

# Consumption Euler equation with non-separable preferences over consumption and leisure and collateral constraints

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## Abstract

I derive and estimate an aggregate consumption Euler equation which allows to distinguish the importance of collateral constraints from non-separability of consumption and leisure as an explanation of excess sensitivity of consumption to current income. At the same time, preferences are consistent with long-run labour supply facts. Estimation results suggest that during a severe financial distress both non-separability and collateral constraints are needed to capture excess sensitivity of consumption to current economic conditions. During more tranquil times, evidence on collateral effects is more limited and non-separability is enough to make the consumption Euler equation agree well with the data.

JEL: E21, E32, E44

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## 1 Introduction

It is widely known and agreed that the simplest form of consumption Euler equation is only weakly consistent with the aggregate consumption data. Empirical failure is commonly associated to excess sensitivity of consumption to current income. Possible explanations provided in the large literature include financial market imperfections in the form of interest rate differentials, credit rationing and collateral constraints (Flavin, 1985, Hubbard and Judd, 1986, Hayashi, 1987, Jappelli and Pagano 1989, Iacoviello, 2004), as well as non-separable preferences and durability of goods and habits (Browning, 1991, Attanasio, 1995, Basu and Kimball, 2002, Kiley, 2007).

In this paper I derive and estimate an aggregate consumption Euler equation which features both non-separability and financial market imperfections in the form of binding collateral constraints. Contribution to the theoretical literature is that the form of

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consumption Euler equation allows to distinguish the importance of collateral constraints from non-separability of consumption and leisure as an explanation of excess sensitivity of consumption to current income. I thus circumvent the problem pointed out by Attanasio (1995) in Campbell and Mankiw's (1989) regression that a coefficient of income growth can be interpreted as a fraction of individuals subject to liquidity constraints only if consumption and leisure are separable in the utility function. At the same time, the form of utility function used in this paper is consistent with the long-run labour supply facts, namely that there is no strong trend in hours worked per person but there is a strong trend in real wage. In other words, income and substitution effects should roughly cancel each other.

In order to illustrate the empirical fit of the consumption Euler equation developed in this paper, I estimate the resulting (linearized) Euler equation using aggregate data from Finland during 1987Q1 - 2008Q2 period and also from the subsample 1995Q1 - 2008Q2. The Finnish data should be very informative on the importance of collateral effects, since the sample includes a period of a dramatic drop and recovery of private consumption in the aftermath of the house price bubble and economic recession in the early 1990s. The recession years were characterised by heightened financial distress and deteriorating financial conditions as major banks defaulted. The estimation sample also includes a period of more tranquil times without a major financial distress (since mid 1990s until early 2008) allowing to assess the relative importance of collateral constraints under widely different financial market conditions<sup>1</sup>. Finally, I compare the estimation results to the more simple consumption Euler equations that have been presented and estimated in Hall (1988), Campbell and Mankiw (1989), Basu and Kimball (2002) and Iacoviello (2004) for the US data.

Estimation results show that both non-separability of consumption and leisure and collateral effects are necessary in order to capture a dramatic drop and recovery of consumption growth in Finland in the early 1990s. During more tranquil times since mid 1990s evidence on collateral constraints is more limited and non-separability is enough to make the consumption Euler equation agree well with the consumption growth data. Furthermore, I find no support for the rule-of-thumb consumption behavior once the non-separability is accounted for<sup>2</sup>. Estimated values for the intertemporal elasticity of substitution (IES) and consumption share of collateral constrained households seem also reasonable and accurately determined. Both IES and consumption share of collateral constrained households are around 0.6 in the estimation of whole sample, including the period of financial distress. In the subsample after mid 1990s collateral constraints

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<sup>1</sup>Honkapohja and Koskela (1999) have earlier argued that direct financial restraints in general played an important role in cutting aggregate demand during the recession years in Finland, yet they did not find strong direct evidence on the impact of house prices on consumption.

<sup>2</sup>Iacoviello (2004) finds strong evidence of collateral effects in the U.S. data, while Basu and Kimball (2002) find that for the last two decades, the Euler equation based on non-separable preferences explains the aggregate consumption growth data very well in the U.S. Also Kiley (2007) has found evidence for non-separability in the U.S. data. Basu and Kimball (2002) argue furthermore that after accounting for the effects of predictable movements in labour implied by non-separability the evidence of excess sensitivity of consumption to predictable changes in income is substantially reduced, if not disappeared. Also Ham and Reilly (2002) provide evidence on non-separability of consumption leisure choice.

become less important, consumption share of collateral constrained households dropping to less than half, while intertemporal elasticity of substitution increases to almost 0.7.

The rest of the paper is organised as follows. Section two develops the model and derives linearized aggregate Euler equation. Section three presents the empirical results and section four concludes. Appendix provides detailed derivations.

## 2 The Model

The economy consists of two types of households, un-constrained and constrained ones. Both households have preferences over consumption, leisure and housing. Housing is separable in consumption and leisure, and all agents can trade houses, the consumption goods and riskless real bonds. As for leisure and consumption, I impose cancellation between non-zero income and substitution effects, by choosing the utility function where the real wage is proportional to consumption times some function of quantity of labour. Convenient form of utility function which delivers this is King-Plosser-Rebelo (1988) form also used in Basu and Kimball (2002). Otherwise, the model can be seen as an extension to Iacoviello (2004).

### 2.1 Un-constrained household

Un-constrained household maximises standard lifetime-utility. The problem reads as

$$\max_{\{C_t^u, H_t^u, N_t^u\}} E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_t^u)^{1-\gamma}}{1-\gamma} e^{(\gamma-1)v(N_t^u)} + \kappa f(H_t^u) \right)$$

*s.t.*

$$B_t^u + Y_t^u + W_t N_t^u = R_{t-1} B_{t-1}^u + C_t^u + Q_t (H_t^u - H_{t-1}^u)$$

where  $C_t^u$  is consumption of un-constrained households,  $Q_t$  is real price of real estate,  $H_t^u$  is housing,  $R_t$  is gross real interest rate,  $B_t^u$  are real bonds (- $B_t$  is lending),  $W_t$  is real wage,  $N_t^u$  is employment and  $Y_t^u$  is random endowment. I assume that  $f'(H_t^u) > 0$ ,  $f''(H_t^u) < 0$  and  $\kappa$  is a positive constant.  $v(N_t)$  denotes disutility from labour with usual properties  $v'(N_t^u) > 0$ ,  $v''(N_t^u) > 0$ .  $\gamma$  is the risk aversion parameter. Housing is treated like durable consumption that never depreciates. Straightforward maximization reduces to the following consumption Euler equation:

$$(C_t^u)^{-\gamma} e^{(\gamma-1)v(N_t)} = \beta R_t E_t \left[ (C_{t+1}^u)^{-\gamma} e^{(\gamma-1)v(N_{t+1})} \right] \quad (1)$$

Linearizing (1) yields:

$$-\gamma \hat{c}_t^u + (\gamma - 1) \tau^u \hat{n}_t^u = -\gamma \hat{c}_{t+1}^u + \hat{r}_t + (\gamma - 1) \tau^u \hat{n}_{t+1}^u + \mathcal{O}_t \quad (2)$$

where  $\hat{\cdot}$  denotes a percentage deviation of corresponding variable from the steady state.  $\mathcal{O}_t$  denotes higher order terms due to first order linearization. When linearizing (1), I

have used the facts that

$$v'(N^u)N^u = \left( \frac{WN^u}{C^u} \right) \equiv \tau^u. \quad (3)$$

where  $N^u$  denotes steady state (optimal) level of labour supply. Furthermore, using the approximation that

$$\Delta v(N_t^u) \approx \tau^u (n_t^u - n_{t-1}^u) = \tau^u \Delta n_t^u \quad (4)$$

equation (2) reduces to

$$\hat{c}_t^u = E_t \hat{c}_{t+1}^u - s \hat{r}_t - (1-s) \tau^u E_t \Delta n_{t+1}^u \quad (5)$$

where  $s \equiv 1/\gamma$  is the intertemporal elasticity of substitution. Linearization is accurate as long as there is no strong trend in labour supply and we are not too far from the steady state, since approximation to  $v(N_t^u)$  is applied around some constant (optimal) value of  $N^u$ .

This is in principle the same Euler equation as the one derived in Basu and Kimball (2002). As discussed by Basu and Kimball, there is a non-trivial linear restriction between the coefficient on the real interest rate  $\hat{r}_t$  and the coefficient of the employment growth. Restriction comes from the facts about long-run labour supply<sup>3</sup>. A low value of the elasticity of intertemporal substitution means that the marginal utility of consumption falls rapidly along with the higher level of consumption. Without any interaction between consumption and labour in the utility function i.e. with separable preferences, a decline in the marginal utility of consumption would lead households to want more leisure unless the real wage increased markedly. When consumption and labor are complements, like here, an increased level of consumption, and thus decline in the marginal utility of consumption, makes labour more pleasant. This makes the association between consumption and real wage stronger when compared to that implied by separable preferences.

