sup>4</sup>A similar mechanism is present also in the recent model by Michelacci and Lopez-Salido (2007), where old jobs cannot upgrade their technology in the same phase as new jobs.

## 2 Model

We consider a discrete-time economy where there are three labour market states for both workers and firms. Workers may be either unemployed, or be employed in a *new* or an *old* match. Analogously, firms may either have an open vacancy, or have an occupied job in a *new* or an *old* match. Firm-worker matches are formed in a search market. All firm-worker pairs are initially new but may become old at an exogenous transition rate. Both new and old matches are subject to exogenous and endogenous job destruction. New and old matches differ wrt. their production function. Consequently their reservation productivity and job flow dynamics differ.

### 2.1 Match productivity

The productivity of a match depends on two factors: aggregate technology  $z_t$  which is common to all matches and on match-specific productivity  $a_{it}$  for which a value is drawn from a stationary distribution  $F(a_{it})$  in each period. The stochastic shocks to  $z_t$  take place at the beginning of each period.

New and old jobs differ along two dimensions. First, newly created jobs are more responsive to aggregate technology shocks than continuing matches. Second, in line with the empirical evidence we allow the average productivity of old jobs to be higher than that of new jobs.

Formally, match output in a newly created and continuing matches is given by

$$a_{it}x^N(z_t) = a_{it}z_t \quad (1)$$

$$a_{it}x^O(z_t, \lambda) = a_{it}(z_t^\gamma - \lambda) \quad (2)$$

respectively.  $a_{it}$  is match specific productivity and  $z_t$  is the common aggregate technology shock that follows an  $AR(1)$  process. Parameters  $\gamma$  and  $\lambda$  capture the relative responsiveness of old and new matches to aggregate technology shocks and the average productivity difference between new and old matches. Note that the empirical evidence suggests that  $\gamma < 1$  and  $\lambda < 0$ . The parameter  $\gamma < 1$  summarizes a number of reasons why continuing jobs may fail to fully incorporate the latest vintage of aggregate technology to their production process. For example, adoption of new technologies or managerial innovations may require costly organizational changes in a firm, changes in working practices, costly software updates etc. We abstract from the specifics of such obstacles/costs as our focus is on understanding the implications of this type of heterogeneity. The parameter  $\lambda$  captures increasing productivity with tenure that may be attributed to e.g. learning by doing or learning of match quality.

## 2.2 Matching and job flows

Unemployed workers and open vacancies are matched in a search labour market characterized by matching frictions. The number of matches in each period is determined by a matching function

$$m(u_t, v_t) = Au_t^\alpha v_t^{1-\alpha} \quad (3)$$

which is increasing in the number of unemployed workers  $u_t$  and open vacancies  $v_t$  and where  $0 < \alpha < 1$ . We thus assume that the matching function satisfies the standard properties.<sup>5</sup> The probability of an open vacancy getting filled and the probability of a worker moving from unemployment to employment are given by

$$q^f(\theta_t) = \frac{m(u_t, v_t)}{v_t} \quad (4)$$

$$q^w(\theta_t) = \frac{m(u_t, v_t)}{u_t} \quad (5)$$

respectively, and where we denote labour market tightness  $\theta_t = \frac{v_t}{u_t}$ . The hazard rate  $q_t^f$  is decreasing and  $q_t^w$  is increasing in  $\theta_t$ .

After being matched in period  $t$ , a firm-worker pair enters the next period  $t + 1$  as a new match. In the beginning of that period, before production starts, it becomes immediately *old* with probability  $\phi$  or remains *new* with probability  $1 - \phi$ . For already existing matches the same transition rule applies: matches that have remained *new* until that date become *old* with probability  $\phi$  or remain *new* with probability  $1 - \phi$  in the beginning of each period. Old matches remain old, and cannot become new.

Once the distribution of match types is determined, a fraction  $\rho^x$  of both types is destroyed by an exogenous shock. The surviving firm-worker pairs observe the aggregate productivity shock  $z_t$  and their match specific productivity realization  $a_{it}$ , after which they decide whether to start production or separate endogenously. There is a reservation productivity  $\tilde{a}_t^j, j = N, O$  for both match types such that all new matches with productivity  $a_{it} > \tilde{a}_t^j$  start production and all matches with a lower match specific realization are destroyed endogenously. The endogenous separation rate for matches of type  $j$  is then

$$\rho_t^{nj} = \Pr \left[ a_t \leq \tilde{a}_t^j \right] = F(\tilde{a}^j). \quad (6)$$

where  $F(\cdot)$  denotes cumulative distribution function of match specific productivity realizations. Note importantly that the reservation productivities  $\tilde{a}_t^N$  and  $\tilde{a}_t^O$  are not

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<sup>5</sup>The standard matching function is assumed to be homogenous and increasing in both of its arguments, concave and to have constant returns to scale.

necessarily the same, although we assume that the match specific productivity draws are from the same distributions.

The total separation rate for matches of type  $j$  is

$$\rho_t^j = \rho^x + (1 - \rho^x) \rho_t^{nj}. \quad (7)$$

The separated workers return to the pool of searching unemployed workers within the same period.

We next turn to the job flow equations. The number of new matches that *enter* a period is given by

$$n_{t+1}^N = m(u_t, v_t) + (1 - \phi) (1 - \rho_t^N) n_t^N, \quad (8)$$

where  $n_{t+1}^N$  is the measure of employed new workers at the beginning of period  $t + 1$  before production takes place. This consists of those workers that were matched in the previous period  $m(u_t, v_t)$  and new workers of the previous period who remained new and survived from job destruction in the previous period. Notice that if  $\phi = 1$ , the measure of new workers at the beginning of period  $t + 1$  consists of new matches only, i.e.  $n_{t+1}^N = m(u_t, v_t)$ .

The number of old matches that *enter* a period is given by

$$n_{t+1}^O = (1 - \rho_t^O) [n_t^O + \phi n_t^N]. \quad (9)$$

where  $n_t^O$  is a measure of employed old workers at the beginning of period  $t + 1$  before production takes place. This consists of those who were old and survived from job destruction in the previous period. It also contains those who became old at the beginning of period  $t + 1$  (when entering period  $t + 1$ ). Alternatively, these are workers who were new in period  $t$  and became old at the end of the period after the end of production. Once more, notice a special case where  $\phi = 1$ . In this case,  $n_{t+1}^O = (1 - \rho_t^O) [n_t^O + n_t^N]$ . The overall number of matches that *enter* a period is given by

$$n_{t+1} \equiv n_{t+1}^N + n_{t+1}^O. \quad (10)$$

The number of searching workers  $u_t$  in period  $t$  differs from the number of unemployed workers  $1 - n_t$  in the beginning of period  $t$  as some of the employed workers separate from their matches and start searching for a new job within the same period. A measure of workers who search in period  $t$  (and thus are not involved in production) is

$$u_t = 1 - n_t + (1 - \phi) \rho_t^N n_t^N + \rho_t^O (n_t^O + \phi n_t^N) \quad (11)$$

where  $1 - n_t$  is the number of unmatched workers in the beginning of the period.  $(1 - \phi) \rho_t^N n_t^N$  is the number of new matches at the beginning of the period that remain

new and are subject to job destruction at rate  $\rho_t^N$  and start to search, and  $\rho_t^O(n_t^O + \phi n_t^N)$  is the number of matches in the beginning of the period which are destroyed at rate  $\rho_t^O$  and start to search. This consists of those that were already old at the beginning of period  $t$ ,  $n_t^O$ , and those that were new, but became old at the beginning of period  $t$ ,  $\phi n_t^N$ .

Next we turn into the net job creation and destruction rates. In each period  $q_t^f v_t$  new vacancies are filled. A fraction  $\rho^x$  of the new and previously existing matches are destroyed exogenously at the beginning of the period. The rate of turnover is then  $q_t^f \rho^x n_t$  and the net job creation rate can be expressed as

$$j c_t = \frac{q_t^f v_t}{n_t} - q_t^f \rho^x. \quad (12)$$

The net job destruction rate is given by

$$j d_t = \frac{(1 - \phi)\rho_t^N n_t^N + \rho_t^O(n_t^O + \phi n_t^N)}{n_t} - q_t^f \rho^x \quad (13)$$

where the first term on the RHS is the aggregate job destruction rate and  $q_t^f \rho^x$  are the exogenously destroyed matches that re-match within the same period.<sup>6</sup>

### 2.3 Value functions and match surplus

Match surplus is a key element in determining job creation and destruction. The surplus is the difference between the asset values of being matched and the outside values and is given by

$$S_t^j(a_{it}) = [J_t^j(a_{it}) - V_t] + [W_t^j(a_{it}) - U_t] \quad (14)$$

$J_t^j(a_{it})$  and  $W_t^j(a_{it})$  are the asset values for a firm and worker respectively of being matched and  $V_t$  and  $U_t$  are the asset values of being idle for the firm and the worker, that is, having an open vacancy for the firm and being unemployed for the worker.

The asset value to a firm of a filled new job with match specific productivity realization  $a_{it}$  is given by

$$J_{it}^N = a_{it} x^N(z_t) - w_{it}^N(a_{it}) + E_t \beta \left\{ (1 - \rho^x) \left[ \phi \int_{\bar{a}_{t+1}^O}^{\infty} J_{t+1}^O(a) f(a) da \right. \right. \quad (15) \\ \left. \left. + (1 - \phi) \int_{\bar{a}_{t+1}^N}^{\infty} J_{t+1}^N(a) f(a) da \right] + \rho^x V_{t+1} \right\}.$$

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<sup>6</sup>In the definitions of job destruction and job creation, we follow Trigari (2004) and Den Haan et. al. (2000).

The value consists of the current payoff, given by the real value of match output  $a_{it}x^N(z_t) = a_{it}z_t$  net of the wage cost  $w_{it}^N(a_{it})$ , and the expected future payoff of the match which is discounted according to the discount factor  $\beta$ . With probability  $\phi$  the match becomes old and with probability  $1 - \phi$  it remains new. The match survives exogenous job destruction with probability  $(1 - \rho^x)$ . For a surviving match that remains new or becomes old, a productivity realization below the respective reservation productivity  $\tilde{a}_{t+1}^N$  or  $\tilde{a}_{t+1}^O$  leads to endogenous separation. A match with a productivity realization above the respective reservation productivity starts producing as either a new or an old match. In case of separation the firm gets the asset value of an open vacancy  $V_{t+1}$ .

The asset value of an old job is given by

$$J_{it}^O = a_{it}x^O(z_t) - w_{it}^O(a_{it}) + E_t\beta \left[ (1 - \rho^x) \int_{\tilde{a}_{t+1}^O}^{\infty} J_{t+1}^O(a) f(a) da + \rho^x V_{t+1} \right] \quad (16)$$

where match output and wage are determined by  $a_{it}x^O(z_t, \lambda) = a_{it}(z_t^\gamma - \lambda)$  in period  $t$  and otherwise the equation has the same interpretation as the one for a new job.  $\lambda$  is the vintage parameter and  $\gamma$  determines a relative sensitivity of new matches to technology shocks when compared to old matches. For an old match the expected future payoff of the match is analogous to that of a new job, except that for old matches the future value is always that of an old match, as there is no transition from old matches back to new matches.

The value of an open vacancy satisfies

$$V_t = -\kappa + E_t\beta \left\{ q_t^f (1 - \rho^x) \left[ \left( \phi \int_{\tilde{a}_{t+1}^O}^{\infty} J_{t+1}^O(a) f(a) da \right. \right. \right. \quad (17)$$

$$\left. \left. \left. + (1 - \phi) \int_{\tilde{a}_{t+1}^N}^{\infty} J_{t+1}^N(a) f(a) da \right) - V_{t+1} \right] + V_{t+1} \right\}$$

where  $\kappa$  is the periodical search cost and the expected payoff of search is given by the second RHS term. With a probability  $q_t^f$  the firm matches with a worker, and with probability  $\phi$  the match becomes old and with probability  $1 - \phi$  it remains new. Endogenous separation and job values are given as above. If the firm doesn't match it gets the asset value of an open vacancy  $V_{t+1}$ .

Workers may either be unemployed and searching for a job or employed in a new or old match. The asset value of working in a *new* job with match specific productivity  $a_{it}$

is

$$W_{it}^N = w_{it}^N(a_{it}) + E_t \beta \left\{ (1 - \rho^x) \left[ \phi \int_{\bar{a}_{t+1}^O}^{\infty} W_{t+1}^O(a_{t+1}) f(a) da \right. \right. \quad (18)$$

$$\left. \left. + (1 - \phi) \int_{\bar{a}_{t+1}^N}^{\infty} W_{t+1}^N(a_{t+1}) f(a) da \right] + \rho^x U_{t+1} \right\}$$

The worker receives a wage of a new job  $w_{it}^N(a_{it})$  in period  $t$ , depending on the production function  $a_{it}x^N(z_t) = a_{it}z_t$ . In the next period, with probability  $\phi$  the match becomes old and with probability  $1 - \phi$  it remains new. If the match survives exogenous job destruction, the old or new match with a productivity realization below the reservation productivity  $\bar{a}_{t+1}^O$  and  $\bar{a}_{t+1}^N$  respectively will separate endogenously. A match with a productivity realization above the respective reservation productivity will produce as either new or old matches with the value  $W_{it+1}^j(a_{it+1})$ ,  $j = N, O$ . In case of separation the worker gets the asset value of unemployment  $U_{t+1}$ .

