Liquidity and Economic Fluctuations[*]

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Abstract

This paper studies the relationship between liquidity and economic fluctuations when firms endogenously select their leverage level to finance investment opportunities on which they have private information. The dynamic of leverage decision is the driving force delivering all the main results. First, the analysis provides a theoretically based measure of the liquidity of the economy, i.e. the cross sectional variance in firm debt yields. Second, it shows why liquidity, investment and the dispersion of leverage levels observed in the bond markets are procyclical. Third, it argues that the theoretical implications of the model are consistent with two novel US financial market regularities: (1) the negative correlation between the cross sectional variance in firm debt yields and the cyclical component of GDP and (2) the positive correlation between cross sectional variance in firm debt ratings and the cyclical component of GDP. The approach also relates the illiquidity of the economy to the existence of pooling equilibria in the security space. This analysis shows how these kinds of equilibria become surprisingly robust to off-equilibrium perturbations.

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1 Introduction

This paper studies the relationship between market liquidity and economic fluctuations when bond issuances are endogenously selected by firms. This perspective differs from recent related works[1] by proposing a central role for the dynamic of security design due to the presence of asymmetric information. I provide a theoretical explanation in which liquidity, investment and the variety of financial contracts observed in financial markets are all positively related to economic growth. I also compare the predictions of the model with empirical evidence from the US. In particular I present two significant stylized facts: (1) a negative relationship between the cyclical component of GDP growth and the dispersion in the "yields at maturity"[2] on publicly traded debt - the latter of which I interpret as a measure of liquidity in the economy; (2) a positive relationship between the cyclical component of GDP growth and the dispersion in *ratings* on publicly traded bonds - a measure of the variety of securities present in financial markets. Both may be interpreted, in light of the proposed theory, as illustration of the fact that liquidity and the variety of financial contracts are both procyclical.

As in Eisfeldt (2004) I define the liquidity of the economy by the ease of translating future values of assets into present market prices[3]. Thus an illiquid economy is characterized by the fact that all assets are mispriced so that good (poor) quality securities face a market price below (above) their actual discounted payments[4]. Though there may be several reasons for the existence of a wedge between the actual discounted value and the market price of a security, here I focus *exclusively* on informational asymmetries[5].

I consider an economy with adverse selection where privately informed entrepreneurs finance their investment through the issuance of bonds with the possibility of default. There are two

[1] Eisfeldt (2004), Eisfeldt and Rampini (2006).

[2] The "yield at maturity" is the interest paid on issued debt at maturity and includes the interest rate, whether the bond was at par or below par and the market price at the time of issuance.

[3] Quoting Holmstrom and Tirole (2001), footnote 1: "Liquidity [...] does not [only] refer to the ease with which assets can be resold, but rather to [...] the value of financial instruments used to transport wealth across time [...]."

[4] The reader may recognize some similarity with Diamond and Dybvig (1983), Caballero and Krishnamurthy (2001) (2002), Krishnamurthy (2003).

[5] Johnson (2004) stresses that the finance literature highlights at least three distinct sources for (il)liquidity: search costs, inventory risks and asymmetric information. Here I focus on a reinterpretation of the last. Morris and Shin (2004) base their notion of liquidity on private information, though not about the value of the traded object as I do here.

distinctive features of this economy. The first is that entrepreneurs have private information about the quality of the investment opportunity they own. There are two types of technologies that can be ranked on the basis of their expected production and variance. All agents are identical except for the investment opportunity some receive. I label the agents endowed with investment opportunities as "entrepreneurs". Within the group of entrepreneurs, some receive the high productivity technology while the others receive the low productivity one. The agents without an investment opportunity are labelled "consumers".

The second distinctive feature of this economy is that entrepreneurs can choose what kind of security they issue[6]. In reality firms issuing securities can offer a wide array of contingent payments. Here firms design securities differing in the likelihood and extent of default. Since consumers can observe how many bonds a firm issues - but not *their* probability of default -, I model the entrepreneur's selection of a specific security type by the decision of how many bonds he issues[7]. Increasing the number of issued bonds raises the total future payments due to bondholders. This increase translates in turn into higher probability of default because it increases the probability that future production will not be enough to pay for all outstanding bonds.

Consumers lend funds to entrepreneurs by purchasing bonds in competive markets. They can only distinguish one firm from the other on the basis of the number of bonds a firm issues, i.e. the security type it chooses. Capital markets are consistently organized since all firms issuing the same number of bonds - and thus selecting the same security type - trade their assets in a single and separate market. The market price of each security reflects the uncertainty associated with the mix of entrepreneurs issuing a specific security. Entrepreneurs decide what kind of security to issue comparing the market price with their expected payments across different security types. High productivity entrepreneurs try to signal their quality by adjusting the number of bonds they issue, while low productivity entrepreneurs wish to mimick them. This generates a trade-off between the security choice necessary to accomodate the informational asymmetry (the leverage effect) and the degree of subsidization on bond prices from good to bad entrepreneurs (the price effect). This trade-off is ultimately responsible for the emergence of a particular type of equilibrium in the security

[6] This study does not model financial intermediation or banks. The purpose here is to study the liquidity of traded financial instruments.

[7] This is in line with Leland and Pyle (1977) and Myers and Majluf (1983) where the decision to issue securities signals information to the market.

space.

I interpret the liquidity of the economy by the type of equilibrium - pooling or separating - that appears in the security space. When entrepreneurs of different productivity issue the same security we are in a pooling equilibrium. The pooling equilibrium is illiquid because better entrepreneurs find it relatively more difficult to borrow funds: because of the informational asymmetry between entrepreneurs and consumers, better technology owners bear a relatively higher cost in financing their investment and so are adversely selected. Bonds issued by high (low) quality entrepreneurs are underpriced (overpriced) since they are traded at a price strictly below (above) their expected payment. The economy displays a "liquidity premium" since the expected rate of return paid by better bonds is larger than the one paid by worse ones. When, instead, entrepreneurs with different technologies issue different securities, then a separating equilibrium appears in the security space. The separating equilibrium is liquid because pricing is accurate since private information is fully revealed: consumers identify the quality of each security - i.e. its expected delivery, as a result, security pricing reflects the actual vale of future payments for each bond. The rates of retun on different securities converge and the better technologies are adversely selected to a lesser extent.

There are two interesting implications of the proposed perspective. The first result, which is the main contribution here, is that the equilibrium in the security space, and therefore economy-wide liquidity, depends on the state of the economy. If the state of the economy is sufficiently bad, then the probability that investment opportunities succeed becomes sufficiently small and illiquid pooling equilibria emerge. In contrast, when the economy picks up, liquid separating equilibria arise. The intuition is simple. On the one hand, by attempting to differentiate themselves, high productivity entrepreneurs distort their issuing decision and increase the number of bonds they issue. On the other hand, low quality entrepreneurs mimick the security decision of high quality entrepreneurs to have their bonds overpriced (positive *price effect*). At the same time, they pay the cost of distorting their leverage decision by issuing a suboptimally large number of bonds (negative *leverage effect*). When the state of the economy is bad, this is convenient because good and bad technologies are very different and the "price effect" is stronger than the "leverage effect". But as the economy improves, the probability that investment opportunities succeed increases and so bonds' expected payments become closer. Therefore, the subsidization in bonds' prices - the price effect - becomes less attactive to low quality entrepreneurs and they prefer to abandon the pooling equilibrium without distorting their issuing decision. I show that this is sufficient to guarantee the emergence of a liquid separating equilibrium.

Notice that this result is far from obvious: in the presence of security endogeneity, procyclical liquidity may seem counterintuitive. One is tempted to argue that, when the economic outlook looks grim, high quality firms will want to avoid the additional liquidity cost connected with subsidizing low quality bonds in the pooling equilibrium. This intuition is not correct if one accepts the basic assumption that the payment of a bond is directly related to the quality of the technology. When productivity slows down (picks up), bonds's payments become more (less) different and it becomes relatively less (more) costly for low quality entrepreneurs to mimick good entrepreneurs.

The second result - which is important from a modelling point of view - is that when the economy displays pooling equilibria they are *robust*. There is an informal consensus that, for economies with asymmetric information, pooling equilibria are difficult to sustain because they are sensitive to how optimistic agents' beliefs about off-equilibrium actually are[8]. This could be a problem in the present context because pooling equilibria are central to the analysis of liquidity. I show that pooling equilibria are robust to optimistic off-equilibrium expectations, *provided* that securities' payments are positively related to the performance of the issuing firm, so that the average payment of a bond, for any given security type, increases with the quality of the technology of the issuing firm. This assumption simply means that better firms issue more valuable bonds, given the security type they issue. The intuition behind the *robustness* of pooling equilibria is simple: the positive correlation between bonds' value and technology's quality makes the dimension of separation, the number of bonds, relatively more costly for good quality entrepreneurs. Good quality entrepreneurs may attempt to distinguish themselves by issuing more bonds thereby distorting their leverage decision. But since bonds issued by good technology entrepreneurs are more valuable, the total payments of good quality entrepreneurs increase relatively faster than that of bad quality securities as *more bonds are issued*.

The theory discussed has two direct empirical predictions that find support in the US data.

[8]See Dubey and Geanakoplos (2003) and Martin (2004) for a discussion about the "fragility" of pooling equilibria in the context of insurance models.

