Real Exchange Rate in Emerging Economies: The Role of Different Investment Margins

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Abstract

In this paper we analyze the role of various investment margins in explaining the trend real exchange rate appreciation observed in European transition countries. We present a model that introduces vertical investment margin in addition to the standard horizontal investment assumed in the literature. We show using simulations for a large set of parameters that the horizontal investment margin is insufficient for replicating the observed trend real exchange rate appreciation and thus the vertical investment is needed.

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1 Introduction

The real exchange rate is an important policy variable, for instance determining part of the monetary policy conditions in open economies and conditioning the optimal timing for monetary integration. Although it traditionally receives special attention, the recent trend real exchange rate appreciation in majority of the Central and Eastern European countries constitutes a puzzle for international macroeconomics. Since the dominant part stems from the real exchange rate for tradable goods, see Égert et al. (2007) and Cincibuch and Podpiera (2006), it leaves a very limited scope for the mainstream theoretical explanation of the real exchange rate appreciation, i.e. the Harrod-Balassa-Samuelson type of convergence. In addition, in absence of the Harrod-Balassa-Samuelson effect on real exchange rate, as documented for instance by Mihaljek and Klau (2006) and Flek et al. (2003), the mainstream theory, represented by Obstfeld and Rogoff (1996), would predict the real exchange rate depreciation for the converging economy instead of its appreciation.

We associate the deficit of the mainstream models for explanation of the experience of converging countries in an implicit assumption that along the transition path products of all countries have comparable qualities. Consequently, a product expansion in a converging economy results in a decrease in the price of products produced in the economy because of the downward sloping demand curve and apparently leads to its real exchange rate depreciation. The changing quality of production might be a plausible explanation of the real exchange rate appreciation, especially in tradables. This can be seen from the difference between Slovenia and Visegrad-4 countries during 1995-2005. While the proportion of medium and high-tech products was high and constant in Slovenia and Slovenia did not experience real exchange rate appreciation, the Visegrad-4 countries started at low proportion and have experienced significant but gradual shift towards high-tech products and recorded trend real exchange rate appreciation, see Fabrizio et al. (2007).

Recent models by Melitz (2003) and Ghironi and Melitz (2005) introduce a limited role for quality of basket of products in the explanation of the real exchange rate appreciation. In their model, the number of basket varieties can be taken as a metaphor for the quality of the basket of products through the love-for-variety. The love-for-variety attribute of consumers' utility makes consumers pay more for a basket with a higher content of varieties and an increase in varieties overestimates observed price indices. If the number of varieties expands in the converging economy relatively more to its advanced counterpart, the real exchange rate based on the observed consumer price indices (as opposed to the rate computed using welfare-theoretic indices) will appreciate for the converging economy.

The strength of this effect is, however, conditional on two aspects. First, the effect will be stronger for converging economy with relatively lower initial content of varieties, i.e. the greater the gap in development, the stronger the effect from varieties. Second, the strength of the effect will depend on the degree of initial openness and the dynamics of integration of the converging economy, since a significant part of the increases in varieties is facilitated by imported varieties into the converging economy. Nevertheless, the real exchange rate appreciation seems to be strictly linked neither to the state of development nor to the degree of openness or speed of opening up. Slovenia for instance, started its convergence to the Euro Area countries at much higher level of gross domestic product per capita than countries in the group of Visegrad-4, had similar degree of openness, and experienced roughly the same speed of integration via foreign trade, however, unlike the Visegrad-4 which appreciated by 30 to 50% over a decade 1995-2005, it did not experience the real exchange rate appreciation.

Besides, as we show in this article using simulations for reasonable range of parameters, the effect of quality (variety) is insufficient for explanation of the typical pace of the real exchange rate appreciation of transition economies, especially in Central and Eastern Europe. Therefore, we propose an extension to the model by Ghironi and Melitz (2005) by introducing a direct vertical quality investment margin, since exogenous quality shocks are shown by Dury and Oomen (2007) to lead to appreciation of the quality-unadjusted real exchange rate. In essence, the content of quality in the product baskets in our model is besides the effect of varieties given by optimal decisions of companies to invest into quality. The content of quality is again the source of the observed real exchange rate appreciation as in Ghironi and Melitz (2005). We show using simulations with the extended model for a reasonable range of parameters that the endogenous mechanism on quality decisions in a converging transition economy readily generates the observed pace of the real exchange rate appreciation.

The rest of the paper is organized as follows. Section 2 describes the proposed model extension. Section 3 uses numerical techniques popular in the global sensitivity analysis literature to assess potential of various investment margins over a large set of parameters to deal with the real exchange-rate experience of emerging economies. Section 4 concludes.

2 Description of the Model

This section presents the workhorse model used throughout this paper. The two countries are modeled in discrete time that runs from zero to infinity. The home country is populated by a representative competitive household which has recursive preferences over discounted streams of momentary utilities. The momentary utility is derived from consumption. A similar household inhabits the foreign country. Production takes place in heterogenous production entities called firms.

2.1 Firms

In the domestic country, there is a large number of firms, which are owned by the domestic household. In each period there is an unbounded mass of potential, ex-ante identical, entrants. The number of potential entrants is determined by the zero-profit condition. Firms differ by location [1] and vintage.

Firms ex-post entry differ by an idiosyncratic variation of the total factor productivity: when a firm enters, it draws a shock z from a distribution $G(z)$. At the end of each period, there is an exogenous probability that a firm is hit by an exit shock. This probability is δ and is assumed to be independent on aggregate as well as individual states. Hit firms shut down.

The production function maps two inputs into two outputs. The first input is fixed and we label it as 'capital'; the second input is variable and is labeled as 'labor'. The variable input is available in inelastic supply in each country and is immobile between countries.

The first outputs is quality h and if the firm j uses k_j units of capital, then the quality of its product is given simply as $h_j = k_j$. Capital investment can be thus considered as an improvement in quality. The second output is the physical quantity of goods produced x. The production function is given as follows: $x_{jt} = z_j A_t \ell(l_{jt}, k_j)$. The production function ℓ is strictly increasing in the first argument (labor), but strictly decreasing in the second argument (capital) [2]. This implies that investments in quality increase the labor inputs needed to produce physical quantities. One may think that the production of a more sophisticated good requires more labor or more skilled labor. Thus, quality investment is costly for two reasons: first, it requires fixed input k_j; and second, more labor is required to produce better goods.

[1] Since the paper explores the potential of various investment margins to explain the observed pace of real exchange rate appreciation, it abstracts from the cross-border asset ownership. Thus domestic firms are those firms, which are owned by the domestic agent and are located at the domestic country, analogously for foreign firms. See Brůha, Podpiera, Polák (2007) or Brůha, Podpiera (2007a,b) for applications with cross-border firm ownerships.

[2] We require that the function ℓ is strictly decreasing in capital. If the function ℓ were not decreasing in capital, the linearity of h_j in k_j would imply endogenous growth, as in Young (1998) or Baldwin and Forslid (2000).

The production of physical quantities is increasing at the level of firm total factor productivity $A_t z_j$, which has two components: (a) the idiosyncratic component z_j, which is i.i.d. across firms and which follows the distribution $G(z)$ introduced above, and (b) the common component A_t. Domestic firms enjoy at time t productivity A_t, while foreign firms enjoy productivity A_t^*.