The value of working in an *old* job with match specific productivity  $a_{it}$  is

$$W_{it}^O = w_{it}^O(a_{it}) + E_t \beta \left[ (1 - \rho^x) \int_{\bar{a}_{t+1}^O}^{\infty} W_{t+1}^O(a_{t+1}) f(a) da + \rho^x U_{t+1} \right] \quad (19)$$

with an analogous interpretation to equation (16) above. Notice that if matches are similar in all respects, equation (18) and (19) deliver the same. In particular, transition probability has no effect on determination of wages.

The value of unemployment  $U_t$  is given by

$$U_t = b + E_t \beta \left\{ q_t^w (1 - \rho^x) \left[ \phi \int_{\bar{a}_{t+1}^O}^{\infty} W_{t+1}^O(a_{t+1}) f(a) da \right. \right. \quad (20)$$

$$\left. \left. + (1 - \phi) \int_{\bar{a}_{t+1}^N}^{\infty} W_{t+1}^N(a_{t+1}) f(a) da - U_{t+1} \right] + U_{t+1} \right\}.$$

where  $b$  is the flow utility of non-market activities and the term in brackets is the asset value of search on the labour market. With a probability  $q_t^w$  the worker matches with a firm, and with probability  $\phi$  the match becomes old and with probability  $1 - \phi$  it remains new. Endogenous separation and the asset values of being matched in an old and new match are given analogously as above. An unmatched worker continues to receive the asset value of unemployment  $U_{t+1}$ .

## 2.4 Wage determination

We assume that wages are negotiated each period and separately for new and old matches. In both match types, the total intertemporal match surplus is shared through a Nash-bargaining process between the firm and the worker, giving rise to two separate Nash wage equations. Accordingly, the individual wage rate satisfies

$$w_{it}^j = \arg \max \left[ W_t^j(a_{it}) - U_t \right]^\eta \left[ J_t^j(a_{it}) - V_t \right]^{1-\eta}, \quad j = N, O \quad (21)$$

where the parameter  $\eta$  represents the worker's share of the match surplus. The first order condition is

$$\eta \left( J_{it}^j - V_t \right) = (1 - \eta) \left( W_{it}^j - U_t \right) \quad (22)$$

Substituting from the value equations and using the free entry condition  $V_t = 0$  in the first order condition we arrive to the familiar individual Nash wage equation for new and old matches<sup>7</sup>

$$w_{it}^j = \eta \left( a_{it} x^j(z_t) + \kappa \theta_t \right) + (1 - \eta) b, \quad j = N, O \quad (23)$$

The wage depends both on idiosyncratic and aggregate conditions. Equation (23) also reflects the fact that wages are bargained after the realization of the idiosyncratic productivity  $a_{it}$ . The Nash wage equation implies that the wage dynamics between the new and old matches differ to the extent that match specific productivity  $a_{it} x^j(z_t)$  differ. Our model thus captures the different wage dynamics in the old and new matches by relying on different match specific productivity dynamics, instead of assuming that the wage determination mechanism between new and old matches differs.

We define aggregate wages for each match type (new and old) as

$$w_t^j = \int_{\tilde{a}_t^j}^{\infty} \frac{f(a^j)}{1 - F(\tilde{a}_t^j)} w_t^j(a) da, \quad j = N, O. \quad (24)$$

Finally, aggregating over old and new matches we get the aggregate economy wide wage

$$w_t = \frac{(1 - \rho_t^N) (1 - \phi) n_t^N w_t^N + (1 - \rho_t^O) [n_t^O + \phi n_t^N] w_t^O}{(1 - \rho_t^N) (1 - \phi) n_t^N + (1 - \rho_t^O) [n_t^O + \phi n_t^N]}, \quad (25)$$

where  $(1 - \rho_t^N) (1 - \phi) n_t^N + (1 - \rho_t^O) [n_t^O + \phi n_t^N]$  is a measure of workers involved in production, of which  $(1 - \rho_t^N) (1 - \phi) n_t^N$  is a measure of workers in the new matches after destruction and transition and  $(1 - \rho_t^O) [n_t^O + \phi n_t^N]$  is a measure of workers who after transition are in old matches and have survived job destruction. In essence, the

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<sup>7</sup>See appendix for details.

aggregate wage is a weighted average (with time varying weights) of the wages in new and old matches. Consequently, the aggregate wage dynamics contain the composition effect due to fluctuations in the share of workers in old and new jobs. Note that in the special case where  $\phi = 1$ , equation (25) implies that  $w_t = w_t^O$ . This is natural, since in this case all new matches become old before the endogenous decision to continue with the match takes place, and thus before the wages are bargained.

## 2.5 Job creation and destruction

### 2.5.1 Job creation condition

Free entry of firms to the market implies that firms enter until the value of posting a vacancy is driven to zero in equilibrium. Setting  $V_{t+i} = 0$  in (17) and substituting equation (15) produces the job creation condition

$$\frac{\kappa}{q_t^f} = E_t \beta (1 - \rho^x) \left[ \phi \int_{\bar{a}_{t+1}^O}^{\infty} J_{t+1}^O(a) f(a) da + (1 - \phi) \int_{\bar{a}_{t+1}^N}^{\infty} J_{t+1}^N(a) f(a) da \right]. \quad (26)$$

This equation states that expected search costs are equal to expected value of a filled job. The expected value of a filled job takes into account the transition probability of new job becoming old immediately.

The job creation condition can be expressed more explicitly as a function of endogenous reservation productivities of the two job types. Using the free-entry condition and the relevant wage equations (23) and (19) in the value equations for a new and old job (15) and (16) yields

$$J_{it}^N = (1 - \eta) (a_{it} x^N(z_t) - b) - \eta \kappa \theta_t \quad (27)$$

$$+ E_t \beta (1 - \rho^x) \left[ \phi \int_{\bar{a}_{t+1}^O}^{\infty} J_{t+1}^O(a) f(a) da + (1 - \phi) \int_{\bar{a}_{t+1}^N}^{\infty} J_{t+1}^N(a) f(a) da \right],$$

$$J_{it}^O = (1 - \eta) (a_{it} x^O(z_t) - b) - \eta \kappa \theta_t + E_t \beta (1 - \rho^x) \int_{\bar{a}_{t+1}^O}^{\infty} J_{t+1}^O(a) f(a) da \quad (28)$$

Evaluating these expressions at  $a_{it} = \tilde{a}_t^j$ , noting that  $J_t(\tilde{a}_t^j) = 0$ , and then subtracting the resulting equations from (27) and (28) respectively yields

$$J_{it}^j = (1 - \eta) x^j(z_t) (a_{it} - \tilde{a}_t^j). \quad (29)$$

Substituting (29) into the job creation condition (26) we arrive to an alternative expression for job creation condition which expresses the job creation condition as a function

of the reservation productivities

$$\begin{aligned} \frac{\kappa}{q_t^f} = & E_t \beta (1 - \rho^x) (1 - \eta) \left[ \phi x^O (z_{t+1}) \int_{\tilde{a}_{t+1}^O}^{\infty} (a_{it+1} - \tilde{a}_{t+1}^O) dF (a_{it+1}) \right. \\ & \left. + (1 - \phi) x^N (z_{t+1}) \int_{\tilde{a}_{t+1}^N}^{\infty} (a_{it+1} - \tilde{a}_{t+1}^N) dF (a_{it+1}) \right]. \end{aligned} \quad (30)$$

Naturally, there is only one job creation condition in the model, since all the new jobs are new matches.

### 2.5.2 Job destruction condition

Jobs are endogenously destroyed when the realization of match productivity makes the value of the match go to zero such that

$$S_{it}^j(\tilde{a}_t^j) = 0, \quad j = N, O \quad (31)$$

This condition implicitly determines the reservation productivities for old and new jobs. Because new and old jobs differ by productivity dynamics also the reservation productivities, and thus job destruction rates, for old and new matches are distinct. Due to the assumption of Nash bargaining this reservation productivity can equally be determined by the value of  $\tilde{a}_t^j$  at which match surplus is zero for either the firm or the worker.

Setting (27) and (28) to equal zero and substituting (29) we obtain the following job destruction conditions for new and old jobs

$$\tilde{a}_t^N x^N (z_t) - b - \frac{\eta}{1 - \eta} \kappa \theta_t \quad (32)$$

$$\begin{aligned} & + E_t \beta (1 - \rho^x) \left[ \phi x^O (z_{t+1}) \int_{\tilde{a}_{t+1}^O}^{\infty} (a_{it+1} - \tilde{a}_{t+1}^O) dF (a_{it+1}) \right. \\ & \left. + (1 - \phi) x^N (z_{t+1}) \int_{\tilde{a}_{t+1}^N}^{\infty} (a_{it+1} - \tilde{a}_{t+1}^N) dF (a_{it+1}) \right] \\ = & 0. \end{aligned}$$

$$\tilde{a}_t^O x^O (z_t) - b - \frac{\eta}{1 - \eta} \kappa \theta_t + \quad (33)$$

$$\begin{aligned} & E_t \beta (1 - \rho^x) x^O (z_{t+1}) \int_{\tilde{a}_{t+1}^O}^{\infty} (a_{it+1} - \tilde{a}_{t+1}^O) dF (a_{it+1}) \\ = & 0. \end{aligned}$$

We can relate the two reservation productivities by setting the LHS of (32) and (33) equal and cancelling terms. This yields

$$\begin{aligned}
E_t \beta (1 - \rho^x) (1 - \phi) & \left[ x^N(z_t) \int_{\tilde{a}_{t+1}^N}^{\infty} (a_{it+1} - \tilde{a}_{t+1}^N) dF(a_{it+1}) \right. \\
& \left. - x^O(z_t) \int_{\tilde{a}_{t+1}^O}^{\infty} (a_{it+1} - \tilde{a}_{t+1}^O) dF(a_{it+1}) \right] \\
= & \tilde{a}_t^O x^O(z_t) - \tilde{a}_t^N x^N(z_t)
\end{aligned} \tag{34}$$

This relation will be used in the calibration of reservation productivities.

## 2.6 Aggregate output

Aggregate output  $Q_t$  is determined by output produced by workers in the new and old matches. Integrating over the production of all matches yields:

$$Q_t = (1 - \rho_t^N) (1 - \phi) n_t^N x^N(z_t) H(\tilde{a}_t^N) + (1 - \rho_t^O) [n_t^O + \phi n_t^N] x^O(z_t) H(\tilde{a}_t^O) \tag{35}$$

where  $H(\tilde{a}_t^j) = \int_{\tilde{a}_t^j}^{\infty} a_{it} \frac{f(a_{it})}{1 - F(\tilde{a}_t^j)} da_{it}$ ,  $j = N, O$ , is the conditional expectation of productivity realizations in new and old matches, and where we have used the fact that  $(1 - \rho_t^j) = (1 - \rho^x)[1 - F(\tilde{a}_t^j)]$ . Note that although the match specific productivity draws arrive from the same distribution  $F(\cdot)$ , possibly differing reservation productivities imply that the average productivity of the match types may differ. This is the case when we allow for vintage structure and set  $\lambda \neq 0$ .<sup>8</sup> Finally, aggregate income  $Y_t$  defined as total production net of vacancy costs is:

$$Y_t = Q_t - \kappa v_t. \tag{36}$$

## 3 Calibration

We study the model's properties by linearizing the respective equilibrium conditions around their deterministic steady state and then evaluating the model's performance by means of impulse responses and stochastic simulations.<sup>9</sup> In particular, we compare the main unconditional moments produced by different versions of the model to those of the

<sup>8</sup>In principle, one could also assume that the distributions for the first and subsequent period matches are different. This approach has been taken for instance in Mortensen and Nagypal (2007)

<sup>9</sup>Linearised equations and the deterministic steady state equations are provided in appendix A.1 and A.2.

quarterly US data during 1951-2003. Our main interest is to contrast the performance of the model with heterogenous matches to the standard model. The standard model is obtained by setting  $\phi = 1$ ,  $\lambda = 0$  and  $\gamma = 1$ . Our strategy is to calibrate the standard model following typical values from the literature (e.g. Walsh (2003, 2005), Trigari (2004), Krause and Lubik (2003) and den Haan et al. (2000)). Table 1 summarizes the calibration of the standard model.

