Macroeconomic Effects of Immigration in a New Keynesian Model

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Abstract

This paper analyses the impact of immigration inflows on the macroeconomic variables of a developed economy under a standard New Keynesian framework. We highlight four important results. First, immigration inflows destabilize the host economy from its long run equilibrium. In the short-run an inflow of immigrants may create a positive output effect, but in the long-run the final outcome depends on the "deep" parameters of the model, and on whether immigration alters the composition of total workforce permanently. Second, the model confirms existing results on the re-distributional effects of immigration. Immigration inflows seem to be welfare improving for domestic agents as long as domestic consumption and labor market outcomes for natives are positively affected by immigration shocks. Newcomers compete with existing immigrants for job opportunities and affect negatively only the labor market of foreign-born agents. Third, potential shocks on immigrants' preference behavior (remittances shock) cause both an increase of total output and a decline of inflation rate, partly explaining why immigration could be considered a disinflationary force of the host country. Finally, our results demonstrate that in the short-run the demand side effect of immigration dominates any supply side effect and hence raise inflation, but in the long-run immigration can be proved a disinflation force in case newcomers are more productive than native agents.

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1 Introduction

The consequences of international migration on the macroeconomic position of the host countries have constituted an issue of major concern for monetary policy makers, since official data suggest an enlargement of labor force movements from developing and poor countries to developed nations, such as the US, European Union members, even Japan, a traditionally closed country. In a globalized world where there is free movement of goods and production factors, immigration appears to follow an upward trend. Each year around a million of people migrate legally to the US, while European Union accepts approximately 2.8 million of foreigners. Immigration can be further decomposed to legal and illegal components.

In general, immigration is classified according to three criteria: first, the type of workers; second, the duration of residence of foreigners in the host country; third, immigrants' political status (Ethier, 1986). According to the first criterion, there are immigration flows of either high-skilled or low-skilled workers. The second criterion separates immigrants between workers of permanent and temporary residence. Usually, immigrants who intend to live permanently in the host country are high-skilled laborers, who migrate in order to find a job that matches their capabilities. In contrast, immigrants of temporary residence are low-skilled workers who migrate either because of poor living perspectives in their home countries or because of the seasonal demand for unskilled labor existing in developed economies. The political status of foreigners distinguishes immigration between legal and illegal. Illegal immigration mainly concerns unskilled workers from poor countries, who find crossing the borders as the only way to improve their lives.

Although potential combinations of the above categories give numerous types of immigration flows, recent studies (Dench et al. 2006, Blanchflower et al. 2007) suggest that in UK, a representative developed country of Europe, labor force inflows usually concern unskilled, low-paid workers, who enter either legally or illegally, but for short-run employment residence of one or two years. The welfare gap between developed and developing countries tends to support the low educational attainment or skill abilities of immigrants. Indeed, the lack of knowledge of native language puts a barrier between natives and foreigners. Other studies (Saleheen and Shadforth, 2006) find that immigrants tend to be more educated than native workers. However, due to their special characteristics, such as the limited information set and the low knowledge of domestic language, immigrants are mainly employed in low or semi-skilled, low-paid jobs. In addition, immigrants face a lower probability to find a job that matches their abilities in comparison with natives, and, thus, they hesitate to search continuously for better employment opportunities. Literature on European immigration concludes that foreigners, independently of their educational level or skill potentials, are employed in low-skilled positions.

Also, it is observed that Europeans tend to prefer high-skilled, high-paid jobs that reduces the supply of domestic unskilled labor. Employers in order to satisfy demand for unskilled workers look forward to unskilled immigration inflows from poor countries. As a result, in the host country a significant portion of low-skilled workforce is composed by seasonally employed immigrants.

In contrast, for the US economy there is no general agreement about the skill composition of immigration inflows. On the one hand, Card (2005) adopts empirical data that show concentration of immigrants in occupations that require low educational level, but, on the other, Peri (2006a,b) shows employment concentration of immigrants on both skilled and unskilled jobs.

In any case, immigration alters the magnitude and the skill composition of host countries' total workforce permanently or temporarily. Both the time varying magnitude and heterogeneity of total population, caused by immigration inflows, poses the issue of whether immigration creates policy implications for policymakers. Furthermore, central banks' contention that immigration is placed among the disinflation forces of the developed economies motivated us to integrate immigration inflows into a standard New Keynesian model that is widely being used by these institutions.

On the one hand, immigration may be desirable by central banks, as long as they expect immigrants to fill existing gaps in domestic labor markets. On the other hand, immigration inflows may be unexpected disturbances that hit host economies, causing them to deviate from their long-run equilibrium time paths. In the second case, policymakers should undertake measures to re-stabilize the economy back to its equilibrium. Apart from affecting the aggregate demand and supply, the different economic behavior of immigrants from that of natives may affect the traditional channels through which monetary policymakers ease exogenous shocks' consequences. Foreigners, who enter the country for one or two years of residence, neither import in the host country capital holdings nor have the optimizing behavior of natives. The temporary living in the host country prevents immigrants from obtaining the full information set that domestic agents possess. The limited information set, a potentially illegal status, as well as the narrow financial abilities prevent immigrants from participating in contingent nominal asset markets or from investing in capital holdings. We predict that this heterogeneity of total population may change the efficiency of central banks' policy.