We assume that the final output of the firm is given by the product of quality and quantity as follows: $q_{jt} = h_j x_{jt}$. The final quality-quantity bundle is what is sold at the market. This assumption follows the standard approach of growth theoreticians, for example Young (1998). Thus, the production of the final bundle can be described as $q_{jt} = z_j A_t f(k_j, l_{jt})$, where f is given as $f(k_j, l_{jt}) \equiv k_j \ell(l_{jt}, k_j)$. We assume that the final bundle production function is increasing in both arguments and is homogeneous of degree one. We explicitly distinguish the quality-quantity bundle from the physical quantity since the explanation for the observed real exchange rate appreciation is based on a dichotomy between quality -adjusted and -unadjusted prices.

The quality investment is a sunk factor, set at the time of entry, while labor can be freely adjusted. Given a realization of the productivity shock z_j, the probability of the exit shock δ, and a chosen production plan, the value of a firm is determined by expected present value of the stream of profits.

To make reading the paper easier, we introduce the following convention. Location of firms is distinguished by superscript d - for the *domestic* country and f - for the *foreign* country. Firms produce differentiated goods. The good produced by the firm located in the destination market is denoted by the d superscript, while goods imported are denoted by the m superscript. The consumption markets are distinguished by the * superscript: goods consumed by the domestic household are without superscript, while goods consumed by the foreign household do have one. Thus p_{jt}^d will denote the price of a good produced by a firm j located in the domestic country at time t sold to the domestic market, p_{jt}^m is the price of a good j imported to the domestic market from the foreign country, while p_{jt}^{m*} would be a price of a good from the domestic country to the foreign household. We further assume that prices are denominated in the currency of the market.

According to the introduced convention, $\mathbb{P}_{j\tau t}^d$ denotes the t-period *real operating profit* of a domestic firm of vintage τ and is given as follows:

$$\mathbb{P}_{j\tau t}^d = \left[\kappa_{jt} \frac{p_{jt}^d}{P_t} + (1 - \kappa_{jt}) \frac{\eta_t}{1 + \mathbf{t}} \frac{p_{jt}^{m*}}{P_t^*} \right] A_t z_j f(k_j, l_{jt}) - \mathbb{W}_t l_{jt},$$

where $0 \le \kappa_{jt} \le 1$ is the share of product sold in the domestic markets, P_t is the domestic price level, P_t^* is the foreign price level, η_t is the *real exchange rate*, which is linked to the nominal exchange rate s_t as $\eta_t = s_t P_t^* / P_t$, $\mathbf{t} \ge 0$ represents unit iceberg exporting costs, and $\mathbb{W}_t$ is the *real wage*. Firms of different vintage and different ownership have different levels of invested capital, that is why $\mathbb{P}^d_{j\tau t}$ will be naturally different along these dimensions. Similar definitions apply to the remaining types of firms as well.

Note that prices such as p^d_{jt} are prices of the final quantity-quality bundles and therefore derived indexes P_t, P^*_t, η_t are related to aggregations of these final bundles. The prices related to physical quantities are then given by $\wp^d_{jt} \equiv k_j p^d_{jt}$. The discussion about distinct roles of prices of quality-quantity bundles and of prices defined on physical quantities is left to subsection 2.3.

Firms may export only if special fixed costs are invested. If a firm at the time of entry decides to invest the fixed export costs, then it becomes eligible to export in all subsequent periods, otherwise it is in all periods not eligible to export. Exporting decisions of *eligible* firms are taken on a period-by-period basis. Unit iceberg exporting costs $\mathbf{t}$ represents transportation costs, policy barriers such as tariffs, while the fixed costs may represent expenditures associated with acquiring necessary expertise such as legal, business, or accounting issues of the foreign markets.

Capital is the sunk factor and each firm decides how much capital to acquire at the time of entry: this means that the firm decides the quality of its product at the time of entry, while produced quantities are variable during its lifetime. We assume that real investment costs take the following form: $(k + c^\xi)$, $\xi \in \{e, n\}$. We assume that:

$$c^e > c^n > 0,$$

where the superscript refers to eligibility, i.e. e – *eligible* or n – *noneligible*: eligible firms pay larger fixed costs. The cost structure implies – as in Melitz (2003) – that in equilibrium there is a cut-off productivity value $\overline{z}$, such that firms with lower idiosyncratic productivity $z_j < \overline{z}$ will not invest to become eligible, while firms with a sufficiently high productivity level $z_j \geq \overline{z}$ will do.

Since capital is invested *after* learning z_j, the invested amount of capital k_j will be different for firms with different z_j. We assume that firm's manager maximizes the expected discounted stream of profits. Thus, the value of the profit stream of the domestic firm of vintage τ, enjoying the idiosyncratic productivity level z_j is (in real terms):

$$V^d_\tau(z_j) = \max_{\xi, k, \{l_\tau\},} \sum_{t=\tau}^{\infty} (1-\delta)^{t-\tau} \mu^t_\tau \mathbb{P}^d_{j\tau t} - (c^\xi + k), \tag{1}$$

where $\mathbb{P}^d_{j\tau t}$ is the t-time real operating profit of a firm of vintage τ, enjoying the productivity level z_j under the optimal production plan (derived later in Subsubsection 2.1.2), and the effective discount factor is given as $(1-\delta)^{\tau-t}\mu^t_\tau$, where μ^t_τ is the marginal rate of intertemporal substitution between dates τ and t. The rate of the intertemporal substitution is defined in more details in Subsection 2.2. The value of the firm owned by the foreign household is defined analogously.

To summarize the sequencing, the timing proceeds first with the households' decision about the number of new entrants. Then, each new entrant draws a productivity level from the distribution G and it decides the amount of invested capital and whether to invest for export eligibility. Then, labor demand and production (of both entrants and incumbents) take place. At the end of the period, some firms experience the exit shock and shut down.

Firms located in the same country differ along two dimensions: idiosyncratic productivity level z_j and vintage τ. The vintage affects incentives to invest provided that macroeconomic conditions change (such as A_t increases). This implies that firms of different vintage invest different amounts of capital, even if they experience the same idiosyncratic productivity level. Therefore we shall define the time-varying distribution measure over domestic firms as $\Gamma_t^d(j,\tau)$; the counterpart of foreign firms is denoted by $\Gamma_t^f(j,\tau)$.