The quarterly discount factor is set to  $\beta = 0.99$ . Job flows are determined by the matching and separation probabilities of firms and workers. The quarterly rate of filling vacancies is set to  $\bar{q}^f = 0.71$ , following den Haan et al. (2000). The job finding probability of the workers is set endogenously to  $\bar{q}^w = 0.61$ . This implies that labour market tightness  $\bar{\theta}$  is 0.87. Shimer (2005) reports monthly job finding probability to be 0.45 in the US. If we aggregate the monthly job finding probability of 0.45 to a quarterly frequency, we get  $\bar{q}^w = 0.83 = (1 - (1 - 0.45)^3)$ . This is somewhat higher than our value of 0.61. For the matching function, we set  $\alpha = 0.4$ . This is in accordance with the empirical studies of the matching function<sup>10</sup>. As for the worker's bargaining power and value of leisure, we use a standard calibration of  $\eta = \alpha$ . This internalizes the search externality.<sup>11</sup> The size of the labour force is normalized to one and the employment rate is set to  $\bar{n} = 0.94$ , which implies an unemployment rate of 6 percent, close to true mean in the US data. For the exogenous job destruction rate we use the value calibrated by den Haan et al. (2000)  $\rho^x = 0.068$  and this is the same for both new and old matches.<sup>12</sup>

Since our model makes a distinction between new and continuing matches, we need to determine the relative share of new and old matches in the steady state. We calibrate this share such that at given values of  $\phi$  and  $\lambda$ , the aggregate job destruction rate is consistent with the empirical value of 0.1. We do this by employing the aggregate (steady state) job destruction rate given by (51). After fixing the ratio of old matches to employment  $\bar{n}^O/\bar{n}$ , we use the equation

$$\frac{\bar{n}^O}{\bar{n}} = \frac{(1 - \bar{\rho}^O) \phi}{\bar{\rho}^O + \phi(1 - \bar{\rho}^O)} \quad (37)$$

to compute  $\bar{\rho}^O$  and thus reservation productivity  $\bar{a}^O$  for old jobs at given  $\phi$ . After finding

<sup>10</sup>See e.g. Petrongolo and Pissarides (2001) and Blanchard and Diamond (1989).

<sup>11</sup>This is also known as Hosios (1990) efficiency condition: Workers bargaining power  $\eta$  is equal to elasticity of matching function with respect to unemployment. This makes bargaining efficient, in the sense that it maximizes the present value of market and non market income net of vacancy costs in the standard model. See Shimer (2005) for details.

<sup>12</sup>In the steady state, a measure of old matches which is destroyed must be equal to the measure of new matches that become old minus the measure which is destroyed. This secures that steady state distribution of old and new matches is well defined and constant in the steady state.

$\bar{\rho}^O$  we infer  $\bar{n}^N$  from the aggregate constraint  $\bar{n}^N = \bar{n} - \bar{n}^O$ . Using a linearized version of (34) and  $\bar{a}^O$  computed earlier we can then find  $\bar{\rho}^N$  i.e. the reservation productivity of the new matches  $\bar{a}^N$ . Note that in the standard case, where  $\phi = 1$ , (37) implies that  $\bar{n}^O = (1 - \bar{\rho}^O) \bar{n}$ . This is natural, since it states that, in the steady state, the measure of old matches must be equal to  $\bar{n}$  minus those destroyed. We assume that  $F(\tilde{a}^j)$ ,  $j = N, O$  is log normal c.d.f. with support  $\mu_{\ln A} = 0$  and  $\sigma_{\ln A} = 0.12$ .<sup>13</sup> These values are roughly consistent with den Haan et al. (2000) and Walsh (2005) and Krause and Lubik (2005).

Once reservation productivities for new and old matches have been found, we infer  $m(u_t, v_t)$  from the steady state equation (3) and then compute  $\bar{v}$  and  $\bar{q}^w$  from (4) and (5). The level parameter  $A$  in the matching function is then computed from the following steady state condition:

$$\bar{n}^N = \frac{A\bar{\theta}^{1-\alpha}\bar{u}}{1 - (1 - \phi)(1 - \bar{\rho}^N)} \quad (38)$$

Note again that in the standard case where  $\phi = 1$ , this reduces to  $\bar{n}^N = A\bar{\theta}^{1-\alpha}\bar{u}$ . The periodical search cost  $\kappa$  and the value of leisure  $b$  are inferred from the steady state job creation condition (52) and the job destruction condition for old jobs (54), respectively.

Finally, the log of the aggregate productivity shock  $z_t$  is assumed to follow the first order autoregressive process. We estimate the  $AR(1)$  coefficient and the standard error of innovations using the US data (see appendix B for details of the data). The point estimate for the first order autocorrelation coefficient is 0.78 with an unconditional standard deviation of 0.014 for the HP(1600) filtered productivity process. Innovations have a standard error of 0.0088.<sup>14</sup> These values are basically the same as those reported in Hagedorn and Manovskii (2008).

As for the explicit values of the parameters  $\phi$ ,  $\gamma$  and  $\lambda$ , recall that the standard model is reproduced by setting  $\phi = \gamma = 1$  and  $\lambda = 0$ . In this case, all new matches are converted to old before the endogenous decision to continue with the match takes place. In addition, all matches are similar in their responsiveness to productivity shocks and the steady state level of match productivities are the same.

To calibrate the heterogeneity, we draw on different literature. First, our preferred value for  $\gamma$ , the responsiveness of old matches to aggregate technology shocks, involves matching the responsiveness of wages of new and old jobs to aggregate unemployment rate roughly in accordance with the empirical literature summarized for instance in

<sup>13</sup>Mortensen and Pissarides (1994) used uniform distribution for  $F(\tilde{a})$ .

<sup>14</sup>Labour productivity is measured in terms of log real non-farm output per log total non-farm employment

Parameter	Value	Steady state	Value
$\beta$	0.99	$\bar{n}$	0.94
$\eta$	0.60	$\bar{n}^O/\bar{n}$	0.9
$\alpha$	$\eta$	$b/\bar{w}$	0.63
$\mu_{\ln A}$	0	$\kappa v/\bar{y}$	0.009
$\sigma_{\ln A}$	0.12	$\bar{v}$	0.13
$\rho_x$	0.068	$\bar{u}$	0.15
$\gamma$	1	$\bar{\rho}^O$	0.10
$\phi$	1	$\bar{\rho}^N$	0.10
$\lambda$	0	$\bar{q}^f$	0.71
$\sigma_\epsilon$	0.0088	$\bar{q}^w$	0.61
$\rho_z$	0.78	$\bar{\rho}$	0.10
$A$	0.65		

Table 1: Parameters and Steady State Values in the Standard Model

Pissarides (2008). In our preferred calibration, the difference between the elasticity of wages to unemployment of new and old matches is roughly 1.3, which is a plausible value in the light of the empirical evidence summarized in Pissarides (2008). This is achieved by setting  $\gamma = 0.5$ . Note that, in general,  $\gamma < 1$  captures the basic idea of Caballero and Hammour (1998) and Campbell (1995) that the new firms receive productivity associated with the latest technology.

In order to calibrate the productivity level parameter  $\lambda$ , we exploit the following features. On the one hand,  $\lambda < 0$  implies that in the steady state, the new jobs have a higher destruction rate than the old jobs. On the other hand,  $\lambda < 0$  reduces an incentive to replace old matches with new ones in the face of positive and persistent productivity shocks. Combining  $\lambda = -0.03$  with  $\phi = 0.1$  and  $\gamma = 0.5$  results in the standard error of aggregate job separation rate which is reasonably close to its empirical value of 2.78. At the same time, the separation rate of new jobs is roughly 20 % higher than that in the old jobs, also a reasonable number. Finally, as for the calibration of  $\phi$ , we rely on Baldwin (1995), who has found that about half of the new entrants die within the first decade, while those who survive reach average productivity in about a decade. This seems to support the fact that transition from new to old is a very slow process, i.e. that realistic values for  $\phi$  should be closer to zero than 1. Since the empirical evidence does not give us a direct way to calibrate exact values for  $\phi$ , we let  $\phi \in (1, 0.3, 0.1)$ . Our

preferred value for  $\phi$  is 0.1.

## 4 Equilibrium responses to technology shocks

After a persistent technology shock, the standard Mortensen and Pissarides (1994) model suggests that vacancies and thus also net job creation reacts on impact, while output and unemployment follow a hump-shape pattern. Labour market tightness or the vacancies to unemployment ratio,  $v/u$ , reacts also on impact. Due to the hump shaped pattern of the unemployment rate, the model with endogenous job destruction has difficulties to produce a negative correlation between vacancies and unemployment, *i.e.* that after a positive technology shock the vacancy rate goes up and the unemployment rate drops contemporaneously. Our model with heterogeneous matches exhibits less such difficulties.

### 4.1 Responsiveness of new and old matches

We start the discussion of the model's performance by considering the heterogeneity in responsiveness of new and old matches to aggregate productivity shocks. We set  $\gamma = 0.5$  s.t. old matches are less responsive than new hires and study two cases where the transition rate  $\phi = 0.5$  and  $\phi = 0.3$ , while keeping  $\lambda = 0$  in both cases. In each case, we re-calibrate the share of old matches to total employment such that  $\bar{\rho} = 0.1$ . Notice also that in the steady state the reservation productivity of old and new matches are the same. This is due to the fact that  $\lambda = 0$  so the steady state average productivity level of new and old matches is equal *i.e.* we abstract from the vintage structure. We discuss the role of vintage later on.

#### 4.1.1 Impulse responses

Figure 1 draws the impulse responses. In response to aggregate productivity shocks, the model with heterogeneous matches ( $\gamma = 0.5$  with  $\phi = 0.3$  and  $\phi = 0.1$ ) shows a clearly stronger response of vacancies and job creation when compared to the standard model. The response of vacancies also shows clearly more persistence. On the contrary, the response of job destruction becomes muted.

Comparing the relative responses of job creation and job destruction, it is clear that heterogeneity shifts employment adjustment increasingly to the job creation margin (compare the case where  $\phi = 0.3$  and  $\phi = 0.1$  to the standard model in Figure 1. After a positive technology shock, a temporary productivity difference between new and old

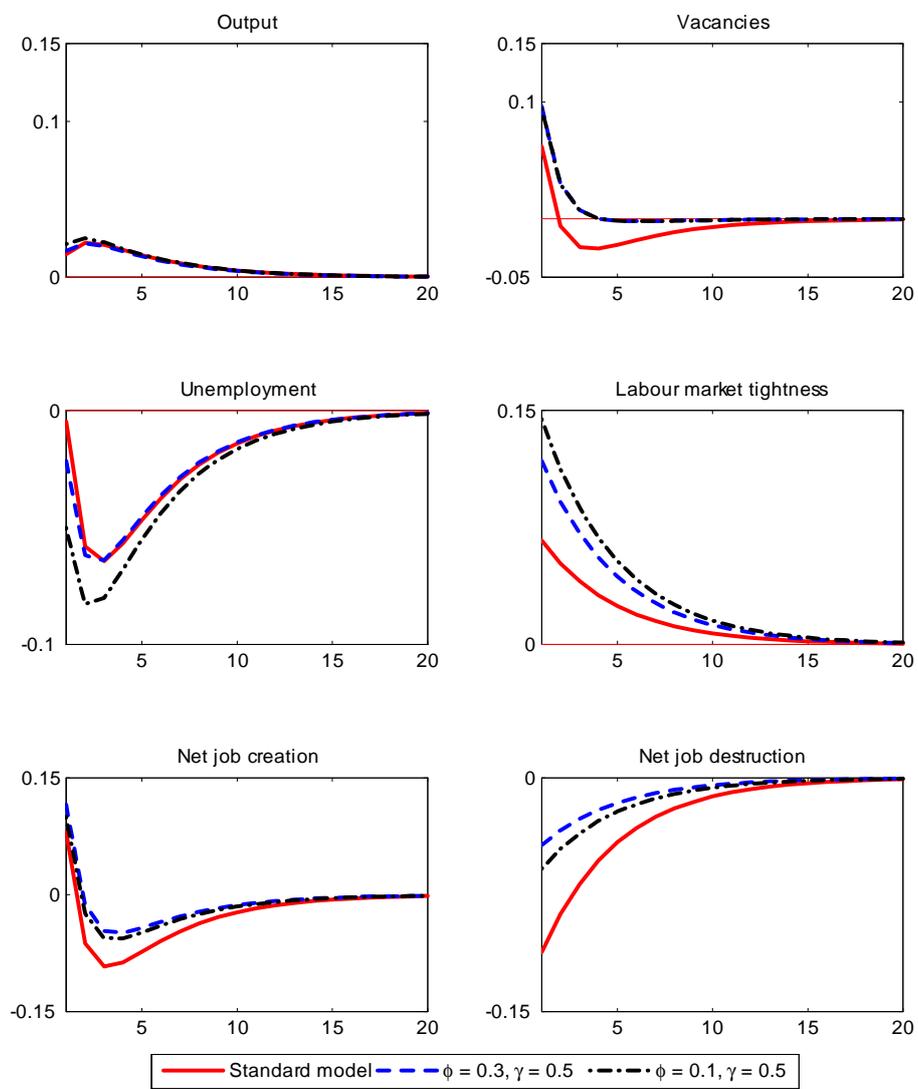


Figure 1: Equilibrium responses to persistent technology shock in different model specifications

jobs creates an incentive for firms to create new productive vacancies and destroy the old matches that are temporarily less productive. As a result, job destruction decreases less and job creation increases more, relative to the standard model. This shifting of the adjustment margin towards job creation becomes stronger as the probability of transition from new to old jobs is smaller.