The first prediction is that we should observe a negative correlation between the growth rate of GDP and the cross sectional variance in bond yields issued by different firms in a given quarter - which will be the measure of economy-wide liquidity. The second is that we should also observe a positive correlation between GDP growth and the variety of financial contracts across firms. Using the variance of bond *ratings* as a proxy for the variety of financial contracts we find that the data support this prediction of the model. When the economy grows less, an illiquid pooling equilibrium emerges. In this equilibrium, firms issue the same security - i.e. security variety is small - and the rates of return for bonds issued by entrepreneurs of different qualities diverge. I identify this dispersion as the measure of the economy-wide "liquidity premium". When the economy accelerates and a liquid separating equilibrium arises, firms instead differentiate themselves by issuing different securities, and the rates of return converge reducing the economy-wide liquidity premium thus enhancing liquidity.

The focus of this paper is macroeconomic though this line of enquiry has the benefit of bringing together different strands of the economic literature. Even though the study proposed here relates most closely to Eisfeldt (2004), the employed setup owns much to Geanakoplos and Zame (2002) and Geanakoplos (2003). This paper is in line with Eisfeldt (2004) and Eisfeldt and Rampini (2006) in so far it addresses the relationship between liquidity and economic fluctuations. It differs from the abovementioned works because it considers the issue of security design and the consequential endogeneity of the asset structure as a crucial mechanism.

The problem of how to embed an endogenous asset structure in general equilibrium has recently been addressed by Geanakoplos and Zame (2002), Geanakoplos (2003). They differ from this study in two aspects: (1) they do not model the role of asymmetric information and (2) they do not study the behavior of liquidity over time. Dubey and Geanakoplos (2002), from which I borrow the definition of equilibrium, address the issue of informational asymmetry in a context not related to liquidity and show that only separating equilibria are robust in competitive markets. Here I show that pooling equilibria may also be robust to optimistic off-equilibrium beliefs.

Since Leland and Pyle (1977) and Myers and Mayluf (1984) economists are aware of how firms may strategically use their decision to issue securities to disclose private information. Demarzo and Duffie (1999) and Demarzo (2003) have recently revived this perspective in the context of security design targeted at providing liquid securities. They both differ from the present study since they do not study the relationship between liquidity and economic fluctuations.

Kiyotaki and Moore (2001a) and (2001b) and Caballero and Krishnamurthy (2001) and (2002) study the relationships between liquidity and asset pricing in an economy with exogenously given illiquid assets. My work stresses instead the fact that the liquidity of an asset should be considered the result of an optimizing behavior and not assumed from the outset.

An outline of the paper is as follows. Section 2 lays out the setup and equilibrium definition. Sections 3 discusses the equilibrium in economies with symmetric information. Section 4 studies the economies with informational asymmetry while section 5 relates the theoretical predictions to two empirical regularities in the US financial market. Section 6 concludes.

2 The Economy: Setup

In the discussion that follows the reader may refer to the following figure:

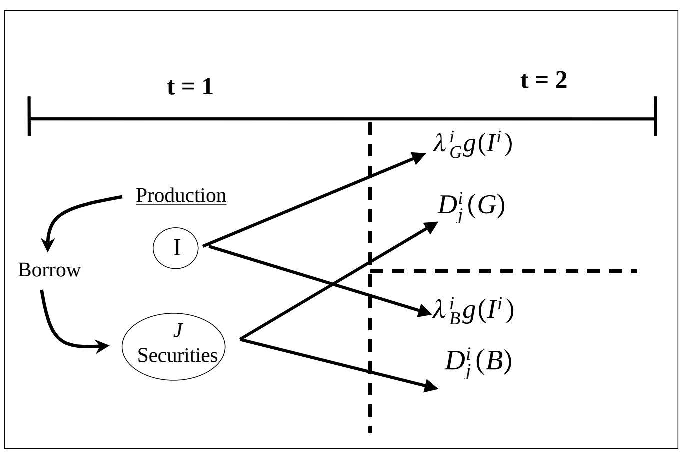

Figure 1

Time: the economy lasts two periods: $t = 1, 2$. Two contingencies, $s \in S = \{G, B\}$, can be realized at $t = 2$ with probabilities $\alpha_s = \Pr(s)$;

Commodity Space: there is a single perishable good in each period. Let $c_t^h(s)$ denote the amount consumed by agent h at time t in state s;

Agents and Endowment: there is a continuum of consumers h uniformly distributed on the unit interval. There is also measure η of entrepreneurs i endowed with an investment technology. Consumers have individual endowment $(w, 0) \in \Re^2_+$ where $w > 0$ while entrepreneurs have $\left(w^i, 0\right) \in \Re^2_+$, $i \in \{L, H\}$, $w^H > w^L$.

Preferences: in order to abstract from risk, all agents in the economy have linear preferences:

$$\begin{gathered} V[c] = c_1 + E_s[c_2(s)] \\ s \in \{G, B\} \text{ at } t = 2 \end{gathered} \tag{1}$$

Technology: there are 2 types of technologies, labeled by $i = \{H, L\}$[9], available at $t = 1$; $t = 2$ production, $g_i(I; s)$, is stochastic:

$$\begin{gathered} g_i\left(I^i; s\right) = \lambda^i_s g\left(I^i\right), \ s \in \{G, B\} \\ \lambda^i \in \left\{\lambda^H, \lambda^L\right\} \subset R^2 \\ g(0) = 0, \ g'(.) > 0, \ g''(.) < 0 \\ \lim_{x \longrightarrow 0} g'(x) = +\infty \end{gathered} \tag{2}$$

The production function is standard neoclassical, while I is the units of capital invested. Moreover I assume:

$$\lambda^H = \left[\lambda^H_s = \lambda^H, \ \forall s\right]$$

$$\lambda^L = \begin{bmatrix} \lambda^L_G = \lambda^L \\ \lambda^L_B = 0 \end{bmatrix}$$

$$\begin{gathered} \lambda^H > \lambda^L >> 1 \\ E_s\left(\lambda^L\right) > E_s\left(\lambda^H\right) \end{gathered} \tag{3}$$

"H" is the most productive technology since it gives higher expected production at lower variance[10]:

$$E_s[g_H(I; s)] > E_s[g_L(I; s)], \ \forall I$$

Moreover I will write:

$$g'_i(I; s) = \frac{dg_i(I; s)}{dI}$$

[9] H (L) stands for high (low) variance.

[10] In order for the arguments in the paper to go through, I only require that the best technology has higher mean and smaller variance. The structure in the paper is assumed for simplicity only.

I assume **perfect positive correlation** (i.e. aggregate uncertainty) in technologies' payoffs[11]. This is equivalent to assuming that there are two aggregate states but that each production process is subject to a technology-specific state contingent shock. The two technologies are distributed in the aggregate according to measure $\eta(.)$:

$$\lambda^i \backsim (\eta(H), \eta(L) = 1 - \eta(H))$$
$$1 > \eta(H) > 0$$

Security Structure: there are (finitely) many financial contracts, $j \in \Re^{N \cdot 10^{100}}$, generated by the entrepreneur's choice of how many securities she issues. Each security is therefore characterized by a vector D_j^i of state contingent deliveries, $D_j^i(s)$, which depends on the number, j, of bonds issued and the quality, i, of the technology in the issuing firm. With a little abuse in the definition, I will often refer to the financial contract corresponding to the issuance of j securities as "secutity j". Securities translate into deliveries at $t = 2$ in the following way:

$$D_j^i(s) = \min\left\{1, \min\left(\frac{g(I^i)}{j}, \frac{\lambda^i g(I^i)}{j}\right)\right\} \tag{4}$$

The delivery vector of a security depends on the default decision of the issuing entrepreneur. Given the chosen financial contract j, the entrepreneur decides whether to default and surrender $g(I^i)$ of his production - whenever $\lambda^i g(I^i) > 0$ - or to honor his initial promise and pay 1 unit of the numeraire per issued bond. Default is thus strategic in this setup. Moreover, in (4) I also assume that, in the event of default, creditors, i.e. security holders, can only seize up to $g(I^i)$ even if actual production is $\lambda_s^i g(I^i)$. The usual explanations for this kind of assumption - e.g. inappropriability of entrepreneurs' human capital, costly post default screening - apply here.

More interestingly, since better technologies default less often - given the chosen security-, (4) assumes that the security's expected payment is an increasing function of the technology's productivity/quality. This is equivalent to assuming that more productive firms issue on average more valuable securities and, at the same time, that the payment linked to a security depends on the economic performance of a firm - its production here. These assumptions seem realistic and will play a central role in establishing a relation between economic fluctuations and liquidity.

[11] This assumption simplifies notation and is made without loss of generality, differently from Holmstrom and Tirole (1998).

Information Structure: At $t = 1$ every entrepreneur is privately informed about the productivity of her investment project. Since the issuer-entrepreneur knows the quality of the technology, she knows the actual payoff/value of the bonds she issued. Thus entrepreneurs are privately informed about the actual quality of the security they are selling.

Security Holdings and Prices: the specific type of financial contract is identified observing the number of securities, (j), issued by the firm. Consumers observe the number of securities a firm issues - which is public information - but not their qualities, which depends on the firm's technological endowment, the entrepreneur's private information. Consumer h purchases $a^h(j)$ units of financial contract j while entrepreneur i issues security j (or j^i when notation requires it) defined by the issuance of number j of bonds. $q(j)$ denotes the price of security of type "j" while $q \in \Re^{N \cdot 10^{100}}$ labels the vector pricing all admissible financial contracts in the economy.