While there is extensive literature on immigration which is concerned about the impact of immigration on domestic labor market outcomes (wages and employment opportunities of natives) [1], there is almost no research on the macroeconomic impacts of immigration flows in a business cycle framework. Barwell (2007) tackles the issue of immigration out of the limited context of labor market and offers a coherent and detailed report on the consequences of international migration to the macroeconomic position of a developed economy, that monetary police committee should be aware of. Briefly, Barwell suggests that an inflow of immigrants boost both aggregate demand and supply, as long as immigrants are consumers and workers, respectively. From a supply side perspective, an inflow of immigrants is expected to increase the total population and workforce of the host economy, boosting in this sense the aggregate labor supply. Especially, immigrants' job search behavior, such as the willingness to work more hours than natives, their low working age as well as their intensive search for employment opportunities, boost even farther the labor supply, and may positively affect supply capacity. Capital stock is expected to be more valuable, triggering in turn investment spending. From a demand side perspective, immigration not only affects domestic investment positively, due to the decrease of the capital-labor ratio, but also boosts total consumption. Im-

[1] A traditional debate in the context of labor market concerns the consequences of immigration inflows on domestic wages, and on employment opportunities of natives (displacement effects). There is extensive literature that examines the impact of immigration on domestic labor market, giving special attention to the degree of substitutability between natives and foreigners. There are three strands of the relevant literature: first, immigration inflows reduce the wages of native born workers, as long as immigration increases the aggregate labor supply relatively to total demand. Second, there is no strong evidence that immigration affects negatively the outcomes of domestic labor markets. Third, immigration expands the employment opportunities and wage earnings of natives.

migrants' consumption adds to domestic consumption spending, raising in that way aggregate demand. Barwell (2007:54) points out that the key issue for monetary policymakers, who are interested in controlling inflationary pressures, is found on how net immigration flows affect the balance between aggregate demand and supply.

Taking into account Barwell's directions, we believe that the limited participation of immigrants to contingent asset markets, as well as their restricted or even forbidden access to credit markets, under the same terms that hold for domestic households, should also be considered by monetary policy committees. The temporary residence of immigrants in the host country is equivalent to a limited information set or an unwillingness to participate in domestic, real or nominal asset markets. Such immigrants ignore interest rate movements caused by the policy makers. Monetary policy committees, that use the interest rate as a policy tool, should reconsider policy effectiveness in stabilizing the economy to various exogenous shocks, given that a portion of aggregate demand becomes irrelevant to interest rate movements and thus may remain uncontrolled.

Theoretically, the model of Canova-Ravn (2000), which examines immigration effects on the host country, constitutes a starting point for future research on immigration at the macro-economy, in a business cycle framework. Among other things, immigration inflows should not be ignored by the policy makers as a potential source of economic fluctuations for the host country. Canova-Ravn focused on low-skilled immigration, examining the implications of different degrees of substitutability between native and foreign workers. Under a real business cycle context, Canova-Ravn observed that immigration shocks create welfare redistribution in favor of natives, but recessionary short-term effects on the macro-economy.

The present paper complements the work of Canova-Ravn (2000), by investigating the macroeconomic consequences of immigration inflows in a rather more realistic framework with monopolistic competition and nominal price rigidities. We develop a standard New Keynesian model with no transaction frictions (cashless economy), but with nominal and real asset holdings (bonds and capital). We decompose the demand side of the host country between native-born and foreign-born households, which are composed by skilled and unskilled workers. Domestic households are optimizing agents who participate in nominal asset and real capital markets, but foreign-born households are rule-of-thumb immigrants who are unable to save in terms of bond holdings or in capital stock. Their behavior is justified by their restricted information set or their unwillingness to save in domestic assets due to their short-run residence. In this sense, both capital and asset markets of the host country are considered segmented.

We also adopt the recent attitude of immigration literature that immigrants and natives are imperfect substitutes, because they differ in educational attainment, skill levels, and language proficiency. Even in case natives and foreigners have the same educational level, immigrants' professional experience, cultural origin, characteristics and motivation, distinguish them from the native-born workers (Peri 2006a,b). Hence, we incorporate in the present analysis a simplified production function used by Ottaviano-Peri (2005, 2006), in which skilled and unskilled labor are composed by imperfectly substitutable natives and immigrants.

Central bank exercises monetary policy by using as a policy instrument the nominal interest rate, following a modified Taylor (1993) rule. For simplicity purposes there are no transaction frictions and, thus, no money balances in the host country. Finally, as long as there is no government sector, there are no border enforcement policies that could control and limit immigration inflows. Hence, immigration inflows are considered as exogenous shocks that hit the host economy unexpectedly.

By investigating the impact of immigration inflows on the optimal paths of host countries' macroeconomic magnitudes, we derive three basic results: First, an immigration shock constitutes by itself a destabilizing force for the host economy, with potentially permanent effects on its macroeconomic position. Under the context of nominal price rigidities, immigration inflows may create a positive output effect in the short-run. In the long-run, economy may return back to steady-state equilibrium or deviate permanently. Long-run effects depend on the "deep" parameters of the model, as well as on whether immigration alters the composition of total workforce permanently. Nominal variables such as inflation and interest rate are only temporarily affected showing a hump-shaped behavior. Second, the model verifies the re-distributional effects of immigration in favor of domestic households, identified by existing studies (Canova-Ravn, 2000). In fact, with respect to disaggregated variables, domestic consumption and labor market outcomes for natives are positively affected by immigration shocks, as long as newcomers compete with existing immigrants for job opportunities and affect negatively only the labor market of foreign-born agents. Third, the analysis suggests that even though the standard New Keynesian model does not verify the deflationary effect of immigration, potential shocks on immigrants' preference behavior, called them remittances shocks, create a positive response of total output and a downward movement of inflation rate. The industrious behavior of immigrants, pointed out by immigration literature, may justify the bottom line adopted by central banks that immigration may be placed among the disinflation forces in host economies.

The remainder of the paper develops as follows. Section 2 describes a standard New Keynesian model with two types of agents, natives and immigrants, and analyzes the equilibrium conditions. Section 3 provides the baseline calibration analysis, and section 4 presents the main results of the paper, that may offer useful insights for central banks. Specifically, under the context of impulse response analysis, section 4 describes how immigration inflows destabilize the host country, as well as how shocks on immigrants' preference behavior may be considered as positive supply disturbances that create deflationary pressures. Finally, section 5 concludes and offers directions for further research.

