Revising the Salop-equilibrium in the case of attention restricted consumers (Preliminary version)*

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Abstract

I investigate the effects of exogenously restricted consumer attention on equilibrium price, diversity and advertising in a Salop-type model. Limited attention is modelled as an upper bound on how many different products can be considered by a consumer. The positive analy-sis reveals that limited attention increases prices by isolating a firm's demand function from competition. As higher prices increase the marginal revenue of advertising also equilibrium advertising is increased. In the no-entry equilibrium this leads to increased profits and redistribution from consumer rent to firms. In the free-entry equilibrium limited attention thus increases equilibrium diversity over zero-profit conditions. Conditions are provided that lead to an attention economy - namely low advertising and setup costs but also high consumer sensitivity which are compatible with a modern economy. The market outcome is generally inefficient as under fairly general conditions too much diversity and too few ads are supplied and limited attention tends to intensify this problem. As a logical extension I also consider the effect of the possibility of spamming the consumers on the equilibrium variables.

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1 Introduction

When a modern economist is confronted with a situation involving an economic choice of any dimension he usually thinks of the problem in terms of maximizing a value function subject to a set of constraints. However both aspects of the standard solution concept have been widely critizised empirically and theoretically. In a seminal paper Simon (1955) revises the notion of the "'rational economic man"' by introducing the idea of additional "'internal constraints"' which account for the fact that besides the traditional budget constraints representing economic scarcity of physical resources an agent also faces physiological and psychological limitations to which he must obey. This reflection and subsequent research patchwork the idea of bounded rationality. Much work has been done to establish theoretical and empirical evidence on what the implications of bounds on rationality might be for economic models (see Conlisk (1996) for a survey). Recently, attention was directed towards cognitive processes of decision making by asking questions like "'how much time should I spend on evaluating a given mass of information before deciding"' (see Gabaix and Laibson, 2004) recognizing the existence of deliberation costs in decision-making. A conceptually different but complementary viewpoint of such an approach is embededded in the idea of limited attention. Recent work in psychology highlights the importance of limitations on perceiving multiple stimuli for making decisions, storing information, planning actions and other mental processes (Pashler, 1998). That attention - since it is limited - must be of importance for economics already can be seen from the expression that "'one should PAY attention to something"'. But it is perhaps not sheer irony that has prevented economists from adding an attention constraint to economic problems already earlier. In an economic setting, attention plays a central role if the information is sufficiently dense as then the attention itself gets scarce (Simon, 1971) and therefore by a standard economic argument very valuable. Thus it is reasonable to expect the attention problem to be of central importance in an information-rich environment - which clearly characterizes a modern economy after IT- and mass media revolution. Today the struggle for attention is seen as a major concern of modern business (Davenport and Beck, 2001) especially since the massiv increase of the internet usage. It is important to make a clear distinction between bounded rationality which refers to the mental process of how a decision is made (people make errors, use approximations or "‘heurisitcs"’ etc., see Payne (1993)) and limited attention which in this paper means that agents have a finite attention capacity and thus only are capable of considering a certain fixed subset of all possible information. But in deciding among the alternatives of this subset they are fully rational in the typical microeconomic sense that they have a rational preference ordering and solve the conditional utility maximization problem correctly. Research has been done by endogenizing the attention problem of an imperfectly informed agent and showing that a certain amount of inattention is optimal (Reis (2006a and 2006b), also Gifford (2005)) and exploring the implications of rational inattention on macroeconomic topics such as the permant income hypothesis (Sims, 2003). However, these models focus basically on the receiver side of information in determining how much attention optimally should be allocated to what type of information or how frequently agents should update their information. Looking at the sender side recent work by Falkinger (2007) emphasizes that attention is not to be regarded as an economic good as there are no markets for attention but that an economic interaction between a sender and a receiver previously requires that the attention of the receiver is directed towards the sender, i.e. that the sender must win a competition for the attention before competing for other economic resources. This simple setup however should make clear that in the presence of limited attention there can exist attention-caused strategic incentives in oligopolistic markets for the senders (in my model these simply will be firms). In a further, very rich model Falkinger (2008) endogeneously determines wheter an economy exhibits limited attention or not in a standard Dixit-Stiglitz model of monopolistic competition. One conclusion of the paper is that an increase in the degree of monopoly power which is fully exogeneous due to the CES-preferences of the buyers increases the probability of an economy to be an attention economy. In this paper I will argue that in a market for differentiated goods rational firms that account for the attention constraint of their consumers the presence of limited attention can increase market power ad extremis up to monopoly level and that therefore this channel is a consequence AND a cause of limited attention which - if unregulated - might be socially very costly. To keep ideas as tractable as possible and to fix economic reasoning to a situation well imaginable I adopt the standard model of product differentiation by Grossmann and Shapiro (1984) which builds on the circular Salop model (Salop, 1979). Thinking of the signals as informative ads emitted by competing firms is a very natural and also important way to explore the strategic incentive effects of limited attention. In this sense this contribution plays a double role: I explore the incentive effects of limited attention on the competitive behaviour of rivaling firms and since informative advertising is the particular signal the findings of this paper may also help to explain some of the empirical paradoxes encountered in investigating the effects of advertising on sales and market concentration (see Bagwell, 2007, for a survey).

2 The Attention Economy

Falkinger (2007) divides the economy in senders and receivers of information. My paper focuses on a concrete economic situation where the senders are advertsing firms, the ads are the informative signal and the receivers are the (potential) consumers. Concerning the senders of information (the firms) Falkinger (2007) identifies two characteristic elements. The first one is the radiation capacity (ρ) of a firm which is exogeneously given and measures the maximal audience size (the range) that can be adressed by a firm. The second element is signal strenght (σ_T), which is the choice variable of each firm. The aggregate signal strenght of the mass of all those firms of which a particular consumer receives a signal constitutes the signal exposure and limited attention means that after a certain threshold value of exposure the attention of the consumers declines with increasing exposure. My paper builds on this preliminary work. Of central importance is how limited attention enters the model. The idea builds on Falkinger (2008) and is kept very simple to illustrate and discuss the main economic effect of limited attention in an oligopolistic scenario as clearly as possible. The number of ads[1] received by a particular consumer i constitute his information set I_i. Limited Attention (LA) is modelled as an upper bound $R \geq 1$ and is fully exogenous and for simplicity the same for all consumers[2]. I call the set of information conditional on which the consumer effectively optimizes his choice set C_i. If $I_i \leq R$ then the attention of this particular consumer is not constrained meaning that $I_i = C_i$

[1] Note that Falkinger (2007 and 2008) uses continuous measures wheras I model the economy in discrete terms.

[2] As firms are risk neutral and consumers and ads will be distributed randomly only the average R in the economy matters for the firm's decision.

and he considers all of the ads he received. If $I_i > R$ then $C_i = R$ as the attention constraint gets binding. In this paper I think of a particular way on how consumers act if they receive too many ads: I assume that people ignore all ads that are in excess completely. The ads that remain in C_i are drawn from I_i by a random process. This will affect the probability of a firm to make a sale to a customer. Since there is market power due to the imperfect competition stemming from heterogenous tastes of consumers it is precisely the interplay of strategic price-setting and advertising under limited attention that will explain the main results of this paper.

Another important distinction is how sensitive consumers react to the intensity of a signal. In the basic model consumers treat all ads received from the SAME firm as a single ad (as a single information unit). This can be interpreted as consumers being invariant to spamming-effects: they recognize instantaneously and without loss of attention if two ads are referring to the same firm and treat them as one[3]. Put differently, I normalize the signal strenght of a firm to one meaning that each firm will never deliberately send more than one ad to a specific consumer (since ads are costly). In this setting firms only can choose their potential market size, i.e. how many consumers they want to inform as a fraction of all potential consumers. I will show that the presence of LA will give market power to the firm as their expected markets get stronger leading to higher prices as in the absence of LA and in the no-entry-equilibrium to higher profits which finally implies a larger amount of diversity in the free-entry equilibrium. This effect gets particularly strong if the attention bound is low (i.e. close to $R = 1$). As an extension I examine what happens if consumers are not invariant towards spamming. Again to get the results as clear-cut as possible I assume that in this case consumers treat each ad as a novum. Since limited attention in my model works only through fixing the maximal number of ads that a consumer wants to consider in detail this means that he might end up with many ads of the same firm in his choice set which could never occur in the basic model. In this sense firms must trade off the number of consumers they want to inform (as before) versus how strong a particular consumer should be spammed. Thus firms can optimize between the reach and the intensity of their signal. Note however that in either case consumers are fully rational in choosing among

[3]If ads where mails then this could be interpreted as a spamming-filter: after I have received one ad of a firm I program the filter to spam out all further ads of this particular firm

the perceived products (i.e. among those products in the choice set).

3 The basic model

As mentioned above I use the extension of the Salop model by Grossmann and Shapiro (further referred to as GS) as workhorse. Consumers and firms are located uniformly around the circumference of the unit circle. δ represents the number of consumers and n the number of firms. In the no-entry-equilibrium n is exogeneous whereas the zero-profit-condition endogenizes n in the free-entry-equilibrium.[4]. Consumption is a zero-one-decision and consumers have a linear utility function[5]:

$$u = v - tw - p \tag{1}$$

v is the value of the product to the consumer and it is the same for all consumers, t measures transportation costs and w is the distance between the location of the firm and the location of the ideal product. p is the price and u is net utility. Consumers choose the product out of their information set that gives them the highest net utility. Uninformed consumers do not consume at all[6].

Firms can set their price and choose the number of ads. An ad generates a sale if the firm's product gives the receiving consumer the highest net utility. Ads are produced with the strictly convex cost function[7] $C(m)$ where $C' > 0$ and $C'' > 0$. In the simulation part I make use of the following specific cost function:

$$C(m) = \theta m^{\alpha} \tag{2}$$

[4]As usual the coice of location is not modeled and it is always assumed that firms are located symmetrically around the circle.

[5]Quadtratic transportation costs do not alter the main results qualitatively

[6]As in GS no search procedure is pictured. In the discussion about LA that is born by this paper it is the authors belief that including search would not alter (but possibly amplify) the role that LA plays in the equilibrium analysis. A special focus is put on equilibria with quasi-full information making search obsolete.

[7]This is also in accordance with Falkinger (2007 and 2008)

with $\theta > 0$ and $\alpha > 1$.

Ads are distributed randomly to consumers. Firms thus cannot target a specific consumer group (e.g. their prime segment) since they cannot identify the particular consumer. Since firms know the overall market size[8] δ it is more natural to work directly with the advertising intensity function $\phi \in [0, 1]$ of the firm[9] which is

$$\phi = \begin{cases} \frac{m}{\delta} & m < \delta \\ 1 & m \geq \delta \end{cases} \tag{3}$$

Using the cost function with (3) we get

$$C(m) = C(\phi\delta) \equiv A(\phi) \tag{4}$$

with $A' > 0$ and $A'' > 0$. For the illustrative technology (4) is

$$A(\phi) = \theta\,(\phi\delta)^{\alpha} \tag{5}$$

As GS and Salop I will only examine the symmetric equilibrium. To find the equilibrium we must first calculate a firms expected demand function from which its reaction function can be deduced.

Note that since consumers are distributed uniformly around the unit circle ϕ measures both the probability of reaching a certain consumer and the fraction of the whole population informed by a firm. Limited Attention is modelled as an upper bound on how many ads a consumer observes. In the present model this bound is fully exogeneous and quantified by the parameter R. $R = 4$ means that a consumer pays attention to at most 4 ads. If he recieves e.g. 6 ads he throws away 2 immediately. In order to derive the expected demand curve which my firm

[8] This logically corresponds to the radiation capacity in Falkinger (2007) which is exogenously fixed and the same for all firms

[9] Note that the specific distribution function is somewhat different than the CRIR-function of GS as the idea of adressing a fraction of the population by mailing technology seems more appropriate than the reader type distribution. However, technically it is also appealing as it gives raise to a constant elasticity of advertising. As calculations and simulations show the results of the main section also holds for the CRIR function.

faces[10] I basically proceed as GS by dividing the mass of consumers into n groups, where group 1 encompasses all those consumers for which I produce the first favourite product in terms of net utility (i.e. given my price p and $\bar{p}$) under perfect information. Group k are therefore all those consumers for which I produce the kth favourite product. A firm derives its demand curve simply by calculation the probability of making a sale in a given group and summing over all groups. In GS since there are no bounds on the attention this means that I only have to consider those subsets of the ad distribution for which in a given group k the consumers receive my ad and not the ads of the $k-1$ better firms. With LA however I may have the possibility to make a sale in this group even if all better firms also have reached this consumers because he might throw away precicely those ads. Therefore under LA it is necessairy to account for the complete distribution of ads in deriving a firm's demand function. This calculation is not fully trivial at first glance. Therefore I first present a full analysis of the easiest case where $R = 2$ and $n = 3$ and carefully develop the basic intuition. (The case where $R = 1$ is special as it gives raise to full monopoly power which is not the case for any $R > 1$. Its analysis is postponed to the appendix).