## 2.2 Constrained households

There is a fraction  $\zeta$  of households who are borrowing constrained. At each point of time, the amount they can agree to repay in the following period cannot exceed a fraction  $m \leq 1$  of next period's expected value of real estate holdings  $(Q_{t+1}H_t^c)^4$ . One may think of  $m \leq 1$  as representing liquidation costs in the case of default. Formally, constrained households' real obligations  $R_t B_t^c$  are limited by

$$R_t B_t^c \leq m E_t [Q_{t+1} H_t^c] \quad (6)$$

This type of collateral constraint can be rationalized by limited enforcement, the idea being that the creditors protect themselves from the threat of repudiation by collateralizing part of the household's real estate holdings. Important feature of (6) is that

<sup>3</sup>Notice that the parameter restriction does not depend on an exact value of the labour supply elasticity indicating the size of the income and substitution effects.

<sup>4</sup>This is a form of collateral constraint used by Kiyotaki and Moore (1997).

the expected movements in the price of collateralised asset (real estate) affect on the borrowing. Potentially, the prices of collateralized assets could also be affected by the size of the credit limits as emphasised by Kiyotaki and Moore (1997).

For simplicity, and following Iacoviello (2004), I assume that constrained households do not discount the future. Otherwise, the constrained households share the preferences with the un-constrained households. Their optimization problem is then the following:

$$\begin{aligned} \max \left( \frac{(C_t^c)^{1-\gamma}}{1-\gamma} e^{(\gamma-1)v(N_t^c)} + \kappa f(H_t) \right) \\ \text{s.t.} \\ C_t^c + Q_t(H_t^c - H_{t-1}^c) + R_{t-1}B_{t-1}^c = B_t^c + W_t^c N_t^c + Y_t^c \\ R_t B_t^c \leq m E_t(Q_{t+1}) H_t^c \end{aligned} \quad (7)$$

The first order conditions for  $B_t^c$  and  $H_t^c$  and  $N_t^c$  yield:

$$(C_t^c)^{-\gamma} e^{(\gamma-1)v(N_t^c)} = R_t \phi_t \quad (8)$$

$$-Q_t (C_t^c)^{-\gamma} e^{(\gamma-1)v(N_t^c)} = \kappa f'(H_t^c) + m E_t(\phi_t Q_{t+1}) \quad (9)$$

$$W_t^c = C_t^c v'(N_t^c) \quad (10)$$

In the absense of discounting, the marginal utility of future consumption does not enter into the unconstrained agent's consumption Euler equation. Instead, *current* marginal utility of consumption is affected by the shadow value of the borrowing constraint ( $\phi_t$ ). There is a distortion towards housing demand, since housing can be used as collateral. Furthermore, also the intratemporal decision is affected indirectly by the shadow value of borrowing constraint, since consumption enters into the intratemporal condition (see 10). In the steady state, the constrained households borrow up to the limit and it is assumed that the constraint holds also in the neighbourhood of the steady state. Increase in housing prices relaxes the borrowing constraint, leads to higher borrowing and thus increases consumption of constrained households. Opposite is of course true when house prices fall. Due to non-separability, increasing consumption makes labour more pleasant and thus non-separability has a tendency to further amplify the impact of house prices on consumption.

Linearizing and combining the first order conditions appropriately (see mathematical appendix for details) delivers the following consumption equation for the constrained households:

$$\hat{c}_t^c = s[\theta \hat{h}_t^c + \omega(\hat{r}_t - E_t \hat{q}_{t+1}) + (1 + \omega)\hat{q}_t] + (1 - s)\tau^c \hat{n}_t^c \quad (11)$$

where  $1 + \omega \equiv \frac{1}{1-m\beta}$  and  $\theta \equiv -\frac{f''(H^c)H^c}{f'(H^c)}$ .  $1 + \omega$  is the inverse of the downpayment needed to purchase one unit of housing, while  $\theta$  is related to long-run demand elasticity of housing services.  $\hat{h}^c$  and  $\hat{q}$  denote housing demand and real house price in percentage deviation from the steady state.

I have linearized  $v(N_t^c)$  around the optimal (trend) level of labour supply just like in the case of un-constrained households. Thus  $\tau^c$  is defined correspondingly as above

in equation (3). The borrowers' consumption is a positive function of house prices, with a coefficient that is equal to the inverse of the downpayment times the intertemporal elasticity of substitution  $s$ . Consumption depends positively also on the measure of labour supply due to the non-separability. With  $s = 1$  and  $\tau = 0$ , this equation is in principle the same as the one derived in Iacoviello (2004).

### 2.3 Derivation of aggregate Euler equation

Having derived the consumption Euler equations for the un-constrained and constrained households, the final step is to obtain the aggregate Euler equation which can then be estimated. Recall for convenience the following Euler equations for un-constrained and constraint agents:

$$\begin{aligned}\hat{c}_t^u &= E_t \hat{c}_{t+1}^u - s\hat{r}_t - (1-s)\tau^u E_t \Delta n_{t+1}^u \\ \hat{c}_t^c &= s\theta \hat{h}_t^c + s\omega(\hat{r}_t - E_t \hat{q}_{t+1}) + s\omega \hat{q}_t + (1-s)\tau^c \hat{n}_t^c\end{aligned}$$

Making rational expectations assumption explicit *i.e.* that

$$\begin{aligned}E_t \hat{c}_{t+1}^u &= c_{t+1}^u + \epsilon_t^{cu} \\ E_t \Delta n_{t+1}^u &= \Delta n_{t+1}^u + \epsilon_t^{\Delta n} \\ E_t \hat{q}_{t+1} &= \hat{q}_{t+1} + \epsilon_t^q \\ E_t \hat{n}_{t+1} &= \hat{n}_{t+1} + \epsilon_t^n,\end{aligned}$$

and substituting rational expectations assumptions into the corresponding Euler equations yields

$$\Delta \hat{c}_{t+1}^u = s\hat{r}_t + \tau^u(1-s)\Delta n_{t+1}^u + \epsilon_t^u + \alpha_u \epsilon_{t-1}^u$$

$$\hat{c}_t^c = s\theta \hat{h}_t^c + s\omega \hat{r}_t - s\omega \Delta q_{t+1} + (1-s)\tau^c \hat{n}_t^c + \epsilon_t^c + \alpha_c^c \epsilon_{t-1}^c$$

$\epsilon_t^i$  are forecast error terms which contain forecast errors related to future consumption, labour and real housing prices.

Let  $\lambda$  denote consumption share of constrained households. Noticing then that aggregate consumption can be expressed in log first differenced form

$$\Delta c_t = \lambda \Delta c_t^c + (1-\lambda) \Delta c_t^u, \quad (12)$$

substituting  $\Delta c_{t+1}^u$  and  $\hat{c}_t^c$  in (12), and manipulating appropriately the resulting equations (see mathematical appendix for details), we find that

$$\Delta c_t = \lambda s \theta \Delta h_t^c + \lambda s \omega [\Delta r_t - \Delta \Delta q_{t+1}] + (1-s)\tau \Delta n_t + (1-\lambda)s\hat{r}_t + \epsilon_t \quad (13)$$

where  $\epsilon_t$  is a combination of forecast errors  $\epsilon_t^i$ . Re-organising slightly and using the fact that

$$\Delta r_t - \Delta \Delta q_{t+1} = r_t^h - r_{t-1}^h$$

where  $r_t^h$  denotes housing real interest rate I finally arrive to the following consumption Euler equation:

$$\Delta c_t - \tau \Delta n_t = \lambda s [\theta \Delta h_t^c + \omega \Delta r_t^h - \hat{r}_t] + s(\hat{r}_t - \tau \Delta n_t) + \epsilon_t \quad (14)$$

On the one hand, in comparison to Basu and Kimball (2002), there is a new term  $\lambda s [\theta \Delta h_t^c + \omega \Delta r_t^h - \hat{r}_t]$ . This comes from the presence of collateral constraint households and it captures the sensitivity of consumption growth to price fluctuations of collateral or collateral effects in short. On the other hand, in comparison to Iacoviello (2004), there is also the term  $\tau \Delta n_t$  on both sides of equation (14)<sup>5</sup>. This captures the effects of non-separability. Equation (14) thus nests both Basu and Kimball (2002) and Iacoviello (2004) specifications. Naturally, it also nests the standard Euler equation with  $\lambda = 0$ ,  $\tau = 0$ . In that case, only the real interest rate appears on the right hand side of equation (14). Finally, notice that (14) allows to distinguish between the importance of collateral constraints and non-separability, since both  $\lambda$  and  $s$  can be identified separately.