2 The Model

2.1 The demand side

The demand side of the economy is composed by two types of agents; natives and immigrants. Natives are considered *optimizing* households, who smooth intertemporally their consumption spending, by allocating their savings both in terms of nominal assets (bonds) and capital holdings (real assets). On the contrary, immigrants are considered *rule-of-thumb* consumers who participate neither to real nor to nominal contingent asset market of the host country. Because of their temporary residence, immigrants have restricted information set about the available saving opportunities offered by domestic asset markets. Alternatively, immigrants may be unwilling to invest in nominal assets or capital holdings of the host country as long as they intend to return back to their home countries. In conformity with immigration literature, there are both *skilled* and *unskilled* natives and immigrants, but the latter do not compete directly with natives for the same employment opportunities. Natives receive skilled and unskilled wages from their employment, an interest from bond assets, dividends from firm ownership, and rents from capital holdings. On the contrary, immigrants receive only labor income. The participation constraint of immigrants in a complete contingent nominal or real asset market constitutes the main characteristic of the present framework. Because of migrants' inability or unwillingness to invest in domestic capital stock or nominal assets, domestic asset and capital markets are considered segmented.

2.1.1 *Domestic Households*

Domestic households derive utility from consumption $C_{d,t}$, and disutility from labor supply $H_{d,t}$. Natives own the total capital stock of the host country, make investment decisions and participate in a perfect market of contingent securities B_t. During every period, the representative domestic household chooses consumption $C_{d,t}$, labor $H_{d,t}$, bond holdings $B_{d,t}$, and investment spending $X_{d,t}$, in order to maximize the expected, discounted, present value of its lifetime utility subject to a sequence of flow budget constraints and the capital accumulation equation. Lifetime utility is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\alpha_{d,t} \ln(C_{d,t}) + \ln\left(1 - H_{d,t}\right)\right]$$

where E_0 denotes the rational expectations operator using information up to time t = 0, $\beta \epsilon (0, 1)$ is the subjective discount factor, $1 - H_{d,t}$ represents leisure time. The momentary utility function is log-separable in its arguments to make the model consistent with the balanced growth properties. The aggregate domestic labor index is assumed to take the form

$$H_{d,t} = \left[\left(H^s_{d,t}\right)^{\frac{\sigma+1}{\sigma}} + \left(H^u_{d,t}\right)^{\frac{\sigma+1}{\sigma}}\right]^{\frac{\sigma}{\sigma+1}}$$

where $\sigma > 0$ and $H^s_{d,t}$, $H^u_{d,t}$ denote the total skilled and unskilled labor offered in the production sector by natives. The above labor index implies that domestic household offers labor in every sector even if skilled and unskilled wages are not equal. In other words, we have a composite household with a portion of its members offering skilled labor while the rest one being unskilled. In case of $\sigma \to \infty$, skilled and unskilled labor would be perfect substitutes and the household would devote all labor to the sector paying the highest wage. The term $\alpha_{d,t}$ is an exogenous preference weight on consumption that follows a first order autoregressive process of the form

$$\ln(\alpha_{d,t}) = (1 - \rho_{\alpha_d}) \ln(\overline{a}_d) + \rho_{\alpha_d} \ln(\alpha_{d,t}) + \varepsilon_{\alpha_d,t}$$

with $0 \leq \rho_{\alpha_d} < 1$, and $\varepsilon_{\alpha_d,t}$ be a normally distributed error with standard deviation $\sigma_{\alpha_d} > 0$. The preference shock $\alpha_{d,t}$ affects the marginal rate of substitution between consumption and leisure, and, in general, has been proved a source of business cycle dynamics (Ireland, 2004). The period budget constraint takes the form

$$C_{d,t} + X_{d,t} + \frac{B_{d,t}}{P_t} \leq \left(\frac{W^s_{d,t}}{P_t}\right) H^s_{d,t} + \left(\frac{W^u_{d,t}}{P_t}\right) H^u_{d,t} + r^k_t K_{d,t} + (1 + i_{t-1}) \left(\frac{B_{d,t-1}}{P_t}\right) + D_{d,t}$$

where $C_{d,t}$, $X_{d,t}$, $B_{d,t}/P_t$, $K_{d,t}$, and $D_{d,t}$ represent consumption, investment, bond holdings, capital, and profits in real terms, respectively. Also, r_t^k and i_t denote the rental price of capital and the nominal interest rate. The budget constraint shows that, during each period, total wealth of domestic household is composed by labor income, rents of capital, dividends from firm ownership, and interests from bond holdings. Notice that there are no money balances in the optimization problem, since by assumption there are no transaction frictions in the present framework. Also, for simplicity purposes we have ignored any quadratic cost of capital adjustment, which is only expected to make the final results more persistent. The capital accumulation equation takes the benchmark form

$$K_{d,t+1} = (1-\delta)K_{d,t} + X_{d,t}$$

where $0 < \delta < 1$ gives the capital depreciation rate. We transform the above problem, in per capita terms, by dividing each variable with the total number of native agents in the host country, given by $N_{d,t}$. The problem becomes as follows:

$$\max_{\{c_{d,t}, h^s_{d,t}, h^u_{d,t}, k_{d,t+1}, \frac{b_{d,t}}{P_t}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\alpha_{d,t} \ln(c_{d,t}) + \ln(1-h_{d,t})\right]$$

subject to

1. the sequence of period t budget constraints:

$$c_{d,t} + x_{d,t} + \frac{b_{d,t}}{P_t} \leq w^s_{d,t}\phi_t h^s_{d,t} + w^u_{d,t}(1-\phi_t)h^u_{d,t} + r^k_t k_{d,t} + (1+i_{t-1})\left(\frac{b_{d,t-1}}{P_t}\right)\frac{1}{g^d} + d_{d,t}$$