3.1 Solution for $n = 3$ and $R = 2$

In deriving the symmetric equilibrium I necessarily have to solve for the reaction function of my firm. The most general form of the profit function that will be considered in this paper is

$$\Pi = (p - c)D\left(p, \phi, \kappa, \bar{p}, \bar{\phi}, \bar{\kappa}, n; R\right) - C(\phi, \kappa) - F \tag{6}$$

p, ϕ, and κ are my choice variables. I take $\bar{p}$, $\bar{\phi}$ and $\bar{\kappa}$ of my opponents as given.[11]

[10] To keep things simple I call the particular firm under consideration "'my"' firm

[11] This is the same reduced form approach as in Salop (1979) and also Grossman and Shapiro (1984). Concerning symmetric equilibrium all results derived could also be obtained by admitting $\bar{p}_2, \bar{p}_3, ..., \bar{p}_{n-1}$. The reason for this is that my price affects only the group size of those consumers for whom I am the first best or the least best alternative. Digging deeper I find that the driving force behind this result is linear transportation costs. With quadratic transportation costs my pricing also affects the size of the consumer groups for which I offer a moderate good. But even then it can be shown that in the symmetric equilibrium values of ϕ and p are not affected if a firm would take into account different rival prices.

p denotes the price, $\phi \in [0,1]$ advertising intensity and κ spamming intensity (which is not considered in the main part of the paper). In order to solve problem (6) in the Salop model the task is to derive an expression for the demand function.

3.1.1 Setup

The simplest way to derive the demand function is to divide the set of all consumers into n groups, where the kth group indicates that for all consumers in this group my firm produces the kth best good in terms of net utility and under perfect information. With perfect information I can only make a sale to those consumers in the first best group which then leads to the usual Salop equilibrium. However, if consumers are informed only by means of advertising firms this changes the equilibrium outcome as a firm can potentially make a sale to a badly informed consumer far away. This is the extension considere by GS. This means - especially when n is large - that consumers may receive many ads. Limited attention goes even one step further and states that if the number of ads a consumer receives is larger than a certain exogenous threshold number the consumer does not consider any further ads.

To solve this problem I require a more general solution concept as in GS. For each group $k = 1, 2, ..., n$ my firm wants to know the probability of making a sale to a member of the group. Call this probability $P[k]$. This probability is composed of the probability to reach a member of this group and the probability of not beeing ignored due to limited attention. Thus

$$P[k] = \phi P[k \,|\phi] \tag{7}$$

Remember that groups are defined over net utility. If we know $\phi P[k \,|\phi]$ for each k we can then calculate expected demand. When deriving the demand curve we can work without loss of generality with the reduced form where the (expected) prices of my opponents are all the same, i.e. $\bar{p}_2 = \bar{p}_3 = \bar{p}$ and also $\bar{\phi}_2 = \bar{\phi}_3 = \bar{\phi}$ as we are interested in symmetric equilibrium only.

3.1.2 Derivation of $P[k\,|\phi]$

Consider a consumer which is reached by my ad and also by the ads of j inferior firms (in net utility). If the number of ads exeeds R the consumer ignores the spare ads randomly. Let $z(j,R) \in (0,1]$ be the probability that my ad is not ignored and I am the best firm among j competitors also not ignored. If $j+1 \leq R$ then $z=1$. I impose $z_j \leq 0$ and $z_R \geq 0$. If there are more competitors my probability z to win the situation can never increase (for given R) but vice versa never decreases if the consumers is less attention constrained (for given j). By definition, $z(j,R)$ is also the winning probability of a conditional competition if I am the best firm among all received ads. Analogously, let $\bar{z}(j,R)$ be the conditional winning probability of the opponent who is the best firm among all his competitiors (including my firm!). For $n=3$ and $R=2$ the conditional winning probability for a consumer in $k=1$ is therefore

$$\begin{aligned} P[1\,|\phi] &= (1-\bar{\phi})^2 + 2\bar{\phi}(1-\bar{\phi}) + \bar{\phi}^2 z(2,2) \\ &= 1 + \bar{\phi}^2\left(z(2,2)-1\right) \end{aligned} \tag{8}$$

Note that without LA we have $z(2,2)=1$ and the GS solution with $P[1\,|\phi]=1$ applies. Thus for given $\bar{\phi}$ the conditional winning probability in the primary group is diminished under limited attention. For the groups $k=2,3$ I find

$$\begin{aligned} P[2\,|\phi] &= (1-\bar{\phi})^2 + \bar{\phi}(1-\bar{\phi})z(1,2) + \bar{\phi}^2\left(1-\bar{z}(2,2)\right)z(1,2) \\ &= 1-\bar{\phi}+\bar{\phi}^2\left(1-\bar{z}(2,2)\right) \\ P[3\,|\phi] &= (1-\bar{\phi})^2 \end{aligned} \tag{9}$$

Thus for given $\bar{\phi}$ the conditional winning probability in group $k=2$ is higher with limited attention compared to GS and the same in group $k=3$.[12]

[12]This makes clear why $n=3$ and $R=2$ is the first generally interesting case to examine the attention effects as with $n=2$ and $R=2$ and no spamming we could never be in an attention economy.

3.1.3 Equilibrium

With linear transportation costs[13], i.e. consumer utility is $u = v - t\omega - p$ and $\omega \in [0, 1/2]$ is the measure of distance, we have the same group demand structure for perfect information as in the Salop-model[14]:

$$N_1 = \left(\frac{\bar{p} - p}{t} + \frac{1}{3}\right)\delta \qquad N_2 = \frac{\delta}{3} \qquad N_3 = \left(\frac{p - \bar{p}}{t} + \frac{1}{3}\right)\delta \tag{10}$$

By symmetry $\bar{z}(2,2) = z(2,2) \equiv z$. Then demand is given as

$$\begin{aligned} D(\phi, p; \bar{\phi}, \bar{p}, z) &= \sum_{k=1}^{3} \phi P\left[k\,|\phi\right] N_k \\ &= \delta\phi\left(\tfrac{3-\bar{\phi}(3-\bar{\phi})}{3} + \tfrac{\bar{p}-p}{t}\left(\bar{\phi}(2 - \bar{\phi}(2-z)\right)\right) \end{aligned} \tag{11}$$

The profit function is

$$\Pi = (p - c)\delta\phi D(\phi, p; \bar{\phi}, \bar{p}, z) - F - A(\phi) \tag{12}$$

With the demand function we can solve the firms profit maximization problem (always assuming that second-order-conditions hold) and find the symmetric equilibrium by setting $\phi = \bar{\phi}$ and $p = \bar{p}$.

$$p = c + \frac{t}{3\phi}\frac{3 - (3-\phi)\phi}{2 - (2-z)\phi} \tag{13}$$

$$\frac{\delta(p - c)\left(3 - (3-\phi)\phi\right)}{3} = A'(\phi) \tag{14}$$

Note that (14) is independent of z which will be important for the intuitive understanding. (13) and(14) can be summarized into

[13]The results do not hinge on this assumption as all the predictions of the model are also valid with quadratic transportation costs. Linearity is chosen for simplicity.

[14]For a derivation see Grossman and Shapiro (1985). The exogeneity assumption of attention in this paper implies that attention is also independent of observed prices. This is not so much of a problem here as the model is static and we consider the symmetric equilibrium only. In a general setting one could allow individuals to allocate more attention towards a market if the observed prices are very different.

$$\frac{t\delta}{9\phi}\frac{(3-(3-\phi)\phi)^2}{2-(2-z)\phi} = A'(\phi) \tag{15}$$

The LHS of (15) is a strictly decreasing function of z. Thus we can state proposition one

Proposition 1 *In the case where $n = 3$ and $R = 2$*

1. *there is more equilibrium advertising at the firm level*[15]*,*
2. *the equilibrium price is higher and*
3. *equilibrium profits are higher*

in the case of limited attention, i.e. if $z < 1$.

See the appendix for a formal proof. At this point I use graphical illustrations to make the intuition of this result as clear as possible. First note from (14) that if prices where exogenously given advertising intensity would be the same for all $z \in (0, 1]$. But considering (13) we see that the equilibrium price is a decreasing function of z for a given ϕ. Thus prices are higher if the attention constraint binds. As prices increase also ϕ increases by (14). Figure 1 illustrates this finding grafically.

Where does this price effect come from? First note the following facts[16].

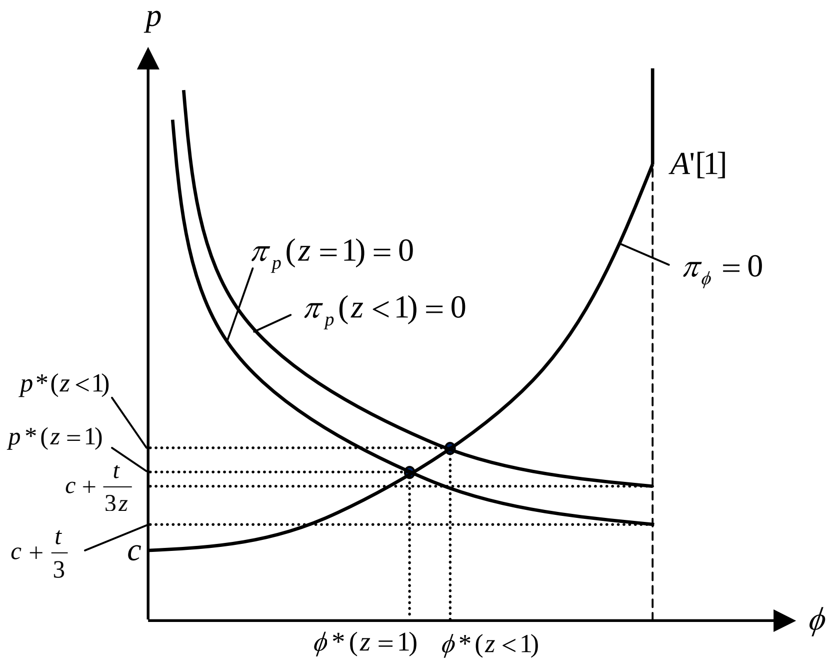
Fact 1

$p_{\bar{p}} = 1/2 > 0$: Prices are strategic complements which is independent of limited attention and is driven only by the usual Salop effect.

Fact 2

[15] As $n = 3$ is fixed also on the economy level

[16] Proofs are obtained by implicit differentiation of the according FOC.



$\phi_{\bar{\phi}} = \frac{\delta(p-c)(2\bar{\phi}-3)}{3} < 0$ if $\bar{p} = p$. Ads then are strategic substitutes which is independent of limited attention and is the GS effect.

Thus the effect of limited attention is not channeled directly over prices and ads. Where limited attention starts to play its crucial role is in the cross-partial derivatives.