### 3 Evidence

#### 3.1 Data

I estimate the variants of (14) with Finnish data, using the sample since 1987<sup>6</sup>. Basu and Kimball (2002) estimated (14) (with  $\lambda = 0$ ) using instrumental variable estimation and assuming that  $\epsilon_t$  has MA(1) structure. I follow Iacoviello (2004), and also Kiley (2007), by using Hansen's (1982) Generalised Methods of Moments (GMM).

As a dependent variable and as a measure of consumption, I use log change in total private consumption per capita ( $\Delta c$ ). The real short-term interest rate is the difference between the quarterly 3 month money market rate and quarter-on-quarter change in the log private consumption deflator ( $r$ ). The housing real interest rate is the difference between the quarterly 3 month money market rate and quarter-on-quarter change in the log house price index ( $r^h$ ). Real house price is the log house price index (whole economy) deflated by the private consumption deflator ( $q$ ). As a proxy for housing demand (for constrained agents), I use detrended log total residential investment per capita ( $h$ ). Implicitly, I make the assumption that most of the variation in housing demand is due to variation of housing demand of constraint households. As a measure of labour supply growth, I use log difference of hours (total economy) per capita ( $\Delta n$ ). I

<sup>5</sup>Iacoviello (2004) derives aggregate Euler equation in a slightly different way. He replaces the conditional expectation of unconstrained household's consumption with long term interest rate. This would yield to

$$\hat{c}_t = -s(1 - \lambda) [\hat{l}_t + \hat{r}_t] + \omega \lambda s [\hat{r}_t + \hat{q}_t - E_t \hat{q}_{t+1}] + \lambda s \hat{q}_t + \lambda s \theta \hat{h}_t^c + (1 - s) \lambda \tau \hat{n}_t$$

where  $\hat{l}_t$  is long-term interest rate.

<sup>6</sup>This is motivated by relatively late process of financial deregulation in Finland. Although financial market liberalisation started in the early 1980s, it intensified during the second half of the decade. For instance, regulation of lending rates was abolished as late as 1986 and market interest rate *helibor* was introduced in 1987.

have smoothed consumption deflator, house price index and hours worked slightly using HP in order to re-move extra noise in the quarterly series.

In order to calibrate  $\tau$ , I use the average effective tax rate on labour ( $\tau_l$ ). The method for computing average effective tax rate on labour from aggregate National accounts data is described in OECD (2000) and Mendoza et al. (1994). Given ( $\tau_l$ ) I compute  $\tau$  as an average of  $\tau_t = \frac{(1-\tau_l)W_tN_t}{C_t}$  following equation (3). Average value of  $\tau$  for the period of 1987-2008 is roughly 0.5, which is the calibrated constant used in the estimation of the whole sample. Given that there is a reasonable degree of uncertainty about the actual value of  $\tau$ , I check the results for different values of  $\tau$ , ranging from 0.4 – 0.6. As instruments ( $\mathbf{z}$ ) in the GMM estimation, I use three to four lags of each right hand side variable of (14) in levels. In order to take into account the first order moving average term in the errors, I use lags greater than 2 for these variables. As additional instruments, I use log disposable income ( $y$ ) consumption income ratio ( $c - y$ ), as well as lagged world GDP ( $y_w$ ) and household debt to disposable income ratio ( $d - y$ ). Disposable income is calculate following National Accounts definition. I report the results also using different instruments subsets in order to check the robustness of the results. All regressions include also a constant term (not reported). The constant in regressions capture the higher order terms due to precautionary savings motives of the consumers, an approximation error due to linearisation. In all regressions I have used rational expectations hypothesis and replaced expected inflation (both house price and consumer price inflation) with its ex post realizations. This is a strong assumption, since real-ex post interest rate could be an inaccurate measure of real interest rate perceived by the households. Lack of quarterly survey data prevents me from checking the importance of this assumption, unfortunately.

## 3.2 Results

### 3.2.1 Sample 1987Q1-2008Q2

I start by first estimating the following three consumption Euler equations with the sets of orthogonality conditions reported below

$$(I) : E_t\{(\Delta c_t - sr_t)\mathbf{z}_t\} = 0 \quad (15)$$

$$(II) : E_t\{(\Delta c_t - \tau\Delta n_t - s[r_t - \tau\Delta n_t])\mathbf{z}_t\} = 0 \quad (16)$$

$$(III) : E\{(\Delta c_t - \tau\Delta n_t - \lambda s[\theta\Delta h_t^c + \omega\Delta r_t^h - r_t] - s(r_t - \tau\Delta n_t))\mathbf{z}_t\} \\ = 0 \quad (17)$$

The first is the standard Euler equation, the second is the specification with non-separable labour only corresponding to Basu and Kimball (2004), and the last one is the specification with collateral constraint households and non-separable preferences, I refer to the third equation often as encompassing model. Results are provided in Table 1, together with the preferred set of instruments used.

The estimates of the intertemporal elasticity of substitution (IES) are somewhat sensitive to whether the collateral constraint and/or non-separability of consumption

and leisure is assumed. Estimates vary from roughly 0.3 to 0.6. Apart from the standard Euler equation, however, IES is always significantly greater than zero at 5% significance level. The consumption share of collateral constraint households ( $\lambda$ ) is roughly 60%. 95% confidence interval of  $\lambda$ , includes values as low (high) as 0.18(1.08). This seems reasonable although the upper bound is somewhat above one. Mean estimate is somewhat high in comparison to other international studies. A high number, however, could reflect the fact that in the early 1990s financial markets in Finland tightened due to severe economic recession and banking crisis. Moreover, the financial markets were liberalised relatively late in Finland<sup>7</sup>.

The estimated parameter related to curvature of the preferences of housing  $\theta$  and the parameter related to liquidation costs in the case of default ( $\omega$ ) have high standard errors.  $\omega$  estimates to a value which is perhaps unrealistically low given that  $1 + \omega = \frac{1}{1-m\beta}$  is inverse of the downpayment needed to purchase one unit of housing services<sup>8</sup>. Given that  $\theta \equiv -\frac{f''(H^c)H^c}{f'(H^c)}$  a small estimated value of  $\theta$  implies that preferences are roughly linear in housing services<sup>9</sup>.

The results are not particularly sensitive to calibrated values of  $\tau$ . Using  $\tau = 0.4$  changes the estimates of IES to 0.29 and to 0.51 in columns II and III of Table 1 respectively. Using  $\tau = 0.6$  leads to estimates of IES of 0.40 and 0.67 in columns II and III. The estimate for the consumption share of collateral constraint households changes to 0.56 and 0.67 respectively, while the estimates for  $\theta$  and  $\omega$  change only marginally. Higher value of intertemporal substitution in column III suggests that non-separability is somewhat less important once collateral constraint households are included into the model. However, it is still possible to reject the hypothesis that  $s = 1$  in column III at 95% level.

Judging the goodness-of-fit on the basis of correlation between actual and fitted series, the results suggest that encompassing model in column III, which combines collateral constraints and non-separable utility, adds predictive power with respect to other models. The correlation between actual consumption growth and dynamic forecasts resulting from the three Euler equation estimations shows that correlation is clearly highest in the model with collateral constraint households included. Correlation ranges from -0.46 to 0.57 in columns I, II and III respectively. Also the root mean squared error (RMSE) of one-step ahead predictions is the lowest in the specification which includes collateral constraint households. This could be due to the fact that the model with collateral constraint households captures better the consumption growth pattern around the 1990s recession. The visual inspection of Figure (1) confirms this. Figure (1) compares the dynamic forecasts of the models resulting from columns II and III in Table 1 to actual

<sup>7</sup>Financial deregulation started in the early 1980s, but major changes, such as abolition of regulation of lending rates took place during the second half of the decade. For details see for instance Honkapohja and Koskela (1999).

<sup>8</sup>Since typically  $\omega$  is calibrated in the DSGE models with collateral constraint households, I have experimented by fixing the value of  $\omega$  to a more reasonable level. Higher values of  $\omega$  tend to result into negative values for the consumption share of collateral constraint households, and smaller IES.

<sup>9</sup>To see this, assume for instance that  $f(H) = \frac{1}{\delta}H^\delta$ . Then,  $\theta = (\delta - 1)$ . A small value of  $\theta$  means that  $\delta$  is very close to unity, and thus preferences are roughly linear in housing services.

consumption growth. Clearly, the encompassing model captures a dramatic drop and recovery of consumption growth during the aftermath of housing price collapse in the early 1990 much better when compared to the model of non-separable preferences alone. This is precisely from where the better fit comes.

Interestingly, after the crisis period from 1995 onwards, the two models from column II and column III have some difficulties to capture volatility of consumption, yet the model with collateral constraint does somewhat better. One reason for this could be related directly to King-Plosser-Rebelo (1988) utility function, which has typically been argued to be unsuccessful in generating business cycle movements consistent with the data of small open economies in particular. The difficulty arises from the fact that these preferences typically yield much too low standard deviation of consumption in general equilibrium models ( and a counterfactual procyclical trade balance, as shown by Correia, Neven and Rebelo, 1995 and Schmitt-Grohe and Uribe, 2003). The fact that the encompassing model seems to do somewhat better shows that inclusion of collateral effects improves the fit in this dimension too.