2. the capital accumulation equation:

$$g^d \cdot k_{d,t+1} = (1-\delta)k_{d,t} + x_{d,t}$$

where, $c_{d,t} = C_{d,t}/N_{d,t}$, $h_{d,t} = H_{d,t}/N_{d,t}$, $h^s_{d,t} = H^s_{d,t}/N^s_{d,t}$, $h^u_{d,t} = H^u_{d,t}/N^u_{d,t}$, $b_{d,t} = B_{d,t}/N_{d,t}$, $w^s_{d,t} = W^s_{d,t}/P_t$, $w^u_{d,t} = W^u_{d,t}/P_t$, $k_{d,t} = K_{d,t}/N_{d,t}$, $x_{d,t} = X_{d,t}/N_{d,t}$, $d_{d,t} = D_{d,t}/N_{d,t}$ give the corresponding per capita variables, and $\phi_t = N^s_{d,t}/N_{d,t}$, $g^d_t = N_{d,t+1}/N_{d,t}$ give the fraction of skilled domestic agents on native - born population and the growth rate of domestic population, respectively. As long as we are interested in immigration inflows, both the ratio ϕ_t and the growth rate of domestic population are assumed constant ($\phi_t = \phi$, $g^d_t \equiv g^d$). In addition, the aggregate domestic labor index in per capita terms takes the form

$$h_{d,t} = \left[\left(\phi_t h^s_{d,t}\right)^{\frac{\sigma+1}{\sigma}} + \left\{(1-\phi_t)h^u_{d,t}\right\}^{\frac{\sigma+1}{\sigma}}\right]^{\frac{\sigma}{\sigma+1}} \tag{2.1}$$

where $h^s_{d,t}$, $h^u_{d,t}$ give the number of labor hours offered by skilled and unskilled domestic workers, respectively.

Solving the above optimization problem we derive the following first order conditions:

$$\frac{\alpha_{d,t}}{c_{d,t}} = \left(\frac{\beta}{g^d}\right)(1+i_t)E_t\left[\frac{\alpha_{d,t+1}}{c_{d,t+1}}\left(\frac{P_t}{P_{t+1}}\right)\right] \tag{2.2}$$

$$\frac{\alpha_{d,t}}{c_{d,t}} = \left(\frac{\beta}{g^d}\right) E_t \left[\frac{\alpha_{d,t+1}}{c_{d,t+1}} \left(r^k_{t+1} + 1 - \delta\right)\right] \tag{2.3}$$

$$\left(\frac{1}{1-h_{d,t}}\right) \left(\frac{\phi_t h^s_{d,t}}{h_{d,t}}\right)^{\frac{1}{\sigma}} = \left(\frac{\alpha_{d,t}}{c_{d,t}}\right) w^s_{d,t} \tag{2.4}$$

$$\left(\frac{1}{1-h_{d,t}}\right) \left(\frac{(1-\phi_t) h^u_{d,t}}{h_{d,t}}\right)^{\frac{1}{\sigma}} = \left(\frac{\alpha_{d,t}}{c_{d,t}}\right) w^u_{d,t} \tag{2.5}$$

$$c_{d,t} + x_{d,t} + \frac{b_{d,t}}{P_t} = w^s_{d,t}\phi_t h^s_{d,t} + w^u_{d,t}(1-\phi_t)h^u_{d,t} + r^k_t k_{d,t} + (1+i_{t-1})\left(\frac{b_{d,t-1}}{P_t}\right)\frac{1}{g^d} + d_{d,t} \tag{2.6}$$

$$g^d \cdot k_{d,t+1} = (1-\delta)k_{d,t} + x_{d,t} \tag{2.7}$$

Condition (2.2) gives the Euler equation of bond holdings which shows that the marginal cost of spending one unit of consumption on bond holdings should be equal with the expected discounted present value of the gross nominal return of bonds in terms of marginal utility units. Accordingly, condition (2.3) gives the Euler equation of capital holdings which states that the marginal cost of spending one unit of consumption on capital stock should be equal with the expected discounted present value of the return of investment (the gross rate of return on capital net of depreciations) in terms of marginal utility units. A combination of the above Euler equations gives the relationship between the real interest rate and the gross nominal interest rate. In log-linearized form this relationship is given by

$$E_t\left(\hat{R}_{t+1}\right) = \hat{r}_t - E_t\left(\hat{\pi}_{t+1}\right)$$

where $\hat{R}_t$ is the gross real interest rate ($R_t = r^k_t + 1 - \delta$), $\hat{r}_t$ the gross nominal interest rate, and $\hat{\pi}_t$ the gross inflation rate in log-linearized terms. The latter condition reflects the arbitrage opportunities between capital ad bond holdings (real versus nominal assets) offered to domestic households.

Conditions (2.4) and (2.5) determine the optimal skilled and unskilled labor supply of domestic households as they equate the marginal rate of substitution between skilled - unskilled labor and consumption with the ratio of their prices, namely the real skilled and unskilled wage, respectively. Finally, conditions (2.6) and (2.7) give the time period budget constraint and the capital accumulation equation, which are necessary for deriving the market clearing condition in good and capital markets. Note that in a symmetric equilibrium where bond market clears ($\frac{b_{d,t}}{P_t} = \frac{b_{d,t-1}}{P_t} = 0$), domestic agents do not hold any nominal asset, and thus bond holdings are dropped out from (2.6).

2.1.2 *Immigrants*

There are two strands on immigration literature about the skill composition of foreign-born population: on the one hand, it is supported that immigration inflows do not alter the skill composition of domestic workforce, and thus skill composition of immigrants resembles that of natives (Peri 2006a,b, Dustmann et al. 2005); on the other hand, empirical studies adopt data that evidence low average schooling of immigrants, which is translated to unskilled immigration inflows (Borjas 2003, Card 2005).

We generalize the present analysis by considering both skilled and unskilled immigration shocks and assuming that foreign-born households are composed by both skilled and unskilled immigrants. Immigrants are imperfect substitutes of domestic workers, even within the same educational or skill group, and thus do not compete with natives for the same job opportunities.