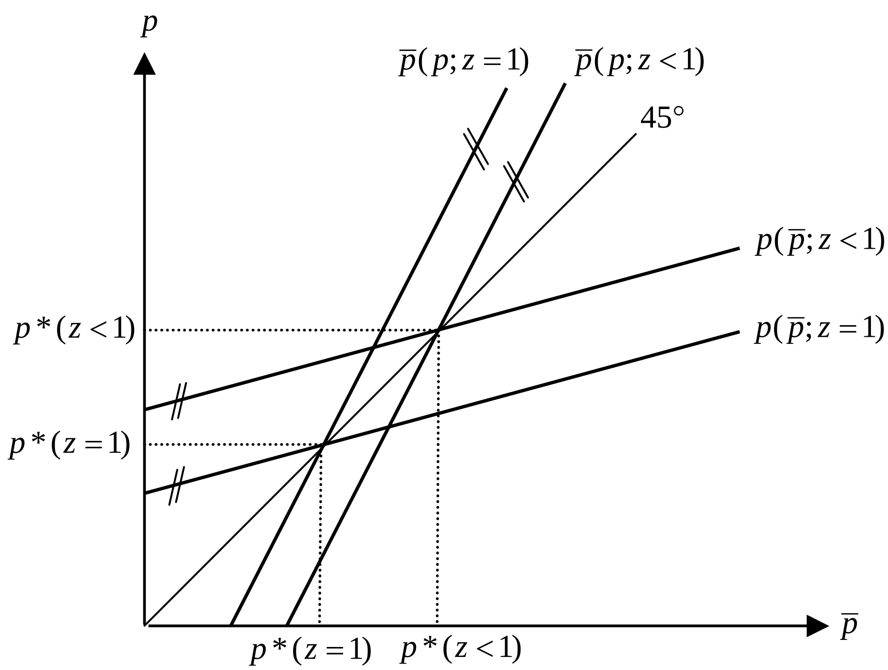
Fact 3

$p_{\bar{\phi}} < 0$ if $z \in [2/3, 1]$ and $p_{\bar{\phi}z} > 0$. Higher marginal advertising of my opponents means that more consumers are informed about the different products which reinforces price competition. Limited attention has the effect of reducing this effect as some consumers do not consider the additional ads at all.

Fact 4

$\phi_{\bar{p}} > 0$ and $\phi_{\bar{p}z} > 0$. Marginally lower prices of my opponents reduce my demand as it shifts the marginal consumer towards my opponents. Under limited attention this effect also is reduced by the fact that not all ads are considered that are received.

Taken together Fact 3 and Fact 4 clearly illustrates that limited attention reduces price competition by firms as demand is less strongly affected by a change of the choice variables. Put differently my firm can raise its price and therby looses less consumers as in the absence of limited attention as some consumers throw away the right ads (namely those of firms who would make the sale due to my price increase in the absence of limited attention). This has its counterpart in the fact that the price elasticity of demand at $\bar{p} = p$ decreases with z leaving the firm with a more inelastic demand and enables it to set strictly higher prices for any given value of $\bar{p}$. This is illustrated in the next figure:



The higher prices lead to a higher marginal products of advertising and therefore also to more advertising in equilibrium.

Finally, I show that equilibrium profits are higher with limited attention which is important as this triggers firm entry and higher equilibrium diversity later on. Using (14) and symmetry in (12) leads to

$$\Pi = \phi A'(\phi) - F - A(\phi) \tag{16}$$

which is an increasing function of ϕ.[17]

3.2 Equilibrium for $R \geq 2$ and $n \geq 1$

To generalize the analysis from the previous section to the n-firm setting where n is exogenously fixed at the moment we need to derive the conditional winning probability, $P[k\,|\phi]$ for $k = 1, 2, ..., n$ as a function of R. We could continue in the same manner as before for $n > 3$ and $R > 2$ and the baseline result - limited attention increases prices, advertising and profits relative to a situation where limited attention is absent - still holds as long as limited attention is actually existent ($R < n$). What we then observe is that the effect of limited attention on the market variables decreases as attention gets less constrained.[18] This is intuitively clear as with a larger R the attention effects is present for less consumers leaving the individual firm ceteris paribus with less market power. In order to formalize this result and keep calculations feasible I need to work with the exact values of the z-function and approximate the demand function in a way similar to GS.

3.2.1 Derivation of $P[k\,|\phi]$

Suppose $n > R$. Then we can split the consumers who receive my ad into those who receive $y < R$ ads and those who receive $R \leq y \leq n-1$ ads from my opponents. The first group are those consumers for which attention does not bind as they do not receive excessive ads. For those consumers my conditional sale probability is (for a given k)

$$\sum_{y=0}^{R-1} \bar{\phi}^{y}(1-\bar{\phi})^{n-1-y} \begin{pmatrix} n-k \\ y \end{pmatrix} \tag{17}$$

[17] $\Pi = \text{Rev}(\phi) - F$ with $\text{Rev}(\phi) = A(\phi)\,(\varepsilon_A(\phi) - 1)$ and $\varepsilon_A(\phi)$ is the elasticity of the cost function. With a constant elasticity (as is the case with the special technology: $\varepsilon_A(\phi) = \alpha$) we get a nice quantitativ prediction: $\frac{Rev'(\phi)\phi}{\text{Rev}(\phi)} = \alpha$. Thus if ϕ increases by one percent then revenue increase by $\alpha > 1$ percent.

[18] Formally: For $R' > R$ and fixed n with $n > R'$ we get in the equilibrium that $p(R') < p(R)$, $\phi(R') < \phi(R)$ and $\Pi(R') < \Pi(R)$

As before k indexes my kth best group. The binomial coeffcient quantifies how many possibilites there are that a consumer from group k receives exactly y ads from my opponents. If $R > k = 2$, $n = 10$ and $y = 3$ we only make a sale if the $y = 3$ ads are from inferior firms. As there are $n - k = 8$ inferior firms in this group and I must draw $y = 3$ of them we get $\begin{pmatrix} 8 \\ 3 \end{pmatrix} = 56$ possible situations. Note that whenever $R \leq n$ (17) fully describes my sale probabilities and

$$P[k\,|\phi] = \sum_{y=0}^{R-1} \bar{\phi}^y (1-\bar{\phi})^{n-1-y} \begin{pmatrix} n-k \\ y \end{pmatrix} = (1-\bar{\phi})^{k-1} \tag{18}$$

which corresponds to the GS solution for $P[k\,|\phi]$.

The second group of consumers consists of those who ignore some ads. Again for each $y = R, R{+}1, ..., n{-}1$ we need the probability of a certain situations which is given by $\bar{\phi}^y(1-\bar{\phi})^{n-1-y}$ and combined with the number of possible situations. This is more complicted as we must take into account if the particular opponents are superior or inferior to my firm for a given group k. The main point is again that -different than in the absence of limited attention - I can make a sale even if there is a superior firm in the choice set. Let the index $s = 0, 1, ..., k-1$ denote the number of superior firms in the choice set. Thus for given k, y and s I have $\begin{pmatrix} k-1 \\ s \end{pmatrix}$ possibilities to draw s superior firms from the total of $k-1$ superior firms. The other $y-s$ firms in the choice set must then be inferior firms. There are $\begin{pmatrix} n-k \\ y-s \end{pmatrix}$ possibilities to draw $y-s$ inferior firms from the total of $n-k$ inferior firms.[19] Finally, I win the conditional competition if all superior firms are ignored but I am not.
This probability is given as

$$z\,(y-s, R) \coprod_{i=0}^{s-1} (1 - \bar{z}\,(y-i, R)) \tag{19}$$

[19]Suppose $n = 10$, $k = 4$,$s = 2$ and $y = 4$. Then there actually are 3 superior firms out of which 2 have reached the consumer. There are 3 possibilities that this might occur. Simultaneously out of 6 possible inferior firms the consumer was reached by 2 inferior firms. There are 15 possibilities for this event. Thus we get $3 * 15 = 45$ possible combinations.

It is the probability that for a given number s and y no superior opponent wins but I do.[20] If we sum the weighted probabilties for a given y over all $s = 0, 1, ..., k-1$ and take the sum over all $y = R, R+1, ..., n-1$ we exactly calculate the conditional sale probability in a given group k that results from limited attention. Thus the overall sale probability in a group k is

$$\begin{aligned} P[k\,|\phi] = & \sum_{y=0}^{R-1} \bar{\phi}^y (1-\bar{\phi})^{n-1-y} \binom{n-k}{y} \\ & + \sum_{y=R}^{n-1} \sum_{s=0}^{k-1} \bar{\phi}^y (1-\bar{\phi})^{n-1-y} \binom{k-1}{s} \binom{n-k}{y-s} z\,(y-s, R) \prod_{i=0}^{s-1} \left(1 - \bar{z}\,(y-i, R)\right) \end{aligned} \tag{20}$$

If (20) is evaluated at $n = 3, R = 2$ we get the demand function from the first part. To simplify the awesome expression in (20) I use the exact ignoring probabilities[21]

$$z\,(j, R) = \bar{z}\,(j, R) = 1 - \frac{\binom{j}{R}}{\binom{1+j}{R}} = \frac{R}{1+j} \tag{21}$$

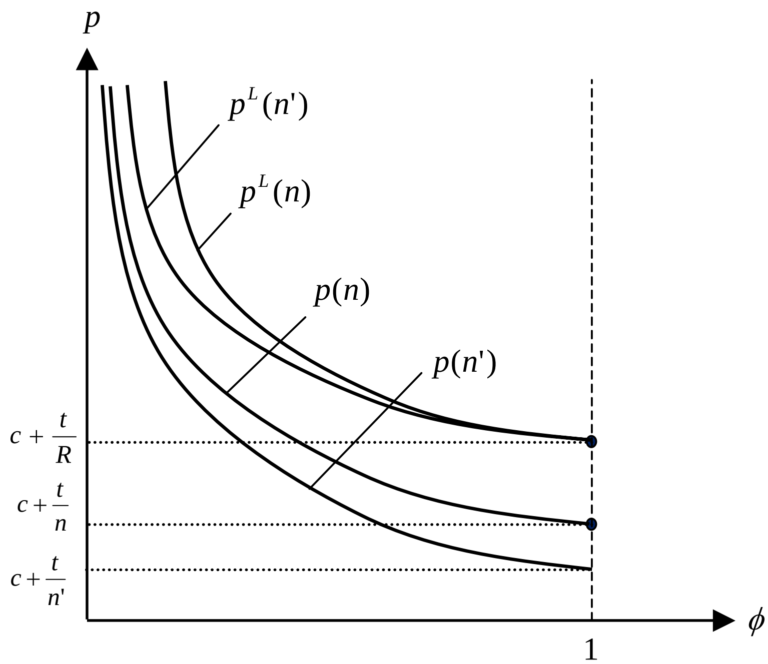
The best firm looses against j inferior competitors if the consumer keeps R ads from the inferior firms.

Using this representation we could write out the demand function for any (R, n) combination but clearly this will lead to very cumbersome expressions. However, if those calculations are done we will see that - as is expected - our main findnings from the last sectaion prevail: we get higher prices, higher advertising and also higher profits in the attention case but these variables will generally be decreasing in n. The intuition of this shift is the same for both the case with and without attention constraints. Intuitively, more firms reduce prices for a given ϕ as local

[20]If $s = 0$ then $\prod_{i=0}^{-1} \left(1 - \bar{z}\,(y-i, R)\right) = 1$

[21]This actually requires that $R \leq 1 + j$. Note however that we can use expression (21) without further restrictions in the probability chain (19).

markets are narrowed under perfect information and therefore also under imperfect information. This increases price competition and deflects the $p(\phi)$ locus downswards (see next figure).



At the same time the equilibrium ϕ is reduced for given p: as more firms are advertising it gets harder to win the conditional competition. As advertising is costly and must be traded off against its marginal benefit which is decreased the optimal response of the firms is to reduce the amount of advertising. Note that as prices and advertising intensity are determined simultaneously the reduction of ϕ means that less information of a single firm is dispersed in the economy which per se would increase prices due to more market power stemming from imperfect information. To generalize the results from the last section we need the following lemma.

Lemma 1 *Let $n \geq 3$ and $R \geq 2$ be finite integers. Define $d(p, \phi)$ and $d^L(p, \phi)$ to be the expected demand evaluated at $\bar{\phi} = \phi$ and $\bar{p} = p$ where L indexes the presence of limited attention. Further $d_p(p, \phi)$, $d_\phi(p, \phi)$, $d_p^L(p, \phi)$ and $d_\phi^L(p, \phi)$ are the respective derivatives evaluated at $\bar{\phi} = \phi$ and $\bar{p} = p$. Then*

1. $d^L(p,\phi) = d^L(\phi) = d(\phi) = d(p,\phi)$ *and* $d^L_\phi(p,\phi) = d^L_\phi(\phi) = d_\phi(\phi) = d_\phi(p,\phi)$

2. $0 > d^L_p(\phi) > d_p(\phi)$.