**Table 1. Estimation results for the whole sample**

Parameter	I	II	III
$\lambda$			0.63*** (0.23)
$s$	0.29* (0.15)	0.35*** (0.07)	0.61** (0.21)
$\theta$			0.003 (0.01)
$\omega$			0.11 (0.13)
{	$c_{t-2}..c_{t-4}$ $r_{t-2}..r_{t-4}$	$c_{t-2}..c_{t-4}$ $r_{t-2}..r_{t-4}$ $y_{t-2}..y_{t-4}$ $n_{t-2}..n_{t-4}$	$c_{t-2}..c_{t-4}$ $r_{t-2}..r_{t-4}$ $y_{t-2}..y_{t-4}$ $n_{t-2}..n_{t-4}$
<b>z</b>	$y_{t-1}^w..y_{t-4}^w$	$(c-y)_{t-2}..(c-y)_{t-4}$ $y_{t-1}^w..y_{t-4}^w$	$(c-y)_{t-2}..(c-y)_{t-4}$ $y_{t-1}^w..y_{t-4}^w$ $r_{t-2}^h..r_{t-4}^h$ $h_{t-2}..h_{t-4}$ $(d-y)_{t-1}..(d-y)_{t-4}$
}			
j-stat (p-value)	0.07 (0.32)	0.07 (0.43)	0.14 (0.40)
s.e.	0.010	0.009	0.009
RMSE	0.0108	0.009	0.0074
Corr	-0.46	0.19	0.57
HAC	Bartlett, NW	Bartlett,NW	Bartlett,NW
$\tau$	0 (rest.)	0.5	0.5

This Table reports GMM estimates of the structural parameters ( $\lambda, s, \theta, \omega$ ) in equations 15-17 together with the list of instruments (z). Dependent variable is  $\Delta c_t$ . Heteroskedasticity and autocorrelation corrected standard errors are reported in parenthesis. j-stat reports minimised value of an objective function and p-value associated with Hansen (1982) test for overidentifying restrictions (in parenthesis). RMSE is one-step-ahead root mean squared prediction error. Corr denotes ordinary correlation coefficient between dynamic forecast and actual consumption growth. HAC reports options used for computing weighting matrix of the objective function and the last row reports  $\tau$  used in the estimation. The estimation period is 1987Q1-2008Q2.

### 3.3 Subsample 1995Q1-2008Q2

I next estimate the three consumption Euler equations using the sample 1995Q1-2008Q2. Results are provided in Table 2. First, there is some evidence that intertemporal elas-

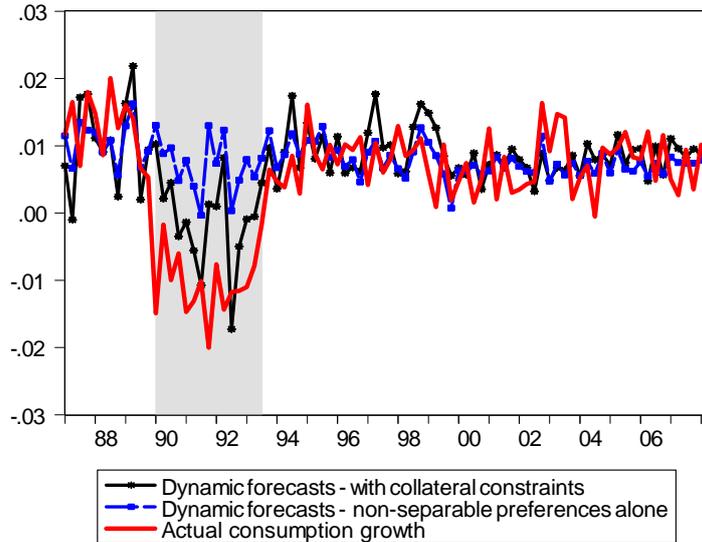


Figure 1: Dynamic forecasts and actual consumption growth - comparisons

ticity of substitution had increased during the latter sample with respect to the whole sample. Columns II and III suggest that the IES is 0.58 in the Basu and Kimball specification, and about 0.67 in the specification including collateral constraint households. IES is highly significant, except when using standard formulation (see column I). Share of collateral constraint households is now 0.47, slightly lower than in the estimation of the whole period. This seems reasonable given that the Finnish economy since 1995 until early 2008 has not been subject to any major turmoil and financial conditions have remained rather stable. The parameters directly related to housing are not significant, although  $\omega$  is now somewhat more reasonable. All in all, it seems that the intertemporal elasticity of substitution does not change widely across the estimation periods and specifications once non-separability is accounted for. Moreover, estimating the model with  $\tau = (0.4, 0.6)$  does not markedly change the results. Higher values of  $\tau$  imply somewhat higher values of IES.

The difference between the Basu and Kimball specification and the encompassing model is now rather small in terms of empirical fit. The correlations between actual and dynamic forecasts are 0.28 and 0.23 respectively. Moreover, the difference between RMSEs is negligible. Consequently, it seems that during a more tranquil period collateral effects are less important and the consumption equation derived from the model with non-separability alone captures the fluctuations in consumption growth reasonably well.

**Table 2. Estimation results for the subsample 1995Q1-2008Q2**

Parameter	I	II	III
$\lambda$			0.47*** (0.23)
$s$	0.22 (0.15)	0.58*** (0.09)	0.67*** (0.18)
$\theta$	–	–	0.01 (0.02)
$\omega$	–	–	0.41 (0.39)
{	$c_{t-2}..c_{t-4}$ $r_{t-2}..r_{t-4}$	$c_{t-2}..c_{t-4}$ $r_{t-2}..r_{t-4}$ $y_{t-2}..y_{t-4}$ $n_{t-2}..n_{t-4}$	$c_{t-2}..c_{t-4}$ $r_{t-2}..r_{t-4}$ $y_{t-2}..y_{t-4}$ $n_{t-2}..n_{t-4}$
<b>z</b>	$y_{t-1}^w..y_{t-4}^w$	$(c-y)_{t-2}..(c-y)_{t-4}$ $y_{t-1}^w..y_{t-4}^w$	$(c-y)_{t-2}..(c-y)_{t-4}$ $y_{t-1}^w..y_{t-4}^w$ $h_{t-2}..h_{t-4}$ $r_{t-2}^h..r_{t-4}^h$ $(d-y)_{t-1}..(d-y)_{t-4}$
}			
j-stat (p-value)	0.13 (0.46)	0.15 (0.22)	0.21 (0.86)
s.e.	0.004	0.004	0.004
RMSE	0.0056	0.0043	0.0044
Corr	-0.30	0.28	0.23
HAC	Bartlett, NW	Bartlett,NW	Bartlett,NW
$\tau$	0 (rest)	0.43	0.43

This Table reports GMM estimates of the structural parameters ( $\lambda, s, \theta, \omega$ ) in equations 15-17 together with the list of instruments ( $z$ ). Dependent variable is  $\Delta C_t$ . Heteroskedasticity and autocorrelation corrected standard errors are reported in parenthesis. j-stat reports minimised value of an objective function and p-value associated with Hansen (1982) test for overidentifying restrictions (in parenthesis). RMSE is one-step-ahead root mean squared prediction error. Corr denotes ordinary correlation coefficient between dynamic forecast and actual consumption growth. HAC reports options used for computing weighting matrix of the objective function and the last row reports  $\tau$  used in the estimation. The estimation period is 1995Q2-2008Q2.

Tables 3-4 show the results from the estimation of Basu and Kimball specification and the encompassing model using alternative instrument sets. Overall, the results from Basu and Kimball specification are rather robust to different instruments sets. IES varies from the lowest value of 0.32 to the highest of 0.40 during the whole sample, and from

0.32 to 0.57 in the subsample 1995Q2-2008Q2. As for the encompassing model, the variability is greater. IES substitution varies from 0.40 to 0.97 in the whole sample and from 0.81 to 1.67 in the subsample 1995Q2-2008Q2. Consumption share of collateral constraint households varies almost equally a lot, from 0.34 to 0.80 and from 0.41 to 0.92 in the whole sample and subsample, respectively. As for  $\omega$  and  $\theta$ , the standard errors are in general large and the magnitudes are typically comparable to the preferred specification in Table 2. While the results from encompassing model are quite sensitive to instrument sets, general conclusions from Tables 1 and 2 still remain valid.