2.1.1 Market Structure

The final good Q in the domestic country is composed of a continuum of quality-quantity bundles (goods), some of which are produced in the domestic country and some are imported. There is imperfect substitution among these goods, the substitution is modeled using the standard constant-elasticity-of-substitution (CES) function with the parameter $\theta > 1$. The aggregate good in the domestic country is defined as:

$$Q_t = \left(\int_{\Omega^\xi} \left(q_{jt}^d\right)^{\frac{\theta-1}{\theta}} \, \mathrm{d}\Gamma_t^\xi(j,\tau) + \int_{\Omega_e^\xi} \left(q_{jt}^m\right)^{\frac{\theta-1}{\theta}} \, \mathrm{d}\Gamma_t^\xi(j,\tau) \right)^{\frac{\theta}{\theta-1}},$$

where, q_j is the output of firm j, Ω^d denotes the set of domestic products. The analogous convention holds for sets of foreign firms. If a set is labeled by the subscript e, it reads as a subset of eligible firms. Thus, $\Omega_e^f \subset \Omega^f$ is the subset of goods produced by *eligible* foreign firms. The final good in the foreign country is defined analogously. The market structure implies the following definition of the aggregate price index:

$$P_t = \left(\int_{\Omega^\xi} \left(p_{jt}^d\right)^{1-\theta} \, \mathrm{d}\Gamma_t^\xi(j,\tau) + \int_{\Omega_e^\xi} \left(p_{jt}^m\right)^{1-\theta} \, \mathrm{d}\Gamma_t^\xi(j,\tau) \right)^{\frac{1}{1-\theta}}, \qquad (2)$$

where p_{jt} is the price of products of firm j at time t. Note that the final good Q_t represents both physical quantities as well as qualities and that the price indexes P_t, P_t^* aggregate both: available quantities and qualities. In that sense, these are quality-adjusted price indexes. If one wants to construct counterparts of empirical price indexes, one has to aggregate prices $\wp_{jt}^d$, rather than p_{jt}^d.

2.1.2 Optimal Plans

In this part, we derive optimal production and investment plans using the backward induction. We derive it for a domestic firm, which is easily generalized for a foreign firm. This part of the paper shows the backward induction for general neoclassical production function. The parametric example of model equations for the Cobb-Douglas production function is given in Appendix A.

Let us assume the problem of maximizing the value of a domestic firm. Since there are no labor adjustment costs, labor decisions are made on a period-by-period basis. Standard results of monopolistically competitive pricing suggest that prices are set as a mark-up over marginal costs. Simultaneously with prices, firms also decide κ_j.

Now, let us take the perspective of a non-eligible firm of vintage τ and productivity level A_t. Its real operating profit $\mathbb{P}^{dn}_{j\tau t}$ in a period t is given – conditional on non-eligibility status, aggregate productivity, idiosyncratic productivity z_j, – as a solution to the following program:

$$\mathbb{P}^{dn}_{j\tau t} = \max_{l_{jt}} \left\{ \frac{p_{jt}}{P_t} A_t z_j f(k_j, l_{jt}) - \mathbb{W}_t l_{jt} \right\} = \max_{l_{jt}} \left\{ \left[A_t z_j f(k_j, l_{jt}) \right]^{\frac{\theta-1}{\theta}} Q_t^{\frac{1}{\theta}} - \mathbb{W}_t l_{jt} \right\}, \tag{3}$$

The second equality in (3) follows from the CES market structure. Similarly, the real operating profit of an eligible firm $\mathbb{P}^{de}_{j\tau t}$ of vintage τ in a period t is given by:

$$\mathbb{P}^{de}_{j\tau t} = \max_{l_{jt}} \left\{ \left(\kappa_{jt} \frac{p_{jt}}{P_t} + (1-\kappa_{jt}) \frac{\eta_t}{1+t} \frac{p^*_{jt}}{P^*_t} \right) A_t z_j f(k_j, l_{jt}) - \mathbb{W}_t l_{jt} \right\} = \tag{4}$$

$$= \max_{l_{jt}} \left\{ \left(\kappa_{jt} Q_t^{\frac{1}{\theta}} + (1-\kappa_{jt}) \frac{\eta_t}{1+t} Q_t^{*\frac{1}{\theta}} \right) \left[A_t z_j f(k_j, l_{jt}) \right]^{\frac{\theta-1}{\theta}} - \mathbb{W}_t l_{jt} \right\}.$$

Then the expected present value of operating profit streams is given as follows

$$\mathbb{P}^{d\xi}_{j\tau} = \sum_{t=\tau}^{\infty} \mu^t_\tau (1-\delta)^{t-\tau} \mathbb{P}^{d\xi}_{j\tau t}$$

with $\xi \in \{n, e\}$. The expected present values depend on idiosyncratic productivity z_j, invested capital k_j, and the future path of productivities, real wages and demands.

The optimal investment decision of a firm, which enjoys a productivity level z_j, maximizes the value of the firm given as $\mathbf{V}^{d\xi}_\tau(k_j|z_j) = \mathbb{P}^{d\xi}_{j\tau} - \left(c^\xi + k_j\right)$, for $\xi \in \{n, e\}$. The maximization of $\mathbf{V}^{de}_\tau(k_j|z_j)$ (resp. $\mathbf{V}^{dn}_\tau(k_j|z_j)$) yields the optimal demand for quality investment (capital) for eligible (resp. non-eligible) firms, and the value of the firm is:

$$V_\tau^{d\xi}(z_j) = \max_{k_j \geq 0} \mathbf{V}_\tau^{d\xi}(k_j | z_j),$$

where $\xi \in \{e, n\}$. Value functions $V_\tau^{dn}(z_j)$, $V_\tau^{de}(z_j)$ implicitly define the cut-off value $\overline{z}$, which is the lowest idiosyncratic shock, which makes the the export-eligibility investment profitable. Thus it is defined as

$$\overline{z}_\tau^d = \min_{z_j}(V_\tau^{de}(z_j) \geq V_\tau^{dn}(z_j)).$$

The value of a firm is given by

$$V_\tau^d(z_j) = \max_{\xi \in \{n,e\}} V_\tau^{d\xi}(z_j) = \begin{cases} V_\tau^{de}(z_j) & \text{if } z_j \geq \overline{z}_\tau^d \\ V_\tau^{dn}(z_j) & \text{if } z_j < \overline{z}_\tau^d \end{cases},$$

and the expected value of a new entrant $\mathcal{V}_\tau^d$ is:

$$\mathcal{V}_\tau^d = \int_{z_L}^{z_u} V_\tau^d(z)\, G(\mathrm{d}z), \tag{5}$$

This completes the backward induction.

The, just derived, optimal production plan induces a measure over firms. Denote $\widetilde{\mathbb{P}}_{\tau,t}^d$ the t-time expected[3] real operating profit of a domestically-owned firm, which enters in time τ, $\widetilde{\mathbb{P}}_{\tau,t}^d = \int_{z_L}^{z_u} \mathbb{P}_{j\tau t}^d\, G(\mathrm{d}z_j)$, and $\widetilde{c}_\tau^d$ the expected real investment costs under such measure. Then:

$$\mathcal{V}_\tau^d = \sum_{\sigma \geq 0} \mu_\tau^{\tau+\sigma}(1-\delta)^\sigma \widetilde{\mathbb{P}}_{\tau,\tau+\sigma}^d - \widetilde{c}_\tau^d,$$

and

$$\widetilde{c}_\tau^d = G(\overline{z}_\tau^d)c^n + (1 - G(\overline{z}_\tau^d))c^e + \int_{z_L}^{\overline{z}_\tau^d} k_j^{opt,n}\, G(\mathrm{d}z) + \int_{\overline{z}_\tau^d}^{z_U} k_j^{opt,e}\, G(\mathrm{d}z).$$

The first two terms correspond to the expected fixed costs, while the last two terms correspond to the expected costs of capital investment.

[3] Expectation is taken with respect to the measure given by the optimal production plan.

The home country is populated by a representative competitive household who has recursive preferences over discounted stochastic streams of period utilities. The period utilities are derived from consumption of the aggregate good. Leisure does not enter the utility, so labor is supplied inelastically. The aggregate labor supply in the domestic country is $\mathcal{L}$, while $\mathcal{L}^*$ is the aggregate labor supply in the foreign country. Households can trade bonds denominated in the foreign currency.