This is clearly visible in Figure 2, which shows the equilibrium responses of employment, wage and job destruction in new and old matches in the different model specifications: An aggregate productivity shock generates a temporary but persistent productivity difference between new and old matches. This makes employment adjustment increasingly procyclical (a-cyclical) in the new matches( old-matches), as transition from new to old matches becomes more sluggish. At the same time, the employment response in the old matches becomes increasingly muted. At the aggregate level, the employment response is also muted, since most of the variation in aggregate employment comes from employment variation of old matches. At the same time, however, the shift of the adjustment margin from destruction to creation amplifies quite strongly the response of unemployment, especially when the transition from new to old jobs is slow (see Figure 1). In response to a positive productivity shock, the destruction rate of new jobs reacts strongly counter-cyclically, leading to a large drop in the flow of workers from new jobs to unemployment, and thus in the measure of searching workers. This effect outweighs the procyclical reaction of the destruction of old jobs.

The feature that job destruction of old matches becomes pro-cyclical when  $\phi$  is smaller is a "natural" property of the model (see the low-middle panel of Figure 2). A low  $\phi$  implies that the expected time of remaining new (and consequently more productive in case of positive productivity shock) is relatively long. This means that the expected surplus of new matches is relatively high compared to the expected surplus of old matches and thus a high surplus differential makes it beneficial to destroy old matches and create new matches. With a higher transition rate  $\phi$  jobs become old at a higher rate (faster), so the difference in expected surplus between new and old matches is smaller.

To demonstrate this further, consider the extreme high value  $\phi = 1$  where matches transit immediately to being old before production starts.<sup>15</sup> In this case newly created and older matches all have equal productivity and react to productivity shocks in an analogous way. All matches then have equal expected surplus and there is no reason for replacing old jobs with new ones. In other words, when all matches are homogeneous, a productivity shock will increase the expected surplus of all matches equally, implying a lower reservation productivity  $\tilde{a}_{it}$  for all matches. Higher expected surplus induces more

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<sup>15</sup>This case is effectively the benchmark Mortensen-Pissarides model.

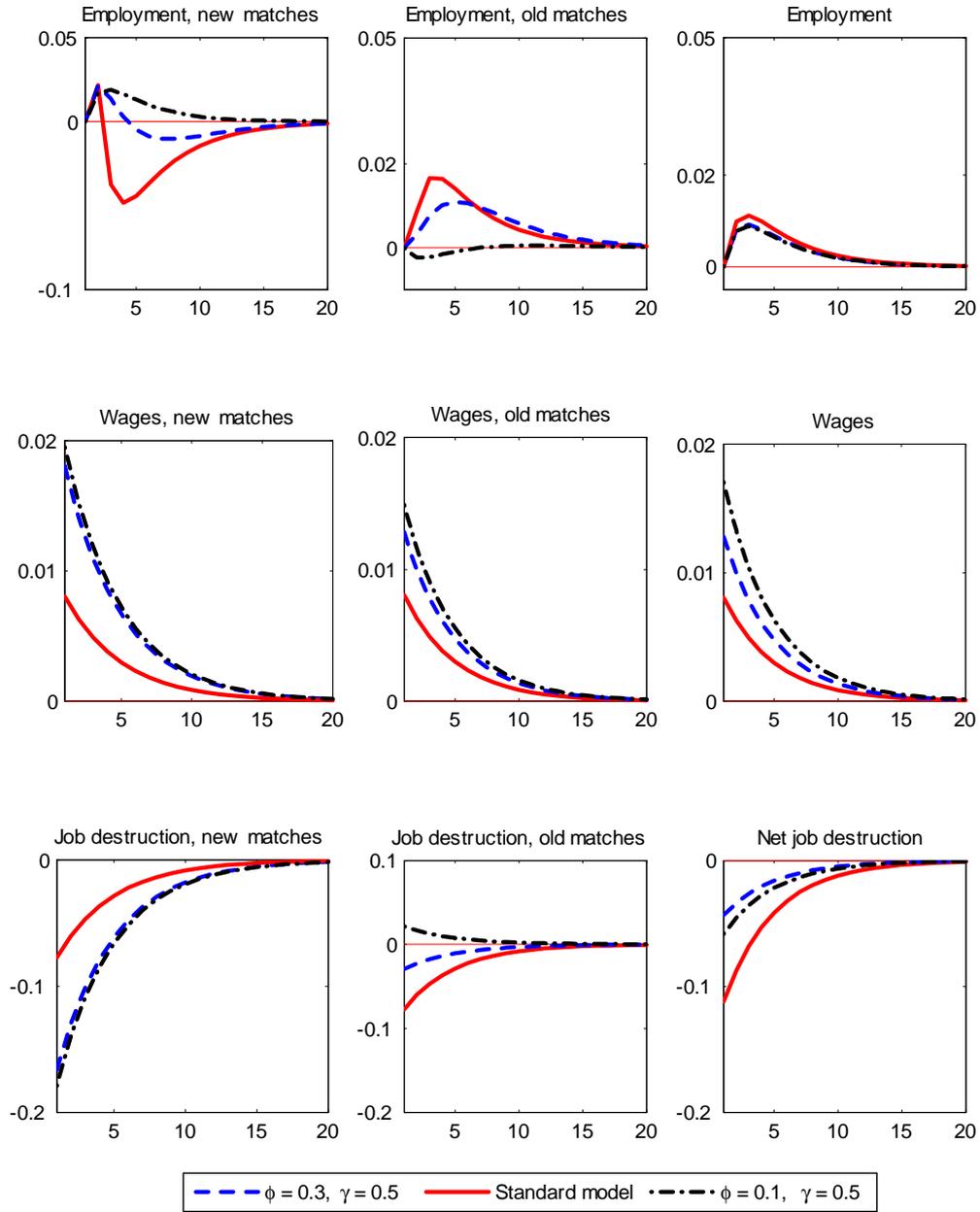


Figure 2: Equilibrium responses to persistent technology shock in different model specifications - new and old matches

job creation and the lower reservation productivity reduces job destruction, so job creation is procyclical and job destruction is countercyclical. As the transition probability  $\phi$  decreases, the expected duration of a match remaining new (high productivity and highly shock responsive) increases and the expected surplus of new matches increases relative to that of old matches. A firm with an old match will now observe the match specific productivity and implied surplus of the current match and the expected value of posting a new vacancy. If the latter value is higher, the firm will destroy the current old match and post a vacancy to search for a new match. The lower is the transition probability  $\phi$  and therefore the higher the difference in match surplus between new and old matches, the higher is the reservation value for the match specific productivity for old matches that leads to job destruction and creation of a new vacancy. Thus the model produces a "creative destruction" effect that increases as the transition rate decreases.

Figure 2 also shows that the wages in the new matches are more volatile than the wages in the old matches when the heterogeneity is allowed for. This is a direct consequence of new matches being more responsive to productivity fluctuations than the old matches, and the fact that Nash bargaining takes place separately to the new and old. Note also that temporary shifts in the composition of new and old matches drive partly the fluctuations of aggregate wages. The positive productivity shock increases the number of new matches with more responsive wages contributing to a stronger response of the aggregate wage in the economy. At the same time, wage fluctuations in continuing matches are moderate. This is consistent with the findings of Haefke et al. (2008) who argue that the relevant wage data for the search model are the wages of new hires, not aggregate wages. They show that wages for newly hired workers respond strongly, even one-for-one, to changes in labour productivity. Also Carneiro et al. (2008) cast some doubt on whether wage stickiness is primary explanation for the unemployment volatility puzzle. Using matched longitudinal employer-employee data from Portugal, they find that the real wage of continuously employed workers is moderately procyclical, while entering worker's real wage is strongly procyclical during 1986-2005. They find that a one point increase in the unemployment rate decreases wages of newly hired male workers by around 2.5 % and by just 1.5 % for workers in continuing jobs. In other words, the elasticity of wages to unemployment of newly hired workers is roughly 1.7 times larger than in the continuing jobs. In our model, the difference between the elasticity of wages to unemployment of new and old matches is roughly 1.3 in the calibration of the model where  $\phi = 0.1$ , and  $\gamma = 0.5$ . This is in the ball park of Carneiro et al. (2008), and also with the evidence summarized in Pissarides (2008).

### 4.1.2 Fluctuations and correlations

Much of the literature has already explored the quantitative performance of the search models (for discussion, see e.g. Shimer (2005) and Yashiv (2006)) by studying the model's performance by means of stochastic simulations. This literature has found that the standard matching model is not able to generate enough fluctuations in labour market variables, when the main driving force of those fluctuations are productivity shocks. The standard matching model produces fluctuations in labour market variables that are 2 – 3 times smaller than they should be (see also Table 2). Furthermore, the standard matching model has difficulties to match dynamic cross-correlations between labour market variables and output and unemployment. The standard model fails in particular with respect to cross-correlations between vacancies and output and vacancies and unemployment: It generates too high correlation between job destruction and output and unemployment, but far too little negative (positive) correlation between vacancies and unemployment rate (output) (See Figures 3-4). A similar failure of the standard model was found also in Merz (1995) and Andolfatto (1996), who enriched the standard search model with a real business cycle framework. As Krause and Lubik (2007) point out, employment adjustment in the standard model takes place through a strong drop in separations rather than through increased job creation because firms can instantaneously and costlessly adjust employment at the separation margin.<sup>16</sup> On the other hand, job creation is time consuming and costly. Therefore firms increase employment by keeping even less productive workers instead of engaging in time consuming and costly search.<sup>17</sup>

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<sup>16</sup>The fact that job separations, or job destruction, appear only moderately cyclical and volatile in comparison to job creation, a feature emphasized by Shimer (2005), has led several authors to abstract from models with endogenous job destruction and revert to models with exogenous job destruction (Trigari and Gertler, 2006 etc.). However, Elsby, Michaels, and Solon (2007) argue for an important role of counter-cyclical inflow into unemployment, or separations, over the business cycle in the US. They argue that "complete understanding of cyclical unemployment requires explanation of counter cyclical unemployment inflow rates as well as procyclical outflow rates [cf. Elsby et al. (2007), p. 23]".

<sup>17</sup>Empirically, labour market variables fluctuate much more than productivity and employment (and output). For example, fluctuations in unemployment rate have been about ten times larger than fluctuations on employment rate and job finding rate in the US during 1951-2003. Fluctuations in vacancies have been about 4 times fluctuations in job separations and about 2 times fluctuations in unemployment rate and job finding rate. The job finding rate, employment, labour market tightness, and vacancies are highly pro-cyclical, while unemployment and job destruction are counter-cyclical. Furthermore, fluctuations in job destruction are less persistent than fluctuations in output or productivity.

	Data	Standard Model	Het. Match I	Het. Match II	Het. Match III
		$\gamma = 1$	$\gamma = 0.5$	$\gamma = 0.5$	$\gamma = 0.5$
		$\phi = 1$	$\phi = 0.3$	$\phi = 0.1$	$\phi = 0.1$
		$\lambda = 0$	$\lambda = 0$	$\lambda = 0$	$\lambda = -0.03$
Job finding	5.25	1.06	1.85	2.03	1.87
Job destruc.	2.76	4.47	1.71	2.06	3.00
Employment	0.65	0.51	0.43	0.36	0.37
Lab mkt tigh.	12.6	2.63	4.63	5.07	4.67
Wage	0.43	0.32	0.55	0.60	0.52
Unemploym.	6.13	2.93	2.90	3.25	3.51
Vacancies	6.83	2.57	3.13	2.88	2.32

Table 2: Volatility of selected variables in the data and in different model specifications. Volatilities are measured by standard errors of HP(1600) filtered series, and relative to output.