Individual Budget Constraint $B^i(q)$, $B^h(q)$: since the issue of securities requires an investment technology, only entrepreneurs can issue securities. Entrepreneur i budget constraint, $B^i(q)$, is:

$$\begin{array}{ll} c_1^i + I^i \leq w + q(j)j & t = 1 \\ c_2^i(s) \leq \lambda_s^i g(I) - D_j^i(s)j & t = 2 \end{array}$$

Consumer h budget constraint, $B^h(q)$, is:

$$\begin{array}{ll} c_1^h \leq w - \sum_j q(j)a^h(j) & t = 1 \\ c_2^h(s) \leq \sum_j \sum_i D_j^i(s)a^h(j)^h & t = 2 \end{array} \tag{5}$$

Securities' payoff depends on their *actual* value. Since consumers are all identical I assume, without loss of generality, they hold the same portfolio, i.e. the average one. The budget constraint is thus standard in the interpretation.

2.1 Definition of the Equilibrium

An equilibrium in this economy is defined by consumption allocations $c = [c_1^h, c_1^i, c_2^h = \left\{c_2^h(G), c_2^h(B)\right\},$ $c_2^i = \left\{c_2^i(G), c_2^i(B)\right\}]$, asset holdings $a = [j^i, a^h(j)]$, $\forall h, i$, and asset prices q satisfying the followings:

1. **Individual Optimum**

$$\left(a^{h,i}, c^{h,i}\right) \in \arg\max \left\{ \begin{array}{c} V\left[c^{h,i}\right] \\ \text{s.t. } \left[B^{h,i}(q)\right] \end{array} \right\} \text{ at given } q, \forall h, i$$

2. **Market Clearing Conditions**

$$\int_0^1 a^h(j)dh - \int_0^\eta j^i \eta(i)di = 0, \ \forall j \text{ at } t = 1 \qquad \text{(Security Markets)}$$

$$\begin{array}{c} \int_0^1 c_1^h dh + \int_0^\eta c_1^i di + \int_0^\eta I^i di = w \int_0^{1+\eta} dh, \ t = 1 \\ \int_0^1 c_2^h(s)dh + \int_0^\eta c_2^h(s)di = \int_0^\eta g_i\left(I^i; s\right) di; \ t = 2, \ s \in \{G, B\} \end{array} \qquad \text{(Goods Market)}$$

2.2 Security Space: Delivery Vector and Number of Securities

Each security is priced according to the *expected* consumption it delivers. This expectation may be affected in equilibrium by the presence of asymmetric information. The payment implications of choosing the kind of security, i.e. the number of bonds a firm issues, can be conveyed through figure 2 below. This is necessary to characterize the security space, J, of the economy.

Given our assumptions and the fact that technology H is more productive than technology L, the expected delivery of their securities behave as shown in figure 2, given an arbitrary constant price $q(\bar{j})$, where $\bar{j}$ denotes the number of assets after which entrepreneurs defaults in all contingencies given $q(\bar{j})$[12].

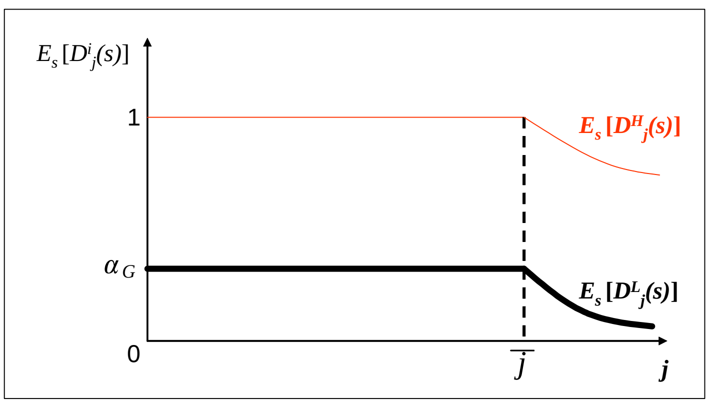

Figure 2

[12] $\bar{j}$ solves $\frac{g\left(I^i = w + q(\bar{j})\bar{j}\right)}{\bar{j}} = 1$.

3 The Economy with Symmetric Information

For the sake of clarity I will briefly discuss the symmetric information benchmark. Then I will turn to the properties of the economy with private information. In order to do so - as a first step -, it is convenient to analyze the relationship between the payoffs to securities of different qualities to characterize the relevant security space.

3.1 The Equilibrium

In the *symmetric* information benchmark, when firms borrow, securities' quality is "transparent" (i.e. anyone knows the actual worth of any bond traded in the market). Consumers/savers always know the actual expected payment of the bond they are purchasing. Since preferences are risk neutral, a security's price is a linear function of its payments[13]:

$$q(j,i) = E_s\left[D_j^i(s)\right];\ \forall i \tag{6}$$

where $q(.)$ is indexed by both i and j since consumers can observe the quality of the issuing firm. In order for the problem to be interesting I assume that access to the credit market is beneficial to entrepreneurs:

$$E_s\left[g_i'(w;s)\right] >> 1;\ \forall i \tag{7}$$

Let

$$R^i(j) = \frac{E_s\left[D_j^i(s)\right]}{q(j,i)}$$

be the bond expected rate of return for security j issued by firm i, i.e. the ratio between expected delivery, $E_s\left[D_j^i(s)\right]$, and market price, $q(j,i)$. Since the focus here is to measure economy-wide liquidity, I define the following[14]:

Definition 1 *The "economy liquidity premium" is defined as:*

$$LP = R^H(j^H) - R^L(j^L)$$

i.e. as the difference between the rates of return of the security issued by entrepreneurs H endowed with better technology, j^H and of the security issued by entrepreneurs L with the worse technology, j^L.

[13] Consumer's utility maximization and standard no arbitrage reasoning delivers so.

[14] This definition is equivalent to Eisfeldt (2004).

Given (2) and (4), we have the following profit function $\pi^i(j)$:

$$\pi^i(j) = \sum_{s\in S} \alpha_s \cdot \left\{ \left[\lambda_s^i g\left(I^i\right) \right] - jD_j^i(s) \right\} \tag{8}$$

where $I^i = w + q(j)j$. Notice that (8) depends asymmetrically on the states in which entrepreneur i defaults and on the states in which he does not. When he defaults, $g\left(I^i\right)$ production is seized by creditors and the entrepreneur's profits become only $\left(\lambda_s^i - 1\right) g\left(I^i\right)$[15]. The isoprofit curves corresponding to (8) can be drawn as in the following figure:

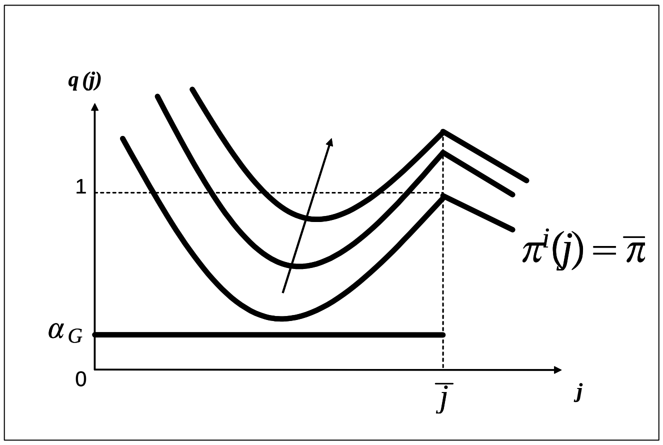

Figure 3

where the reader may observe, perhaps not surprisingly, that given security j profits increase in $q(j)$, the price that bondholders pay to entrepreneurs when purchasing a bond. Given the assumption of risk neutrality, entrepreneurs maximize expected profits $\pi^i(j)$. Exploiting (8) and derivating with respect to the number of bonds being issued[16], j:

$$\left\{ \sum_{s\in S^i} \left[\alpha_s \lambda_s^i\right] + \sum_{s\in S\setminus S^i} \alpha_s \left(\lambda_s^i - 1\right) \right\} \cdot g'\left(I^i\right) = \frac{\sum_{s\in S^i} \alpha_s}{q(j,i) + j\frac{\partial q(j,i)}{\partial j}} \tag{9}$$

where $S^i = \left\{ s \in \{G, B\} \,\middle|\, D_j^i(s) = 1 \right\}$ denotes the contingency/ies where firm i does not default. The interpretation is standard: securities are issued until the level of investment, I^i, equalizes the marginal benefit of investing - marginal productivity - to its marginal cost. The marginal cost is the ratio between the payment to lenders in no default states, $\sum_{s\in S^i} \alpha_s$, and the benefit of

[15] To be precise L entrepreneurs earn *nothing* when they default in the bad contingency because we assumed $\lambda_B^L = 0$.

[16] Appendix.

issuing an additional security, $q(j,i)$, *plus* the effect that the marginal security has on the price of all infra-marginal securities, $j\overset{(-)}{\frac{\partial q(j,i)}{\partial j}}$.

Given my technological assumptions, when security $j \leq \overline{j}$ is chosen, entrepreneurs endowed with better technology H never default while the ones with worse technology L default with positive probability, i.e. $\{s = G\} = S^L \subset S^H$. A direct implication of our assumptions and (9) are:

Proposition 2 *In the equilibrium of the economy with symmetric information the level of investment undertaken by an entrepreneur increases with technological quality, i.e.* $\left(I^H\right)^* > \left(I^L\right)^*$.