The domestic household maximizes

$$\max U = \sum_{t=0}^{\infty} \beta^t u(C_t),$$

subject to

$$B_t = (1 + r^*_{t-1}) B_{t-1} + \frac{-1}{\eta_t} (C_t - \mathbb{W}_t \mathcal{L}) + \frac{1}{\eta_t} \left(\Xi^d_t - \tilde{c}^d_t n^d_t \right) - \frac{\Psi_B}{2} B^2_t + \mathcal{T}_t, \quad (6)$$

where B_t is the real bond holding of the domestic household, C_t is consumption, r^*_{t-1} is the real interest rate of the internationally traded bond, Ψ_B represents portfolio adjustment costs, as in Schmitt-Grohe, Uribe (2003) to stabilize the model[4], and $\mathcal{T}_t$ is the rebate of these costs in a lump-sum fashion to the household. The flow of real operating profits from all domestic firms is denoted as Ξ^d_t and is given by

$$\Xi^d_t = \sum_{\sigma \leq t} (1 - \delta)^{t-\sigma} n^f_\sigma \widetilde{\mathbb{P}}^d_{\sigma,t}.$$

Because of the law of large numbers and of perfect foresight, the *ex-ante* expected values of the key variables for household decisions (such as investment costs or profit flows) coincide with *ex-post* realizations.

The number of new domestically located entrants owned by the domestic household in time t is n^d_t, and is determined by the expected zero-profit in equilibrium.

[4] In a strict sense, the model is stable even without portfolio adjustment costs (i.e., under $\Psi_B = 0$). The model is deterministic and therefore it would not exhibit unit-root behavior even under $\Psi_B = 0$. On the other hand, if $\Psi_B = 0$, then the model would exhibit steady state dependence on the initial asset holding. Therefore we use nontrivial adjustment costs $\Psi_B > 0$ to give up the dependence of the steady state on the initial asset holding.

The first-order conditions for the domestic household are standard ones:

$$(1+\Psi_B B_t) = \frac{\eta_{t+1}}{\eta_t}(1+r_t^*)\mu_t^{t+1}, \tag{7}$$

$$\lim_{t\to\infty} B_{t+1} = 0, \tag{8}$$

$$\tilde{c}_t^d = \sum_{v\geq 0} (1-\delta)^v \, \mu_t^{t+v} \widetilde{\mathbb{P}}_{t,t+v}^d, \tag{9}$$

where the marginal rate of substitution is defined as usually as:

$$\mu_{t_1}^{t_2} \equiv \beta^{t_2-t_1} \frac{u'(C_{t_2})}{u'(C_{t_1})}.$$

It is worth to note that although there is an idiosyncratic variance at the firm level, the model is deterministic at the aggregate level, thus the dynasty problem is deterministic too. Therefore the marginal rate of substitution does not involve the expectation operator. The household problem in the foreign country is defined symmetrically.

Bonds are denominated in the foreign currency and since the model is deterministic, this is a completely innocent assumption. The international bond market equilibrium requires that $B_t + B_t^* = 0$.

2.3 Notes on Price Indexes

As mentioned above, prices p_{jt} and the corresponding price indexes P_t, and P_t^* are quality-adjusted prices. Therefore, the real wages $\mathbb{W}_t$ and $\mathbb{W}_t^*$ and the real exchange rate η_t are measured in the terms of qualities. These measures correspond to real-world price indexes only if the latter are quality-adjusted perhaps using a hedonic approach, which is rarely a case for transition countries (Ahnert and Kenny, 2004, p. 28). To get indexes closer to real-world measures, we have to define aggregate indexes over $\wp_{jt}$. Denote such indexes as $\mathcal{P}_t$ and $\mathcal{P}_t^*$.

We follow Brůha and Podpiera (2007) and use a straightforward approximation to $\mathcal{P}$:

$$\mathcal{P}_t = \mathcal{K}_t P_t,$$

where $\mathcal{K}_t$ is the total amount of invested capital by firms selling its products in the domestic country:

$$\mathcal{K}_t = \int_{\Omega^d} k_{j\tau} \, \mathrm{d}\Gamma_t^d(j,\tau) + \int_{\Omega_e^f} k_{j\tau} \, \mathrm{d}\Gamma_t^f(j,\tau).$$

Nevertheless, $\mathcal{P}_t$ might differ from the CPI-based real-world indexes by one more term. The market structure based on the CES aggregation implies the *love-for-variety* effect, which means that the welfare-theoretical price index differs from the 'average' price by the term $\nu^{\frac{1}{\theta-1}}$, where ν is the number of available varieties and θ is the parameter of substitution in the CES function (see Melitz, 2003 for rigorous definition and derivation of the average price). Therefore, we distinguish the following definitions of the real exchange rate:

Quality-adjusted theoretically-consistent RER η_t is the real exchange rate, which enters the decisions of agents in the model.

Quality-unadjusted theoretically-consistent RER is the real exchange rate defined over physical quantities and is related to the quality-adjusted theoretically-consistent RER as $\frac{\mathcal{P}_t^*/P_t^*}{\mathcal{P}_t/P_t}\eta_t$.

Quality-adjusted CPI-based RER is related to its theoretically consistent counterpart as $\left(\frac{\nu_t^*}{\nu_t}\right)^{\frac{1}{\theta-1}}\eta_t$, where ν_t and ν_t^* is the number of varieties available at time t in the domestic and foreign country, respectively.

Quality-unadjusted CPI-based RER is probably the correct counterpart of the *measured real exchange rate* and is defined as $\left(\frac{\nu_t^*}{\nu_t}\right)^{\frac{1}{\theta-1}}\frac{\mathcal{P}_t^*/P_t^*}{\mathcal{P}_t/P_t}\eta_t$.

The quality-adjusted theoretically consistent real exchange rate η_t depreciates during the transition and the reason is the downward-sloping demand curve. On the other hand, the three remaining indexes may appreciate under some conditions, see Section 3 for discussion and intuition.

The distinction among various definitions of real exchange rate is reflected also in comparison of the economic performance of countries. If one wants to compute a model counterpart of the ratio of GDP per capita in PPP, one has to use $\frac{Y_t}{\eta_t Y_t^*}\frac{\mathcal{L}^*}{\mathcal{L}}$, where $Y_t = Q_t + \eta_t X_t$, $Y_t^* = Q_t^* - X_t$ are the model counterparts of real GDP (in the currency of the respective country) and X_t is the value of *net* real exports of the domestic country expressed in the foreign currency. On the other hand, if one wants to compute a model counterpart of the ratio of the nominal GDP using the nominal exchange rate (which is the same as a ratio of real GDP using the measured real exchange rate), one has to use $\frac{Y_t}{\eta_t Y_t^*}\frac{\mathcal{L}^*}{\mathcal{L}}\left(\frac{\nu_t}{\nu_t^*}\right)^{\frac{1}{\theta-1}}\frac{\mathcal{P}_t/P_t}{\mathcal{P}_t^*/P_t^*}$.

2.4 General Equilibrium

As usual, the general equilibrium is defined as a time profile of prices such that all households optimize and all markets clear. Since there are no price rigidities, only the relative prices matter. The general equilibrium requires that the market-clearing conditions hold.