	Data	Standard Model	Het. Match I	Het. Match II	Het. Match III
		$\gamma = 1$	$\gamma = 0.5$	$\gamma = 0.5$	$\gamma = 0.5$
		$\phi = 1$	$\phi = 0.3$	$\phi = 0.1$	$\phi = 0.1$
		$\lambda = 0$	$\lambda = 0$	$\lambda = 0$	$\lambda = -0.03$
Job finding	0.75	0.89	0.93	0.95	0.95
Job destruct.	-0.58	-0.89	-0.94	-0.95	-0.94
Employment	0.70	0.86	0.83	0.76	0.79
Lab mkt tigh.	0.83	0.91	0.93	0.95	0.95
Wages	0.29	0.89	0.93	0.95	0.95
Unemploym.	-0.78	-0.91	-0.94	-0.98	-0.99
Vacancies	0.85	0.23	0.51	0.56	0.42

Table 3: Contemporaneous correlations with output in different model specifications

	Data	Standard Model	Het. Match I	Het. Match II	Het. Match III
		$\gamma = 1$	$\gamma = 0.5$	$\gamma = 0.5$	$\gamma = 0.5$
		$\phi = 1$	$\phi = 0.3$	$\phi = 0.1$	$\phi = 0.3$
		$\lambda = 0$	$\lambda = 0$	$\lambda = 0$	$\lambda = -0.03$
Job finding	0.80	0.58	0.63	0.55	0.61
Job destruct.	0.48	0.58	0.63	0.55	0.59
Employment	0.92	0.76	0.78	0.75	0.77
Lab mkt tigh.	0.89	0.58	0.69	0.55	0.61
Wages	0.81	0.58	0.63	0.55	0.60
Unemploym.	0.87	0.78	0.85	0.83	0.84
Vacancies	0.91	0.18	0.25	0.15	0.18
Output	0.81	0.83	0.81	0.75	0.78

Table 4: Autocorrelations in different model specifications

In the previous section, the impulse response analysis shows that match heterogeneity amplifies the response of vacancies to productivity shocks. This can also be seen from Table 2. Figures 3-4 show that the model with heterogeneous matches does also a better job in terms of dynamic cross-correlations. In particular, the model matches much better the pattern of dynamic cross-correlations between vacancies and unemployment and vacancies and output than the standard model. The model with heterogeneous matches produces much higher contemporaneous correlation between vacancies and output (and unemployment), without compromising the fit in the other dimensions.<sup>18</sup>

From Table 2, columns Het Match I - Het Match III, we can also confirm that the model with heterogeneous matches generates more fluctuations in vacancies and unemployment, compared with the standard model.<sup>19</sup> This higher volatility in vacancies is

<sup>18</sup>Introducing convex vacancy costs would also help to match the persistence in vacancy creation, as well as strong pro-cyclical (counter-cyclical) correlation between vacancies and output (unemployment). See for instance Gertler and Trigari (2006). Convex vacancy costs, however, strongly increases job destruction even beyond the standard model and what is observed in the data.

<sup>19</sup>We have compared the standard model and the model with heterogeneous matches (and vintage), also with the model with convex vacancy costs. Convex vacancy costs are supported by the empirical literature, such as Yashiv (2000a, 2000b). Convex vacancy costs help to match the high correlation between vacancies and output and unemployment. However, introduction of convex vacancy costs into the model strongly reduces the volatility of vacancy creation, since convex vacancy costs makes firms to smooth vacancy creation over time. Moreover, job destruction becomes more volatile, being a natural

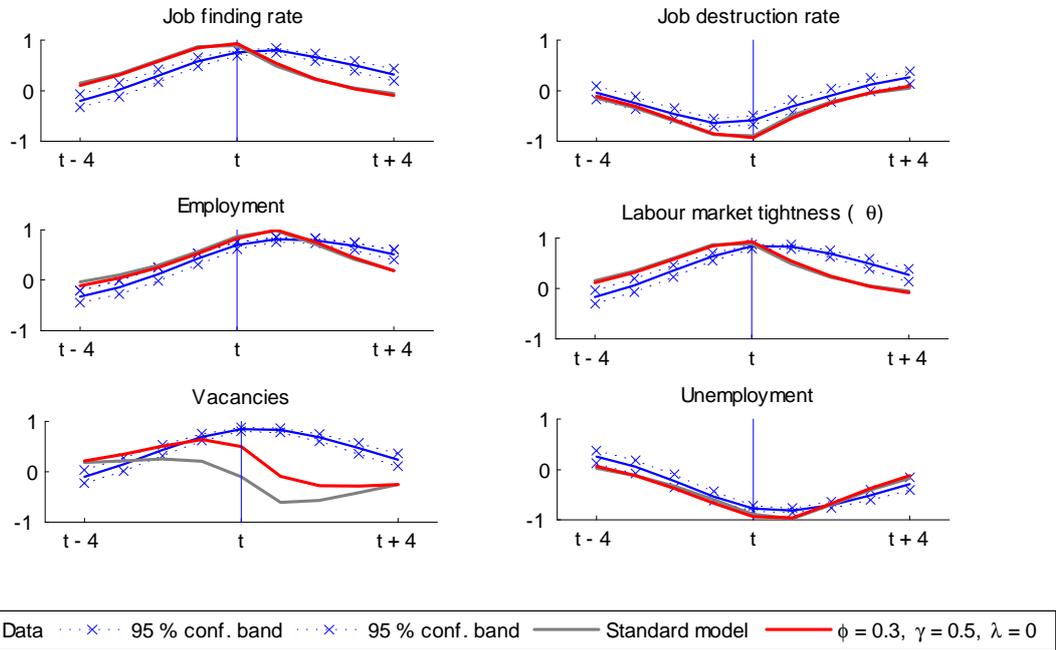


Figure 3: Dynamic cross-correlations of selected labor market variables and output at  $t \pm i$ . Note: Blue (dark) lines correspond to the data, red (semi-light) lines correspond to the model with heterogeneous matches and grey (light) lines correspond to the standard model.

due to the shifting of the adjustment margin from job destruction to job creation discussed earlier. There is also an increase in the volatility of labour market tightness and job finding rate in the model with heterogeneous matches. Due to the increased relative importance of job creation margin, however, the model with heterogeneous matches produces less fluctuations in job destruction, bringing the volatility of the job destruction to a value even below the data. Increased contemporaneous correlation between vacancies and output is also clearly visible in Table 2. The standard model produces contemporaneous correlation of vacancies and output of 0.23, while the model with heterogeneous matches brings this correlation up to roughly 0.5. In the US quarterly data, this correlation is 0.85. A similar pattern is visible in the contemporaneous correlation between vacancies and unemployment as well<sup>20</sup>.

consequence of lower volatility of vacancy creation. The comparison is available on request from the authors.

<sup>20</sup>Note that an alternative way to respond to unemployment volatility puzzle is provided in Hagedorn and Manovskii (2008). They propose to calibrate the value of unemployment to a much higher value

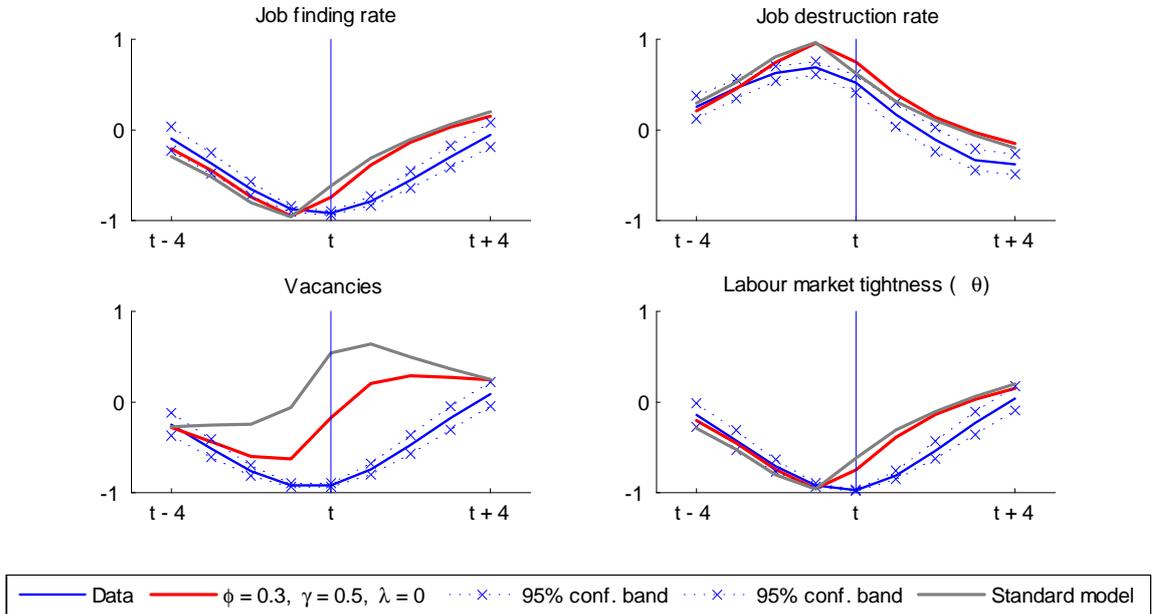


Figure 4: Dynamic cross-correlations of selected labour market variables and unemployment at  $t \pm i$ .

Note: Blue (dark) lines correspond to the data, red (semi-light) lines correspond to the model with heterogeneous matches and grey (light) lines correspond to the standard model.

## 4.2 Average productivity difference

We have demonstrated above that the model with heterogeneous matches is able to fit better the dynamic cross correlations between vacancies and output and unemployment than the standard model. The model also generates a well behaving Beveridge curve despite of endogenous job destruction as well as more fluctuations in vacancies, in the job finding rate and in the labour market tightness. However, this comes at some cost, by <sup>1</sup>than one implied by the unemployment benefits. Moreover, they suggest to calibrate the value of bargaining power of workers to very low value. One problem with their approach is that the steady state unemployment rate becomes very sensitive to assumed values of  $b$ , as pointed out by Costain and Reiter (2008). Their approach also does not lead into strong procyclical (countercyclical) relationship between output (unemployment) and vacancies. It merely helps to match the volatility of vacancies and unemployment.

reducing strongly the cyclical fluctuations in job destruction (See Tables 2-3). Note that empirical evidence suggest that productivity differences are persistent, suggesting that in our model the transition probability  $\phi$  should be calibrated to a relatively low value, perhaps even lower than 0.1. At the same time, however, a small value of  $\phi$  makes the adjustment at the creation margin stronger, such that that destruction of old matches becomes eventually pro-cyclical. While this is not totally implausible due to cleansing type arguments, pro-cyclical aggregate job destruction is not consistent with the data.

Introducing vintage structure i.e. allowing for the average productivity to differ between matches provides a possible empirically justified remedy for this problem, as it is consistent with the microlevel evidence. The empirical evidence points to that fact that the productivity of new jobs is below that of the already existing jobs on average. In our model, this *reduces* an incentive to replace old matches with new ones in the face of positive and persistent productivity shocks (see Table 2, Het. Match III). While this reduces somewhat the volatility of vacancies, the model with heterogeneous matches and long run productivity difference captures better the key correlation structure and relative volatility observed in the data in general. Furthermore, the empirical findings also support the view that the job destruction probability is higher in the new matches relative to older ones. This feature is captured by our model, since with  $\lambda < 0$ , endogenous separation rate of new jobs is higher than that of the old jobs: when  $\gamma = 0.5$ ,  $\phi = 0.1$ ,  $\lambda = -0.03$  an endogenous separation rate for the new matches is 0.11, while for the old and more productive matches, endogenous separation rate is 0.09. In other words, job separation rate of the new matches is about 20 % higher than in the old matches.

## 5 Introducing nominal rigidities

In order to gain further understanding on macroeconomic consequences of match heterogeneity, we extend the model by allowing for nominal rigidities. The search framework has been found a useful tool to model labour markets in the standard New Keynesian setup, which otherwise features Walrasian labour markets. The basic setup is laid down for instance in Walsh (2005).

The model consists of a continuum of households, who purchase consumption goods, and supply one unit of labour inelastically. The standard dynamic optimization problem gives rise to a consumption Euler equation which determines the evolution of a stochastic discount factor and of consumption. Under the assumption of perfect capital markets, the stochastic discount factor is used to value the future expected asset values of employment, unemployment, jobs and vacancies.

Apart from specifying the household's consumption, and thus aggregate demand, the key additional ingredients in the model are price setting and monetary policy. Price setting takes place at separate sectors typically referred to as a retail or a final goods sector. While wholesale firms produce to competitive markets using labour as the only input, the final good firms compete at monopolistic markets. Final good firms simply bundle the intermediate goods and sell directly to the households. In order to capture nominal price stickiness in pricing of the final goods, Walsh (2005) follows Christiano, Eichenbaum and Evans (2001), and assumes that only a fraction  $1 - \omega$  of the firms can optimize their price each period. The remaining firms index their prices to the most recent aggregate rate of inflation. This specification delivers the standard New Keynesian Phillips curve

$$\pi_t = \frac{\beta}{1 + \beta} E_t \pi_{t+1} + \frac{1}{1 + \beta} \pi_{t-1} - \frac{\kappa}{1 + \beta} \hat{\mu}_t \quad (39)$$

for the aggregate inflation rate.  $\hat{\mu}_t$  is the deviation of price markup (mark-up of final over wholesale prices) from its optimal steady state value and  $\kappa \equiv (1 - \omega)(1 - \omega\beta)/\omega$ .