Lemma 1 says that in the symmetric equilibrium expected demand under both regimes is the same as is the first derivative[22] w.r.t. ϕ. These two findings together generalize the result from Proposition 1 that the equilibrium condition for ϕ is the same under both regimes. Again, the driving force is reduced price competition: under limited attention demand responds less to price changes as the attention isolates the markets more leading to lower demand elasticities and higher equilibrium prices.

First, I show that with limited attention firms set higher prices ceteris parbius in a price-symmetric equilibrium for given $(\phi, \bar{\phi})$. The condition for optimal pricing is[23]

$$p = c + \frac{D(\phi, \bar{\phi})}{-D_p(\phi, \bar{\phi}, R)} = c + \frac{t}{n\bar{\phi}} \frac{\left(1 - (1-\bar{\phi})^n\right)}{\left(1 - (1-\bar{\phi})^{n-1} - f(\bar{\phi}, n, R)\right)} \tag{22}$$

where

$$f(\bar{\phi}, n, R) = \begin{cases} \sum_{y=R}^{n-1} \left(\bar{\phi}^y (1-\bar{\phi})^{n-y-1} \begin{pmatrix} n-1 \\ y \end{pmatrix} z(y,R) \right) & \text{if } R < n \\ 0 & \text{else} \end{cases} \tag{23}$$

A specially intersting case occurs when $\phi = 1$ exogenously or endogenously[24]. Then $p^L = c + t/R$ and $p = c + t/n$. If every consumer is informed about every firm limited attention still protects the firms demand from the opponents action and prices and profits remain above the Salop level. To obtain a simpler formal description of these results and further investigate the equilibrium

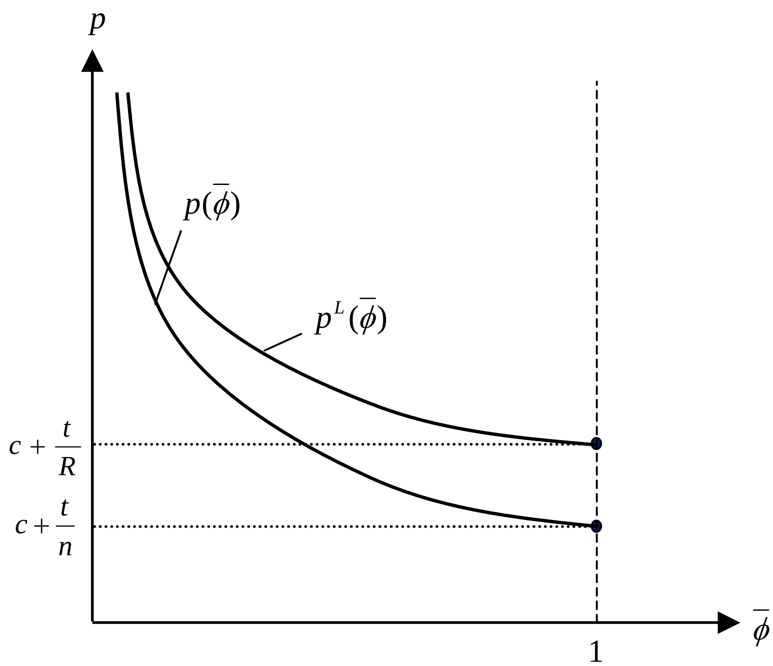
[22]This is a consequence of the linear demand structure which itself is a consequnce of the uniformity of distribution: for an optimizing firm ϕ enters linearly as the probability of reaching a certain group. Limited attention only affects the conditional winning probability and therefore the return on a certain ϕ which for symmetric prices and given $\bar{\phi}$ is the same under both regimes.

[23]For a derivation see the appendix.

[24]This follows e.g. if advertising costs are sufficiently low or consumer valuation t is high.

with endogenous firm entry I derive the approximated demand curve by assuming that almost everybody gets informed i.e. $(1-\bar{\phi})^n \cong 0$.[25]

To consider the equilibrium more in detail we would need to show that the curves resemble those of the introductary section. But as the sum in the denimoninator of (22) is an object not easy to handle (see the appendix for an approximation) I base the following statements on intuition and numerical evaluations[26]. We see that the attention price is strictly higher and places the curve on a higher level but the shape of the curve is dominated by the $1/(n\bar{\phi})$-term.

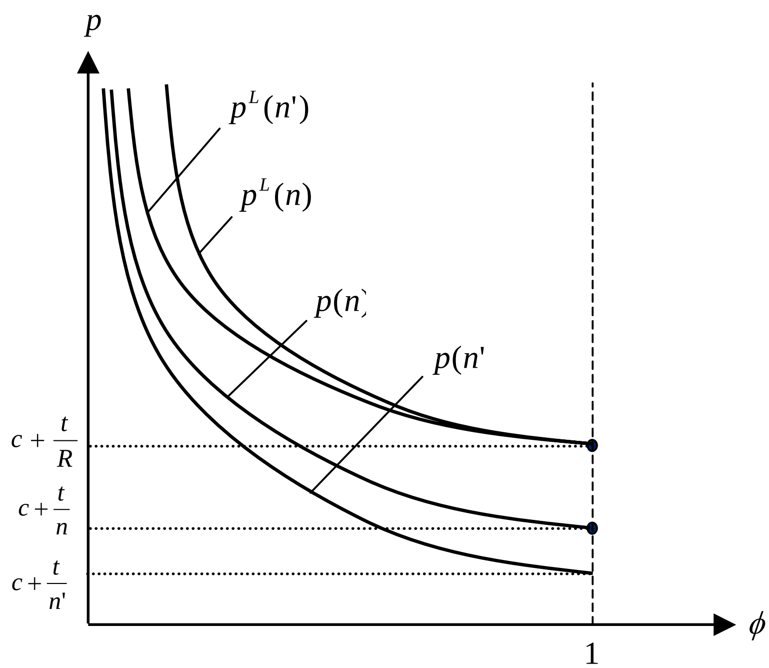


Concerning an increase of n we can expect the price-setting curve under limited attention to react in a similar way as the unlimited price-setting curve, namely that more firms intensify price competition in the usual Salop way. Interesting is the case where $\bar{\phi}=1$ as then without limited attention equilibrium markups, $p-c$, decrease proportionally with n while in the case of limited

[25]We could also explore what happens when $n \to \infty$. In this case of course supply collapses as profits could never be positive. However will withou LA $p \to c$ we get $p^L \to c + t/R$.

[26]The assumed properties hold for any R, n-simulation I have done. A general proof of the result would involve hypergeometric function objects which makes it mathematically complex without revealing any crucial new insights.

attention the markup remains constant. Intuitively, this occurs as $P[1 \left|\phi; \bar{\phi} = 1\right] = \frac{R}{n}$. Suppose I increase my price marginally which would lead some consumers to switch to their next best firm without limited attention. The chance that the consumer keeps the ad of precicely that firm is R/n which decreases proportionally to the number of firms. Suppose n doubles. Then the Salop effect states that the markup is halved. The limited attention effect at the same time doubles the probability to keep a consumer after a marginal price change and the effect cancels which illustrates nicely how limited attention again protects the market demand of a firm. The comparison between the two regimes is illustrated in the following figure.



Now I generalize the findings of the initial example concerning the equilibrium ϕ-function. The FOC is

$$(p - c)D_\phi = A'(\phi) \tag{24}$$

Note that (see appendix for details)

$$
\begin{aligned}
D_\phi &= \delta \tfrac{\bar{p}-p}{t}\left(P\left[1\left|\phi\right.\right]-P\left[n\left|\phi\right.\right]\right)+\tfrac{\delta}{n}\sum_{k=1}^{n}P\left[k\left|\phi\right.\right] \\
&= \delta \tfrac{\bar{p}-p}{t}\left(P\left[1\left|\phi\right.\right]-P\left[n\left|\phi\right.\right]\right)+\tfrac{\delta(1-(1-\bar{\phi})^n)}{n\bar{\phi}}
\end{aligned}
\tag{25}
$$

First note that for given $\bar{p} = p$ and given $\bar{\phi}$ we have $\phi'(\bar{\phi}) < 0$. Ads are strategic substitutes which is independent of limited attention. If the oppsoing firms advertise more this reduces the marginal return of my ads and therefore - as ads are costly - reduces my optimal advertising intensity. The same holds for a ceteris paribus increase of n: more firms lead to better information of consumers and again reduce my marginal revenue of advertising. Limited attention only influences the advertising decision (for fixed $\bar{p}, p$) if $\bar{p} \neq p$.[27] Consider e.g. $\bar{p} > p$. From (25) we see that this results in a higher advertising response of my firm as $P\left[1\left|\phi\right.\right]-P\left[n\left|\phi\right.\right] > 0$ (see appendix). The higher price of my opponents gives me more sale opportunities in my prime group and thus marginal return on advertising is increased. Prices and ads are strategic complements. It is interesting to note that this effect is reduced under limited attention as additional advertising under limited attention results in a smaller increase in the expected number of consumers. There is a stabilizing effect of limited attention on demand in making demand less responsive to price changes.

[27] The price difference must be sufficiently small to guarantee that marginal revenue is a strictly decreasing function of $\bar{\phi}$.

The following equation determines ϕ in the symmetric equilibrium:

$$(p-c) * \frac{\delta(1-(1-\phi)^n)}{n\phi} = A'(\phi) \tag{26}$$

As the LHS of (26) is a decreasing function of ϕ we get a positive relationship between price and equilibrium advertising which is illustrated in the next figure.

3.3 Approximate Equilibrium

Taken together (26) and (22) characterize the equilibrium p and ϕ. If we use the approximations (see the appendix) then without limited attention

$$p = c + \frac{t}{n\phi} \qquad \frac{(p-c)\delta}{n\phi} = A'(\phi) \tag{27}$$

Thus for the optimal ϕ:

$$\frac{t\delta}{(n\phi)^2} = A'(\phi) \tag{28}$$

With limited attention ($R < n\phi$) we get:

$$p = c + \frac{t}{R} \qquad \frac{(p-c)\delta}{n\phi} = A'(\phi) \tag{29}$$

and

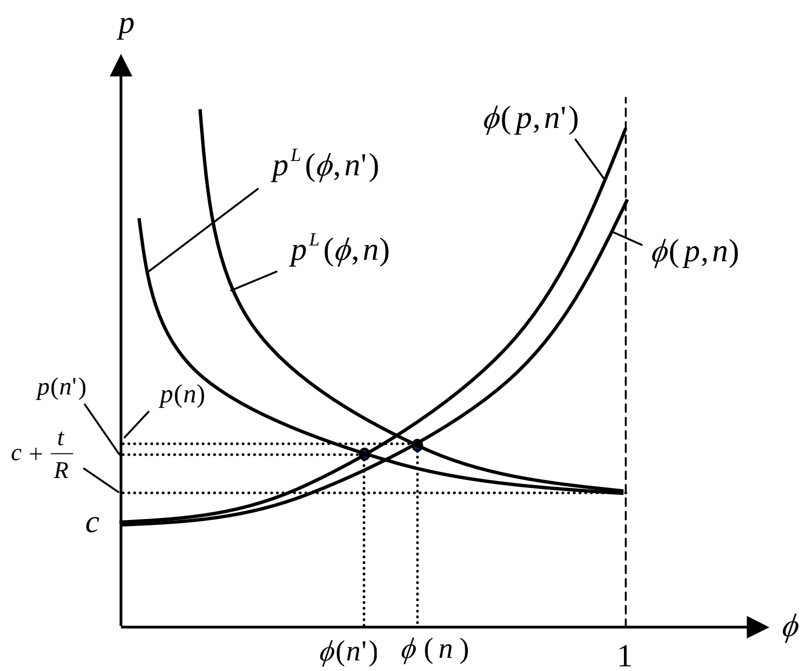
$$\frac{t\delta}{R(n\phi)} = A'(\phi) \tag{30}$$

With $R < n\phi$ obviously prices and advertising intensities are higher in the attention economy for a given R. Also we observe that if attention decreases ($dR < 0$) then prices and advertising is further increased. Further note that in both econonomies more firms lead to lower equilibrium advertising. Compared to the non-approximated equations the only qualitativ different result when using the approximation is that with limited attention the price is independent of n while it is an eays exercise in comparative statics to show that the equilibrium price without limited attention decreases in n. Note that the approximation is exact in the case where $\phi = 1$[28] and

[28] One can always get $\phi = 1$ by assuming e.g. the advertising is free or sufficiently cheap.

is very good if n very large and $\phi > 0$. In the general case we observe p to depend negatively on ϕ. All numerical evaluations show that in an equilibrium with $\phi \in (0,1)$ higher n implies lower prices and lower advertising. The following figure depicts a typical situation for limited attention.