The aggregate resources constraints are given as follows:

$$C_t + n_t^d \tilde{c}_t^d = Q_t, \tag{10}$$

the labor market equilibrium requires:

$$\int_{z_L}^{z_U} l_{jt}\, \mathrm{d}\Gamma_t^d(j,\tau) = \mathcal{L}, \tag{11}$$

where l_{jt} is the labor demand by individual firms, and $\mathcal{L}$ is the aggregate, inelastic, labor supply. Analogous market clearing conditions hold in the foreign country.

The international bond market equilibrium requires that

$$B_t + B_t^* = 0. \tag{12}$$

The last equilibrium condition is the balance-of-payment equilibrium, which requires that:

$$B_{t+1} = (1 + r_t^*)B_t + X_t, \tag{13}$$

where X_t is the value of *net* real exports of the domestic country expressed in the foreign currency.

The appendix summarizes *steady-state* model equations. The reader is referred to the appendix A.2 in Brůha and Podpiera (2007a) for description of the recursive form of a variant of the model, which can be used for dynamic simulations. Papers by Brůha and Podpiera (2007b) and Brůha, Podpiera and Polák (2007) applied the dynamic solutions for policy questions (economic integration and monetary conditions in a small economy, respectively).

3 Quantitative Analysis

We use numerical techniques frequently applied in the global sensitivity analysis literature to assess potential of various investment margins over a large set of parameters to deal with the trend real exchange rate appreciation of transition economies. In particular we run the simulations for the model Ghironi and Melitz (2005), which we reach by calibrating $\alpha = 0$ and subsequently perform the same set of simulations for the extended model introduced in this paper for a range of reasonable values of α.

We define an a priori set of possible values of structural parameters and use a scheme to draw a combination of the parameters. For each combination, we compute a change in the steady-state values of the observable real exchange rate[5]. These steady-state values correspond to (i) the initial situation of the

[5] For the model without an explicit quality investment, the observable RER means the CPI-based RER, for the model with quality, it means the quality-unadjusted CPI-based RER, see Section 2.3 for more discussion.

emerging economy being relatively underdeveloped compared to its advanced counterpart and (ii) to the steady state of equal output per capita in both countries. The experiment is calibrated so that initially the transition economy attains 60% of the output per capita relative to its advanced counterpart [6] .The expected real exchange rate appreciation is then computed as the change in the steady-state value of the real exchange rate for each draw. The set of parameter values is a multidimensional cube given in Table 1.

The numerical ranges for parameters are predominantly motivated by relevant empirical micro and macroeconomic evidence. The range for parameter δ was chosen relatively large since the exit rate for firms in advanced economies might be smaller (0.1 in the U.S) but might be higher for less developed economy. The range for the parameter θ is derived from Rotemberg and Woodford (1992), who use $\theta = 6$ and Ghironi amd Melitz (2005) who opt for a value of 3.8 (based on empirically found mark-ups for the U.S. by Bernard et al., 2003). Iceberg cost was considered between 2.5 and 15% reflecting the empirical evidence between 6 and 11% during 1980-2000, as reported by Gust et al.(2007). The ranges for the remaining parameters for entry and export eligibility costs were derived indirectly considering the empirical variability of the share of consumption to output observed for the Central and Eastern European transition countries, i.e. 0.7-0.8.

The rest of the parameters is set as follows: the final steady state value of the productivity parameters: $A^* = A = 10$, and the rate of the intertemporal rate of substitution $\beta = 0.95$ [7] . When computing the steady state, it is not necessary to specify the parametrization of the momentary utility function u, it is sufficient to assume the usual properties of u, i.e. that it is increasing and concave.

The remaining part of the calibration exercise is the choice of the distribution of idiosyncratic productivity shocks $G(z_j)$. We report the results for uniform distribution on [0 1]. Our choice is motivated by its properties: it is a limit of the Pareto distribution and has bounded support, which implies that the value of a new entrant remains bounded. Nevertheless we replicated the exercises for the Pareto (used in Ghironi and Melitz, 2005) [8] and exponential [9]

[6] For each draw we find the initial productivity A so that the initial ratio 60% is obtained.

[7] In the steady state, the parameters β and δ are separately unimportant. They matter through the product $1 - \beta(1 - \delta)$ and thus it makes sense to fix one and to let vary the other.

[8] The parameter of the Pareto distribution k is sampled from a uniform distribution on [2 6.5] subject the restriction $k > \theta - 1$.

[9] The exponential distribution shares the unbounded support with the Pareto distribution, but similarly to the uniform distribution it ensures the finiteness of the new-entrant expected value for any $\theta > 1$.

distributions. The results of our experiment show that the outcome is robust to the choice of the distribution $G(z_j)$.

As a sampling scheme, we use both a Latin-hypercube sampling and Halton sequences[10] and we sample 10 000 parameter combinations for each. Both sampling strategies give almost identical results and therefore we report the results for Halton-sequences sampling only.

For the set of simulations with the extended model, i.e. a model with explicit quality investment margin, we define the parameter space $\alpha \in [0.05\ 0.50]$. Similarly to the first simulation, we sampled 10 000 different sets of the parameters and computed the change in the steady-state value of the real exchange rate.

Given the sample size, we also approximate the negative of the percentage change in real exchange rate[11] H as follows:

$$H \cong H_0 + \sum_i H_i \log(\pi_i/\overline{\pi}_i) + \sum_{ij} H_{ij} \log(\pi_i/\overline{\pi}_i) \log(\pi_j/\overline{\pi}_j),$$

where H_0, H_i and H_{ij} are parameters of approximation and π_i is a formal argument for the structural parameters (δ, θ, c^n, *etc*). By $\overline{\pi}_i$ we denote the lower bound on the respective parameter. Thus H_0 is the change in the real exchange rate for the lowest possible parameter values and H_i and H_{ij} denotes elasticities.

Based on the sample of 10 000 drawings we approximate the function H in the least-square sense. The results of the projection of the real exchange rate appreciation on parameters are displayed in Tables 2 and 3. Table 3 gives the full projection onto all logs of coefficients and their cross-products. Nevertheless, a majority of them do not contribute to the explanation of the real exchange rate appreciation much and therefore we use a non-parametric specification test to decrease the number of terms in the projection. The resulting parsimonious specification is given in Table 2 (the difference of the sums of square residuals between the full and parsimonious projection is in both cases less than 0.5%).

The parsimonious specification reveals that some parameters are unimportant for explanation of the change in the real exchange rate. This is the case for the probability of the exit shock δ and for the investment cost c^n. Calibration of the two parameters is nevertheless important if one wants to fit the GDP composition.

[10] Both strategies are popular in the sensitivity-analysis literature.
[11] This means that a positive value of H means the appreciation of the currency of the emerging country.

We find the following: the appreciation of the observed real exchange rate is consistent with low values of θ or with a combination of high values of trade costs c^e/c^n, $\mathbf{t}$. Low values of θ make the love-for-variety effect stronger, while high-values of trade costs make the economy more closer (it is more difficult for firms to become export eligible). If the converging economy is initially almost close, it *ceteris paribus* implies that the relative expansion of new varieties (both of domestic origin and imported) is stronger, which translates to the higher observed appreciation. The other mechanism than low intratemporal substitution and initial closeness seems to be of order less important.