Proof. Appendix. ■

Again, I can provide a simple graphical illustration of (9):

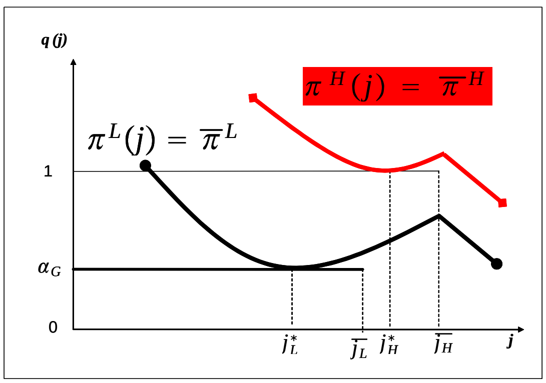

Figure 4

where $\overline{j_i}$ is the number of bonds after which entrepreneur i starts defaulting [17]. The reader may observe that the profit maximizing security is chosen so that isoprofit and expected payments are tangent. By no arbitrage condition, different technologies must guarantee the same rate of return on the bonds they issue:

$$R^L(j_L^*) = \frac{E_s\left[D^L_{j_L^*}(s)\right]}{q(j_L^*,L)} = \frac{E_s\left[D^H_{j_H^*}(s)\right]}{q(j_H^*,H)} = R^H(j_H^*)$$

j_i^* is the security issued by entrepreneur i

and so no adverse selection takes place.

I summarize the main results by the following proposition given without proof:

[17] $\overline{j_i}$ solves $\frac{g\left(I^i = w + q(\overline{j_i})\overline{j_i}\right)}{\overline{j_i}} = 1$

Proposition 3 *In the economy with no private information, the following hold:*

1. *equilibrium investment* $\left(I^i\right)^*$ *is increasing in the quality of technology:* $\left(I^H\right)^* > \left(I^L\right)^*$*;*
2. $LP = 0$*, i.e. the rate of return is equalized across technologies;*
3. *the equilibrium is pareto optimal.*

4 The Economy with Asymmetric Information

I construct the equilibrium of the economy by backward induction. At $t = 2$, production realizes, financial contracts are settled and security payments made. At settlement date, entrepreneurs either pay their contractual obligations, one unit of consumption per bond, or they surrender a share of production $\left(\frac{g(I^i)}{\lambda_s^i g(I^i)} = \frac{1}{\lambda_s^i}\right)$. Consumers holding bonds receive payments corresponding to the quality and kind of securities they own. Everyone consumes his net wealth.

At $t = 1$ entrepreneurs raise funds through capital markets by issuing securities. Private information plays a crucial role:

Axiom 4 ***(Private Information)*** *At* $t = 1$ *the entrepreneur issuing a security is privately informed about its actual payment.*

Assumption (4) is a direct consequence of the assumption that entrepreneurs are privately informed about the quality of the investment technology they own. The buyer of a security can *only* observe the number of bonds issued by a firm and so the security type, not their quality. In any single security market, consumers form expectations about the value of the average bond. This expectation is the crucial determinant of asset prices, $q(j)$. Therefore security prices are indexed by j only.

Entrepreneurs take into account the role of private information when they decide which security they issue. The central question becomes how much separation across technologies, if any, will be present in equilibrium, i.e. whether the equilibrium in the security space is going to be pooling or separating. I turn to this issue now.

4.1 Equilibrium Liquidity

4.1.1 Equilibrium Securities: the Basic Trade-Off

In equilibrium, each security is priced according to the expected consumption it delivers. Because of asymmetric information, buyers do not know the actual delivery of the bond they are buying and so they must form expectations about its average quality/payment. Let me start observing that - since consumers maximize (1) under (5) - security prices are:

$$q(j) = E_i \left[E_s \left[D_j^i(s) \right] \middle| \, j^H, j^L \right] \tag{10}$$

Since buyers can *only* observe j, the number of bonds a firm issues, the price is indexed by j only. The equations above formalize the assumption of rational expectations: the equilibrium price must be equal to the expected delivery of the *average* bond traded in the market for security[18] j. The average security in turn depends on the relative shares of good and bad entrepreneurs issuing it.

The ratio of expected delivery over market price may therefore change across different securities, providing different incentives for the sellers of less valuable securities, entrepreneurs (L), to mimic entrepreneurs (H), sellers of more valuable ones. In general, the easier it is to reproduce the behavior of the good technology entrepreneurs, the more depressed the market price, $q(j)$, is and the higher the illiquidity of good securities issued by entrepreneur H.

Optimally, holders of the good technology H could issue the same number of bonds as the others without experiencing default. But the holders of the bad technology L, may try to mimic them, to sell overpriced bonds. Entrepreneurs H, anticipating this "shading", may decide to distort their security choice j. This distortion implies a trade-off for entrepreneurs with bad technology L: on the one hand, they unambiguously benefit from a subsidized market price (a positive *price effect*); on the other hand, they have to mimic the distorted security decision of entrepreneur H (a negative *leverage effect*).

The buyer of a bond thus faces a potential "Lemons market" problem: he knows that entrepre-

[18] It could not be different since, if $q(j) > E_i \left[E_s \left[D_j^i(s) \right] \middle| \, I_t^i \right]$, no one would buy security j and, if $q(j) < E_i \left[E_s \left[D_j^i(s) \right] \middle| \, I_t^i \right]$, everyone would buy security j and an excess demand would appear.

neurs L issuing bonds whose actual value is below the market price may try to choose that security to realize a capital gain. This tends to drive down the equilibrium market price of traded financial contracts. The lower market price implicitly imposes a premium charged upon entrepreneurs H endowed with good technology. A pooling equilibrium may nonetheless survive, even if the holders of good securities are paying a liquidity premium, because the security choice distortion necessary for entrepreneurs H to differentiate themselves may be too costly. Since all agents are risk neutral, the liquidity premium I derive is due *only* to asymmetric information.

4.1.2 Equilibrium Securities: the Formal Analysis of Equilibrium and Off-Equilibrium Pricing[19]

To characterize the equilibrium of this economy one needs to be careful in defining security prices. Within the proposed competitive analysis in the presence of private information, this raises some important issues in the definition of prices of securities that are not actively issued in equilibrium. In order to choose the optimal security, entrepreneurs must be able to compare prices for all possible securities they may issue. Although establishing prices for equilibrium securities is conceptually simple, rational expectations provide no guidance in the determination of prices for securities that are not traded . This may lead to a paradox: every agent may expect the price of all off-equilibrium securities to be zero, simply because no one is trading them. But these "pessimistic" off-equilibrium expectations would make it possible to support almost any allocation as an equilibrium. In order to get around this problem, I apply the methodology proposed by Dubey and Geanakoplos (2002) in the context of the insurance model *a la* Rothschild and Stiglitz (1976). They address this feature by imposing a tremble “on the market”: introducing an external agent of positive measure forced to issue securities that would otherwise be absent, they are able to precisely define off-equilibrium prices.

I establish an *external agent* of measure $\varepsilon = \{\varepsilon_j\}_{j\epsilon J}$ issuing every off equilibirum security as if he were an entrepreneur of type H endowed with the better technology. The price of off-equilibrium security j' would then be equal to $E_s\left[D_{j'}^H\right]$, if only the external agent were to issue it. Therefore the external agent pins down security prices in all markets. I denote the economy where the external

[19] This subsection adresses few technical issues within the present context in the definition of the equilibrium pricing functional. The uninterested reader may skip to section 4.2 without impairing his understanding of the main results.

agent is introduced as the "ε-economy".

In order to determine the equilibrium of the ε-economy, one has to check whether entrepreneurs find it profitable to issue the same original security or to deviate to another once the external agent is introduced. In practice I am asking each agent whether an entrepreneur would "change his mind", once the external agent enters the economy. Taking into account the optimizing behavior of all agents, one can, by rational expectations, compute security prices in the given ε-economy - i.e. the equilibrium prices of the economy, $q_\varepsilon(j)\ \forall j$, where the external agent is forced to issue all securities and entrepreneurs H and L choose $\left(j^H\right)^{\varepsilon_j}$ and $\left(j^L\right)^{\varepsilon_j}$ optimizing. In the same spirit of (10), I have:

$$q_\varepsilon(j) = \frac{\left[\mu(H,j)^{\varepsilon_j} + \varepsilon_j\right] E_s\left[D_j^H(s)\right] + \mu(L,j)^{\varepsilon_j} E_s\left[D_j^L(s)\right]}{\left[\mu(H,j)^{\varepsilon_j} + \varepsilon_j\right] + \mu(L,j)^{\varepsilon_j}}$$

where $\mu(i,j)^{\varepsilon_j}$ labels the measure of entrepreneur i issuing security j if the external agent has measure ε_j. Thus the equilibrium of the ε-economy is defined:

Definition 5 *An equilibrium of the ε-economy is defined by consumption allocations $c_\varepsilon^h = [c_{1\varepsilon}^h,$ $c_{2\varepsilon}^h = \left\{c_{2\varepsilon}^h(G), c_{2\varepsilon}^h(B)\right\}]$, asset holdings $a_\varepsilon^h(j)$ and issuances $j_\varepsilon^i, \forall h$, asset prices $q_\varepsilon \in \Re^{N\cdot 10^{100}}$ satisfying:*

1. ***Individual Optimum***

$$\left(a_\varepsilon^h, c_\varepsilon^h\right) \in \arg\max \left\{ \begin{array}{cc} & V^h\left[c_\varepsilon^h\right] \\ s.t. & \left[B^h\left(q_\varepsilon\right)\right] \end{array} \right\} \text{ at given } q_\varepsilon, \forall h$$