What the simulations also reveal is that the price and advertising difference between the two regimes is increased as n becomes larger as shown in the next figure. This is intuitive as with increasing n the approximation gets better and attention prices tend to react less to a further increase of n due to the fact that firm demand reacts less elastic with attention restriced consumers.



For the next section it is helpful to understand the comparative static properties of the equilibrium w.r.t changes of n. If $n \leq R$ the price curve in the last figure for the ARE is the same as for the unrestriced economy. If n is increased beyond R then limited attention occurs and as was shown above the attention prices are strictly higher for any given ϕ whereas the ϕ-curve always is the same for both economies if $\bar{p} = p$. The following figure[29] illustrates how the price depends

[29] As Salop and others I allow for a certain fuzziness by admitting n to take on non-integer values which simplifies

on n for given $(\phi, \bar{\phi})$. I work with the approximations here but any simulation I conducted confirms the qualitativ prediction of how n influences equilibrium prices and advertising. We have

$$p = \begin{cases} c + \frac{t}{n\phi} & if \quad R \geq n \\ c + \frac{t}{\phi R} & if \quad R < n \end{cases} \tag{31}$$

For a fixed $R \geq 2$ the attention p-curve will differ from the (hypothetical) non-attention curve iff $R < n$.

the formal analysis.

3.4 Profits

Deriving the profit level of a firm in equilibrium for given n is reasonable as in the long-run equilbrium n is endogenously determined by zero profit conditions. But the way how limited attention will effect diversity is already conceivable: as local markets are more inelastic this raises market power of firms which leads to higher profits and in the long run to more diversity.

To see this in detail assume a price-symmetric situation with fixed prices: $\bar{p} = p$. Then the profit level before optimization is

$$\Pi = (p - c)D - F - A(\phi) \tag{32}$$

where $D = \frac{\delta\phi}{n\bar{\phi}}\left(1 - (1 - \bar{\phi})^n\right)$.

The reaction function of my firm can be written as

$$(p - c)\frac{D}{\phi} = A'(\phi) \tag{33}$$

The level of profits then is given as

$$\Pi = \phi A'(\phi) - F - A(\phi) \tag{34}$$

and is increasing in ϕ.[30] Thus any situation leading to a higher optimal choice of ϕ also increases the level of profits. Thus equilibrium profits are increased by limited attention through the following channel: limited attention leads to inelastic local markets which enables firms to set higher prices and markups which increases equilibrium advertising as its marginal revenue is increased. This is the source of higher profits.

[30] $\Pi'(\phi) = \phi A''(\phi) > 0$.

3.5 Free-entry equilibrium

Using the approximate versions the long-run equilibrium[31] which determines p, ϕ, n endogenously is given by the three equations

$$p = \begin{cases} c + \frac{t}{n\phi} & if \quad R \geq n \\ c + \frac{t}{R} & if \quad R < n \end{cases} \tag{35}$$

$$\frac{(p-c)\delta}{n\phi} = A'(\phi) \tag{36}$$

$$\phi A'(\phi) = F + A(\phi) \tag{37}$$

Note that (37) determines the equilibrium ϕ which is independent of R. Two questions naturally arise: provided that the equilibrium n is smaller than the fixed R: what is the effect of limited attention on prices and diversity? The second question is under what conditions we can expect limited attention to occur.

For the first question note that the equilibrium can be depicted grafically in the ususal p, ϕ diagram:

FIGURE HERE

The equilibrium lies at the point of intersection of all three curves (n determines the position of $\Pi_p = 0$ and $\Pi_\phi = 0$.). In the figure the equilibrium for the case of a non-ARE is plotted. Assuming that for the corresponding n of this solution we have $n > R$ then we know from the preceeding analysis that the attention p-curve is strictly above the non-restriced curve as depicted in the figure. Thus the former equilibrium point could never be a solution for the ARE. As with the approximation the attention p-curve is independent of n the only way the equilibrium equations are satisfied is over a left-shift of the ϕ-curve as depicted due to an increase in n.[32] Thus with limited attention as in the last section we get higher prices but identical

[31] This is for analytical simplicity only. Any simulation I conducted reveal the basic results to be true.

[32] If we used the true system instead the price curve would be shifted downwards simultaneously as n is increased. The true intersection therefore is between the two equilibria on the vertical $\Pi = 0$-curve in the figure. Thus the approximation overestimates the equilibrium price and also the equilibrium diversity.

advertising per firm. The reason is that the higher profits for a given n which are earned with limited attention induce more firm entry which reduce the marginal revenue of advertising and therefore advertising itself. As local markets with limited attention remain more price-inelastic we end up with higher n and higher p. Advertising at the firm level is the same for both regimes but as n is increased in the attention economy we can expect overall advertising to be higher. Note that working with the approximation gives as a lower bound for p as the true p-curve is falling in ϕ. We therefore can expect the true equilibrium price to satisfy $p \geq c + t/R$.

The long-run comparative statics of R are also obvious from the figure. As R decreases the attention p curve moves away from the ϕ-curve and everything else stays equal. But then to establish the attention equilibrium the ϕ-curve must rotate more which implies an even larger n at a higher p.

The second question is when can we expect an economy to be attention restriced. The answer is simple: whenever the equilibrium of the non-attention case contains an n that is larger than R. The approximate equilibrium without limited attention can be reduced to

$$\phi A'(\phi) = F + A(\phi) \qquad \sqrt{\frac{t\delta}{A'(\phi)}} \frac{1}{\phi} = n \tag{38}$$

If the resulting n is larger than R then we obviously are in an ARE. Thus any parameter constellation that results in a relatively large n endogeneously leads to limited attention. As can be deduced from (38) a high value of t and low values of ϕ, generated by low setup- and advertising costs increase n. To get the intuition we can examine this problem grafically. The next figure plots an oligopolistic equilibrium with choosing $n = R$. If the monopolistic equilibrium is on the vertical line through ϕ_0 then n will be increased which leads to an attention equilibrium while at ϕ_1 n would be reduced leading to a nonrestriced equilibrium. Whenever the intersection of the ϕ- and p-curve are to the right of the vertical $\Pi = 0$ we can expect an attention equilibrium to occur.

A higher t shifts the p-curve up: if consumers react more sensitive towards product differentiation firms can charge higher prices which leads to higher marginal revenue of advertising and to higher profits. Thus more firms must enter the market and chances that $R < n$ are higher.

On the other hand lower setup-costs have a one-to-one positive effect on equilibrium profits and therefore more firms must enter the market (this corresponds to a left-shift of the $\Pi = 0$-locus). Finally, we also have a higher chance of being in an attention economy if the ϕ-locus is rotated downwards which occurs for lower (marginal) advertising costs[33]. Lower advertising costs lead to more equilibrium advertising and higher profits which again induces more firm entry. I want to stress especially the last point as relevant for modern economies. There is no doubt that advertising costs have been reduced over the past decades due to more competitive media and new technologies such as search-bound internet advertising which is seen as most important for future advertising markets (The Economist, 29.11.2008, p.61f). In my model marginal advertising costs have a double interpretation. (1)They measure how expensive it is to produce a further ad technically. (2) They measure how difficult it is to reach a further consumer. Especially the internet technology matches with lower advertising costs in both interpretations. Thus beyond all aspects that advertising contain (persuasion or information) my model suggests that the competition for attention is getting a more dominating force in modern economies.

Corollary 1 *Let R, R' and $n \geq 3$ be integers and $R < R' \leq n$. Then in the symmetric equilibrium*

1. $p' < p$, $\phi' < \phi$ *and* $\Pi' < \Pi$.

Next suppose that R, n and n' are integers and $n < n'$. Then

1. $p' < p$, $\phi' < \phi$ *and* $\Pi' < \Pi$.

[33]If $\tilde{A}'(\phi) < A'(\phi)$ for all ϕ then equilibrium advertising will be increased. This can be e.g. accomplished by a lower value of θ in the technological example.

4 Welfare analysis

The bottom line of the preceeding positive analysis is that limited attention increases equilibrium diversity and advertising compared to a situation were attention is less restricted or ad extremum fully supplied to the market. But what can be said about the welfare implications of the market outcome if we take an utilitarian planer as benchmark? From GS we already know that the market equilibrium generally is inefficient: there is excessive diversity (n too large) but advertising is undersupplied[34]. To understand the role of limited attention I proceed as follows. First, I show that welfare in an ARE can never exeed welfare in an unrestriced economy and I illustrate that there is welfare loss if attention gets more restriced. Then I investigate therev special cases to gain the central intuition. The next step is to find analytically tractable approximations that contain the main results which are verfied and quantified using simulation methods in the end.

4.1 Transportation costs

The welfare function considered is given as the sum of consumer and producer surplus and therefore can be written as

$$W = \delta(v-c)\left(1-(1-\phi)^n\right) - nA(\phi) - nF - T \tag{39}$$

where T indicates average aggregate transportation costs. T is calculated as

$$T = \delta t \sum_{k=1}^{n} \frac{2k-1}{4n} \phi P\left[k\,|\phi\right] \tag{40}$$

Without LA we get

$$T^{NL} = \frac{t\delta\left(2-\phi+(1-\phi)^n(\phi(1-2n)-2)\right)}{4n\phi} \tag{41}$$

[34] In their paper, GS argue that there approximation is valid for a large fimr number. However, if advertising gets more expensive (as is the case with increasing values of θ) simulations reveal that - contradiciting the GS conjecture - diversity may also be undersupplied by the market with values of $n > 100$.

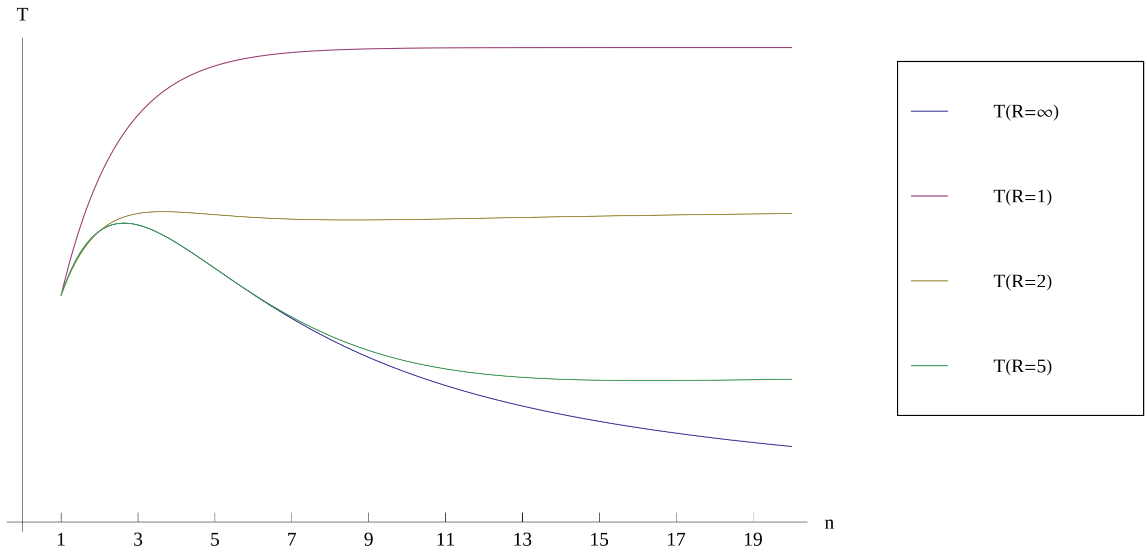
First, note that for any given set of parameters

$$T = \begin{cases} T^{NL} & n \leq R \\ T^{L} & n > R \end{cases} \tag{42}$$

and that for any given (n, ϕ) $T^{NL} \leq T^{L}$.