Monetary policy is specified by a Taylor type of rule, where the short-term nominal gross interest rate  $R_t$  is given by

$$R_t = R_{t-1}^{\rho_R} \left( \frac{P_t}{P_{t-1}} \right)^{\phi_\pi(1-\rho_R)} \exp(\epsilon_t^r). \quad (40)$$

$\rho_R$  is the degree of interest rate smoothing,  $\phi_\pi > 1$  is the response coefficient for inflation and  $\epsilon_t^r$  is a serially uncorrelated, mean zero stochastic process representing an unanticipated interest rate shock.

Without going further into the details of the complete model specification<sup>21</sup>, we consider the importance of nominal rigidities in determining the equilibrium responses of output and labour market variables to productivity and interest rate shocks in different model specifications. In calibrating the model, we follow closely Walsh (2005), except that we set habit persistence parameter to zero. We assume a CRRA utility function with the coefficient of relative risk aversion equal to 2. The steady state price markup for retail firms is set equal to 1.1. The degree of price rigidity is determined by the share of firms who do not optimally adjust their price  $\omega$ . We set this fraction to equal 0.5. We set the response coefficient for inflation in the policy rule  $\phi_\pi$  equal to 1.1 which implies a 110 basis points long-run nominal response to a 100 basis point increase in inflation. Finally, we set  $\rho_R = 0.9$  which is roughly consistent with the empirical evidence on high inertia displayed by central bank policy rules (Walsh 2005).

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<sup>21</sup>Complete specification is available on request from the authors.

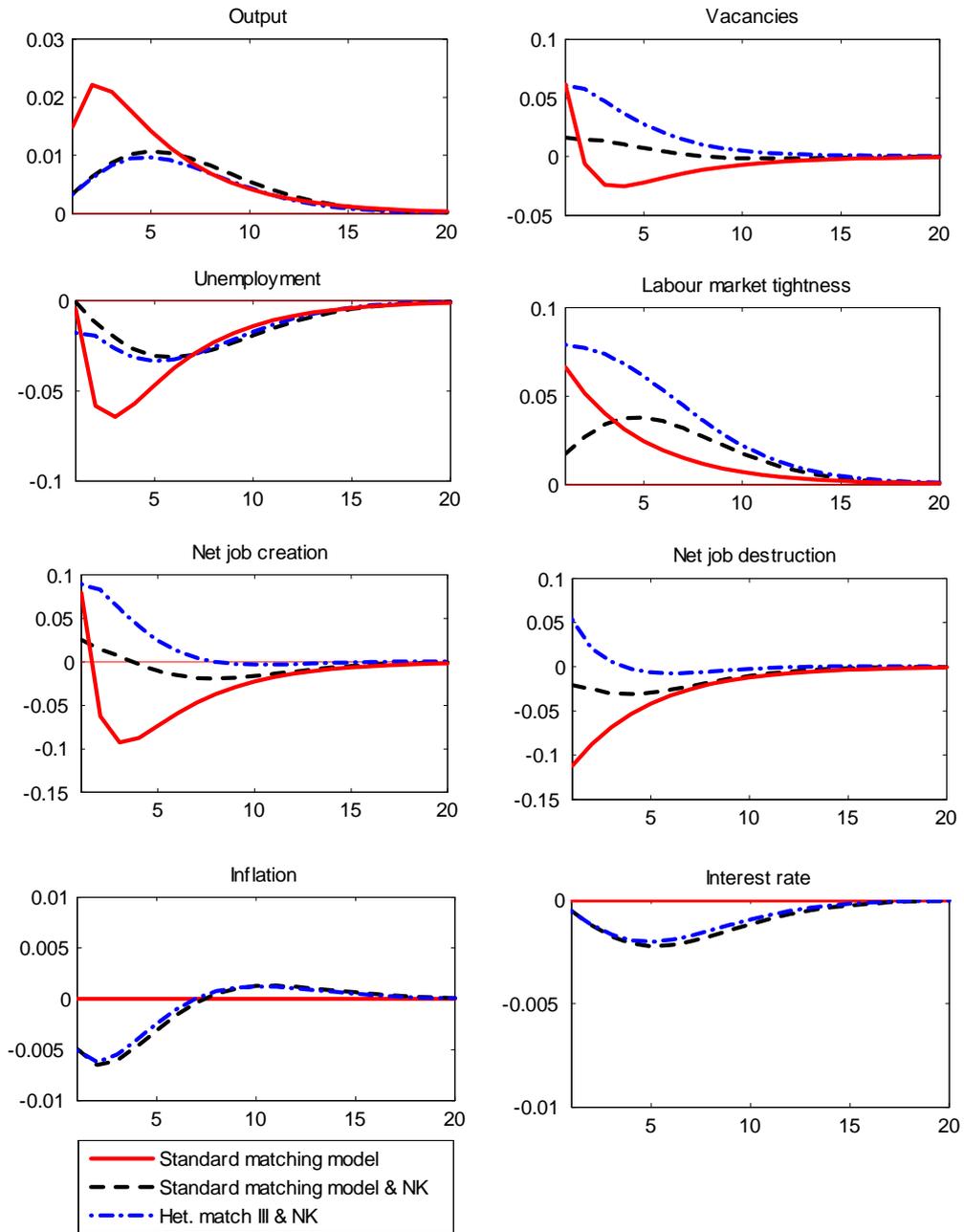


Figure 5: Equilibrium responses to productivity shocks in different model specifications with nominal rigidities

Figure 5 shows the equilibrium responses to a productivity shock in different model specifications. The Figure suggests that nominal rigidities dampen the responses of labour market variables and output to productivity shocks. The responses of output and unemployment show at the same time somewhat more persistence and the peak effects occurs clearly later than in the model without the nominal frictions.

Enriching the standard search model with nominal frictions with the heterogeneous matches improves the model's behavior in the same way as discussed earlier. In response to productivity shocks, responses of the key labour market variables become stronger and more persistent. This is particularly true for vacancies and labour market tightness. As for inflation, output and interest rates, heterogeneity does not have quantitatively important implications. This is primarily due to the fact that in this setup, search frictions *per se* has no implications on price setting behavior of the firms, since vacancy posting decisions and price setting decisions of individual firms occur separately.

What about the transmission of monetary policy? In Figure 6 we consider the impact of an unanticipated change in the interest rate in the standard search model with nominal frictions and the one with heterogeneous matches. We draw the same conclusion as regards the productivity shocks, namely that quantitatively heterogeneity does not have important implications for the transmission of interest rate changes at the aggregate level. However, inspection of dynamics of employment, wages and job destruction in the new and old matches separately reveals some differences. Notably, allowing for heterogeneous matches leads to a more muted employment response of new matches when compared with the standard model. This is mirrored by a stronger impact of job destruction in the new matches. The impact of heterogeneity on the dynamics of old matches is small: this also drives the results at the aggregate level, given that most of the dynamics in the aggregate labour market variables arise from the old matches.

## 6 Concluding remarks

The current labour market matching literature has overlooked the match heterogeneity<sup>22</sup>, and overemphasized the role of wage rigidity as a possible remedy for the difficulty of standard matching models to fit to key moments of the data. In this paper, we have developed a matching model with two types of firm-worker pairs, labelled as new and old. In accordance with the empirical evidence, we have assumed that new matches are more sensitive to productivity fluctuations upon job creation than already existing matches, and extended the model to the case where already existing matches are on

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<sup>22</sup>There are few exceptions, however, such as Mortensen and Nagypal (2007) and Reiter (2006).

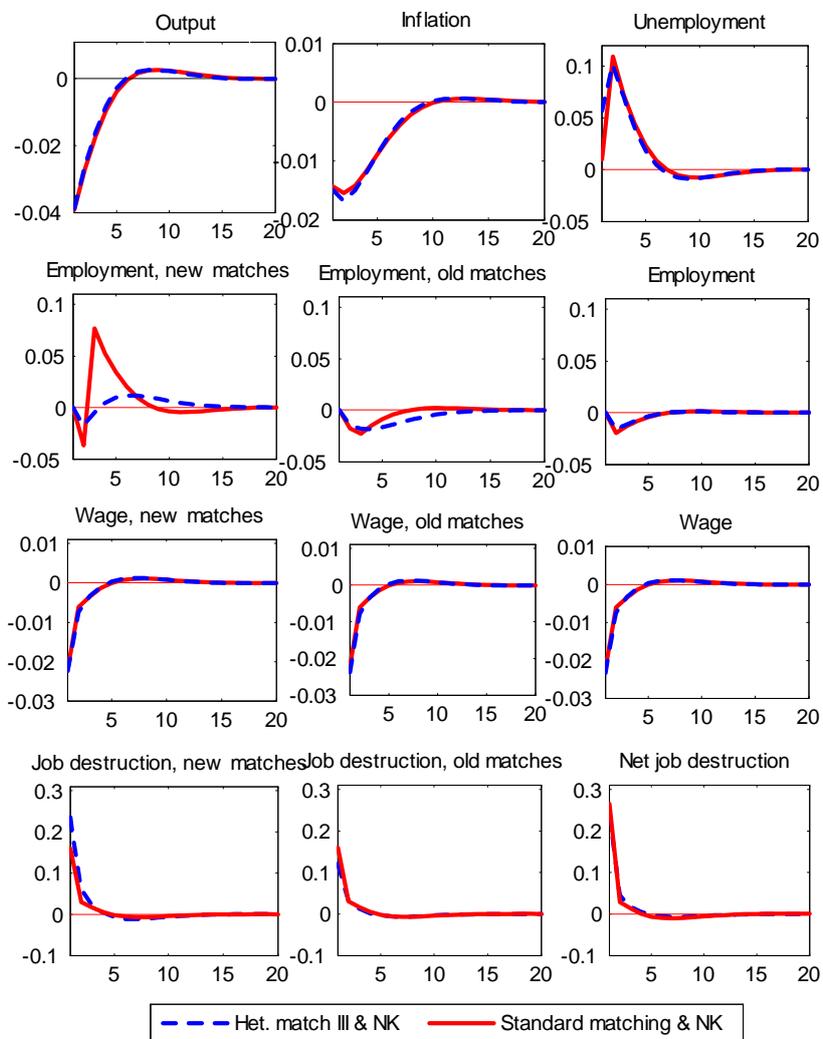


Figure 6: Equilibrium responses to unanticipated interest rate shock in different model specifications with nominal rigidities

average more productive than the new matches. This type of heterogeneity solves a number of problems in the standard matching model. In particular, our model produces a well behaving Beveridge curve despite endogenous job destruction and it narrows the gap between the volatility of the model's labour market variables and actual data. Furthermore, the model captures the dynamic correlations between labour market variables and output (and unemployment) better than the standard matching model, without a need to rely on wage rigidity.

In our model wages of new hires are more responsive to aggregate technology shocks compared to wages of existing hires, consistently with the findings of Haefke et al. (2008), Carneiro et al. (2008), and other studies summarized in Pissarides (2008). We show that persistent productivity differences across matches generated in the model shift the employment adjustment from the job destruction margin towards the job creation margin. In our model, an aggregate productivity shock creates a temporary but persistent productivity difference between the two types of jobs. After a positive and persistent technology shock, this creates an incentive for firms to create new productive vacancies and destroy the old matches that are temporarily less productive. Although employment adjustment does take place through the job destruction margin, this effect makes job destruction less important relative to the standard model. As a result, the model produces a well behaving Beveridge curve, despite job destruction being endogenously determined. Also the volatility of the vacancies and unemployment increases.

Finally, we incorporated nominal frictions into the model following Walsh (2005) and studied transmission of productivity and interest rate shocks in the extended model. As for the interest rate shocks, it turned out that heterogeneity *per se* does not have quantitatively important implications for the transmission of interest rate changes in the model at the aggregate level. An obvious, but not necessarily straightforward, extension of our framework would be to allow price setting and vacancy posting decisions to occur within a single firm, following Krause and Lubik (2005), Kuester (2007), and Thomas (2008). In these models, search frictions give rise to real rigidity, which leads into more sluggish wage and price responses.