2. ***Market Clearing Conditions***

$$\int_0^1 a_\varepsilon^h(j)dh - \int_0^\eta j_\varepsilon^i \mu(i,j)^{\varepsilon_j} di = 0,\ \forall j \text{ at } t = 1 \qquad \text{(Security Markets)}$$

$$\begin{array}{c} \int_0^1 c_{1\varepsilon}^h dh + \int_0^\eta c_{1\varepsilon}^i di + \int_0^\eta I_\varepsilon^i di = w \int_0^{1+\eta} dh,\ t = 1 \\ \int_0^1 c_{2\varepsilon}^h(s)dh + \int_0^\eta c_{2\varepsilon}^h(s)di = \int_0^\eta g_i\left(I^i; s\right) di;\ t = 2,\ s \in \{G, B\} \end{array} \qquad \text{(Goods Market)}$$

To check whether an equilibrium is sustained, one has to control that it is the limit a sequence of ε-economies as the measure of the external agent goes to zero, i.e. $\varepsilon_j(n) \to 0, \forall j$ for $n \longrightarrow +\infty$. If, as the external agent gets smaller and smaller, more and more entrepreneurs leave off-equilibrium securities and return back to issue the security/ies of the equilibrium one wants to support, I say that the original security/ies is/are an equilibrium surviving the “tremble”. In principle, the larger is the set of expected payments of external agent for which an equilibrium survives, the more robust

I will say it is. For the sake of simplicity, we have explicitly considered only the case most likely to break any equilibrium, where the external agent behaves as a good quality entrepreneur. If an equilibrium survives this "optimistic" tremble, it survives any other tremble where the external agent partly behaves as an entrepreneur of lower quality. I will formally state this defining an equilibrium to be robust if it survives the tremble in which the external agent behaves as a good quality entrepreneur:

Definition 6 *An equilibrium is robust if it is the limit, for $\varepsilon_j(n) \to 0$, $\forall j$ when $n \longrightarrow +\infty$, of a sequence of ε-economies in which the external agent issues shares of measure $\varepsilon_j(n)$ paying $E_s\left[D_j^L(s)\right]$ in each off-equilibrium security.*

Reinterpreting the notion of *pool* in Dubey and Geanakoplos (2002) as my notion of *security*, I prove that a pooling equilibrium are surprisingly robust. In the following section I use a constructive approach to identify the conditions under which *pooling* equilibria arise and are robust.

4.2 Liquidity Premium and the State of the Economy

4.2.1 Illiquid Pooling Equilibria

In an equilibrium where entrepreneurs endowed with different technologies issue different securities j^i, i.e. in a separating equilibrium, the rate of return on all securities is $R^i(j^i) = 1$, $\forall i, j$. Instead, if, in equilibrium, all entrepreneurs issue the same security j^*, i.e. if the equilibrium in the security space is pooling, it must be that:

$$R^L(j^*) > 1 > R^H(j^*)$$

Therefore, by the definition of economy liquidity premium (1), a pooling equilibrium is characterized by $LP > 0$ and the economy is said to be *imperfectly liquid* (or *illiquid*). In a separating equilibrium $LP = 0$ and the economy is defined *liquid*.

Because of rational expectations, (10) ensures that security prices reflect the average quality of abond traded in a single security market. The better a technology is, the higher is the interest rate it pays in a pooling equilibrium. Therefore, in equilibrium, good technology entrepreneurs borrow at unfavorable terms and so are adversely selected . Equation (9) states the reference criterion by which the entrepreneur chooses the optimal security to issue. The reader is referred to (9) for the relevant interpretation. Here it suffices to remind that (9) defines equality between marginal benefit of investing and the marginal cost of financing. Since (9) is also relevant in the case of asymmetric information, it is convenient to rearrange it into the following:

$$\left.\frac{\partial q^i(j)}{\partial j}\right|_{\pi^i(j)=\overline{\pi}} = \frac{\sum_{s\in S^i}\alpha_s}{jg'\left(I^i\right)\left[\sum_{s\in S^i}\alpha_s\lambda_s^i + \sum_{s\in S\setminus S^i}\alpha_s\cdot\left(\lambda_s^i - 1\right)\right]} - \frac{q(j)}{j} \tag{11}$$

which gives the slope of the isoprofit curve for entrepreneur i. Given our technological assumption (3) and profit function (8), it is immediate to show that, given any couple $(j, q(j))$ of security type and price, we have the following relationship between the slopes of the isoprofit curve for the different technologies:

$$\begin{array}{l} \frac{\partial q^H(j)}{\partial j} < \frac{\partial q^L(j)}{\partial j} \text{ if } j \leq \overline{j} \\ \frac{\partial q^H(j)}{\partial j} = \frac{\partial q^L(j)}{\partial j} \text{ if } j > \overline{j} \end{array} \tag{12}$$

Given any pooling price $q(j^*)$ one can compute the security level, $\overline{j}$, after which all firms default. (12) implies that for any security $j \leq \overline{j}$, starting from $(\overline{j}, q(j^*))$ the price fall sufficient to keep profits constant as the number of bonds decreases is greater for entrepreneurs endowed with worse technology L (Figure 5). This property is sufficient to deliver the following theorem:

Theorem 7 (Pooling Equilibrium in Slow Growth Economies) *When the probability that the good contingency realizes is sufficiently small,* $0 \leq \alpha_G \leq \alpha_G^*$*, there is a unique robust pooling equilibrium where both types of firms issue security* $j^* = \overline{j}$ *at price*

$$q(j^* = \overline{j}) = \eta(H) + (1 - \eta(H))\alpha_G$$

These economies are illiquid since they display a positive liquidity premium:

$$LP > 0$$

Proof. Appendix. ■

Since the existence of a pooling equilibrium is the foundation of the analysis of liquidity, it is worthwhile to provide intuition for the reason why the proposed setup is conducive to it. I refer the interested reader to the appendix for a formal proof. The intuition for the pooling equilibrium is quite direct once the reader refers to Figure 5 where I draw two solid lines representing isoprofit curves for entrepreneurs H and L going through the pooling equilibrium $(\overline{j}, q(\overline{j}))$, and a dashed line representing the isoprofit of entrepreneur L when she chooses the profit maximizing security at the "fair" price α_G. Entrepreneur H wishes to differentiate himself to avoid subsidizing entrepreneurs L. Attempting to do so, the most he can do is to choose security $\bar{j}$ where he commits to issue more bonds that would be optimal for them to do at the given price $q(\bar{j})$[20]. Figure 5 shows that this is the only choice they have: if entrepreneurs H were to issue security, $j < \bar{j}$, entailing a lower number of bonds, they would require a price strictly higher than the one sufficient for entrepreneurs L to keep their profits at the same level of the pooling equilibrium (by (12)).

At the same time, entrepreneurs L find it profitable to mimick entrepreneurs H, although this requires a distortion in their issuance decision. Entrepreneurs L undertake the costly action of distorting their security choice toward $\bar{j}$ only if this gives them higher profits than what they would earn if they chose their security independently and were recognized by the market for what they are: low quality entrepreneurs. In figure 5, this is represented by the fact that isoprofit tangent to the delivery of low quality entrepreneurs (the dashed line) lies *below* the isoprofit of entrepreneur L going through the pooling equilibrium.

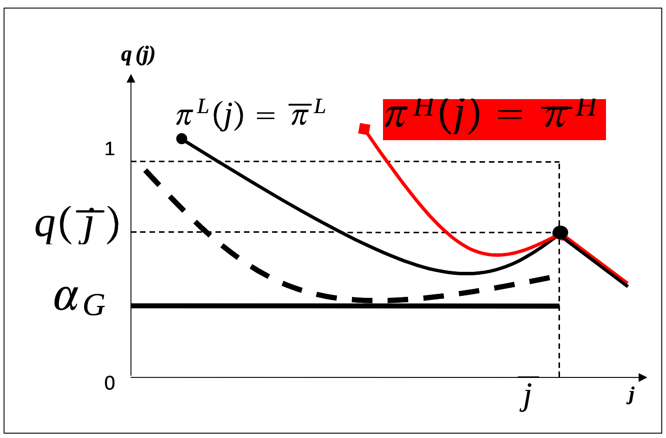

Figure 5

The result may be simply stated: if the probability of the good state of the economy, α_G, is sufficiently small, then technologies are very apart from one another and so low quality entrepreneurs gain a lot from mimicing good quality entrepreneurs. This can be graphically depicted by observing

[20] Notice that it is irrelevant to choose security $j > \bar{j}$ for entrepreneurs H because isoprofits of H and L coincide in that region by (12).

that, in Figure 5, the "height" of the dashed line depends on the probability of the good state, α_G. Whenever the latter is small enough, I say that the positive "price effect" enjoyed by entrepreneurs L - the "subsidy" their bond price receives - dominates the negative "leverage effect" they pay - the distortion in their bond issuance. The theorem above states that good quality entrepreneurs try their best to differentiate themselves and signal the market their quality, but it is really the incentive of bad quality entrepreneurs to mimic them or not that plays the crucial role.

The fact that a sufficiently small probability of the good state of the economy, α_G, delivers a pooling equilibrium which is robust in the sense of definition 6, i.e. robust to perturbations of any mix of good and bad entrepreneurs[21], is of interest in its own right. It is moreover interesting to observe that the equilibrium is independent from the relative measure of productive and unproductive entrepreneurs. The intuition behind the robustness of pooling equilibria in our framework is grounded on two assumptions: (1) good quality entrepreneurs may attempt to distinguish themselves by issuing securities entailing more bonds but, since their bonds deliver a higher payment, their total expected delivery increases relatively faster than the one of bad quality securities as the number of bonds increases; (2) when entrepreneurs default creditors can seize an amount of production that is technology independent, $g(I^i)$. Therefore, it is the positive correlation between a bond's payments and the technology's quality that turns out to be ultimately responsible for the robustness of the illiquid pooling equilibrium. Stating that my pooling equilibrium is robust has important macroeconomic implications: it supports the view that not only may liquidity shortages arise in the economy, but it states that imperfect liquidity, generated by asymmetric information, is robust to the different expectations that entrepreneurs may form about the prices of off-equilibrium securities.