As long as we study mainly short-run immigration inflows, by assumption, immigrants neither invest in capital stock nor save in terms of domestic asset holdings. Their *rule-of-thumb* behavior may be justified by immigrants' limited financial abilities, lack of full information set that natives possess, or potentially illegal political status [2]. In addition, the central motive of immigrants to send a large fraction of their income back to their families, living in their home country, may explain their unwillingness to invest in real or nominal assets in the host-country. For these reasons, we consider immigrants as *rule-of-thumb* agents who consume all of their labor income in the current period. Although immigrants' behavior differs from that of natives, we assume that both representative households have identical preferences. Immigrants derive utility from consumption spending, but disutility from labor supply. In order to ensure a compatible steady-state equilibrium (King et al., 1988), momentary utility function is log-separable in both of its arguments. Immigrants' optimization problem is formulated as follows:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[\alpha_{im,t} \ln(C_{im,t}) + \ln(1 - H_{im,t})\right]$$

subject to a series of time period budget constraints

$$C_{im,t} \leq \left(\frac{W^s_{im,t}}{P_t}\right) H^s_{im,t} + \left(\frac{W^u_{im,t}}{P_t}\right) H^u_{im,t}$$

The aggregate labor index of immigrants is identical to that of natives and is given by

$$H_{im,t} = \left[\left(H^s_{im,t}\right)^{\frac{\sigma+1}{\sigma}} + \left(H^u_{im,t}\right)^{\frac{\sigma+1}{\sigma}}\right]^{\frac{\sigma}{\sigma+1}}$$

where $\sigma > 0$, and $H^s_{im,t}$, $H^u_{im,t}$ denote the total skilled and unskilled labor of immigrants, offered in the production sector. Clearly, the above labor index implies that foreign-born households offer labor in both sectors even in case skilled and unskilled wages are not equal. In other words, we consider a composite foreign-born representative household with a portion of immigrants supplying skilled labor and the rest one being employed in low-skilled jobs. In the extreme case of $\sigma \to \infty$, skilled and unskilled labor would be perfectly substitutable, and immigrants would have devoted all labor to the sector paying the highest wage.

We transform the above problem in per capita terms, by dividing each variable with the measure of total immigrants in the host country given by $N_{im,t}$. The optimization problem of a representative foreign-born household is written as

$$\max_{\{c_{im,t}, h^s_{im,t}, h^u_{im,t}\}} E_0 \sum_{t=0}^{\infty} \beta^t \left[\alpha_{im,t} \ln(c_{im,t}) + \ln(1 - h_{im,t})\right]$$

[2] An analogous assumption was taken by Hazari and Sgro (2003) where wages paid to illegal immigrants were equal to immigrants' consumption spending.

subject to

$$c_{im,t} \leq w^s_{im,t}\lambda_t h^s_{im,t} + w^u_{im,t}(1-\lambda_t)h^u_{im,t} \tag{2.8}$$

where $c_{im,t} = C_{im,t}/N_{im,t}$, $w^s_{im,t} = W^s_{im,t}/P_t$, $w^u_{im,t} = W^u_{im,t}/P_t$, $h^s_{im,t} = H^s_{im,t}/N^s_{im,t}$, $h^u_{im,t} = H^u_{im,t}/N^u_{im,t}$, $\lambda_t = N^s_{im,t}/N_{im,t}$ and

$$h_{im,t} = \left[\left(\lambda_t h^s_{im,t}\right)^{\frac{\sigma+1}{\sigma}} + \left\{(1-\lambda_t)h^u_{im,t}\right\}^{\frac{\sigma+1}{\sigma}}\right]^{\frac{\sigma}{\sigma+1}} \tag{2.9}$$

We treat immigrants' preference weight over consumption $\alpha_{im,t}$ as an exogenous stochastic variable that follows a stationary first order autoregressive process of the form

$$\ln(\alpha_{im,t}) = (1-\rho_{\alpha_{im}})\ln(\overline{a}_{im}) + \rho_{\alpha_{im}}\ln(\alpha_{im,t}) + \varepsilon_{\alpha_{im},t}$$

with $0 \leq \rho_{\alpha_{im}} < 1$, and $\varepsilon_{\alpha_{im},t}$ be a normally distributed error disturbance term with standard deviation $\sigma_{\alpha_{im}} > 0$.

The first order conditions of the representative foreign-born household are the following:

$$\left(\frac{1}{1-h_{im,t}}\right)\left(\frac{\lambda_t h^s_{im,t}}{h_{im,t}}\right)^{\frac{1}{\sigma}} = w^s_{im,t}\left(\frac{\alpha_{im,t}}{c_{im,t}}\right) \tag{2.10}$$

$$\left(\frac{1}{1-h_{im,t}}\right)\left(\frac{(1-\lambda_t)h^u_{im,t}}{h_{im,t}}\right)^{\frac{1}{\sigma}} = w^u_{im,t}\left(\frac{\alpha_{im,t}}{c_{im,t}}\right) \tag{2.11}$$

and the above budget constraint holding as a strict equality, during every period $t = 0, 1, 2, \ldots$ [3]

2.2 Population Dynamics

The total population of the host country is composed by native-born and foreign-born agents: domestic households and immigrants. A fraction (ϕ_t) of domestic households and (λ_t) of immigrants is composed by skilled workers, while the rest one by unskilled agents ($1-\phi_t$ and $1-\lambda_t$, respectively). As we work out the model in per capita terms, it is necessary to develop the population dynamic conditions.