PROOF: to be provided in the appendix

The next figure illustrates how a different R affects transportation costs as a function of ϕ:



From (39) we see that therefore welfare can never be higher in an ARE for any set of parameter than in an unrestriced economy. Put differently, if the concern was only about minimizing transportation costs, a planer would - if this were possible - never choose a situation were the attention restriction gets binding. However minimizing transportation and production costs must be traded off against the utility loss from not realizing a sale. To understand this trade-off in detail we first look at a special cases

4.1.1 The case $\phi = 1$

First consider a situation where full information, $\phi = 1$, is imposed exogenously[35]. Assume parameter values such that the market equilibrium exists and second order conditions of the planer problem are satisfied. Market level diversity is then given as

$$n = \begin{cases} \frac{t\delta}{(F+A(1))R} & n > R \\ \sqrt{\frac{t\delta}{F+A(1)}} & n \leq R \end{cases} \tag{43}$$

where $A(1) \geq 0$. If $A(1) = 0$ this corresponds to the Salop equilibrium. What is the optimal degree of diversity a planer would choose given $\phi = 1$?

From (39) we see that for $\phi = 1$:

$$W_n = -F - A(1) - T'(n) \tag{44}$$

and

$$T(n) = \begin{cases} \frac{t\delta}{4n} & n \leq R \\ \frac{t\delta(1+2n-R)}{4n(1+R)} & n > R \end{cases} \tag{45}$$

But then

$$T'(n) \begin{cases} < 0 & n \leq R \\ \geq 0 & n > R \end{cases} \tag{46}$$

Thus we can conclude that a planer will never choose $n > R$. He will choose $n = n_{NL} \leq R$ if $W(n_{NL}) \geq W(R)$. Note that in the extreme case where $R = 1$ the planer will choose $n = 1$, a pure monopolist to serve the entire marke'! Intuitively this occurs as with $R = 1$ average transportation costs are $t\delta/4$ and independent of n. If consumers only consider one ad anyway then they travel 1/4 on avergage - which is the same as if only a monopolist would serve the market. To minimize setup-costs the planer therefore chooses $n = 1$. The general insight of this extreme case is that when attention is extremly restricted (R very small) and at the same time people are

[35] Endogenously this is the case if advertising were free or sufficiently cheap.

informed of (almost) all firms then advertising is a bad mean for matching firms with consumers.

We also see that the market provides excessiv diversity and that limited attention tends to make things even worse. Suppose in the market equilibrium we have $R < n$ and thus

$$n_m = \frac{t\delta}{(F + A(1))\,R} \tag{47}$$

The planer solution is

$$n = \begin{cases} R & R \leq \sqrt{\frac{t\delta}{4(F+A(1))}} \\ \sqrt{\frac{t\delta}{4(F+A(1))}} & else \end{cases} \tag{48}$$

Thus if the attention constraint is bindning for the planer (e.g. R small) the difference between the social optimal n and the market n gets even larger.

4.2 Approximate solutions

Assuming that $(1-\phi)^n \approx 0$ The GS approximation to T is

$$T \approx \frac{t\delta(2-\phi)}{4n\phi} \tag{49}$$

To derive a similar approximation it is easiest to use $(1-\phi)^n \approx 0$ in (21). Then it can be shown that

$$P\left[k\,|\phi\right] \approx \frac{R}{n\phi} \prod_{i=0}^{R-2} \frac{n-k-i}{n-1-i} \tag{50}$$

Thus[36]

$$T^L \approx \frac{(1+2n-R)t\delta}{4n(1+R)} \tag{51}$$

4.2.1 How do the planer solutions compare?

As can be seen from (51) the approximate transportation costs in the case of the ARE are independent of ϕ and the GS approximation leads to an absurd prediction. To maintain the trade-off between informing people and minimizing transportation costs I suggest the following simplified welfare function:[37]

$$W = \delta(v-c)\left(1-(1-\phi)^n\right) - nF - nA(\phi) - T \tag{52}$$

where I use two approximations (49) and (51) for T.

[36] Note that this is the same as if $\phi = 1$. Intuitively, this occurs as transportation costs quickly converge towards (51) as ϕ increases as limited attention makes the conditional probabilities and thus transportation costs less responsive to advertising.

[37] Compared to the true solution this approximation is very reliable qualitatively as well as quantitatively in calculating the optimal (ϕ, n) under both regimes as simulation results reveal. However, the approximations cannot be used to compare overall welfare levels.

Unrestriced planer solution

Assuming $\phi \in (0,1)$ the FOC for the unrestricted planer problem are

$$(W_\phi) \qquad (v-c)\delta(1-\phi)^{n-1} + \frac{t\delta}{2n^2\phi^2} = A'(\phi) \tag{53}$$

and

$$(W_n) \qquad \frac{t\delta(2-\phi)}{4n^2\phi} - (v-c)\delta(1-\phi)^n Log(1-\phi) = F + A(\phi) \tag{54}$$

which can be summarized to

$$\frac{t\delta}{2n^2\phi}\left(\frac{2-\phi}{\phi} + \frac{1-\phi}{\phi}Log(1-\phi)\right) - (1-\phi)A'(\phi)Log(1-\phi) = F + A(\phi) \tag{55}$$

Restricted planer solution

Assuming $\phi \in (0,1)$ the FOC for the restriced planer problem are

$$(W_\phi) \qquad (v-c)\delta(1-\phi)^{n-1} = A'(\phi) \tag{56}$$

and

$$(W_n) \qquad \frac{(1-R)t\delta}{4n^2(1+R)} - (v-c)\delta(1-\phi)^n Log(1-\phi) = F + A(\phi) \tag{57}$$

which can be summarized to

$$\frac{(1-R)t\delta}{4n^2(1+R)} - (1-\phi)A'(\phi)Log(1-\phi) = F + A(\phi) \tag{58}$$

Let (ϕ_L, ϕ_{NL}) denote the solution of a planer facing an attention economy and (ϕ_{NL}, n_{NL}) is the planer solution of an unrestricted economy.[38] First compare (58) to (55). Note that

$$\frac{d}{dR}\left(\frac{(1-R)t\delta}{4n^2(1+R)}\right) \leq 0 \tag{59}$$

[38] It is helpful for the proofs to look at the two regimes separately.

and assume $R = 1$.

Then for (58):

$$-(1-\phi)A'(\phi)Log(1-\phi) = F + A(\phi) \tag{60}$$

but as $\left(\frac{2-\phi}{\phi} + \frac{1-\phi}{\phi}Log(1-\phi)\right) > 0$ for $\phi \in (0,1)$ we have $\phi_{NL} < \phi_L$ for $R = 1$ as the restriced planer curve lies beneath the unrestricted planer curve. By (59) this also holds for any $R > 1$ as the curve is shifted downwards further. Also note that the same is true if $n_L < n_{NL}$ as $\left(\frac{(1-R)t\delta}{4n^2(1+R)}\right) \leq 0$ if $R \geq 1$.

Next compare (53) to (56). Define $(v-c)\delta(1-\phi)^{n-1} \equiv f(\phi, n)$ and note that $f_\phi < 0$ and $f_n < 0$. Suppose $\phi_L > \phi_{NL}$. Then

$$A'(\phi_{NL}) = \frac{t\delta}{2n_{NL}^2\phi_{NL}^2} + f(\phi_{NL}, n_{NL}) < A'(\phi_L) = f(\phi_L, n_L) \tag{61}$$

But as $\phi_L > \phi_{NL}$ (61) requires that $n_L < n_{NL}$ which completes the proof.

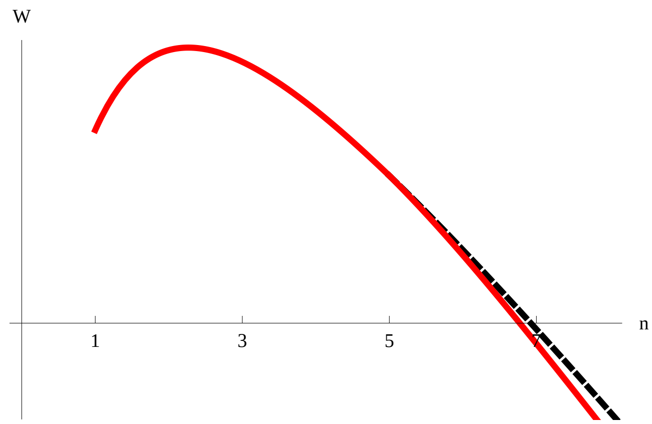
Thus from the approximation we see that $n_L < n_{NL}$ - that is under limited attention the planer wants to reduce diversity[39]. Intuitively, this occurs as transportation costs react less to changes in n under limited attention as can be seen in FIGURE X.

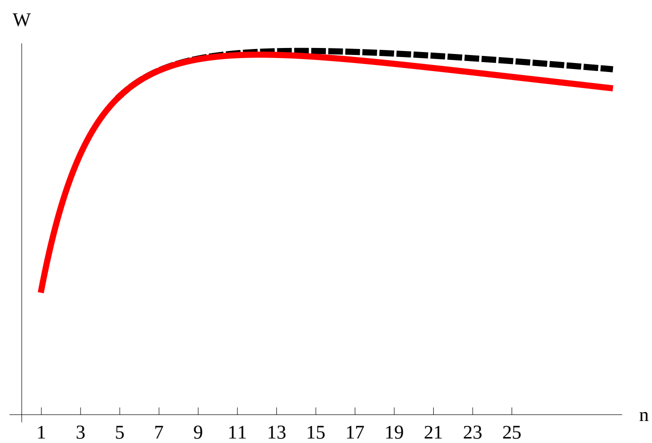
4.2.2 Putting the facts together

The analysis so far has revealed two important facts about welfare maximisation. First, if attention gets limited, $n > R$, then welfare is decreased. For this reason the planer would never choose to be in an attention econcomy. However, the reduction in n eventually leaves more people uninformed which is also considered by the planer. It is clear that if R is relatively small and n_{NL} relatively large (so that $n_{NL} > R$ holds) then n_{NL} is not feasible and the planer chooses (ϕ_L, n_L) with $n_L < n_{NL}$. However if $n_{NL} \leq R$ then n_{NL} is feasible and the planer

[39] Any simulation conducted reveal the same result in general. Also the difference in ϕ is typically small (below 0.1 percent) wheras the difference in n can be dramatic.

chooses ϕ_{NL}, n_{NL} as solution. This two situations are qualitatively illustrated in the following figures for $R = 5$.





The red line indicates the welfare function wheras the dotted black line indicates the (hypothetical) unrestricted planer function. The first figure shows that the planer chooses $n < R = 5$ and thus the same solution as a planer in an unrestricted economy would. In the second figure he would again like to choose the n that maximizes the dotted curve but for this n we see that $n > R$ and it is not feasible. The best this planer can do is to choose the n that maximizes the red function.

The second obesrvation was that the welfare maximum of the ARE w.r.t. n is always located to the left of the welfare maximum of the unrestricted economy. That is we either observe $n_L = n_{NL}$ or $n_L < n_{NL}$ but never $n_L > n_{NL}$. This is the same conclusion as we obtained for

$\phi = 1$.