Notice finally that the level of investment is constant across technologies in the illiquid pooling equilibrium, although the rates of return at which good entrepreneurs borrow is larger than the one at which bad entrepreneurs do. The difference between the terms of borrowing between the two kinds of entrepreneurs, absent in the symmetric information benchmark, is responsible for the instance of the adverse selection in investment we have here. Not surprisingly, this conflicts with

[21] This results complements Dubey and Geanakoplos (2003) where it is argued that, organizing security trade through "pools", separating equilibria always exist and are the only ones robust to "optimistic" off-equilibrium expectations.

the results of proposition 3.

4.2.2 Liquid Separating Equilibria

Theorem 7 has an important corollary necessary to study the relationship between liquidity and economic fluctuations. I have already highlighted that, if the probability of the good state is sufficiently low, the pooling equilibrium is the unique robust equilibrium. It is thus natural to ask the question of what type of equilibrium emerges when the economy improves and the probability, α_G, of the good contingency, G, increases sufficiently. The answer to this question is provided by the following corollary:

Corollary 8 (Separating Equilibrium in High Growth Economies) *When the probability of the good contingency is sufficiently large, $\alpha_G^* < \alpha_G \leq 1$, the economy displays a separating equilibrium where different kinds of firms issue different securities, j^i, at prices*

$$q(j^H) = 1$$
$$q(j^L) = \alpha_G$$

These economies are liquid since they display no liquidity premium:

$$LP = 0$$

I will convey the intuition through Figure 6 below:

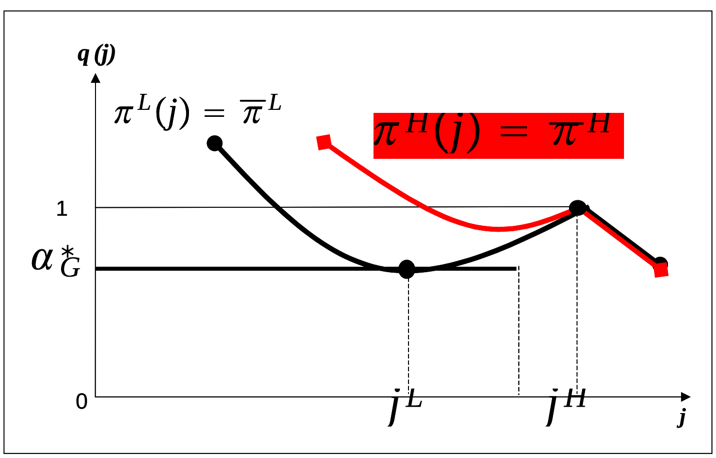

Figure 6

In order to see that a liquid separating equilibrium emerges as the probability of the good state being realized increases, it suffices to argue at the upper bound of the probability distribution over states of the world, i.e. when probability α_G is equal to 1. In this case, for entrepreneur L the "price effect" would be equal to zero while the "leverage effect" would still be negative. Thus there would be no advantage for entrepreneurs L in distorting their security choice since they would gain nothing by doing so. A separating equilibrium would naturally arise. But then, arguing by continuity, a separating equilibrium would survive even if I lowered the probability of the good state by an arbitrary small amount. Thus there is a continuum of values of $\alpha_G << 1$ so that the liquid separating equilibrium emerges.

The intuition remains in line with the previous one: as α_G increases the two technologies become more and more similar. Eventually, entrepreneurs L prefer to leave the pooling equilibrium, be priced for what they really offer and avoid the distortion in the security choice. This is an incentive compatible equilibrium when, as Figure 6 illustrates, the security choice of entrepreneurs H, j^H, and the choice of entrepreneurs L, j^L, lie on the same isoprofit of low quality entrepreneur L.

In conclusion, it is worthwhile to point out that the increase in liquidity as the state of the economy improves is due to the change in the equilibrium security and not to the increase in the average quality of the traded ones, as in Eisfeldt (2004). In the perspective of this paper, it is the dynamic of security design which plays the central role in building a relationship between the liquidity of the economy and its fluctuations. This dynamic also delivers that the level of aggregate investment is procyclical for reasonable parametrizations of the economy.

5 Liquidity and Economic Growth: Evidence in the US

There is a developing line of research in the macroeconomics literature aimed at explaining the relation between the business cycle and liquidity of the economy[22]. Policy analysts seem to agree that market liquidity tends to covary positively with economic growth, even though the measures of liquidity used are often different[23]. This is why it is an important contribution of the proposed theory to define a precise liquidity measure which can be successfully compared with the data.

In an important theoretical contribution in this line of research, Eisfeldt (2004) takes the same

[22] Eisfeldt (2004), Rampini (2003) and Rampini and Eisfeldt (2006).

[23] Measures of liquidity differ on the basis of the macroeconomic or financial focus of the study. Sometimes it is identified, like here, with how accessible credit is, other times by bid-ask spread and/or market depth.

conceptual standpoint adopted here regarding the definition of economy-wide liquidity. A summary of her results is as follows: as the productivity of technology changes along the business cycle, entrepreneurs have the incentive to raise more funds when productivity is higher. The only way open to entrepreneurs to finance their investment is selling claims over the future production of projects *previously initiated*. Since the incentive to invest is stronger for entrepreneurs endowed with better technologies, when the economy faces a high productivity shock, they tend to sell relatively more claims than the owner of worse technologies. Therefore, when the economy faces a higher productivity shock, the quality of the average traded security increases, the market price increases and liquidity turns higher.

This explanation relies on a crucial restriction in the security market, a restriction not present in the economy considered here: entrepreneurs face a security space made up of only one equity-like claim over future production. They cannot use their security choice to signal their private information because this choice can not be observed by financial markets. This restriction is central to the argument of this paper and may, in principle, undermine the explanation of procyclical liquidity proposed in Eisfeldt (2004). In fact, if different types of entrepreneurs were to issue different security types (i.e. different amount of equity) these would always be liquid and there would be no change in equilibrium liquidity as a consequence of productivity shocks.

In reality, firms have different financing instruments resulting in a variety of securities they may issue. Typically, increasing the number of securities a firm issues affects the likelihood and extent of default. Most importantly, financial markets can observe these choices and how much leverage a firm is undertaking. This is precisely the standpoint I considered in this study.

Non Convertible Publicly Traded Corporate Debt, 1970-2005

	Issuances	% of Total
Total	153, 377	(100.0%)
Known Yield		
Total sample	74,166	(52,2 %)
S&P 500	21,812	(15.5%)
Non S&P 500	52,354	(37,2%)

Table1, Source: SCD premium

I am now going to apply the theory of this paper to the empirical evidence in the US bond market to characterize the relationship between liquidity and economic fluctuations. Theorem 7 and its corollary have two empirical implications that can be compared with the data. The first one deals closely with the relationship between economic fluctuations and the liquidity of the economy, as defined in this paper. At least since Kydland and Prescott (1984), the state of the economy can be identified with the probability distribution on future contingencies. This simple fact is translated here by changing α_G - the probability attached to the G(ood) contingency - to adjust to the shift in economic conditions and study how the equilibrium liquidity changes. From this standpoint, higher growth is the resut of economies where α_G is greater and vice versa. In this light, the proposed theory predicts a positive correlation between the state of the economy, which I measure by the cyclical component of GDP, and the liquidity premium of the economy, as in definition 1. It is possible to measure the economy-wide liquidity premium of the economy in US bond market since the proposed theory has identified it by the cross sectional variance of the yields in the bonds issued in any given quarter.

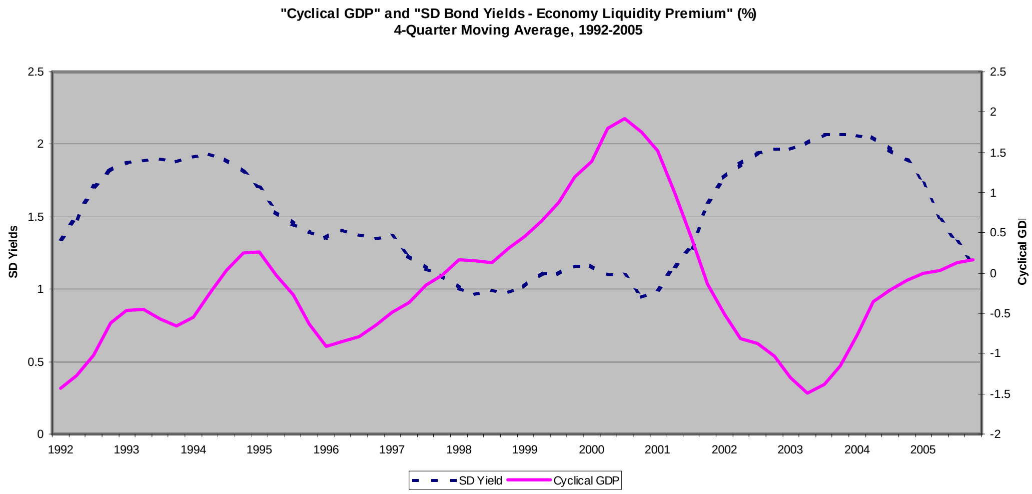

Figure 7

In order to build the relevant measure of liquidity, the first challenge I addressed has been to collect data about all issuances of publicly traded non convertible corporate debt in the US from 1970 to 2005. Second, it has been necessary to collect the "yield at maturity" for each bond issue in the sample. Because of the lack of a single comprehensive dataset, a variety of sources was combined[24]. Eventually, attention was restricted only to those issues where yield data were available, around 52,2% of the original sample as Table 1 shows.