**Table 3. Alternative instruments  
in Basu-Kimball specification**

Instrument set	s	
	87Q1-08Q1	95Q1-08Q2
$r_{t-2}..r_{t-4}$	0.40***	0.56**
$n_{t-2}..n_{t-4}$	(0.14)	(0.29)
<hr/>		
$r_{t-2}..r_{t-4}$	0.33***	0.32*
$n_{t-2}..n_{t-4};$	(0.10)	(0.19)
$c_{t-2}..c_{t-4}$		
<hr/>		
$c_{t-2}..c_{t-4};r_{t-2}..r_{t-4}$	0.32***	0.57**
$n_{t-2}..n_{t-4}$	(0.08)	(0.23)
$y_{t-2}..y_{t-4}$		
<hr/>		
$c_{t-2}..c_{t-4};r_{t-2}..r_{t-4}$	0.36***	0.57**
$y_{t-2}..y_{t-4};n_{t-2}..n_{t-4}$	(0.07)	(0.24)
$(c-y)_{t-2}..(c-y)_{t-4}$		

This Table reports GMM estimates of the IES ( $s$ ) for the two sample periods based on 16 using different sets of instruments ( $z$ ). Heteroskedasticity and autocorrelation corrected standard errors are reported in parenthesis.

### 3.4 Further robustness checks and comparisons

I now proceed to compare the results to Campbell and Mankiw (1989) model with rule-by-thumb consumers. Campbell and Mankiw (1989) specification is achieved by imposing restrictions that  $\tau = 0$  and  $\lambda = 0$ , but augmenting the standard Euler equation with a measure of disposable income ( $\Delta y_t$ ). Furthermore, I test whether after accounting for non-separable consumption-leisure choice, consumption is still sensitive to changes in disposable income. Finally, I also report the estimates of Iacoviello's specification with separable preferences in column III of Table 5. Iacoviello's specification is obtained from (17) by imposing restrictions  $\tau = 0$  and  $s = 1$ <sup>10</sup>. This corresponds to separable

<sup>10</sup>Iacoviello (2004) arrived to aggregate consumption Euler equation in a slightly different way, allowing him also to estimate intertemporal elasticity of substitution even with separable labour. My specification does not allow to identify separately intertemporal substitution and consumption share of constraint household. Therefore, I have imposed an additional restriction that  $s = 1$ . This corresponds to log

Table 4. Alternative instrument sets in encompassing model

Instrument set	s			$\lambda$			$\theta$			$\omega$	
	87Q1-08Q1	95Q1-08Q2	87Q1-08Q2	87Q1-08Q1	95Q1-08Q2	87Q1-08Q2	87Q1-08Q1	95Q1-08Q2	87Q1-08Q2	87Q1-08Q1	95Q1-08Q2
$r_{t-2} \cdot r_{t-4}; n_{t-4}$	0.95**	0.81***	0.34	0.41**	0.08	-0.04	0.35		-0.38		
$h_{t-2} \cdot h_{t-4}; r_{t-2}^h \cdot r_{t-4}^h$	(0.47)	(0.21)	(0.37)	(0.21)	(0.14)	(0.02)	(0.23)		(0.79)		
$yt-2 \cdot yt-4$											
$r_{t-2} \cdot r_{t-4}; n_{t-2} \cdot n_{t-4}$	0.97**	1.64***	0.85***	0.41	-0.004	-0.04	0.58		0.14		
$h_{t-2} \cdot h_{t-4}; r_{t-2}^h \cdot r_{t-4}^h$	(0.38)	(0.39)	(0.15)	(0.26)	(0.006)	(0.03)	(0.53)		(0.11)		
$ct-2 \cdot ct-4; yt-2 \cdot yt-4$											
$dsyt-1 \cdot dsyt-4$											
$r_{t-2} \cdot r_{t-4}; n_{t-2} \cdot n_{t-4};$	0.40*	1.65***	0.78***	0.80***	0.002	0.001	-0.07		-0.27		
$h_{t-2} \cdot h_{t-4}; r_{t-2}^h \cdot r_{t-4}^h$	(0.21)	(0.15)	(0.25)	(0.04)	(0.011)	(0.002)	(0.04)		(0.24)		
$(d-y)_{t-1} \cdot (d-y)_{t-4}$											
$; y_{t-1}^w \cdot y_{t-4}^w; ct-2 \cdot ct-4$											
$ct-2 \cdot ct-4; r_{t-2} \cdot r_{t-4}$	0.97**	1.47***	0.80***	0.92***	-0.00	0.03***	-0.54***		0.14		
$h_{t-2} \cdot h_{t-4}; yt-2 \cdot yt-4$	(0.38)	(0.36)	(0.18)	(0.10)	(0.01)	(0.008)	(0.13)		(0.12)		
$r_{t-2}^h \cdot r_{t-4}^h; n_{t-2} \cdot n_{t-4}$											
$(c-y)_{t-2} \cdot (c-y)_{t-4}$											

This Table reports GMM estimates of the structural parameters ( $\lambda, s, \theta, \omega$ ) based on equation (17) and using different sets of instruments ( $z$ ). Heteroskedasticity and autocorrelation corrected standard errors are reported in parenthesis.

logarithmic utility. The results are provided in Table 5. For convenience, I report below the regression specifications. As earlier, estimations are done using GMM.

$$\begin{aligned}
\text{(I)} & : \quad \Delta c_t - \tau \Delta n_t = s(\hat{r}_t - \tau \Delta n_t) + \alpha \Delta y_t + \epsilon_t^1 \\
\text{(II)} & : \quad \Delta c_t = s \hat{r}_t + \alpha \Delta y_t + \epsilon_t^2 \\
\text{(III)} & : \quad \Delta c_t = \lambda[\theta \Delta h_t^c + \omega \Delta r_t^h] + (1 - \lambda) \hat{r}_t + \epsilon_t
\end{aligned} \tag{18}$$

Column I reports the estimates of Basu and Kimball regression with current disposable income included, while column II reports the estimates of Campbell and Mankiw (1989) specification. Column III reports the results from Iacoviello (2004) specification with the restriction that  $s = 1$  and  $\tau = 0$ . First observation from columns I and II is that real disposable income is either insignificant or it enters with incorrect sign. Second, IES is small and insignificant in the Campbell and Mankiw (1989) specification (see Column II) and disposable income is marginally significant only in the whole sample. Finally, in Iacoviello's specification the share of collateral constraints  $\lambda$  as well as  $\theta$  and  $\omega$  are comparable with the results obtained in Tables 1 and 2. However, the predictive power of Iacoviello's (2004) specification in comparison to non-separable case is weaker. These results thus provide further support that collateral effects are important, yet non-separability combined with collateral constraints delivers a better performance. This is especially true for the whole sample, which contains a period of financial distress in the early 1990s.

## 4 Conclusions

In this paper, I have derived an aggregate consumption Euler equation in which non-separability between consumption and leisure, and collateral constrained households, makes current consumption dependent upon employment as well as upon the development of the asset (house) prices. Form of a consumption Euler equation allowed also to distinguish the importance of collateral constraints from non-separability of consumption and leisure as an explanation of excess sensitivity of consumption to current income while at the same it is consistent with the long-run labour supply facts.

I used data from Finland for the 1987Q1 - 2008Q2 period to study the empirical relevance of the resulting consumption Euler equation. Estimation results indicate clearly that both complementarity of consumption and labour introduced by non-separability, and collateral effects are important feature of aggregate consumption behavior. The model that combines the two is able to explain a major share of the variation in consumption growth during 1987Q1 - 2008Q2 period. This is quite remarkable given that the period includes a dramatic drop and recovery of consumption during and after the 1990s recession. Estimates for intertemporal elasticity of substitution and consumption share of collateral constraints households seem also reasonable and accurately determined. Furthermore, I find no support for the rule-of-thumb consumption behavior, once non-separability is accounted for.

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utility.

**Table 5. Robustness checks – alternative specifications**

Parameter	I			II			III		
	87Q1-08Q2	95Q1-08Q2	87Q1-08Q2	95Q1-08Q2	87Q1-08Q2	95Q1-08Q2	87Q1-08Q2	95Q1-08Q2	95Q1-08Q2
$s$	0.21** (0.09)	0.46*** (0.09)	0.08 (0.17)	-0.06 (0.18)	1 (rest.)	1 (rest.)	1 (rest.)	1 (rest.)	1 (rest.)
$\alpha$	-0.05 (0.06)	-0.15*** (0.02)	0.23* (0.06)	-0.10 (0.06)	-	-	-	-	-
$\lambda$	-	-	-	-	0.87*** (0.11)	0.87*** (0.11)	0.87*** (0.11)	0.87*** (0.11)	0.87*** (0.11)
$\theta$	-	-	-	-	-0.004 (0.005)	-0.004 (0.005)	-0.004 (0.005)	-0.02*** (0.01)	-0.02*** (0.01)
$\omega$	-	-	-	-	0.15** (0.07)	0.15** (0.07)	0.15** (0.07)	0.31 (0.08)	0.31 (0.08)
j-stat (p-value)	0.09 (0.42)	0.17(0.11)	0.07(0.75)	0.12(0.11)	0.12(0.16)	0.12(0.16)	0.12(0.16)	0.14(0.25)	0.14(0.25)
s.e.	0.008	0.005	0.009	0.005	0.01	0.01	0.01	0.004	0.004
RMSE	0.008	0.004	0.009	0.005	0.01	0.01	0.01	0.004	0.004
Corr	0.42	0.17	0.31	-0.24	-0.19	-0.19	-0.19	0.31	0.31
HAC	Bartlett, NW	Bartlett, NW	Bartlett, NW	Bartlett, NW	Bartlett, NW	Bartlett, NW	Bartlett, NW	Bartlett, NW	Bartlett, NW
$\tau$	0.5	0.43	0 (rest.)	0 (rest.)	0 (rest.)	0 (rest.)	0 (rest.)	0 (rest.)	0 (rest.)