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# A Appendix

## A.1 Steady state equations

- Number of new matches that enter a given period

$$\bar{n}^N = \frac{A\bar{u}^\alpha \bar{v}^{1-\alpha}}{1 - (1 - \phi)(1 - \bar{\rho}^N)} \quad (41)$$

- Number of old matches that enter a given period

$$\bar{n}^O = \frac{(1 - \bar{\rho}^O) \phi \bar{n}}{\bar{\rho}^O + \phi(1 - \bar{\rho}^O)} \quad (42)$$

- Aggregate employment

$$\bar{n} = \bar{n}^N + \bar{n}^O \quad (43)$$

- Unemployed job seekers

$$\bar{u} = 1 - \bar{n} + (1 - \phi) \bar{\rho}^N \bar{n}^N + \bar{\rho}^O (\bar{n}^O + \phi \bar{n}^N) \quad (44)$$

- Separation rate for matches of type  $j$

$$\bar{\rho}^j = \rho^x + (1 - \rho^x) \bar{\rho}^{nj}. \quad (45)$$

- Firm's hazard rate

$$\bar{q}^f = \frac{m(\bar{u}, \bar{v})}{\bar{v}} \quad (46)$$

- Worker's hazard rate

$$\bar{q}^w = \frac{m(\bar{u}, \bar{v})}{\bar{u}} \quad (47)$$

- Labor market tightness

$$\bar{\theta} = \frac{\bar{v}}{\bar{u}} \quad (48)$$

- Net job creation rate

$$\bar{j}^{cr} = \frac{\bar{q}^f \bar{v}}{\bar{n}} - \bar{q}^f \rho^x \quad (49)$$

- Net job destruction rate

$$\bar{j}^{dr} = \frac{(1 - \phi) \bar{\rho}^N \bar{n}^N + \bar{\rho}^O (\bar{n}^O + \phi \bar{n}^N)}{\bar{n}} - \bar{q}^f \rho^x \quad (50)$$

- Aggregate job destruction rate

$$\bar{\rho} = \frac{(1 - \phi) \rho^N \bar{n}^N + \bar{\rho}^O (\bar{n}^O + \phi \bar{n}^N)}{\bar{n}} \quad (51)$$

- Job creation condition (determines  $\bar{q}^f$ ).

$$\begin{aligned} \frac{\kappa}{\bar{q}^f} = & \beta (1 - \eta) \left\{ \phi \bar{x}^O (1 - \bar{\rho}^O) \left[ \overline{H(\tilde{a}^O)} - \bar{a}^O \right] \right. \\ & \left. + (1 - \phi) \bar{x}^N (1 - \bar{\rho}^N) \left[ \overline{H(\tilde{a}^N)} - \bar{a}^N \right] \right\} \end{aligned} \quad (52)$$

- Job destruction condition, new jobs (determines reservation productivity for new jobs).

$$\begin{aligned} & \bar{a}^N \bar{x}^N - b - \frac{\eta}{1 - \eta} \kappa \bar{\theta} \\ & + \beta \left\{ \phi \bar{x}^O \bar{\varphi}^O \left[ \overline{H^O(\tilde{a})} - \bar{a}^O \right] \right. \\ & \left. + (1 - \phi) \bar{x}^N \bar{\varphi}^N \left[ \overline{H^N(\tilde{a})} - \bar{a}^N \right] \right\} \\ = & 0 \end{aligned} \quad (53)$$

- Job destruction condition, old jobs (determines reservation productivity for old jobs)

$$\bar{a}^O \bar{x}^O - b - \frac{\eta}{1 - \eta} \kappa \bar{\theta} + \beta \bar{x}^O \bar{\varphi}^O \left[ \overline{H^O(\tilde{a})} - \bar{a}^O \right] = 0 \quad (54)$$

- Average wage, new jobs.

$$\bar{w}^N = \eta \left[ \bar{x}^N \overline{H(\tilde{a}^N)} + \kappa \bar{\theta} \right] + (1 - \eta) b \quad (55)$$

- Average wage, old jobs.

$$\bar{w}^O = \eta \left[ \bar{x}^O \overline{H(\tilde{a}^O)} + \kappa \bar{\theta} \right] + (1 - \eta) b \quad (56)$$

- Average aggregate wage.

$$\begin{aligned} \bar{w} = & \frac{(1 - \bar{\rho}^N) (1 - \phi) \bar{n}^N}{(1 - \bar{\rho}^N) (1 - \phi) \bar{n}^N + (1 - \bar{\rho}^O) [\bar{n}^O + \phi \bar{n}^N]} \bar{w}^N \\ & + \frac{(1 - \bar{\rho}^O) [\bar{n}^O + \phi \bar{n}^N]}{(1 - \bar{\rho}^N) (1 - \phi) \bar{n}^N + (1 - \bar{\rho}^O) [\bar{n}^O + \phi \bar{n}^N]} \bar{w}^O \end{aligned} \quad (57)$$

- Output

$$\bar{Q} = (1 - \bar{\rho}^N) (1 - \phi) \bar{n}^N \bar{x}_t^N \overline{H(\tilde{a}^N)} + (1 - \bar{\rho}^O) \bar{n}^O \bar{x}_t^O \overline{H(\tilde{a}^O)} + \phi \bar{x}^O \bar{n}^N \overline{H(\tilde{a}^O)} \quad (58)$$

- Output, net of vacancy costs.

$$\bar{Y} = \bar{Q} - \kappa \bar{v}. \quad (59)$$

## A.2 Linearized equations

- Number of new matches that enter a given period

$$\hat{n}_{t+1}^N = \frac{A\bar{u}^\alpha\bar{v}^{1-\alpha}}{\bar{n}^N} (\alpha\hat{u}_t + (1-\alpha)\hat{v}_t) + (1-\phi)\bar{\varphi}^N (\hat{\varphi}_t^N + \hat{n}_t^N) \quad (60)$$

- Number of old matches that enter a given period

$$\hat{n}_{t+1}^O = \bar{\varphi}^O (\hat{\varphi}_t^O + \hat{n}_t^O) + \bar{\varphi}^O \phi \frac{\bar{n}^N}{\bar{n}^O} (\hat{\varphi}_t^O + \hat{n}_t^N) \quad (61)$$

- Aggregate employment

$$\hat{n}_{t+1} = \frac{\bar{n}^N}{\bar{n}} \hat{n}_{t+1}^N + \frac{\bar{n}^O}{\bar{n}} \hat{n}_{t+1}^O \quad (62)$$

- Unemployed job seekers.

$$\hat{u}_t = -\frac{\bar{n}}{\bar{u}} \hat{n}_t + \frac{(1-\phi)\bar{\rho}^N\bar{n}^N}{\bar{u}} (\hat{\rho}_t^N + \hat{n}_t^N) + \frac{\bar{\rho}^O\bar{n}^O}{\bar{u}} \hat{\rho}_t^O + \frac{\phi\bar{\rho}^O\bar{n}^N}{\bar{u}} (\hat{\rho}_t^O + \hat{n}_t^N) \quad (63)$$

- Separation rate for new jobs

$$\hat{\rho}_t^N = \frac{(1-\rho^x)\bar{\rho}^{nN}}{\bar{\rho}^N} e_{F,a}^N \hat{a}_t^N \quad (64)$$

where  $e_{F,a}^N = \frac{\partial F(\bar{a}_t^N)}{\partial \bar{a}_{Nt}} \frac{\bar{a}_t^N}{F(\bar{a}_t^N)}$ .

- Separation rate for old jobs

$$\hat{\rho}_t^O = \frac{(1-\rho^x)\bar{\rho}^{nO}}{\bar{\rho}^O} e_{F,a}^O \hat{a}_t^O \quad (65)$$

where  $e_{F,a}^O = \frac{\partial F(\bar{a}_t^O)}{\partial \bar{a}_t^O} \frac{\bar{a}_t^O}{F(\bar{a}_t^O)}$ .

- Job survival rate for new jobs

$$\hat{\varphi}_t^N = -\frac{\bar{\rho}^N}{\bar{\varphi}^N} \hat{\rho}_t^N \quad (66)$$

- Job survival rate for old jobs

$$\hat{\varphi}_t^O = -\frac{\bar{\rho}^O}{\bar{\varphi}^O} \hat{\rho}_t^O \quad (67)$$

- Firm's hazard rate

$$\hat{q}_t^w = -\alpha A \hat{\theta}_t \quad (68)$$

- Worker's hazard rate

$$\hat{q}_t^w = (1 - \alpha) A \hat{\theta}_t \quad (69)$$

- Labor market tightness

$$\hat{\theta}_t = \hat{v}_t - \hat{u}_t \quad (70)$$

- Productivity, new jobs

$$\hat{x}_t^N = \hat{z}_t \quad (71)$$

- Productivity, old jobs

$$\hat{x}_t^O = \frac{\gamma}{1 - \lambda} \hat{z}_t \quad (72)$$

- Net job creation

$$\widehat{jcr}_t = \frac{\bar{v}\bar{q}^f}{jcr\bar{n}} \left( \hat{q}_t^f + \hat{v}_t - \hat{n}_t \right) - \frac{\rho^x \bar{q}^f}{jcr} \hat{q}_t^f. \quad (73)$$

- Net job destruction

$$\begin{aligned} \widehat{jdr}_t &= (1 - \phi) \frac{\bar{\rho}^N \bar{n}^N}{jdr\bar{n}} (\hat{\rho}_t^N + \hat{n}_t^N - \hat{n}_t) \\ &\quad + \frac{\phi \bar{\rho}^O \bar{n}^N}{jdr\bar{n}} (\hat{\rho}_t^O + \hat{n}_t^O - \hat{n}_t) - \frac{\bar{q}^f \rho^x}{jdr} \hat{q}_t^f \end{aligned} \quad (74)$$

- Aggregate job destruction

$$\begin{aligned} \hat{\rho}_t &= (1 - \phi) \frac{\bar{\rho}^N \bar{n}^N}{\bar{\rho}\bar{n}} (\hat{\rho}_t^N + \hat{n}_t^N - \hat{n}_t) \\ &\quad + \frac{\phi \bar{\rho}^O \bar{n}^N}{\bar{\rho}\bar{n}} (\hat{\rho}_t^O + \hat{n}_t^O - \hat{n}_t) \end{aligned} \quad (75)$$

- Job creation (determines  $q^f$ ).

$$\begin{aligned} -\frac{\kappa}{\bar{q}^f} \hat{q}_t^f &= E_t \beta (1 - \eta) \phi \bar{x}^O \bar{\varphi}^O \left[ \overline{H(\tilde{a}_{t+1}^O)} - \bar{a}_{t+1}^O \right] (\hat{x}_{t+1}^O + \hat{\varphi}_{t+1}^O) \\ &\quad + E_t \beta (1 - \eta) \phi \bar{x}^O \bar{\varphi}^O \left[ \overline{H(\tilde{a}_{t+1}^O)} e_{H,a}^O - \bar{a}_{t+1}^O \right] \hat{a}_{t+1}^O \\ &\quad + E_t \beta (1 - \eta) (1 - \phi) \bar{x}^N \bar{\varphi}^N \left[ \overline{H(\tilde{a}_{t+1}^N)} - \bar{a}_{t+1}^N \right] (\hat{x}_{t+1}^N + \hat{\varphi}_{t+1}^N) \\ &\quad + E_t \beta (1 - \eta) (1 - \phi) \bar{x}^N \bar{\varphi}^N \left[ \overline{H(\tilde{a}_{t+1}^N)} e_{H,a}^N - \bar{a}_{t+1}^N \right] \hat{a}_{t+1}^N \end{aligned} \quad (76)$$

- Reservation productivity, new jobs. Job destruction condition, new jobs.

$$\begin{aligned}
& \bar{a}^N \bar{x}^N (\hat{a}_t^N + \hat{x}_t^N) - \frac{\eta}{1-\eta} \kappa \bar{\theta} \hat{\theta}_t \\
& + \beta \phi \bar{x}^O \bar{\varphi}^O \left[ H^O(\tilde{a}) - \bar{a}^O \right] (\hat{x}_{t+1}^O + \varphi_{t+1}^O) \\
& + \beta \phi \bar{x}^O \bar{\varphi}^O \left[ \overline{H^O(\tilde{a})} e_{H,a}^O - \bar{a}^O \right] \hat{a}_{t+1}^O \\
& + \beta (1-\phi) \bar{x}^N \bar{\varphi}^N \left[ \overline{H^N(\tilde{a})} - \bar{a}^N \right] (\hat{x}_{t+1}^N + \varphi_{t+1}^N) \\
& + \beta (1-\phi) \bar{x}^N \bar{\varphi}^N \left[ \overline{H^N(\tilde{a})} e_{H,a}^N - \bar{a}^N \right] \hat{a}_{t+1}^N \\
& = 0
\end{aligned} \tag{77}$$

- Reservation productivity, old jobs. Job destruction condition, new jobs.

$$\begin{aligned}
& \bar{a}^O \bar{x}^O (\hat{a}_t^O + \hat{x}_t^O) - \frac{\eta}{1-\eta} \kappa \bar{\theta} \hat{\theta}_t \\
& + E_t \beta \bar{x}^O \bar{\varphi}^O \left[ H^O(\tilde{a}) - \bar{a}^O \right] (\hat{x}_{t+1}^O + \varphi_{t+1}^O) \\
& + E_t \beta \bar{x}^O \bar{\varphi}^O \left[ \overline{H^O(\tilde{a})} e_{H,a}^O - \bar{a}^O \right] \hat{a}_{t+1}^O \\
& = 0
\end{aligned} \tag{78}$$