In any given quarter firms issue bonds in the united statesThese data have been used to compute the cross sectional variance of yields across different bond issuances in any given quarter. Finally quarterly data for the HP filtered cyclical component of GDP and standard deviation have been smoothed using an annual moving average. The results are represented in Figure 7.

It is immediate to observe a negative correlation between the cyclical component of GDP and the standard deviation of bond yields issued in a given quarter, i.e. the economy-wide liquidity premium. In Table 3 I show that the negative relationship is statistically significant across the subsample 1992-2005 that gathers most of the observations, once we control for the maturity and *Moody's* rating of the issued bonds. In oredr to control for this issue I start from the following

[24] Compustat and SCD Premium dataset have been joined to compute rates of return.

simple regression:

$$r_t^i = \alpha_0 + \alpha_1(1yr - bills)_t + \alpha_2\,(maturity)_t^i + \alpha_3\,(rating)_t^i + \varepsilon_t^i \qquad (13)$$

where the rate of return promised (yield) by bond of firm i in quarter t, r_t^i, is a function of the interest rate paid by 12 months treasury bills, $(1yr - bills)_t$, its maturity, $(maturity)_t^i$, and the Moody's rating, $(rating)_t^i$, it received and an idiosyncratic shock, ε_t^i. Taking variances of both sides in equation (13), I obtain a relationship between the cross sectional variance of bonds' yields - the proposed meaure of liquidity - and all relevant variances and covariances of trasury bills, ratings and maturities. I then test whether cyclical GDP still retains statistically significant explanatory power in the meaure of liquidity I introduced. As it is customary with time series data, serial correlation is an issue that distorts the efficiency of the estimators in this kind of regressions. To correct this problem I apply the Cochrane-Orcutt procedure. Since the procedure delivers estimates displaying Durbin Watson (transformed) statistics very close to 2, the exercise is considered satisfactory.

Liquidity Premium	1992-2005 Var(Yield)	*Cochrane-Orcutt Estimate*
Cyclical GDP	-0.19593	
	(2.08)**	
Var(ratings)	-0.00102	
	(0.310)	
Var(maturity)	-0.00013	Constant Included
	(3.44)***	
Cov(rating,mat)	0.002833	
	(6.39)***	Absolute Value t
t_92_05	-0.009	Statistics in Parentheses
	(1.050)	
Var(1yr-Tbills)	-2.4657	* significant at 10%
	(-1.36)	** significant at 5%
Cov(1yr-Tbills,mat)	0.032448	*** significant at 1%
	(10.77)***	
Cov(1yr-Tbills,rating)	-0.06133	*DW (Original)*
	(2.19)**	1.268307
Observations	56	*DW (Transformed)*
Adjusted R-squared	0.88	1.947812

Table 3

It could be argued that the significance of the relationship between cyclical GDP and the cross sectional dispersion of bond yields issued in a given quarter is due to the fact that different kinds of firms enter the sample at different levels of GDP growth. In order to control for this feature, I compute the correlation coefficient for a set of subsamples where I include only the firms that issue bonds in all the years of the subsample. The calculations confirm the negative relationship between economic growth and bond rates dispersion, although it is not always statistically significant[25].

The provision of a theoretical explanation for the inverse relation between the state of the economy - that I measure by cyclical GDP - and the standard deviation in bonds' yields is, to my knowledge, novel and interesting in its own right. The adopted approach discusses why different bonds pay different yields to focus on the systematic relationship between their cross sectional variance and economic fluctuations. Here I want to stress that the provided framework identifies the dispersion in the rates of returns across different bond issuances as a measure of the liquidity of the economy. I argue this is intuitive: the lack of economy-wide liquidity translates into bonds mispricing, yields dispersion and investment misallocation.

Correlation Coefficient between Cyclical GDP and

Standard Deviation Corporate Bond Yields, 1970-2005

	$1970-2005$
Total sample	-0.06^{**}
S&P 500 sample	-0.19^{***}
Non S&P 500 sample	-0.06^{**}

Table 2, */**/*** means significant at the 10%/5%/1% level, respectively

According to these argumente better entrepreneurs are forced, on average, to pay relatively higher rates of return than worse entrepreneurs. It is interesting to notice that this is especially the case when the economy slows down: it is in the most adverse economic environments that good entrepreneurs single themselves out by paying higher interest rates and thus increasing the overall variance in rates of return.

[25] In the subsamples (1993-2005), (1992-2005) and (1991-2005), the correlation coefficients are, respectively, -0.59 (significant at 10%), -0.60 and -0.62 (all significant at 5%). The firms included in each subsamples are, respectively, 18, 18 and 13. In the (1991-2005) subsample these firms are: American General Finance Corp, Caterpillar Finl Svcs Corp, Consolidated Edison Co of NY, Federal Farm Cr Banks Funding, Federal Home Loan Banks, Federal Home Loan Mortgage, Fannie Mae, Ford Motor Credit Co, General Electric Capital Corp, Huntington Natl Bk, International Lease Finance, Lehman Brothers Holdings Inc, Merrill Lynch & Co Inc, Morgan Stanley Group Inc, Bank of New York, Bear Stearns Cos Inc, Private Export Funding Corp, Tennessee Valley Authority, Toyota Motor Credit Corp.

The second main implication of the proposed framework is that the variety of securities issued in any given quarter will be positively correlated with economic growth. In fact, as the the probability of the good contingency increases, the economy improves. An improving economy eventually abandons the pooling equilibria in the bond space, where all entreprenuers issue the same security type, and start displaying separating equilibria where different kinds of entrepreneurs issue different securities.

Correlation Coefficient between Cyclical GDP and
Standard Deviation of Moody's Ratings, 1970-2005

	1970 – 2005
Total sample	0.19***
S&P 500 sample	0.05*
Non S&P 500 sample	0.18**

Table 4, */**/*** means significant at the 10%/5%/1% level, respectively

Observing a security's type is, in reality, almost an heroic activity: it is not obvious what a sensible measure might be and available data do not seem to provide sufficient information. In Table 4 I proxy security type by the security *rating* an issuance receives by the major rating agency for private bonds, *Moody's.* The data display what the model predicts: a positive correlation between the stae of the economy and the variety in the securities issued at a given time. Once I translate this prediction into the study of the correlation between cyclical GDP and the variance of ratings across different bonds issued in a given quarter, I find the two to be significanlty and positively correlated as Table 3 shows. Employing the constructed dataset, I conclude that the empirical evidence supports the two main empirical predictions of the model.

6 Extensions and Concluding Remarks

The main purpose of this paper has been to propose a theoretical foundation for the question of what determines the economy-wide level of liquidity and how we should think about procyclical liquidity. This has been done in a context where securities are endogenously selected in equilibrium. Under this new perspective it is the dynamic of security design that drives the behavior of liquidity over time. It has been discussed in the context of a general equilibrium characterization of the notion of liquidity and its main determinant: the nature of the optimal security design in the private sector. I measured the liquidity of a security via an informational asymmetry concerning the value of its payments. The informational asymmetry is responsible for the illiquidity of the economy because it may generate a discrepancy between current market prices and actual discounted future deliveries. Then I relate the liquidity of a security to its suitability in financing private investment. In this way the equilibrium level of liquidity becomes a crucial factor in the allocation of credit, and thus private investment, in a production economy.

The analysis yields the following main implications. First, it shows that illiquidity is an equilibrium phenomenon of the economy. Firms take into account the state of the economy when they decide how many bonds they issue and thus how often their securities default and how serious this default is. Even allowing for this freedom, this study argues that firms may tend to issue very similar securities, i.e. undertake similar bond issuances, when economic growth lags. Second, illqiuid pooling equilibria are shown to be robust to off-equilibrium expectations. Even if this economy faces a basic signalling problem, the positive correlation between security value and the underlying technological quality is ultimately responsible for the appearance of robust pooling equilibria. This is interesting in its own right. Third, I relate the theoretical model to the two empirical regularities of the US financial market that I am the first to highlight: (1) the negative correlation between the variance in firm debt yields and economic growth and (2) the positive correlation between variance in firm debt ratings and economic growth. I provide an explanation for these empirical regularities which is rooted in the underlying dynamic of security design. This is, to my knowledge, the first theoretical explanation of the relationship between bond yields variance and economic growth and one that relaxes restrictions on the observability of firm actions which seem not realistic. It is the entrepreneurs'choice and the resulting dynamic of security design - I argue - to be ultimately responsible for the level of liquidity of the economy.

The reader may argue that our analysis is based on restrictive technological assumptions. In this respect I would like to make few brief comments. First, the only requirement we need for the pooling equilibrium is that the dimension of separation - the security choice - must be relatively more expensive for entrepreneurs endowed with the better technology. An alternative way of getting to this result would be to assume an economy with more than two contingencies and in which the worse technology produces less on average but more than the good one in at least one contingency. I have avoided this setup because the construction of the security space is more cumbersome. The setup employed here is noticeably simpler thanks to the assumption of imperfect appropriability of production by creditors. But this assumption, as I already argued, is by no means necessary.

Second, one could argue that there is really no need for the economy to move from pooling to separating equilibria in order to provide a theory of procyclical liquidity. The fact that default rates are typically countercyclical would naturally bring together rates of return in good times and vice versa. This is certainly true, but such a theory, where only pooling equilibria exists, would not be able to explain the *positive* relationship between GDP growth and the variance of ratings across different bonds issued in a given quarter. Moreover, this alternative theory would predict a positive correlation between average ratings and economic growth, which is the opposite of what I findin the data[26].