This Table reports some robustness checks using GMM estimation method. Column I reports the estimates of Basu and Kimball specification with disposable income, while column II shows the results from Cambell and Mankiw (1989) specification. Column III reports the estimates of Iacoviello's specification without imposing non-separable labour. Dependent variable is  $\Delta c_t$ . Instrument sets in column I and III are the same as in Table 1, column II and III respectively. In column II instrument set is  $z = \{c_{t-2} \cdot c_{t-4}; T_{t-2} \cdot T_{t-4}; y_{t-2} \cdot y_{t-4}; (c - y)_{t-2} \cdot (c - y)_{t-4}\}$ .

In more general, the results suggest that during financial market distress, like in Finland in the early 1990s, binding quantity constraints can become an important feature of aggregate behavior of the economy. Interesting and useful extension would be to allow only occasionally binding constraints and then fully account for general equilibrium effects by completing the modelling of housing and production side. This would provide a useful framework to assess, among other things, a transmission of monetary policy under different financial conditions.

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## A Mathematical appendix - detailed derivation

### A.1 Un-constrained household

Dynamic optimisation of un-constrained household is:

$$\max_{\{C_t, H_t, N_t\}} E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{(C_t^u)^{1-\gamma}}{1-\gamma} e^{(\gamma-1)v(N_t)} + \kappa f(H_t^u) \right)$$

*s.t.*

$$C_t^u + Q_t(H_t^u - H_{t-1}^u) + R_{t-1}B_{t-1}^u = B_t^u + W_t^u N_t^u + Y_t^u$$

where  $C_t^u$  is consumption of un-constrained households,  $Q_t$  is relative price of real estate,  $H_t^u$  is housing,  $R_t$  is gross real interest rate,  $B_t$  are real bonds ( $-B_t$  is lending),  $W_t$  is real wage,  $N_t$  is employment and  $Y_t$  is random endowment. I assume that  $f'(H_t) > 0$ ,  $f''(H_t) < 0$ .  $v(N_t)$  denotes disutility from labour with the usual properties  $v'(N_t) > 0$ ,  $v''(N_t) > 0$ .  $\gamma$  is the usual risk aversion parameter. The first order conditions become:

$$\begin{aligned} u_C(C_t^u, N_t^u, H_t^u) &= C_t^{u-\gamma} e^{(\gamma-1)v(N_t^u)} \\ u_H(C_t^u, N_t^u, H_t^u) &= Q_t C_t^{u-\gamma} e^{(\gamma-1)v(N_t^u)} \\ u_N(C_t^u, N_t^u, H_t^u) &= C_t^{u(1-\gamma)} e^{(\gamma-1)v(N_t^u)} v'(N_t^u) \end{aligned}$$

where  $u_j(\cdot), j = c, H, N$ , denotes marginal utility with respect to  $j = c, H, N$ . Intratemporal condition is:

$$W_t = -\frac{u_N(C_t^u, N_t^u, H_t^u)}{u_C(C_t^u, N_t^u, H_t^u)} = C_t^u v'(N_t^u) \quad (19)$$

Optimal choice of consumption implies the Euler equation

$$\begin{aligned} u_C(C_t^u, N_t^u, H_t^u) &= \beta R_t E_t u_C(C_{t+1}^u, N_{t+1}^u, H_{t+1}^u) \\ C_t^{-\gamma} e^{(\gamma-1)v(N_t)} &= \beta R_t E_t \left[ C_{t+1}^{-\gamma} e^{(\gamma-1)v(N_{t+1})} \right] \end{aligned} \quad (20)$$

Linearizing (20) yields:

$$-\gamma \hat{c}_t^u + (\gamma - 1) \tau \hat{n}_t^u = -\gamma \hat{c}_{t+1}^u + \hat{r}_t + (\gamma - 1) \tau \hat{n}_{t+1}^u \quad (21)$$

$\hat{c}_t$  denotes a percentage deviation of consumption from the steady state. When linearizing (20), I have used Taylor series expansion of  $v(N_t)$  around the constant level of optimal choice of labour  $N^*$ . More precisely, we have used the fact that for any function  $F(x)$

$$F(x_t) \approx F(x)(1 + \eta \hat{x}), \quad \eta = \frac{f'(x)x}{f(x)}$$

Since

$$\frac{\partial e^{(\gamma-1)v(N)}}{\partial N} = (\gamma - 1) e^{(\gamma-1)v(N)} v'(N) \quad (22)$$

and therefore

$$\eta = \frac{(\gamma - 1) e^{(\gamma-1)v(N)} v'(N) N}{e^{(\gamma-1)v(N)}} = (\gamma - 1) v'(N) N \quad (23)$$

In order to find  $v'(N)N$ , we evaluate an intratemporal condition at the steady state ie.  $W = Cv'(N)$ . It turns out that

$$v'(N)N = \left( \frac{WN}{C} \right) \equiv \tau. \quad (24)$$

Therefore, combining (23) and (3), we find that  $\eta = (\gamma - 1) \tau$ . Furthermore, notice that

$$\Delta v(N_t) \approx \tau (n_t - n_{t-1}) = \tau \Delta n_t \quad (25)$$

Substituting (4) into (21) yields finally:

$$\tilde{c}_t^u = E_t \tilde{c}_{t+1}^u - s \hat{r}_t - (1 - s) \tau E_t \Delta n_{t+1}^u \quad (26)$$

where  $s \equiv 1/\gamma$ , intertemporal elasticity of substitution.

## A.2 Constrained households

Constrained households solve the following problem:

$$\begin{aligned} & \max \left( \frac{(C_t^c)^{1-\gamma}}{1-\gamma} e^{(\gamma-1)v(N^c)} + \kappa f(H_t) \right) \\ & \text{s.t.} \\ & C_t^c + Q_t(H_t^c - H_{t-1}^c) + R_{t-1}B_{t-1}^c = B_t^c + W_t^c N_t^c + Y_t^c \\ & B_t^c \leq m E_t(Q_{t+1}) H_t^c / R_t \end{aligned}$$

$\phi_t$  is the time  $t$  shadow value of borrowing constraint and assume that household's collateral constraint hold with equality. Forming a Lagrangian and substituting the budget constraint into the maximization problem delivers:

$$\begin{aligned} \mathcal{L}_t^c = & \frac{(B_t^c + W_t^c N_t^c + Y_t^c - Q_t(H_t^c - H_{t-1}^c) - R_{t-1}B_{t-1}^c)^{1-\gamma}}{1-\gamma} e^{(\gamma-1)v(N)} + \kappa^c f(H_t^c) \\ & + \phi_t [m E_t(Q_{t+1}) H_t^c - R_t B_t^c]. \end{aligned}$$

The first order conditions for  $B_t^c$  and  $H_t^c$  and  $N_t^c$  yields:

$$(C_t^c)^{-\gamma} e^{(\gamma-1)v(N_t^c)} = R_t \phi_t \quad (27)$$

$$-Q_t (C_t^c)^{-\gamma} e^{(\gamma-1)v(N_t^c)} = \kappa f'(H_t^c) + m E_t(\phi_t Q_{t+1}) \quad (28)$$

$$W_t^c = C_t^c v'(N^c) \quad (29)$$

Linearizing (27) yields

$$\begin{aligned} -\gamma\hat{c}_t^c + (\gamma - 1)\tau\hat{n}_t^c &= \hat{\phi}_t + \hat{r}_t. \\ \hat{c}_t^c &= \tau^c(1 - s)\hat{n}_t^c - s\hat{\phi}_t - s\hat{r}_t, \quad s = \frac{1}{\gamma} \end{aligned} \quad (30)$$

$v(N_t^c)$  is linearized around the optimal (trend) level of labour supply just like in the case of un-constrained households.  $\hat{\phi}_t$  is Lagrange multiplier in percentage deviation from the steady state.