By definition, the share of domestic households in total population (γ_t) is given by the ratio of their total measure $N_{d,t}$ to the total number of agents of the host country N_t; namely:

$$\gamma_t = \frac{N_{d,t}}{N_t}$$

which gives

[3]Numerous studies, for example Djajić (1987), Jahn and Straubhaar (1998), Rivera-Batiz (1999), Hazari and Sgro (2003), Moy and Yip (2005), underline and adopt the "wage discrimination" against immigrants, which is mainly observed in wage earnings of unskilled, illegal immigrants; namely, $w^u_{im,t} \equiv \beta^w_t w^u_{im,t}$, with discrimination coefficient given by $0 < \beta^w_t < 1$. We incorporated in the present analysis the "wage exploitation" of unskilled immigrants, by considering the exploitation measure, given by $1 \geq \tau_t = 1 - \beta^w_t \geq 0$, as a stochastic variable that follows a white noise process. The impulse responses showed a negative but infinitesimal response of inflation rate.

$$\gamma_t = \left(\frac{g_t^d}{g_t^T}\right)\gamma_{t-1} \tag{2.12}$$

By assumption the gross growth rate of native-born workers ($g_t^d = N_{d,t}/N_{d,t-1}$) is constant over time, that is $g_t^d = g^d$ and equals unity ($N_{d,t} = N_{d,t-1}$). The term $g_t^T = N_t/N_{t-1}$ determines the gross growth rate of the total population. Using the definition of aggregate population, which is the sum of the measure of each class of agents, that is

$$N_t = N_{d,t} + N_{im,t}$$

we derive a relationship between the gross growth rate of aggregate population and the growth rates of the corresponding classes of agents, as follows: [4]

$$g_t^T = g^d\gamma_{t-1} + g_t^{im}(1-\gamma_{t-1}) \tag{2.13}$$

Equivalently, foreign-born population is composed by skilled and unskilled workers. That is,

$$N_{im,t} = N_{im,t}^s + N_{im,t}^u$$

which gives the definition of growth rate of immigrants' population in the host country:

$$g_t^{im} = g_{im,t}^s\lambda_{t-1} + g_{im,t}^u\left(1-\lambda_{t-1}\right) \tag{2.14}$$

The fraction of skilled immigrants on foreign - born population (λ_t) is defined as

$$\lambda_t = \frac{N_{im,t}^s}{N_{im,t}}$$

which may be rewritten as follows:

$$\lambda_t = \left(\frac{g_{im,t}^s}{g_t^{im}}\right)\lambda_{t-1} \tag{2.15}$$

2.2.1 *Temporary immigration shock*

We assume that immigration shock involves an influx of immigrants, either skilled ($N_{im,t}^s$) or unskilled ($N_{im,t}^u$). By considering the number of native agents constant in the short-run ($N_{d,t} = \overline{N}_d$), the number of agents in the total population is written as $N_t = \overline{N}_d + N_{im,t}$, and thus the fraction of natives on total population becomes

$$\gamma_t = \frac{\overline{N}_d}{\overline{N}_d + N_{im,t}} \tag{2.16}$$

Accordingly, by definition, the fraction of skilled workers on foreign-born population is given by

$$\lambda_t = \frac{N_{im,t}^s}{N_{im,t}^s + N_{im,t}^u} \tag{2.17}$$

[4] $N_t = N_{d,t} + N_{im,t} \Rightarrow \frac{N_t}{N_{t-1}} = \frac{N_{d,t}}{N_{d,t-1}}\frac{N_{d,t-1}}{N_{t-1}} + \frac{N_{im,t}}{N_{im,t-1}}\frac{N_{im,t-1}}{N_{t-1}} \Rightarrow g_t^T = g_t^d\gamma_{t-1} + g_t^{im}(1-\gamma_{t-1})$ and $g_t^d = g^d$.

with $N_{im,t} = N^s_{im,t} + N^u_{im,t}$ determining the number of immigrants on the host country, composed by skilled ($N^s_{im,t}$) and unskilled ($N^u_{im,t}$) workers.

Skilled immigrants influx We examine an influx of skilled immigrants separately from unskilled immigration inflows in order to identify any potential difference between these two kinds of temporary shocks on the macroeconomic magnitudes of the host country. We assume that in the short-run the number of unskilled immigrants remains fixed, while skilled immigrants ($N^s_{im,t}$) enter the country unexpectedly. Therefore, we model

$$N^u_{im,t} = \overline{N}^u_{im} \quad \text{and} \quad N^s_{im,t} = \mu_t \overline{N}^s_{im} \tag{2.18}$$

where the exogenous variable μ_t follows a stationary first-order autoregressive process of the form

$$\ln(\mu_t) = \rho_\mu \ln(\mu_{t-1}) + \varepsilon_{\mu,t}$$

with $0 \leq \rho_\mu < 1$, and $\varepsilon_{\mu,t}$ be a normally distributed error disturbance term with standard deviation $\sigma_\mu > 0$.

Unskilled immigrants influx Equivalently, in order to examine an unexpected influx of unskilled immigrants only, we assume that, in the short-run, the number of skilled immigrants remains constant and unskilled foreigners ($N^u_{im,t}$) enter the country. That is,

$$N^s_{im,t} = \overline{N}^s_{im} \quad \text{and} \quad N^u_{im,t} = \mu_t \overline{N}^u_{im} \tag{2.19}$$

where the exogenous variable μ_t follows the stationary first-order autoregressive process described above.

2.2.2 *Permanent immigration shock*

Permanent immigration shocks alter both the skill composition of the foreign-born population in the long-run, and the ratio of natives on total population of the host country. In each case, permanent immigration inflows are introduced by considering as a stochastic variable the growth rate of each foreign-born population rather than the number of the corresponding foreign-born agents.

Skilled immigration shock We introduce in the host country a skilled immigration shock by assuming that the gross growth rate of skilled immigrants $g^s_{im,t}$ is given by

$$g^s_{im,t} = g^u_{im,t} \cdot \mu_t \tag{2.20}$$

and the gross growth rate of unskilled immigrants constant

$$g^u_{im,t} = g^u_{im} \tag{2.21}$$

The term μ_t is an exogenous variable that follows a stationary first order autoregressive process of the form

$$\ln(\mu_t) = \rho_\mu \ln(\mu_{t-1}) + \varepsilon_{\mu,t}$$

with $0 \leq \rho_\mu < 1$, and $\varepsilon_{\mu,t}$ be a normally distributed error disturbance term with standard deviation $\sigma_\mu > 0$.