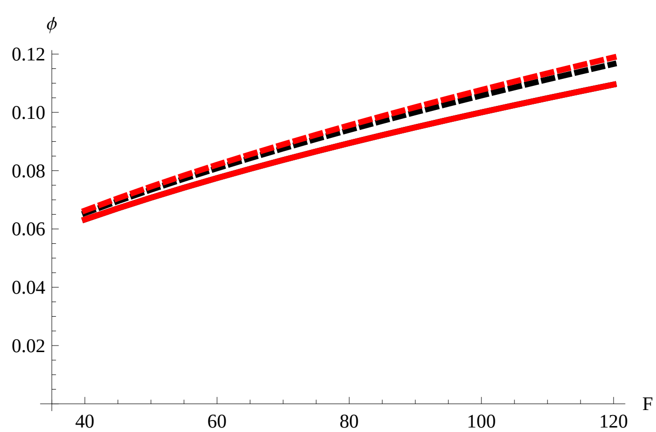
5 Simulation results

To compare market and planer outcome I conducted a wide range of simulations with the true functions. Other than GS I found that the market may either over- or undersupply the socially optimal degree of diversity - even if the number of firms is not small. Advertising costs - influenced by the parameter θ - play a key role. If advertising is very cheap this induces firms to advertise a lot[40]. But more advertising would increase equilibrium profits and therefore leads to firm entry and increased diversity. Already without limited attention the market generates excessive diversity in such a case. But then with limited attention we get

$$n_m^L > n_M^{NL} > n_P^{NL} \geq n_P^L$$

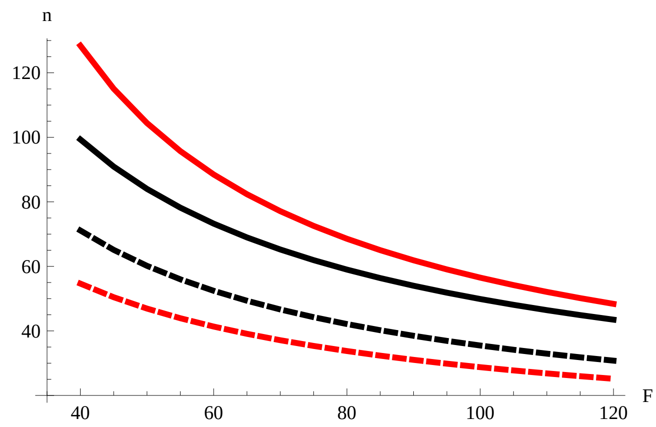
Thus limited attention makes things even worse as the gap between the social optimal degree of diversity and the market counterpart is increased. The next figure presents a typical plot of such a situation[41].

Black lines indicated nonrestriced solutions while red lines indiciate ARE solutions. The dotted lines are the planer solutions.

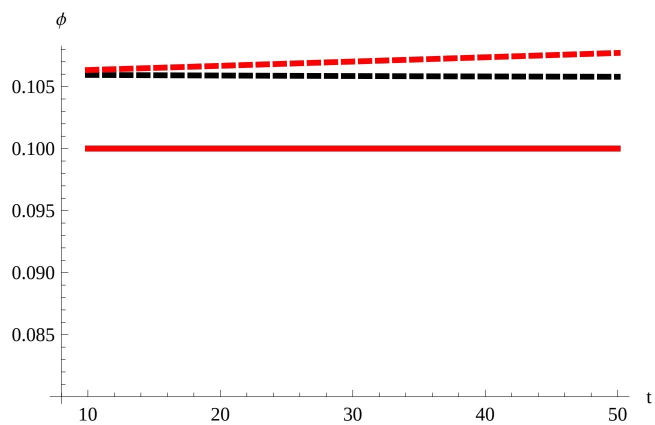


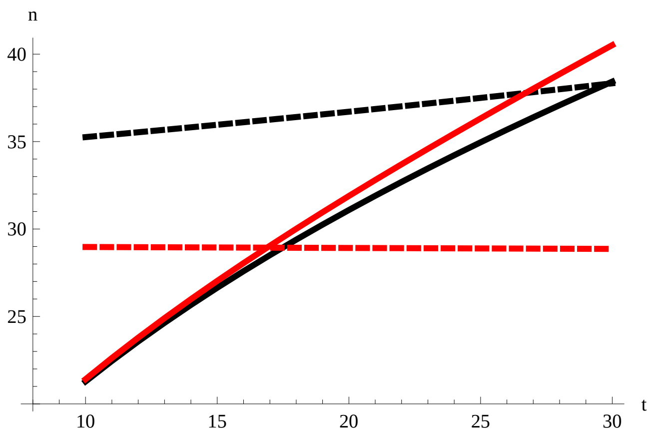
[40] Use the implicit function theorem on $\phi A'(\phi) = F + A(\phi)$ and $A(\phi) = \theta(\delta\phi)^{\alpha}$. The same holds for changes in α but is harder to see.

[41] In the base run simulation I used the following values: $\delta = 1000$, $v = 50$,$t = 50$, $c = 1$, $\alpha = 2$, $R = 5$, $\theta = 0.01$.



However simulations also reveal that if advertising gets expensive the the market may under-provide diversity. This especially occurs when t gets small. In this case limited attention may moderate the effect as is illustrated in the following figures (using $F = 100$):





We see that for low values of t attention effects may moderate the market inefficiency compared to a situation where attention is absent. As t increases however this observation is reversed.

6 Conclusion

In the Salop-model with informative advertising we can exogenously limited consumer attention to have a positive effect on equilibrium prices, advertising and diversity. Under modern conditions, reflected in low costs and more specialized consumers we can generally expect the market to overprovide diversity and underprovide information - a result which is intensified by limited attention.

7 Extension: Spaming irrestistent consumers

This is not completed yet.

Up to now firms could only choose the range of consumers to be informed but not the intensity with which a certain consumer is informed. Now suppose that consumers are not spamming-resistant in the sense that sending more than one ad to a certain consumer increases the change of making a sale to that consumer. The idea is straightforward and at the heart of limitted attention. Suppose a consumer received 7 ads of distinct firms and 10 ads of my firm. Then a spamming-irrestistent consumer buys my product with a higher probability as my ads consume more of his limitted attention. Thus spamming under limited attention of consumers has the impact of overriding the information of other firms. To see the equilibrium consequences I analyse the case of two firms and $R = 2$ which contains much of the intuition. Let $\kappa = \{1, 2, ...\}$ denote how many ads per consumer (i.e. the intensity of advertising) I send. $\kappa = 1$ is the case from before. Similarly, $\bar{\kappa} = \{1, 2, ...\}$ is the number of ads of my opponent per consumer. Advertising costs now depend on the choice of reach, ϕ and also on the choice of intensity κ:

$$Costs = A(\phi) + Q(\phi, \kappa) \tag{62}$$

with $Q(\phi, 1) = 0 \qquad \forall \phi$.
The profit function of my firm then is

$$\Pi = (p - c)D\left(\underset{-}{p}, \underset{+}{\bar{p}}, \underset{+}{\phi}, \underset{-}{\bar{\phi}}, \underset{+}{\kappa}, \underset{-}{\bar{\kappa}}\right) - F - A(\phi) - Q(\phi, \kappa) \tag{63}$$

Q_κ is the marginal cost of intensity and measures how costly it is to spam a particular consumer.[42] For the intensity cost function I assume that

$$\begin{aligned} Q_\kappa(\phi, \kappa) > 0 \qquad \kappa > 1 \\ Q_\phi(\phi, \kappa) \geq 0 \qquad \kappa > 1 \end{aligned} \tag{64}$$

The first assumption states that the marginal cost of producing another ad per consumer be positive. The second assumption states whether the marginal cost of increasing the range depends (positively) on the consumer intensity or not. An interesting case arises if there is independence: this means that the marginal cost of the signal range is independent of the intensity of the signal. This makes sense if we think of modern advertising by internet where it is costly to produce more units of spam but once they are produced can be sent at the same cost to many receivers. In the numerical analysis I used the functional form

$$Q(\phi, \kappa) = \gamma(\delta\phi)(\kappa - 1)^\beta \qquad \beta \geq 1 \tag{65}$$

The true task is to derive the demand function. I proceed in the usual way. With two firms there are two consumer groups. Again I can make a sale to the first group if the consumer receives

[42] As in the last section costly measures both technological costs and also how resistant consumers are.

my ads and keeps at least one of them. For a given reach this probability can now be increased by increasing the intensity. For group 1 we get

$$P\left[1\,|\phi\right] = (1-\bar{\phi}) + \bar{\phi}\left(1 - \frac{\begin{pmatrix}\bar{\kappa} \\ R\end{pmatrix}}{\begin{pmatrix}\kappa+\bar{\kappa} \\ R\end{pmatrix}}\right) \tag{66}$$

For group 2 we get

$$P\left[2\,|\phi\right] = (1-\bar{\phi}) + \bar{\phi}\left(\frac{\begin{pmatrix}\kappa \\ R\end{pmatrix}}{\begin{pmatrix}\kappa+\bar{\kappa} \\ R\end{pmatrix}}\right) \tag{67}$$

Then my demand is for $R = 2$

$$\begin{aligned} D &= \phi P\left[1\,|\phi\right] N_1 + \phi P\left[2\,|\phi\right] N_2 \\ &= \delta\phi\left(1 + \tfrac{\bar{\phi}\bar{\kappa}}{(\kappa+\bar{\kappa})(\kappa+\bar{\kappa}-1)}\left(2\kappa(\bar{P}-P) + t(1-(\kappa+\bar{\kappa}))\right)\right) \end{aligned} \tag{68}$$

If (68) is plugged into (63) and differentiated with respect to p, ϕ and κ we find the marginal products. In the symmetric equilibrium[43] we get three equations determining the three unknowns:

$$p = c + t\left[\left(\frac{1}{2k} - 1\right) + \frac{1}{\phi}\left(2 - \frac{1}{k}\right)\right] \tag{69}$$

$$\frac{(p-c)\delta(2-\phi)}{2} = A'(\phi) + Q_\phi(\phi,\kappa) \tag{70}$$

$$\frac{(p-c)\delta\phi^2}{4k} = Q_\kappa(\phi,\kappa) \tag{71}$$

The first equation again characterises how much the price can be pulled over marginal costs due to the heterogeneity market power effect. The squared bracket measures the attention markup

[43] Second-order conditions for a maximum are satisfied

and again is never smaller than one. Additionaly, this effect is increasing in k but decreasing in ϕ. More spamming for a given equilibrium range generates a stronger local market. Comparing (70) and (71) gives us the equilibrium relation between k and ϕ. For the interesting special case where the spaming costs is independent of ϕ but also for the more general cost function in(65) I find a strictly positive and increasing relationship between ϕ and κ. If the equilibrium range and thus the competition for the attention of new consumers increases so thus equilibrium intensity and thus the competition for all reached consumers. Will spamming occur in equilibrium? It can be shown that $k = 1$ can never be part of a maximum by the second order conditions. Thus spamming will occur. But $k = 1$ would correspond to the La equilibrium before. If the equilibrium relationship between k and ϕ is positive we may conclude that with spamming we also get a higher value of ϕ. What about prices? As the markup can be written as

$$p - c = \frac{\mathbf{2}\left(\mathbf{A}'(\phi) + \mathbf{Q}_{\phi}(\phi, \kappa)\right)}{\delta(2 - \phi)} \tag{72}$$

and the LHS is increasing in ϕ under my assumptions we end up with higher prices in equilibrium. Note that the prices are higher especially when $Q_{\phi} > 0$.

8 Appendix

8.1 Proof of proposition 1

To be supplied in January.

8.2 Proof of Lemma 1

In equilibrium, i.e. with $\bar{p} = p$ and $\bar{\phi} = \phi$:

$$\sum_{k=1}^{n} P^{L}\left[k\,|\phi; R\right] = \sum_{k=1}^{n} P\left[k\,|\phi\right] \tag{73}$$

This must hold as the ignoring probabilities are the same for all consumers: spare ads are disposed of randomly, every informed consumer consumes somewhere and consumers and ads are uniformly distributed. Therefore in the symmetric equilibrium every firm must get the same expected demand. If this were not the case then some consumers would systematically have to ignore the ads of a certain firm which is a contradiction to the disposing process.[44] The argument can even be extended to the case where $\bar{p} = p$ but $\bar{\phi} \neq \phi$. Define an attention situation as a situation where a certain consumer from a group k receives my ad and $y \geq R$ ads of other firms. For $\bar{\phi} = 0$ the proof is trivial. For $0 < \bar{\phi} \leq 1$ note that because of the uniformity assumption the probability for a situation is guided by $\bar{\phi}^y (1 - \bar{\phi})^{n-1-y}$ for each group k. This means that the conditional probability to win an attention situation against firm 2 in group 2 with x other firms in is exactly the same as the probability that firm 2 wins a similar attention situation in group 1 with the same x firms in. As this argument obviously holds for any possible attention situation we may conclude that a situation as

$$\sum_{k=1}^{n} P^L [k \,|\phi; R] \neq \sum_{k=1}^{n} P [k \,|\phi]$$

is never possible as it would require a systematic bias of the attention process concerning my firm which contradicts the last observation.