Despite the two channels, the real exchange rate appreciation in a model without explicit investment in quality is not stronger than 35%, see Figure 1. Even this number is attained for an extreme closeness: the ratio c^e/c^n should be greater than 3 (Ghironi and Melitz, 2005, calibrate this ratio to 1.235). For values of the ratio c^e/c^n lower than 3, the observed appreciation does not exceed 6%.

On the other hand, if we model investment in quality explicitly, one can attain easily the observed real exchange rate appreciation about 30-50% even for reasonable parameter values, see Figure 2.

The simulations show that the model that relies on quality only in the form of expansion of varieties for explanation of the real exchange rate appreciation in converging economies, such as by Ghironi and Melitz (2005), is insufficient for explanation of the recent experience in transition countries in Central and Eastern Europe. Nevertheless, we also show that an extension to the model in the form of additional investment margin (investment in quality) endogenously generates the real exchange rate appreciation in comparable magnitudes that have been observed in these transition countries.

4 Concluding remarks

In this paper we address the recent puzzle of the real exchange rate appreciation and the terms of trade and the real market shares improvements of converging transition countries. In models frequently used in the international macroeconomics the increase in productivity (uniform across sectors) is consistent with the real exchange rate depreciation. This is because output expansion can be sustained as equilibrium only if the corresponding prices decline. However, empirical observations contradict this paradigm since more advanced countries tend to have higher price levels. Also, the converging transition economies experienced both an increase in the real export market shares and the real exchange rate appreciation.

Also models with exogenous share of tradable goods and productivity concentration in tradable good sectors, thus generating the real exchange rate appreciation through the Harrod-Balassa-Samuelson effect, proved to be dissatisfactory due to very weak empirical support at least for the European transition countries. Besides, this type of models would assume constant or declining terms of trade which is also in contradiction with the empirical evidence of the terms of trade improvement for these countries.

Even the latest advance in explaining the real exchange rate appreciation after a uniform productivity increase is not fully satisfactory. In models that combine an endogenous number of varieties with a market structure featuring the love-for-variety effect (type of quality effect) implies the divergence between welfare-theoretic price index and price index based on average prices. Consequently, an increase in productivity in converging economy expands its relative number of varieties and causes the real exchange rate appreciation of the exchange rate defined per unit of physical product. Nevertheless, as we have shown, this effect is quite limited for reasonable range of model parameters and thus it turns out to be insufficient for the recent empirical evidence.

Therefore, we have presented a model that introduces an endogenous vertical investment margin into the consistent general equilibrium with trade and investment costs and have shown that such an extension is needed for satisfactory explanation of the size of the recent trend real exchange rate appreciation for the Central and Eastern European transition economies.

A Steady State under a Particular Functional Form

In this part, we characterize the steady state and we derive and present the steady-state equations of the model for a particular functional form. Since we deal with the steady state, we give up the time subscripts t.

A.1 The Steady State

The steady state is the long-run equilibrium and it is obtained when exogenous parameters (particulary productivity parameters A and A^*) are constant for a sufficiently long period of time. The steady state is characterized by a number of features. The most important ones include:

- zero bond holding $B_{ss} = 0$, which is due to adjustment costs $\psi_B > 0$;
- constant endogenous quantities and prices;
- the marginal rate of the intertemporal substitution $\mu_{t_1}^{t_2} = \beta^{t_2 - t_1}$;
- the steady-state effective discount rate $\mathcal{R} \equiv \sum_{t \geq 0} (1-\delta)^t \mu_\tau^{\tau+t}$ reads as $\mathcal{R} = \frac{1}{1-\beta(1-\delta)}$ and the steady-state interest rate $r_{ss} = \beta^{-1} - 1$;
- net exports are zero;
- the distribution of firms degenerate over the vintage dimension: thus one can write $\Gamma_{ss}^d(j)$ instead of $\Gamma_{ss}^d(j, \tau)$.

A.2 Derivation under a Particular Functional Form

As a benchmark calibration, we use the iso-elastic production function $\ell(l,k) \equiv \left(\frac{l}{k}\right)^{1-\alpha}$ for production of physical quantities. This formulation implies the Cobb-Douglas production function $f(k,l) = k^\alpha l^{1-\alpha}$ for the production of the quality-quantity bundle. The cost function associated with the Cobb-Douglas production function is given as follows:

$$\mathbb{C}(q, \mathbb{W}, A, z_j, k_j) = \mathbb{W} \left[\frac{q}{A z_j k_j^\alpha} \right]^{\frac{1}{1-\alpha}}.$$

The curvature of the momentary utility function u does not matter for the steady-state properties as long as u is strictly increasing and concave.

Note that for $\alpha = 0$ (and taking the relevant limits where necessary), the production function is linear in a single input – labor, which corresponds to the parametrization used by Ghironi and Melitz (2005).

First, we derive the optimal investment decision, and the present value of profit flows for a non-eligible firm[12]. Such a firm will supply the following quantity-quality bundle q_j^d to the domestic market:

$$q_j^d = \left(\left[\frac{\theta - 1}{\theta} (1-\alpha) \mathbb{W}^{-1} \left[A z_j k_j^{\alpha} \right]^{\frac{1}{1-\alpha}} \right]^{\theta} Q \right)^{\frac{(1-\alpha)}{\alpha\theta + (1-\alpha)}},$$

the real turnover is:

$$\frac{p_j^d}{P} q_j^d = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \left[\frac{\theta-1}{\theta} (1-\alpha) \mathbb{W}^{-1} A^{\frac{1}{1-\alpha}} \right]^{\frac{(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} Q^{\frac{1}{(1-\alpha)+\alpha\theta}},$$

and the period real operating profit is given by:

$$\begin{aligned} \mathbb{P}_{j\tau\sigma}^d &= \frac{p_j^d}{P} q_j^d - \mathbb{C}(q_j^d, \mathbb{W}, A, z_j, k_j) = \\ &= z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathbb{W}^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} A^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} Q^{\frac{1}{(1-\alpha)+\alpha\theta}} \mathcal{W}_1, \end{aligned}$$

where we define $\mathcal{W}_1 = \frac{\alpha(\theta-1)+1}{(\theta-1)(1-\alpha)} \left[\frac{\theta-1}{\theta} (1-\alpha) \right]^{\frac{\theta}{(1-\alpha)+\alpha\theta}}$.