Unskilled immigration shock Alternatively, we insert an unskilled immigration shock by assuming that the gross growth rate of unskilled immigrants $g^u_{im,t}$ is given by

$$g^u_{im,t} = g^s_{im,t} \cdot \mu_t \tag{2.22}$$

and the gross growth rate of skilled immigrants is constant

$$g^s_{im,t} = g^s_{im} \tag{2.23}$$

with the exogenous term μ_t described above.

2.3 The supply side

The supply side of the economy is composed by $j\epsilon[0,1]$ intermediate goods firms, each one producing a differentiated product in monopolistically competitive markets. The representative intermediate goods firm hires total labor, and rents capital from natives. Total labor is a CES aggregate of skilled and unskilled one, offered by natives and immigrants. Also, there is a representative final good-producing firm, operating in a perfectly competitive market, that uses the intermediate goods $j\epsilon[0,1]$ as inputs so as to produce the final homogeneous good Y_t.

2.3.1 *Final good-producing firm*

The finished good-producing firm uses $Y_t(j)$ units of each intermediate good $j\epsilon[0,1]$ as inputs, purchased at price $P_t(j)$, to produce and sell Y_t units of the homogeneous final good. The constant returns to scale production technology of the finished good Y_t is of Dixit-Stiglitz (1977) form, given by

$$\left[\int_0^1 Y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}} \geq Y_t$$

In per capita terms, the above production function becomes

$$\left[\int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}} \geq y_t$$

where $y_t = Y_t/N_t$ denotes per capita output, and $\varepsilon > 1$ represents the elasticity of substitution among the intermediate goods $j\epsilon[0,1]$. The higher value ε takes, the more competitive the market of intermediate goods becomes.

The final good-producing firm minimizes the total production cost subject to the above constant returns to scale technology. Specifically, the optimization problem is written as

$$\min_{\{y_t(j)\}} \int_0^1 \left[P_t(j) y_t(j)\right] dj$$

subject to

$$\left[\int_0^1 y_t(j)^{\frac{\varepsilon-1}{\varepsilon}}dj\right]^{\frac{\varepsilon}{\varepsilon-1}} \geq y_t$$

In aggregate terms, the first order condition is given by

$$Y_t(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\varepsilon} Y_t \tag{2.24}$$

and in per capita terms by

$$y_t(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\varepsilon} y_t \tag{2.25}$$

for all $j\epsilon[0,1]$ and $t = 0,1,2,\ldots$ The latter conditions define the demand for the intermediate good $j\epsilon[0,1]$ by the finished good-producing firm, satisfied during every period by the intermediate good-producing firm. Finally, the perfectly competitive nature of the final good market implies a zero profit condition, which yields

$$P_t = \left(\int_0^1 P_t(j)^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}}$$

where P_t is the aggregate price level, and $P_t(j)$ is the price of each intermediate good $j\epsilon[0,1]$.

2.3.2 *Intermediate good-producing firm*

In standard New Keynesian models there is a continuum of intermediate good firms, indexed in the unit interval $j\epsilon[0,1]$, each one producing a differentiated product j. Every firm j uses labor and capital as inputs of production, and charges a price $P_t(j)$ for its product in a staggered fashion. The monopolistic nature of intermediate good sector allows each firm to charge a price $P_t(j)$ without loosing a significant fraction of sales. Staggered price setting can be inserted either by assuming a quadratic cost of nominal price adjustment that every intermediate good firm faces, as Rotemberg (1982) described, or by assuming that each firm $j\epsilon[0,1]$ resets its price whenever receives a random exogenous signal with probability $1-\theta$, as Calvo (1983) described.

Cost minimization problem First, intermediate good-producing firms determine the optimal quantity of production inputs, labor and capital, in order to define their optimal production cost. Note that there are so many intermediate firms so that each one is unable to affect the wages of labor and the price of capital. Hence, real wages and the rental price of capital are taken as given by each firm $j\epsilon[0,1]$. For simplicity purposes, the index $j\epsilon[0,1]$ is ignored by the following specific problem that each representative firm $j\epsilon[0,1]$ faces. Firm minimizes the total real cost of production

$$\min \quad \left\{ \quad w^s_{d,t}H^s_{d,t} + w^u_{d,t}H^u_{d,t} + w^s_{im,t}H^s_{im,t} + w^u_{im,t}H^u_{im,t} + r^k_t K_t \quad \right\}$$

subject to the production technology

$$Y_t = Z_t\left[\zeta H_{s,t}^{1-\rho} + H_{u,t}^{1-\rho}\right]^{\frac{a}{1-\rho}} K_t^{1-a}$$

where K_t denotes the aggregate capital stock, $a\epsilon(0,1)$ is the share of efficient labor H_t^e on total output, $\zeta > 1$ reflects the productivity difference between skilled and unskilled labor, and $\rho \geq 0$ is the inverse of the elasticity of substitution between skilled and unskilled labor. We distinguish labor between skilled and unskilled one, as long as immigrants are concentrated on skilled or unskilled jobs (Peri 2006a, b). Obviously, the production function of intermediate good-producing firm $j\epsilon[0,1]$ is a Cobb-Douglas function of constant returns to scale, with two inputs of production: efficient labor (H_t^e) and capital (K_t). Thus, we may write:

$$Y_t(j) = Z\left[H_t^e\right]^a K_t^{1-a}$$

where $H_t^e = \left[\zeta H_{s,t}^{1-\rho} + H_{u,t}^{1-\rho}\right]^{\frac{1}{1-\rho}}$ defines aggregate labor in efficiency units. In the production function, the term Z_t reflects a stochastic aggregate productivity disturbance that follows by assumption an exogenous, stationary, first order autoregressive process

$$\ln(Z_t) = (1-\rho_z)\ln(\overline{Z}) + \rho_z \ln(Z_{t-1}) + \varepsilon_{z,t}$$

with $0 \leq \rho_z < 1$ and $\varepsilon_{z,t}$ be a normally distributed disturbance term with standard deviation $\sigma_z > 0$.