8.3 Deriving the FOC for p

At the moment this only is a sketch. Demand for my firm is given as

$$D = \left(\frac{\bar{p} - p}{t} + \frac{1}{n} \right) \delta\phi P [1 \,|\phi] + \sum_{k=2}^{n-1} \frac{\phi\delta}{n} P [k \,|\phi] + \left(\frac{p - \bar{p}}{t} + \frac{1}{n} \right) \delta\phi P [n \,|\phi]$$

Thus

$$-D_p = \frac{\delta\phi}{t} \left(P [1 \,|\phi] - P [n \,|\phi] \right)$$

Assume $R > 1$. $P [n \,|\phi] = (1 - \bar{\phi})^{n-1}$ as even if $y \geq R$ I can never win an attention situation since with $R > 1$ at least one superior firm remains in the choice set. But attention effects $P [1 \,|\phi]$:

[44] Note that uniformity of distribution is an indispensable assupmtion for this statement. If e.g. consumers where not uniformly distributed and would cluster around a certain firm this might interact with limited attention: as it is more likely to reach a consumer in the cluster for a given amount of advertising. But then even with equal advertising of all firms the conditional winning probabilities will not be the same for different firms and different groups as the location of the cluster plays a role.

$$P\left[1\left|\phi\right.\right] = \sum_{y=0}^{R-1}\left(\bar{\phi}^y(1-\bar{\phi})^{n-y-1}\begin{pmatrix} n-1 \\ y \end{pmatrix}\right) + \sum_{y=R}^{n-1}\left(\bar{\phi}^y(1-\bar{\phi})^{n-y-1}\begin{pmatrix} n-1 \\ y \end{pmatrix} z\left(y,R\right)\right)$$

Use

$$1 = \sum_{y=0}^{R-1}\left(\bar{\phi}^y(1-\bar{\phi})^{n-y-1}\begin{pmatrix} n-1 \\ y \end{pmatrix}\right) + \sum_{y=R}^{n-1}\left(\bar{\phi}^y(1-\bar{\phi})^{n-y-1}\begin{pmatrix} n-1 \\ y \end{pmatrix}\right)$$

$$\Leftrightarrow \quad \sum_{y=0}^{R-1}\left(\bar{\phi}^y(1-\bar{\phi})^{n-y-1}\begin{pmatrix} n-1 \\ y \end{pmatrix}\right) = 1 - \sum_{y=R}^{n-1}\left(\bar{\phi}^y(1-\bar{\phi})^{n-y-1}\begin{pmatrix} n-1 \\ y \end{pmatrix}\right)$$

Then

$$P\left[1\left|\phi\right.\right] = 1 - \sum_{y=R}^{n-1}\left(\bar{\phi}^y(1-\bar{\phi})^{n-y-1}\begin{pmatrix} n-1 \\ y \end{pmatrix}\right)(1-z(y,R))$$

and hence

$$-D_p = \frac{\delta\phi}{t}\left(1-(1-\bar{\phi})^{n-1} - \sum_{y=R}^{n-1}\left(\bar{\phi}^y(1-\bar{\phi})^{n-y-1}\begin{pmatrix} n-1 \\ y \end{pmatrix}\right)(1-z(y,R))\right)$$

Define for simple reference

$$f(\bar{\phi},n,R) \equiv \sum_{y=R}^{n-1}\left(\bar{\phi}^y(1-\bar{\phi})^{n-y-1}\begin{pmatrix} n-1 \\ y \end{pmatrix}\right)(1-z(y,R)) \tag{74}$$

Then the FOC may be rewritten as

$$D = c + \frac{t}{n\bar{\phi}}\frac{1-(1-\bar{\phi})^n}{1-(1-\bar{\phi})^{n-1}-f(\bar{\phi},n,R)} \tag{75}$$

where $1-(1-\bar{\phi})^{n-1}-f(\bar{\phi},n,R) > 0$ if $R > 1$ and $\bar{\phi} > 0$. This can be seen by noting that $f(\bar{\phi},n,R)$ is ceteris paribus maximal if $z(y,R) = 0$. But then

$$1-(1-\bar{\phi})^{n-1}-f(\bar{\phi},n,R) = \sum_{y=0}^{R-1}\left(\bar{\phi}^y(1-\bar{\phi})^{n-y-1}\begin{pmatrix} n-1 \\ y \end{pmatrix}\right) - (1-\bar{\phi})^{n-1}$$

$$= (1-\bar{\phi})^{n-1} + \sum_{y=1}^{R-1}\left(\bar{\phi}^y(1-\bar{\phi})^{n-y-1}\begin{pmatrix} n-1 \\ y \end{pmatrix}\right) - (1-\bar{\phi})^{n-1} > 0$$

As a consequence $f(\bar{\phi}, n, R) \leq 1 - (1-\bar{\phi})^{n-1}$. Form (75) we immediatly see that limited attention reduces the denominator and therefore increases the equilibrium price for a given $\bar{\phi}$. Also we note that the attention effect on prices vanishes as R approaches n as can be seen in (74) as a higher R reduces the sum size and $z_R \geq 0$. We can also generalize the intuition form the introductory part on how the attention effect works in this model. Setting a higher price always reduces demand in my primary group. But with limited attention some consumers who would jump of to a superior firm if they considered all ads still may consume my product as they eliminated exactly the ad of the superior firm from their choice set. Thus limited attention works as a source of market power and therefore casues higher prices and profits as market demand reacts less elastic to a change of price.

The expression in (74) is not an easy to handle symbolically. An excellent approximation to (75) is given by assuming that $(1-\bar{\phi})^n$ and $(1-\bar{\phi})^{n-1}$ are close to zero and negligible[45] Then

$$p \cong c + \frac{t}{n\bar{\phi}} \frac{1}{1 - f(\bar{\phi}, n, R)}$$

Note that if $\bar{\phi} = 1$ then[46]

$$p = c + \frac{t}{n} \frac{1}{R/n} = c + \frac{t}{R} \tag{76}$$

8.4 Approximation

If $\bar{\phi} = 1$ then

$$f(1, n, R) = 1 - z(n-1, R) = 1 - \frac{R}{n} = \frac{n-R}{n}$$

But then

$$p = c + \frac{t}{n} \frac{1}{1 - \frac{n-R}{n}} = c + \frac{t}{R}$$

This means that even under full information of all opponents which in the absence of limited attention would result in the usual Salop price the attention price is strictly higher following the

[45]This is the same type of large group assumption as in GS.

[46]This also is true without the approximation.

same intuition as developed above.

Further I use $z(y, R) = R/(1+y)$ and then approximate the last expression as[47]

$$1 - f(\bar{\phi}, n, R) \cong \frac{R}{n\bar{\phi}} \tag{77}$$

Thus we receive again expression (76) which we would also receive for $n \to \infty$. The drawback of this approximation is that it requires $R < n\bar{\phi}$ to make sense. The approximation without limited attention is given as

$$p \cong c + \frac{t}{n\bar{\phi}}$$

while the approximation with limited attention is given as

$$p \cong c + \frac{t}{R}$$

Obviously, the two curves intersect at $R = n\bar{\phi}$ where the general solution in (75) reveals that no such intersection exists and the attention curves is stricly on a higher level. Thus approximation is only sensible if $R < n\bar{\phi}$ which also makes sense intuitively. Rational firms only take limited attention into account if it can occur, i.e. if $R < n$. If we work we the suggested type of approximation this condition simplifies to $R < n\bar{\phi}$ as $n\bar{\phi}$ measures how many ads a certain consumer receives on average. The condition $R > n\bar{\phi}$ states that limited attention occurs on average in the economy.

The approximation cleary overestimates prices as ϕ moves away from being one. Note that the original function is very flat around $\phi = 1$ (this originates in the stabilizing effect of limited attention) while the approximation is much steeper. For any quantitativ prediction we should use the true system - but for analytical tractability I work with the approximated curve as it contains the important qualitativ features of the original curve. The only feature missing is the fact that also under limited attention prices would be a decreasing function of n: the attention curve of p

[47] This approximation also uses $(1-\bar{\phi})^n = 0$: write out $1 - f(\bar{\phi}, n, R)$ for any given R and set $(1-\bar{\phi})^n = 0$. This gives $R/(n\bar{\phi})$.

of the approximation will remain constant and the non-attention curve shifts downwards - with the true equations there would also be a downward shift of the attention curve (firms are more protected from competition with LA).

8.5 Proof about FOC ϕ

To show that $\frac{(1-(1-\phi)^n)}{\phi}$ is a strictly decreasing function of $\phi \in (0,1)$ note that the first order MacLaurin series of $(1-\phi)^n$ is given as

$$(1-\phi)^n = 1 - n\phi + R(\phi) \tag{78}$$

where $R(\phi)$ is the remainder term. The MacLaurin series expansion is valid as $\frac{\partial^{n+i}(1-\phi)^n}{\partial(\phi)^{n+i}} = 0$ for $i = 1, 2, 3, ...$ Then

$$\frac{d}{d\phi}\left(\frac{1-(1-\phi)^n}{\phi}\right) < 0 \quad \Leftrightarrow \quad n\phi(1-\phi)^{n-1} < 1-(1-\phi)^n \tag{79}$$

But using (78) in the RHS of (79) we get

$$(1-\phi)^{n-1} < 1 + R(\phi)$$

which holds as $R(\phi) = \frac{\phi^2 n(n-1)(1-\theta\phi)^{n-2}}{2} > 0$ for $0 < \theta < 1$.

The proof for convexity follows the same line of argument but is more cumbersome.

8.6 Proof of Corollary 1

By FOC

$$p = c - \frac{D(p,\phi)}{D_p(p,\phi,R)} \tag{80}$$

We get higher prices under more limited attention if $d_{p,R} < 0$ that is if the marginal loss of demand by a price increase is reduced by reduced attention. As

$$D(p,\phi) = \phi\left(\frac{\bar{p}-p}{t} + \frac{\delta}{n}\right)P\left[1\,|\phi\right] + \phi\left(\frac{p-\bar{p}}{t} + \frac{\delta}{n}\right)P\left[n\,|\phi\right] + \phi\sum_{k=2}^{n-1}\frac{\delta}{n}P\left[k\,|\phi\right]$$

we get

$$D_p(p,\phi) = \frac{\phi}{t}\left(P\left[n\left|\phi\right.\right] - P\left[1\left|\phi\right.\right]\right) \tag{81}$$

Note that $P\left[n\left|\phi\right.\right] = (1-\bar{\phi})^{n-1}$ if $R > 1$. Even with limited attention I can only make a sale to the last group if I am the only firm who informs such a consumer. Next note that

$$P\left[1\left|\phi;R\right.\right] = \sum_{y=0}^{R-1}\left(\bar{\phi}^y(1-\bar{\phi})^{n-y-1}\left(\begin{array}{c} n-1 \\ y \end{array}\right)\right) + \sum_{y=R}^{n-1}\left(\bar{\phi}^y(1-\bar{\phi})^{n-y-1}\left(\begin{array}{c} n-1 \\ y \end{array}\right) z\left(y,R\right)\right)$$

If we increase R by one unit to R' we get $P\left[1\left|\phi;R'\right.\right] > P\left[1\left|\phi;R\right.\right]$ as the first summation part is increased by one summand and every subexpression of the second summation is multiplied by a larger number as $z(j,R) < z(j,R') \leq 1$.

But then by (81) $D_p(p,\phi,R') < D_p(p,\phi,R) < 0$. QED.

By FOC for ϕ we know that this equation is independent of z and also R. But as prices are decreased in R so is the equilibrium marginal revenue of advertising which leads to lower equilibrium ϕ. QED.

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