### A.3 Linearizing asset demand equation for constrained households

Start from the Euler equation (28):

$$Q_t (C_t^c)^{-\gamma} e^{(\gamma-1)v(N_t^c)} = k^c f'(H_t^c) + mE_t(\phi_t Q_{t+1}) \quad (31)$$

Then, notice that the steady state version of (27) gives

$$(C^c)^{-\gamma} e^{(\gamma-1)v(N^c)} = \bar{R}\phi \quad (32)$$

where the steady state interest rate  $\bar{R}$  can be found from steady state version of consumption Euler equation for un-constrained agents (20):

$$\bar{R} = \frac{1}{\beta} \quad (33)$$

Combining (33) and (32) yields

$$\beta (C^c)^{-\gamma} e^{(\gamma-1)v(N^{*c})} = \phi \quad (34)$$

Linearizing  $v(N_t^c)$  around optimal steady state level of labour supply  $N^{*c}$ , we find that LHS of (28) is:

$$Q (C^c)^{-\gamma} e^{(\gamma-1)v(N^{*c})} (1 + \hat{q}_t) (1 - \gamma\hat{c}_t^c) (1 + (\gamma - 1)\tau^c\hat{n}_t^c)$$

where  $\tau^c \equiv \frac{N^{*c}W}{C^c}$ . Linearizing RHS of (28) yields:

$$k^c f'(H^c) (1 - \theta\hat{h}_t^c) + mE(\phi Q) (1 + \hat{\phi}_t) (1 + \hat{q}_{t+1})$$

and where  $\theta \equiv -\frac{f''(H^c)H^c}{f'(H^c)}$ . Combining linearized versions of LHS and RHS yields:

$$\begin{aligned} Q (C^c)^{-\gamma} e^{(\gamma-1)v(N^{*c})} (1 + \hat{q}_t) (1 - \gamma\hat{c}_t^c) (1 + (\gamma - 1)\tau^c\hat{n}_t^c) &= k^c f'(H^c) (1 - \theta\hat{h}_t^c) \\ &\quad + mE(\Phi Q) (1 + \hat{\phi}_t) (1 + \hat{q}_{t+1}) \\ Q (C^c)^{-\gamma} e^{(\gamma-1)v(N^{*c})} [\hat{q}_t - \gamma\hat{c}_t^c + (\gamma - 1)\tau^c\hat{n}_t^c] &= -k^c f'(H^c) \theta\hat{h}_t^c \\ &\quad + mE(\Phi Q) [\hat{\phi}_t + \hat{q}_{t+1}] \end{aligned} \quad (35)$$

Next we notice that the steady state version of (31) implies:

$$\begin{aligned}
(C_t^c)^{-\gamma} e^{(\gamma-1)v(N^c)} &= k^c f'(H^c) + m\Phi \\
(C_t^c)^{-\gamma} e^{(\gamma-1)v(N^c)} &= k^c f'(H^c) + m\beta[(C^c)^\gamma e^{(\gamma-1)v(N^c)}] \\
1 &= \frac{k^c f'(H^c)}{(C^c)^\gamma e^{(\gamma-1)v(N^c)}} + m\beta \\
1 - m\beta &= \frac{k^c f'(H^c)}{(C^c)^\gamma e^{(\gamma-1)v(N^c)}}
\end{aligned}$$

Substituting this into (35), and using (34) yields:

$$\begin{aligned}
Q (C^c)^{-\gamma} e^{(\gamma-1)v(N^c)} [\hat{q}_t - \gamma \hat{c}_t^c + (\gamma-1)\tau^c \hat{n}_t^c] &= -k^c f'(H^c) \theta \hat{h}_t^c + mE(\beta (C^c)^\gamma e^{(\gamma-1)v(N^c)} Q) [\hat{\phi}_t + \hat{q}_{t+1}] \\
[\hat{q}_t - \gamma \hat{c}_t^c + (\gamma-1)\tau^c \hat{n}_t^c] &= -\frac{k^c f'(H^c)}{Q (C^c)^{-\gamma} e^{(\gamma-1)v(N^c)}} \theta \hat{h}_t^c + m\beta E_t [\hat{\phi}_t + \hat{q}_{t+1}] \\
\hat{q}_t - \gamma \hat{c}_t^c + (\gamma-1)\tau^c \hat{n}_t^c &= -(1-m\beta) \theta \hat{h}_t^c + m\beta E_t [\hat{\phi}_t + \hat{q}_{t+1}], \quad Q = 1. \\
\hat{q}_t - \gamma \hat{c}_t^c + (\gamma-1)\tau^c \hat{n}_t^c &= -\theta (1-m\beta) \hat{h}_t^c + m\beta \hat{\phi}_t + m\beta E_t \hat{q}_{t+1} \tag{36}
\end{aligned}$$

where I have normalised  $Q = 1$ . Furthermore, recall from the consumption Euler equation for constrained agents that

$$\begin{aligned}
(C_t^c)^{-\gamma} e^{(\gamma-1)v(N_t^c)} &= \phi_t R_t \\
&\approx \\
-\gamma \hat{c}_t^c + (\gamma-1)\tau^c \hat{n}_t^c &= \hat{\phi}_t + r_t \\
&\Rightarrow \\
\hat{\phi}_t &= -\gamma \hat{c}_t^c + (\gamma-1)\tau^c \hat{n}_t^c - r_t
\end{aligned}$$

Using this to substitute away  $\hat{\phi}_t$  from (36) yields:

$$\begin{aligned}
\hat{q}_t - \gamma \hat{c}_t^c + (\gamma-1)\tau^c \hat{n}_t^c &= -(1-m\beta) \theta \hat{h}_t^c + m\beta [-\gamma \hat{c}_t^c + (\gamma-1)\tau^c \hat{n}_t^c - r_t] + m\beta E_t \hat{q}_{t+1} \\
\hat{q}_t - \gamma \hat{c}_t^c + m\beta \gamma \hat{c}_t^c + (\gamma-1)\tau^c \hat{n}_t^c - m\beta [(\gamma-1)\tau^c \hat{n}_t^c] &= -(1-m\beta) \theta \hat{h}_t^c - m\beta r_t + m\beta E_t \hat{q}_{t+1}
\end{aligned}$$

Multiplying both sides by  $\frac{1}{\gamma}$  and using  $s \equiv \frac{1}{\gamma}$  yields:

$$s\hat{q}_t - \hat{c}_t^c + m\beta \hat{c}_t^c + (1-s)\tau^c \hat{n}_t^c - m\beta(1-s)\tau^c \hat{n}_t^c = -s(1-m\beta) \theta \hat{h}_t^c - sm\beta r_t + sm\beta E_t \hat{q}_{t+1}$$

Solving for  $\hat{c}_t^c$ :

$$\hat{c}_t^c = s\theta \hat{h}_t^c + s \frac{m\beta}{1-m\beta} r_t - s \frac{m\beta}{1-m\beta} E_t \hat{q}_{t+1} + \frac{s}{(1-m\beta)} \hat{q}_t + (1-s)\tau^c \hat{n}_t^c$$

Denoting  $1 + \omega = \frac{1}{1-m\beta}$  and so that  $\omega = \frac{m\beta}{1-m\beta}$  we finally get an expression:

$$\hat{c}_t^c = s\theta \hat{h}_t^c + s\omega(\hat{r}_t - E_t \hat{q}_{t+1}) + s(1+\omega)\hat{q}_t + (1-s)\tau^c \hat{n}_t^c \tag{37}$$

$1 + \omega$  is inverse of the downpayment needed to purchase one unit of housing and where

$$\theta \equiv -\frac{f''(H^c)H^c}{f'(H^c)} > 0, \text{ since } f'(H_c) > 0, f''(H_c) < 0, H^c > 0.$$

Recalling furthermore that  $\Delta\hat{q}_{t+1}$  is a change in the relative, or real price of housing, and that  $\hat{r}_t$  is real ex-ante interest rate expressed in consumption price inflation, we have that

$$\begin{aligned} & \hat{r}_t - E_t(\hat{q}_{t+1} - \hat{q}_t) \\ &= i_t - (p_{t+1}^c - p_t^c) - [(q_{t+1}^c - q_t^c) - (p_{t+1}^c - p_t^c)] \\ &= i_t - [(q_{t+1}^c - q_t^c)] \\ &\equiv r_t^h \end{aligned}$$

where  $r_t^h$  denotes real ex ante housing interest rate. Consequently, we can re-express (37) as:

$$\begin{aligned} \hat{c}_t^c &= s\theta\hat{h}_t^c + s\omega r_t^h + s\hat{q}_t + (1-s)\tau^c\hat{n}_t^c \\ \hat{c}_t^c - \tau^c\hat{n}_t^c &= s[\theta\hat{h}_t^c + \omega r_t^h + \hat{q}_t - \tau\hat{n}_t^c] \end{aligned}$$

#### A.4 Derivation of aggregate Euler equation

In this section, I derive aggregate Euler equation. Recall for convenience the following Euler equations for unconstrained and constrained agents:

$$\begin{aligned} \hat{c}_t^u &= E_t\hat{c}_{t+1}^u - s\hat{r}_t - (1-s)\tau E_t\Delta n_{t+1}^u \\ \hat{c}_t^c &= s\theta\hat{h}_t^c + s\omega(\hat{r}_t - E_t\hat{q}_{t+1}) + s\omega\hat{q}_t + (1-s)\tau^c\hat{n}_t^c \end{aligned}$$