- Average wage rate, new jobs.

$$\hat{w}_t^N = \eta \left[ \frac{\bar{x}^N \overline{H(\tilde{a}^N)}}{\bar{w}^N} (\hat{x}_t^N + e_{H,a}^N \hat{a}_t^N) + \frac{\kappa \bar{\theta}}{\bar{w}^N} \hat{\theta}_t \right] \tag{79}$$

- Average wage rate, old jobs.

$$\hat{w}_t^O = \eta \left[ \frac{\bar{x}^O \overline{H(\tilde{a}^O)}}{\bar{w}^O} (\hat{x}_t^O + e_{H,a}^O \hat{a}_t^O) + \frac{\kappa \bar{\theta}}{\bar{w}^O} \hat{\theta}_t \right] \tag{80}$$

- Aggregate wage

$$\begin{aligned}
\hat{w}_t &= \frac{\bar{\varphi}^N (1-\phi) \bar{n}^N \bar{w}^N (\hat{\varphi}_t^N + \hat{n}_t^N)}{\bar{\varphi}^N (1-\phi) \bar{n}^N \bar{w}^N + \bar{\varphi}^O [\bar{n}^O + \phi \bar{n}^N] \bar{w}^O} \\
& + \frac{\bar{\varphi}^O \bar{w}^O [\bar{n}^O (\hat{\varphi}_t^O + \hat{n}_t^O) + \phi \bar{n}^N (\hat{\varphi}_t^O + \hat{n}_t^N)]}{\bar{\varphi}^N (1-\phi) \bar{n}^N \bar{w}^N + \bar{\varphi}^O [\bar{n}^O + \phi \bar{n}^N] \bar{w}^O} \\
& + \frac{[\bar{\varphi}^N (1-\phi) \bar{n}^N \bar{w}^N] \hat{w}_t^N + \bar{\varphi}^O \bar{w}^O [\bar{n}^O + \phi \bar{n}^N] \hat{w}_t^O}{\bar{\varphi}^N (1-\phi) \bar{n}^N \bar{w}^N + \bar{\varphi}^O [\bar{n}^O + \phi \bar{n}^N] \bar{w}^O} \\
& - \frac{\bar{\varphi}^N (1-\phi) \bar{n}^N (\hat{\varphi}_t^N + \hat{n}_t^N) + \bar{\varphi}^O [\bar{n}^O (\hat{\varphi}_t^O + \hat{n}_t^O) + \phi \bar{n}^N (\hat{\varphi}_t^O + \hat{n}_t^N)]}{\bar{\varphi}^N (1-\phi) \bar{n}^N + \bar{\varphi}^O [\bar{n}^O + \phi \bar{n}^N]}
\end{aligned}$$

- Output.

$$\hat{Q}_t \bar{Q} = (1 - \phi) \bar{\varphi}^N \bar{n}^N \bar{x}^N \overline{H(\tilde{a}^N)} (\hat{\varphi}_t^N + \hat{n}_t^N + \hat{x}_t^N + e_{H,a}^N \hat{a}_t^N) \quad (81)$$

$$\begin{aligned} & + \bar{\varphi}^O \bar{n}^O \bar{x}^O \overline{H(\tilde{a}^O)} (\hat{\varphi}_t^O + \hat{n}_t^O + \hat{x}_t^O + e_{H,a}^O \hat{a}_t^O) \\ & + \bar{\varphi}^O \bar{n}^N \phi \bar{x}^O \overline{H(\tilde{a}^O)} (\hat{\varphi}_t^O + \hat{n}_t^N + \hat{x}_t^O + e_{H,a}^O \hat{a}_t^O) \end{aligned} \quad (82)$$

- Aggregate income net of vacancy costs

$$\hat{y}_t \bar{y} = \hat{Q}_t \bar{Q} - \kappa \bar{v} \hat{v}_t \quad (83)$$

### A.3 Derivation of the wage equation

The match surplus is shared between the firm and the worker according to the parameter  $\eta$  which represents the workers share of the match surplus. The wage rate satisfies

$$w_t^j = \arg \max \left[ W_t^j(a_{it}) - U_t \right]^\eta \left[ J_t^j(a_{it}) - V_t \right]^{1-\eta}. \quad (84)$$

The first order condition is

$$\eta \left( J_{it}^j - V_t \right) = (1 - \eta) \left( W_{it}^j - U_t \right) \quad (85)$$

#### A.3.1 Wage $N$

Substituting the values for a filled job, the value of working, the value of unemployment and  $V_t = 0$  into the first order condition, rearranging and cancelling terms produces

$$\begin{aligned} & \eta \left\{ a_{it} x_t^N - w_{it}^N(a_{it}) \right. \\ & \left. + E_t \beta (1 - \rho^x) \left[ \phi \int_{\bar{a}_{t+1}^O}^{\infty} J_{t+1}^O(a_{t+1}) f(a) da + (1 - \phi) \int_{\bar{a}_{t+1}^N}^{\infty} J_{t+1}^N(a_{t+1}) f(a) da \right] \right\} \\ = & (1 - \eta) \left\{ w_{it}^N(a_{it}) + E_t \beta (1 - \rho^x) \left[ \phi \left( \int_{\bar{a}_{t+1}^O}^{\infty} W_{t+1}^O(a_{t+1}) f(a) da - U_{t+1} \right) \right. \right. \\ & \left. \left. + (1 - \phi) \left( \int_{\bar{a}_{t+1}^N}^{\infty} W_{t+1}^N(a_{t+1}) f(a) da - U_{t+1} \right) \right] \right. \\ & \left. - b - E_t \beta q_t^w (1 - \rho^x) \left[ \phi \int_{\bar{a}_{t+1}^O}^{\infty} W_{t+1}^O(a_{t+1}) f(a) da \right. \right. \\ & \left. \left. + (1 - \phi) \int_{\bar{a}_{t+1}^N}^{\infty} W_{t+1}^N(a_{t+1}) f(a) da - U_{t+1} \right] \right\} \end{aligned} \quad (86)$$

Using the free-entry condition  $V_t = 0$  in the Nash bargaining first-order condition (85) gives the relation  $\eta J_{it}^j = (1 - \eta) (W_{it}^j - U_t)$ . Using this to cancel terms and re-arranging produces

$$w_{it}^N(a_{it}) = \eta a_{it} x_t^N + (1 - \eta) \left\{ b + E_t \beta q_t^w (1 - \rho^x) \left[ \phi \left( \int_{\bar{a}_{t+1}^O}^{\infty} W_{t+1}^O(a_{t+1}) f(a) da - U_{t+1} \right) + (1 - \phi) \left( \int_{\bar{a}_{t+1}^N}^{\infty} W_{t+1}^N(a_{t+1}) f(a) da - U_{t+1} \right) \right] \right\}$$

Use the Nash first order condition to transform the equation into

$$w_{it}^N(a_{it}) = \eta a_{it} x_t^N + (1 - \eta) \left\{ b + E_t \beta q_t^w (1 - \rho^x) \frac{\eta}{1 - \eta} \left[ \phi \int_{\bar{a}_{t+1}^O}^{\infty} J_{t+1}^O(a_{t+1}) f(a) da + (1 - \phi) \int_{\bar{a}_{t+1}^N}^{\infty} J_{t+1}^N(a_{t+1}) f(a) da \right] \right\}.$$

Substitute using the job creation condition (26) to obtain

$$w_{it}^N(a_{it}) = \eta a_{it} x_t^N + (1 - \eta) \left( b + \frac{\eta}{1 - \eta} \frac{q_t^w}{q_t^f} \kappa \right) \quad (87)$$

and by using the properties of the matching function we get

$$w_{it}^N(a_{it}) = \eta a_{it} x_t^N + (1 - \eta) \left( b + \frac{\eta}{1 - \eta} \kappa \theta_t \right). \quad (88)$$

### A.3.2 Wage $O$

Substituting the values for a filled job, the value of working, the value of unemployment and  $V_{t+1} = 0$  into the first order condition, rearranging and cancelling terms produces

$$\begin{aligned} & \eta \left\{ a_{it} x_t^O - w_{it}^O(a_{it}) + E_t \beta (1 - \rho^x) \int_{\bar{a}_{t+1}^O}^{\infty} J_{t+1}^O(a) f(a) da \right\} \quad (89) \\ = & (1 - \eta) \left\{ w_{it}^O(a_{it}) + E_t \beta (1 - \rho^x) \left[ \int_{\bar{a}_{t+1}^O}^{\infty} W_{t+1}^O(a_{t+1}) f(a) da - U_{t+1} \right] \right. \\ & - b - E_t \beta q_t^w (1 - \rho^x) \left[ \phi \int_{\bar{a}_{t+1}^O}^{\infty} W_{t+1}^O(a_{t+1}) f(a) da \right. \\ & \left. \left. + (1 - \phi) \int_{\bar{a}_{t+1}^N}^{\infty} W_{t+1}^N(a_{t+1}) f(a) da - U_{t+1} \right] \right\} \end{aligned}$$

Use the Nash bargaining first-order condition (85) to cancel terms from the first two rows of this equation and rearrange to get

$$\begin{aligned}
w_{it}^O(a_{it}) &= \eta a_{it} x_t^O & (90) \\
&+ (1 - \eta) \left\{ b + E_t \beta q_t^w (1 - \rho^x) \left[ \phi \left( \int_{\bar{a}_{t+1}^O}^{\infty} W_{t+1}^O(a_{t+1}) f(a) da - U_{t+1} \right) \right. \right. \\
&\quad \left. \left. + (1 - \phi) \left( \int_{\bar{a}_{t+1}^N}^{\infty} W_{t+1}^N(a_{t+1}) f(a) da - U_{t+1} \right) \right] \right\}
\end{aligned}$$

Then proceed as in the derivation of  $w_{it}^N(a_{it})$  in the preceding subsection to obtain

$$w_{it}^O(a_{it}) = \eta a_{it} x_t^O + (1 - \eta) \left( b + \frac{\eta}{1 - \eta} \kappa \theta_t \right) \quad (91)$$

## B Data appendix

The data is collected from various US sources. Job finding rate and job separation rate are from Robert Shimer's homepage. Vacancies (help wanted index) are from St. Louis Fed database. Unemployment rate is from BLS database, series LNS14000000. Production is measured as per capita non-farm output, directly from NIPA Tables. Real wage is measured as  $\frac{\text{nominal compensation} \times \text{output}}{\text{hours} \times \text{nominal output}}$ , using series PRS85006043, PRS85006033, PRS85006063, PRS85006053) from BLS. Employment is total non-farm employment, series CES0000000001 from BLS. Unemployment is series LNU03000000 from BLS. Job finding rate, job separation rate, vacancies, employment and unemployment are quarterly averages, computed from monthly data. When computing the moments, all the variables have been transformed in logarithms. Logarithmic variables have been then HP filtered with  $\lambda^{HP} = 1600$ . Figure (7) depicts the key variables.

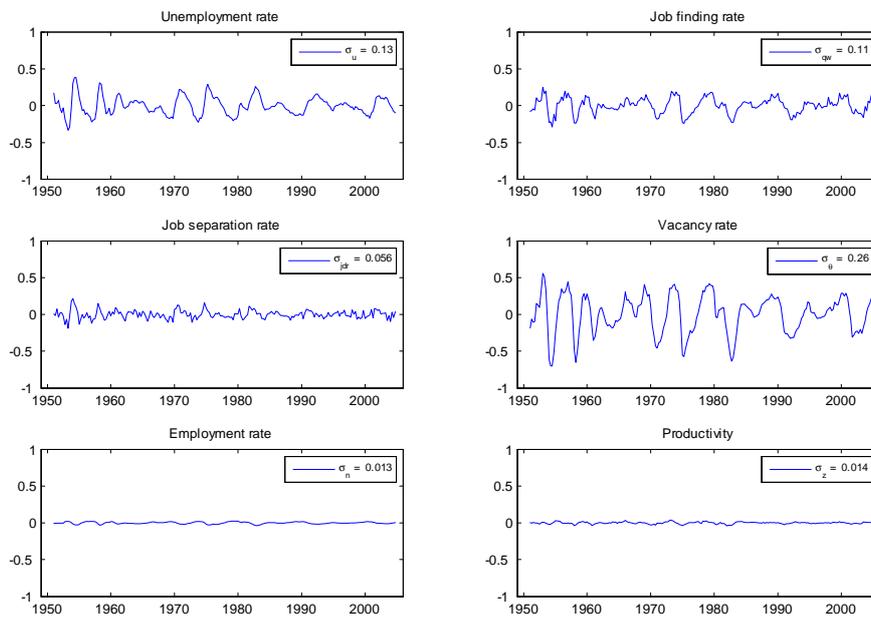


Figure 7: Fluctuations in selected business cycle and labor market variables in the US.