I wish to conclude stressing how fruitful this framework may be in the future providing a general benchmark able to analyze the role of government bonds and financial arrangements in addressing economy-wide liquidity. I argue that a general framework may eventually be used and this is the object of my current research and a companion working paper, Taddei (2004).

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7 Appendix

7.1 Entrepreneur's Profit Maximization

Given assumption (7), maximizing entrepreneur i's utility is equivalent to maximizing $\pi^i(j)$. This reduces to:

$$\max_j \left[\pi^i(j)\right] = \max_j \left[\sum_{s\in S^i} \alpha_s \cdot \left\{\left[\lambda_s^i g\left(I^i\right)\right] - j\right\} + \sum_{s\in S\setminus S^i} \alpha_s \cdot \max\left\{\left(\lambda_s^i - 1\right) g\left(I^i\right), 0\right\}\right] \tag{14}$$

where $S^i = \left\{s \in \{G, B\} \middle| D_j^i(s) = 1\right\}$ denotes the contingency/ies where firm i does not default. Equilibrium investment is:

$$I^i = w + q(j)j$$

The relevant FOC to $\max\left[\pi^i(j)\right]$ is:

$$\begin{aligned} \tfrac{d\pi^i(j)}{dj} &= \sum\nolimits_{s\in S^i} \alpha_s \cdot \left\{\left[\lambda_s^i g'\left(I^i\right)\right]\left(q(j) + j\tfrac{\partial q(j,i)}{\partial j}\right) - 1\right\} + \\ &+ \sum\nolimits_{s\in S\setminus S^i} \alpha_s \cdot \left\{\left[\left(\lambda_s^i - 1\right) g'\left(I^i\right)\right]\left(q(j) + j\tfrac{\partial q(j,i)}{\partial j}\right)\right\} = 0 \end{aligned} \tag{15}$$

Rearranging (15) we can derive the slope of the isoprofit function:

$$\left.\frac{\partial q(j,i)}{\partial j}\right|_{\pi^i(j)=\overline{\pi}} = \frac{\sum_{s\in S^i} \alpha_s}{jg'\left(I^i\right)\left[\sum_{s\in S^i} \alpha_s \lambda_s^i + \sum_{s\in S\setminus S^i} \alpha_s \cdot \left(\lambda_s^i - 1\right)\right]} - \frac{q(j)}{j}$$

Proof of (2). Without loss of generality attention can be restricted to securities $j \leq \overline{j}$. Thus (6) can be written:

$$q(j,i) = \sum_{s\in S^i} \alpha_s \tag{16}$$

and so we have:

$$q(j,H)=1$$

$$q(j,L)=\alpha_G$$

which, observing that $\frac{\partial q(j,i)}{\partial j}=0$ if $j\leq\overline{j}$, implies by (9):

$$E_{s^H}\left[g'_H\left(I;s\right)\right]=1$$

$$E_{s^L}\left[g'_L\left(I;s\right)\right]=\frac{\sum_{s\in S^L}\alpha_s}{q(j,L)}=1$$

and then

$$g'\left(I^L\right)=\tfrac{1}{\alpha_G\lambda^L}$$

$$g'\left(I^H\right)=\tfrac{1}{\lambda^H}$$

but, since $\lambda^H>\lambda^L$ by assumption , $g'\left(I^H\right)<g'\left(I^L\right)$ and $I^H>I^L$. ■

Proof Theorem 7. In order to prove the existence of a pooling equilibrium we must ensure that, at the equilibrium, the relevant local and global incentive compatibility constraints are satisfied for both kinds of entrepreneurs. We prove that the number of bonds $j^*=\overline{j}$ is the pooling equilibrium of the economy where both kinds of firms issue the same security at price $q(j^*)$, which - by rational expectations formalized by (10) - is equal to:

$$q(j^*=\overline{j})=\eta(H)+(1-\eta(H))\alpha_G$$

First one needs to consider that the pooling equilibrium is locally incentive compatible for both types of entrepreneurs. In order to prove so, one may study the isoprofit curve, the locus combining j and $q(j)$ so that profits remain unchanged. We have already found that:

$$\tfrac{\partial q^H(j)}{\partial j}<\tfrac{\partial q^L(j)}{\partial j}\text{ if } j\leq\overline{j}$$

$$\tfrac{\partial q^H(j)}{\partial j}=\tfrac{\partial q^L(j)}{\partial j}\text{ if } j>\overline{j}$$

Therefore we can use Figure 5 to check that $\overline{j},q(\overline{j})$ is indeed the pooling equilibrium we are looking for. All I need to control, to ensure that I am supporting a pooling equilibrium is that there no deviations that are profitable to entrepreneurs H without being so for entrepreneurs L. First notice that there no deviation with this property for security $j\leq\overline{j}$. In fact, any price for security $j\leq\overline{j}$ that is above the isoprofit of entrepreneurs H going through the pooling equilibrium is also above the isoprofit of entrepreneurs L. Since the isoprofit curves of H and L coincide to the right of security $\overline{j}$, there is no deviation that can attarct entrepreneurs of good quality only. All that remain to check is that entrepreneurs L would not rather choose a security where they are recognized for what they are. This is equivalent to check that

$$\pi^L(\bar{j}) \geqslant \pi^L(j^L)$$

that can be worked into to:

$$\lambda^L g\left[w + \bar{j}q(\bar{j})\right] - \bar{j} \geqslant \left[\lambda^L g\left(w + j^L q(j^L)\right)\right] - j^L$$

that is

$$\lambda^L g\left[w + \bar{j}\left(\eta(H) + (1-\eta(H))\alpha_G\right)\right] - \bar{j} \geqslant \lambda^L\left[gw + j^L\alpha_G\right] - j^L$$

which id satisfied for $\alpha_G = 0$. Thus by continuity, we can argue that there is α_G^* below which the pooling is incentive compatible and so is supported ■

I conclude the proof by showing that the pooling equilibrium is robust in the sense that it survives the perturbation where the external agent $\varepsilon_j(n)$ behaves as high quality (L) entrepreneurs on off-equilibrium securities. The crucial condition for the equilibrium to survive is that $q^L(j) \geq q^H(j)$. I will present the argument concisely since the structure is very similar to Dubey and Geanakoplos (2002):

Lemma 9 *When $0 \leq \alpha_G \leq \alpha_G^*$, the pooling equilibrium $j^* = \bar{j}$ is robust in the sense of definition 6.*

Proof. Pick the pooling equilibrium and consider the only active security $\underline{j} = j^{pool}$. By equation (10), given the proportions of good and bad quality entrepreneurs $\eta(L)$ and $\eta(H)$, $\bar{j}$'s price is determined by the delivery vectors $D_{\bar{j}}^i(s)$, $i = H, L$. Now focus on off-equilibrium securities, $j < \bar{j}$:

1. consider an external agent entirely characterized by her delivery vector which I assume identical to $D_j^L(s)$, the delivery vector of good quality entrepreneurs;

2. for each security $j < \bar{j}$, let $\varepsilon_j(n)$ be apositive mass of external agent issuing securities in market j. Given the exogenous mass $\varepsilon_j(n)$, find the equilibrium of the ε-economy as defined in (5) allowing all agents to reoptimize;

3. finally take $\varepsilon(n) \underset{n\longrightarrow+\infty}{\longrightarrow} 0$, i.e. the measure of the external agent to zero. The limit of the sequence, $\varepsilon(\infty) = 0$, replicates the original economy. If the equilibrium of the $\varepsilon(n)$ economy converges to the pooling equilibrium $j^* = \overline{j}$ of the original economy, then we say that the equilibrium survives the external agent "perturbation". If the equilibrium survives this perturbation is robust. Any robust equilibrium survives perturbations defined for delivery rates smaller or equal to the one considered here. Thus the equilibrium survives the perturbation in which the external agent is characterized by any delivery vector between $D_j^H(s)$ and $D_j^L(s)$.

In order to prove the lemma it suffices to show that in the $\varepsilon(n)$ economy entrepreneurs with low quality technology (L) are at least indifferent between issuing $j^* = \overline{j}$ and $j \neq \overline{j}$ while entrepreneurs with high quality technology (H) are at most indifferent, if not worse off. This is equivalent to finding that the price that makes "L" entrepreneurs indifferent to the pooling equilibrium security makes "H" entrepreneurs willing to abandon it, i.e. $q^L(j) > q^H(j)$. To this purpose consider the choice of the entrepreneur who is issuing $j^* = \overline{j}$ and is now facing the introduced perturbation on security $j < \overline{j}$ in the $\varepsilon(n)$ economy.

This suffices to support the pooling equilibrium $j^* = \overline{j}$. This implies that, at the equilibrium prices of the ε-economy, low quality technology holders (L) are (weakly) better off issuing security $j < \overline{j}$ while high quality technology holders are *strictly* worse off by doing so. But this means that, when the measure of high quality entrepreneurs is sufficiently large, the pooling equilibrium $\overline{j}$ survives the perturbation consisting of an external agent behaving as if he were a good quality entrepreneur on off-equilibrium securities.

Since the deviations of low quality entrepreneurs is triggered by the external agent of measure $\varepsilon(n)$ to issue $j < \overline{j} = j^*$, the measure of entrepreneurs L issuing securities different from $\underline{j} = j^{pool}$ converges to zero as $\varepsilon(n) \rightarrow 0$. ■