Second, we derive optimal production decisions of eligible firms is derived. The CES market structure implies that $q_j^d = \left[\frac{\theta-1}{\theta} \left(\frac{MC_j}{P} \right)^{-1} \right]^{\theta} Q$, and $q_j^{*m} = \left[\frac{\theta-1}{\theta} \frac{\eta}{1+\mathbf{t}} \left(\frac{MC_j}{P} \right)^{-1} \right]^{\theta} Q^*$. Some simple, but tedious, algebraic manipulations yield:

$$\kappa_j = \frac{Q}{Q + Q^* \left(\frac{\eta}{1+\mathbf{t}} \right)^{\theta}}.$$

Observe that κ_j does not depend on individual characteristics of firms: z_j and k_j; it depends only on relative demands of both markets and on the real exchange rate corrected for transport costs $\mathbf{t}$. Therefore, all eligible firms will sell the same share of its products to the domestic resp. foreign markets. Thus henceforth we will simply write κ for κ_j. Define

$$\xi \equiv Q + Q^* \left(\frac{\eta}{1+\mathbf{t}} \right)^{\theta} = \frac{Q}{\kappa}.$$

The total production of eligible firms can be written as follows:

$$q_j = \left(z_j^{\theta} k_j^{\alpha\theta} \right)^{\frac{1}{(1-\alpha)+\alpha\theta}} \left\{ \left[\frac{\theta-1}{\theta} (1-\alpha) \mathbb{W}^{-1} [A]^{\frac{1}{1-\alpha}} \right]^{\theta} \xi \right\}^{\frac{(1-\alpha)}{(1-\alpha)+\alpha\theta}},$$

[12] Also, in this part of the paper, we derive expressions only for domestic firms. The expressions for foreign firms are easily derived then.

and real turnovers on the domestic and the foreign markets, respectively is given by:

$$\frac{p_j^d}{P} q_j^d = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \kappa^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \left[\frac{\theta-1}{\theta} (1-\alpha) \mathbb{W}^{-1} A^{\frac{1}{1-\alpha}} \right]^{\frac{(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} Q^{\frac{1}{(1-\alpha)+\alpha\theta}},$$

$$\left(\frac{\eta}{1+\mathbf{t}}\right) \frac{p_j^{m*}}{P^*} q_j^{m*} = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} (1-\kappa)^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \left(\frac{\eta}{1+\mathbf{t}}\right)^{\frac{\theta}{(1-\alpha)+\alpha\theta}} \times$$
$$\times \left[\frac{\theta-1}{\theta} (1-\alpha) \mathbb{W}^{-1} A^{\frac{1}{1-\alpha}} \right]^{\frac{(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} Q^{*\frac{1}{(1-\alpha)+\alpha\theta}}.$$

Real production costs of eligible firms read as follows:

$$\mathbb{C}_j = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} A^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathbb{W}^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} \left\{ \left[\frac{\theta-1}{\theta} (1-\alpha) \right]^{\theta} \xi \right\}^{\frac{1}{(1-\alpha)+\alpha\theta}},$$

thus, the real period operating profit in a given period – say t – is given as:

$$\mathbb{P}_{j\tau t}^d = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} A^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathbb{W}^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} \mathcal{W}_1 \xi^{\frac{1}{(1-\alpha)+\alpha\theta}}.$$

Now, we are able to derive the expected present value of profit stream. We start with an eligible firm $\mathbb{P}_{j\tau}^{de}$, the expected present value satisfies:

$$\mathbb{P}_{j\tau}^{de} = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \underbrace{\mathcal{W}_1 \mathcal{R} A^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathbb{W}^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} \xi^{\frac{1}{(1-\alpha)+\alpha\theta}}}_{\varpi_\tau^e},$$

while the expected present value $\mathbb{P}_{j\tau}^{dn}$ of a non-eligible firm satisfies:

$$\mathbb{P}_{j\tau}^{dn} = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \underbrace{\mathcal{W}_1 \mathcal{R} A^{\frac{(\theta-1)}{(1-\alpha)+\alpha\theta}} \mathbb{W}^{\frac{-(\theta-1)(1-\alpha)}{(1-\alpha)+\alpha\theta}} Q^{\frac{1}{(1-\alpha)+\alpha\theta}}}_{\varpi_\tau^n}.$$

The value of an eligible firm located in the domestic country and owned by the domestic household – which enjoys a productivity level z_j – is determined by capital investment:

$$\mathbf{V}_\tau^{de}(k_j|z_j) = \mathbb{P}_{j\tau}^{de} - (c^e + k_j) \equiv z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \varpi_\tau^e - (c^e + k_j);$$

and similarly for a non-eligible firm

$$\mathbf{V}_\tau^{dn}(k_j|z_j) = \mathbb{P}_{j\tau}^{dn} - (c^n + c^n k_j) = z_j^{\frac{\theta-1}{(1-\alpha)+\alpha\theta}} k_j^{\frac{\alpha(\theta-1)}{(1-\alpha)+\alpha\theta}} \varpi_\tau^n - (c^n + k_j).$$

If firms' managers maximize the value of firms, they chose the following capital level:$k_j^{opt,e} = z_j^{\theta-1}\left[\frac{\alpha(\theta-1)\varpi_\tau^e}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)+1}$, and the value of an eligible firm is:

$$V_\tau^{de}(z_j) = \max_{k_j\geq 0}\mathbf{V}_\tau^{de}(k_j|z_j) = z_j^{(\theta-1)}\left[\varpi_\tau^e\right]^{\alpha(\theta-1)+1}\mathcal{G} - c^e,$$

where $\mathcal{G} = \frac{1}{\alpha(\theta-1)+1}\left(\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}\right)^{\alpha(\theta-1)}$.

Similarly, the value of a non-eligible firm is

$$V_\tau^{dn}(z_j) = \max_{k_j\geq 0}\mathbf{V}_\tau^{dn}(k_j|z_j) = z_j^{(\theta-1)}\left[\varpi_\tau^n\right]^{\alpha(\theta-1)+1}\mathcal{G} - c^n,$$

and the optimal capital investment to quality is $k_j^{opt,n} = z_j^{\theta-1}\left[\frac{\alpha(\theta-1)\varpi_\tau^n}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)+1}$.

Value functions $V_\tau^{dn}(z_j)$, $V_\tau^{de}(z_j)$ implicitly define the cut-off value $\overline{z}$, which is the least idiosyncratic shock, which makes the the export-eligibility investment profitable. Thus it is defined as

$$\overline{z}_\tau^d = \min_{z_j}(V_\tau^{de}(z_j) \geq V_\tau^{dn}(z_j)).$$

A.3 Market-clearing conditions

The steady-state market clearing conditions are as follows:

(1) Zero-profit condition in equilibrium;
(2) Goods-market clearing in both countries;
(3) Labor-market clearing in both countries;
(4) Balance of Payment

Zero expected profits in equilibrium The condition of zero expected profits in equilibrium implies that the expected value of a new domestic entrant be zero. $\int_{z_L}^{\overline{z}} V_\tau^{dn}(z_j)\,\mathrm{d}G(z_j) + \int_{\overline{z}}^{z_U} V_\tau^{de}(z_j)\,\mathrm{d}G(z_j) = 0$. This can be restated as:

$$\mathcal{G}\left\{\left[\varpi_\tau^n\right]^{\alpha(\theta-1)+1}\int_{z_L}^{\overline{z}} z_j^{\theta-1}\,\mathrm{d}G(z_j) + \left[\varpi_\tau^e\right]^{\alpha(\theta-1)+1}\int_{\overline{z}}^{z_U} z_j^{\theta-1}\,\mathrm{d}G(z_j)\right\} = c^n + (c^e - c^n)\left[1 - G(\overline{z})\right].$$

Under the parametrization, this reduces to

$$A^{\theta-1}\mathbb{W}^{-(\theta-1)(1-\alpha)}\left\{Q\int_{z_L}^{\overline{z}} z_j^{\theta-1}\,\mathrm{d}G(z_j) + \xi\int_{\overline{z}}^{z_U} z_j^{\theta-1}\,\mathrm{d}G(z_j)\right\} = \frac{c^n + (c^e - c^n)\left[1 - G(\overline{z})\right]}{\mathcal{G}(\mathcal{W}_1\mathcal{R})^{\alpha(\theta-1)+1}}.$$

The analogous condition holds for foreign entrants.