Make rational expectations (RE) assumption explicit *i.e.* that

$$\begin{aligned} E_t\hat{c}_{t+1}^u &= c_{t+1}^u + \epsilon_t^{cu} \\ E_t\Delta n_{t+1}^u &= \Delta n_{t+1}^u + \epsilon_t^{\Delta n} \\ E_t\hat{q}_{t+1} &= \hat{q}_{t+1} + \epsilon_t^q \\ E_t\hat{n}_{t+1} &= \hat{n}_{t+1} + \epsilon_t^n \end{aligned}$$

$\epsilon_t^i$  is forecast error term. Substituting RE assumptions into corresponding Euler equations yields

$$\Delta\hat{c}_{t+1}^u = s\hat{r}_t + \tau^u(1-s)\Delta n_{t+1}^u + \epsilon_t + \alpha_u\epsilon_{t-1}$$

$$\hat{c}_t^c = s\theta\hat{h}_t^c + s\omega\hat{r}_t - s\omega\Delta\hat{q}_{t+1} + (1-s)\tau^c\hat{n}_t^c + \epsilon_t + \alpha_c\epsilon_t$$

Notice then that aggregate consumption can be expressed in log first differenced form:

$$\begin{aligned} \hat{c}_t &= \lambda\hat{c}_t^c + (1-\lambda)\hat{c}_t^u \\ &\Rightarrow \\ \Delta c_t &= \lambda\Delta c_t^c + (1-\lambda)\Delta c_t^u \end{aligned} \tag{38}$$

Substituting  $\Delta c_{t+1}^u$  and  $\hat{c}_t^c$  in (38), we find that:

$$\begin{aligned}\Delta c_t &= \lambda[s\theta\hat{h}_t^c + s\omega\hat{r}_t - s\omega\Delta q_{t+1} + (1-s)\tau^c\hat{n}_t^c + \epsilon_t + \alpha_c\epsilon_t - c_{t-1}^c] \\ &\quad + (1-\lambda)[s\hat{r}_t + \tau^u(1-s)\Delta n_t^u + \epsilon_t + \alpha_u\epsilon_{t-1}]\end{aligned}$$

$$\begin{aligned}\Delta c_t &= \lambda[s\theta\hat{h}_t^c + s\omega\hat{r}_t - s\omega\Delta q_{t+1} + (1-s)\tau^c\hat{n}_t^c + \epsilon_t + \alpha_c\epsilon_t - c_{t-1}^c] \\ &\quad + (1-\lambda)[s\hat{r}_t + \tau^u(1-s)\Delta n_t^u + \epsilon_{t-1} + \alpha_u\epsilon_{t-2}]\end{aligned}$$

$$\begin{aligned}\Delta c_t &= \lambda[s\theta\hat{h}_t^c + s\omega\hat{r}_t - s\omega\Delta q_{t+1} + (1-s)\tau^c\hat{n}_t^c + \epsilon_t + \alpha_c\epsilon_t - c_{t-1}^c] \\ &\quad + (1-\lambda)[s\hat{r}_t + \tau^u(1-s)\Delta n_t^u + \epsilon_{t-1} + \alpha_u\epsilon_{t-2}]\end{aligned}$$

$$\begin{aligned}\Delta c_t &= \lambda[s\theta\Delta\hat{h}_t^c + s\omega\Delta\hat{r}_t - s\omega\Delta\Delta q_{t+1} + (1-s)\tau^c\Delta\hat{n}_t^c + \Delta\epsilon_t + \alpha_c\Delta\epsilon_t] \\ &\quad + (1-\lambda)[s\hat{r}_t + \tau^u(1-s)\Delta n_t^u + \epsilon_{t-1} + \alpha_u\epsilon_{t-2}]\end{aligned}$$

$$\Delta c_t = \lambda[s\theta\Delta\hat{h}_t^c + s\omega\Delta\hat{r}_t - s\omega\Delta\Delta q_{t+1}] + (1-s)[\lambda\tau^c\Delta n_t^c + \tau^u(1-\lambda)\Delta n_t^u] + (1-\lambda)s\hat{r}_t + \epsilon_t' \quad (39)$$

Problem with this expression is that we have several unobserved variables. In particular, we do not have observations on consumption and employment for unconstrained and constraint agents separately. However, there is a way to simplify the above equation. First, we make use of an aggregate constraint for labour:

$$N_t = N_t^u + N_t^c$$

Linearizing this yields:

$$\hat{n}_t = \lambda_n\hat{n}_t^c + (1-\lambda_n)\hat{n}_t^u$$

where  $\lambda_n$  is average employment share of constraint households. Recall then from the intratemporal condition for labour that for both households

$$W_t = C_t^u v'(N_t^u) \quad (40)$$

$$W_t = C_t^c v'(N_t^c) \quad (41)$$

Equalising (40) and (41), and evaluating these in the steady state delivers an expression for relative consumption shares of the households, which depends on the relative marginal disutility of labour.

$$\begin{aligned} C_t^c v'(N_t^c) &= C_t^u v'(N_t^u) \\ \frac{C^c}{C^u} &= \frac{v'(N^u)}{v'(N^c)} \end{aligned} \quad (42)$$

We also know that  $N^i v'(N^i) = \left(\frac{WN^{ci}}{C^{ci}}\right) \equiv \tau^i$ . Using these to replace  $v'(\cdot)$  in (42) gives

$$\frac{N^c}{N^u} = \frac{C^c \tau^c}{C^u \tau^u} \quad (43)$$

Recall then that  $\lambda$  is consumption share of constraint agents and that  $C = C^u + C^c$ . Thus

$$\frac{\lambda_n}{1 - \lambda_n} = \frac{N^c}{N^u} = \frac{\tau^c C^c / C}{\tau^u C^u / C} = \frac{\tau^c \lambda}{\tau^u (1 - \lambda)} \quad (44)$$

Consider now the term

$$[\lambda \tau^c \Delta n_t^c + (1 - \lambda) \tau^u \Delta n_t^u]$$

in (39). Recalling from (44) that

$$\tau^c \lambda = \frac{\lambda_n}{1 - \lambda_n} \tau^u (1 - \lambda)$$

so that

$$\begin{aligned} [\lambda \tau^c \Delta n_t^c + (1 - \lambda) \tau^u \Delta n_t^u] &= \left[ \frac{\lambda_n}{1 - \lambda_n} \tau^u (1 - \lambda) \Delta n_t^c + (1 - \lambda) \tau^u \Delta n_t^u \right] \\ &= (1 - \lambda) \tau^u \left[ \frac{\lambda_n}{1 - \lambda_n} \Delta n_t^c + \Delta n_t^u \right] \end{aligned}$$

Then, using the fact that

$$\Delta n_t^u = \frac{1}{1 - \lambda_n} \Delta n_t - \frac{\lambda_n}{1 - \lambda_n} \Delta n_t^c$$

so that

$$\begin{aligned} [\lambda \tau^c \Delta n_t^c + (1 - \lambda) \tau^u \Delta n_t^u] &= (1 - \lambda) \tau^u \left[ \frac{\lambda_n}{1 - \lambda_n} \Delta n_t^c + \Delta n_t^u \right] \\ &= (1 - \lambda) \tau^u \left[ \frac{\lambda_n}{1 - \lambda_n} \Delta n_t^c + \left[ \frac{1}{1 - \lambda_n} \Delta n_t - \frac{\lambda_n}{1 - \lambda_n} \Delta n_t^c \right] \right] \\ &= \frac{(1 - \lambda) \tau^u}{1 - \lambda_n} \Delta n_t \end{aligned}$$

Finally recall that by definition  $\frac{(1-\lambda)\tau^u}{1-\lambda_n} = \frac{\tau^c\lambda}{\lambda_n} = \tau$ , and so we get that

$$[\lambda\tau^c\Delta n_t^c + (1-\lambda)\tau^u\Delta n_t^u] = \frac{(1-\lambda)\tau^u}{1-\lambda_n}\Delta n_t = \tau\Delta n_t \quad (45)$$

Consequently, substituting (45) into (39) we arrive into following expression for linearized aggregate consumption Euler equation:

$$\Delta c_t = \lambda[s\theta\Delta h_t^c + s\omega\Delta r_t - s\omega\Delta\Delta q_{t+1}] + (1-s)\tau\Delta n_t + (1-\lambda)sr_t + \epsilon_t \quad (46)$$

Alternatively, replacing  $\Delta r_t - \Delta\Delta q_{t+1} = r_t^h - r_{t-1}^h$ , we finally arrive to

$$\Delta c_t - \tau\Delta n_t = \lambda s[\theta\Delta h_t^c + \omega\Delta r_t^h - \hat{r}_t] + s(\hat{r}_t - \tau\Delta n_t) + \epsilon_t \quad (47)$$

where  $r_t^h$  denotes housing real interest rate and  $i_t$  is nominal short term interest rate. This is equation (14) in the main text