Goods-market clearing This condition simply says that $Q = \mathcal{L}\mathbb{W}$ and $Q^* = \mathcal{L}^*\mathbb{W}^*$ because of the zero expected profits in equilibrium (and realized and expected profits are equal by the law of large numbers).

Labor-market clearing The individual labor demand is obtained as $l_j = \left[\frac{q_j}{Az_j k_j^\alpha}\right]^{\frac{1}{1-\alpha}}$, and integrating yields the aggregate labor demand as follows:

$$[\mathcal{W}_1\mathcal{R}]^{\alpha(\theta-1)} \left[\frac{\alpha(\theta-1)}{\alpha(\theta-1)+1}\right]^{\alpha(\theta-1)} \frac{A^{(\theta-1)}}{\mathbb{W}^{(\theta-1)(1-\alpha)+1}} \left[Q\int_{z_L}^{\overline{z}} z_j^{\theta-1}\,\mathrm{d}G(z_j) + \xi\int_{\overline{z}}^{z_U} z_j^{\theta-1}\,\mathrm{d}G(z_j)\right],$$

which should be equal to $\mathcal{L}$.

The Balance of Payment The balance of payment equilibrium condition can be rewritten as:

$$\eta^{2\theta-1}\frac{\int_{\overline{z}^*}^{z_U} z^{\theta-1}\,\mathrm{d}G(z)}{\int_{\overline{z}}^{z_U} z^{\theta-1}\,\mathrm{d}G(z)} = \left(\frac{n^*}{n}\right)\left(\frac{Q}{Q^*}\right)\left(\frac{A^*}{A}\right)^{\theta-1}\left(\frac{\mathbb{W}}{\mathbb{W}^*}\right)^{(\theta-1)(1-\alpha)}.$$

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Table 1: Sampling scheme

Parameter	Lower bound	Upper bound
δ	0.050	0.750
θ	3.500	7.500
$\mathbf{t}$	0.025	0.150
c^n	2.000	10.00
c^e/c^n	1.050	5.000

Table 2: The projection of the real exchange rate appreciation (the parsimonious specification)

Ghironi- Melitz model		Extended model	
Parameter	Coefficient	Parameter	Coefficient
H_0	0.0137	H_0	0.1367
θ	-0.0035	θ	-0.2034
$\mathbf{t}$	-0.0105	$\mathbf{t}$	0.1089
c^e/c^n	-0.0036	c^e/c^n	0.5220
		α	-0.5181
θ^2	0.0723	θ^2	0.4478
$\theta\mathbf{t}$	-0.0155		
$\theta c^e/c^n$	-0.0703	$\theta c^e/c^n$	-0.2092
$\mathbf{t}^2$	0.0044	$\theta\alpha$	-0.4223
$\mathbf{t}c^e/c^n$	0.0150	α^2	0.2984
$(c^e/c^n)^2$	0.0359		

Notice that although the expansion is given in logs, we write – for sake of brevity – only symbols of coefficients, thus δ means $\log(\delta/0.05)$, while θ^2 means $\log^2(\theta/3.5)$ and so on.

Table 3: The projection of the real exchange rate appreciation (the full model)

Ghironi- Melitz model		Extended model	
Parameter	Coefficient	Parameter	Coefficient
H_0	0.0134	H_0	-0.0058
δ	0.0002	δ	0.0888
θ	-0.0032	θ	0.0612
$\mathbf{t}$	-0.0099	$\mathbf{t}$	0.0608
c^n	-0.0001	c^n	0.1178
c^e/c^n	-0.0037	c^e/c^n	0.4586
		α	-0.5371
δ^2	-0.0002	δ^2	-0.0026
$\delta\theta$	-0.0003	$\delta\theta$	-0.1134
$\delta\mathbf{t}$	0.0003	$\delta\mathbf{t}$	-0.0413
δc^n	0.0000	δc^n	-0.0147
$\delta c^e/c^n$	0.0003	$\delta c^e/c^n$	0.0233
		$\delta\alpha$	0.0143
θ^2	0.0723	θ^2	0.4471
$\theta\mathbf{t}$	-0.0155	$\theta\mathbf{t}$	0.0019
θc^n	0.0003	θc^n	-0.0364
$\theta c^e/c^n$	-0.0703	$\theta c^e/c^n$	-0.2085
		$\theta\alpha$	-0.4307
$\mathbf{t}^2$	0.0044	$\mathbf{t}^2$	0.0305
$\mathbf{t}c^n$	-0.0011	$\mathbf{t}c^n$	-0.0045
$\mathbf{t}c^e/c^n$	0.0150	$\mathbf{t}c^e/c^n$	0.0525
		$\mathbf{t}\alpha$	0.0109
$c^n c^n$	0.0005	$c^n c^n$	0.0138
$c^n c^e/c^n$	0.0000	$c^n c^e/c^n$	-0.0362
		$c^n\alpha$	-0.0423
$(c^e/c^n)^2$	0.0359	$(c^e/c^n)^2$	-0.0212
		$(c^e/c^n)\alpha$	0.0180
		α^2	0.3007

For notes, see Table 2.

Figure 1: The model without explicit quality investments (Ghironi-Melitz)

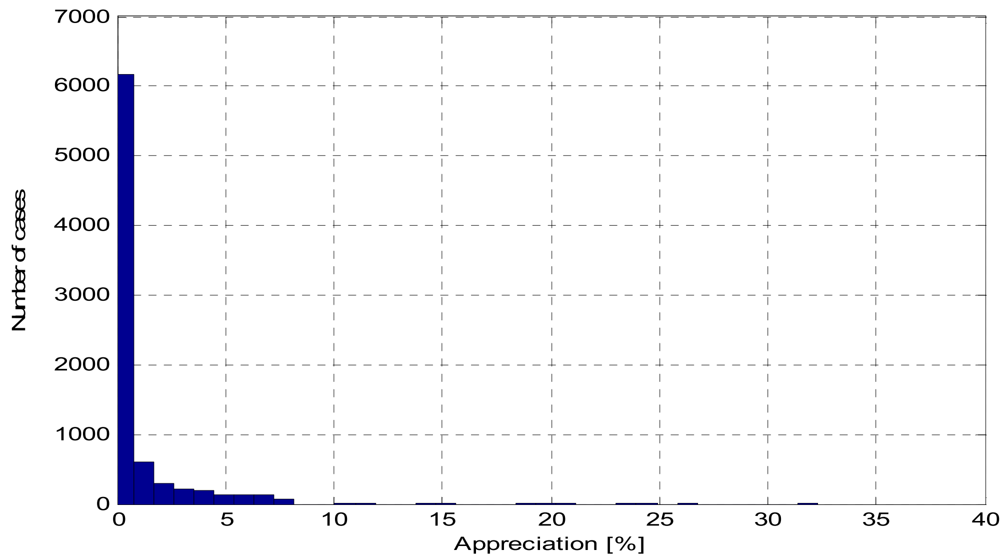

Figure 2: The extended model with explicit quality investments