According to Peri (2006a), immigrants and native workers are imperfect substitutes, because they differ in educational attainment, skill levels, and language proficiency. Even in case natives and foreigners have the same educational level and skill abilities, immigrants' experience, cultural origin, characteristics and motivation, distinguish them from the native-born workers. As a result, we use a simplified version of Ottaviano - Peri (2005, 2006) production function, in which skilled ($H_{s,t}$) and unskilled ($H_{u,t}$) labor are considered as a CES aggregate of natives and immigrants; that is

$$H_{s,t} = \left[\omega_s\left(H_{d,t}^s\right)^{1-\rho_1} + \left(H_{im,t}^s\right)^{1-\rho_1}\right]^{\frac{1}{1-\rho_1}}$$

and

$$H_{u,t} = \left[\omega_u\left(H_{d,t}^u\right)^{1-\rho_2} + \left(H_{im,t}^u\right)^{1-\rho_2}\right]^{\frac{1}{1-\rho_2}}$$

The parameter $\rho_1 > 0$ determines the elasticity of substitution between native-born and foreign-born skilled workers, and $\rho_2 > 0$ determines the substitutability between unskilled natives and unskilled immigrants. The alternative viewpoint that natives and immigrants are perfect substitutes (Card, 2005) can also be adopted by setting $\rho_1 = 0$, and $\rho_2 = 0$. In this case, skilled and unskilled labor become linear functions.

Also, parameters $\omega_s > 0$, and $\omega_u > 0$ reflect productivity differences between natives and immigrants, for skilled and unskilled population, respectively. Ottaviano - Peri (2005) adopt the hypothesis that there is no productivity difference between natives and immigrants within each skill-educational group, which implies $\omega_s = \omega_u = 1$. Nevertheless, a range of values for the coefficients of productivity differences between native-born and foreign-born workers can be examined in order to implement a comparative static analysis.

By writing the above problem in per capita terms we have:

$$\min_{\{h_{d,t}^s, h_{d,t}^u, h_{im,t}^s, h_{im,t}^u, k_t\}} \left\{ w_{d,t}^s \phi_t \gamma_t h_{d,t}^s + \right.$$
$$\left. + w_{d,t}^u (1-\phi_t)\gamma_t h_{d,t}^u + w_{im,t}^s \lambda_t (1-\gamma_t) h_{im,t}^s + w_{im,t}^u (1-\lambda_t)(1-\gamma_t) h_{im,t}^u + r_t^k k_t \right\}$$

subject to

$$y_t = Z_t \left[h_t^e\right]^a k_t^{1-a}$$

where

$$h_t^e = \left[\zeta h_{s,t}^{1-\rho} + h_{u,t}^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{2.26}$$

$$h_{s,t} = \left[\omega_s \left\{\gamma_t \phi_t h_{d,t}^s\right\}^{1-\rho_1} + \left\{(1-\gamma_t)\lambda_t h_{im,t}^s\right\}^{1-\rho_1}\right]^{\frac{1}{1-\rho_1}} \tag{2.27}$$

$$h_{u,t} = \left[\omega_u \left\{\gamma_t (1-\phi_t) h_{d,t}^u\right\}^{1-\rho_2} + \left\{(1-\gamma_t)(1-\lambda_t) h_{im,t}^u\right\}^{1-\rho_2}\right]^{\frac{1}{1-\rho_2}} \tag{2.28}$$

give the per capita efficient labor, the total skilled and unskilled hours, respectively. The first order conditions of the above cost minimization problem are summarized by the following:

$$\frac{w_{d,t}^s}{w_{im,t}^s} = \omega_s \left[\frac{(1-\gamma_t)\lambda_t h_{im,t}^s}{\gamma_t \phi_t h_{d,t}^s}\right]^{\rho_1} \tag{2.29}$$

$$\frac{w_{d,t}^s}{w_{d,t}^u} = \zeta \left(\frac{\omega_s}{\omega_u}\right) h_{s,t}^{\rho_1-\rho} h_{u,t}^{\rho-\rho_2} \gamma_t^{\rho_2-\rho_1} \left[\phi_t h_{d,t}^s\right]^{-\rho_1} \left[(1-\phi_t) h_{d,t}^u\right]^{\rho_2} \tag{2.30}$$

$$\frac{w_{d,t}^s}{w_{im,t}^u} = \zeta \omega_s h_{s,t}^{\rho_1-\rho} h_{u,t}^{\rho-\rho_2} \left[\gamma_t \phi_t h_{d,t}^s\right]^{-\rho_1} \left[(1-\gamma_t)(1-\lambda_t) h_{im,t}^u\right]^{\rho_2} \tag{2.31}$$

$$\frac{w_{d,t}^s}{r_t^k} = \zeta \omega_s \left(\frac{a}{1-a}\right) \left(h_t^e\right)^{\rho-1} k_t h_{s,t}^{\rho_1-\rho} \left[\gamma_t \phi_t h_{d,t}^s\right]^{-\rho_1} \tag{2.32}$$

and the production function

$$y_t = Z_t \left[h_t^e\right]^a k_t^{1-a} \tag{2.33}$$

Pricing decision problem One may now turn to the pricing decision problem of the representative intermediate good-producing firm $j\epsilon[0,1]$ which gives the time pattern of aggregate price level in a symmetric equilibrium. Due to monopolistically competitive markets, each firm is able to choose a unique price $P_t(j)$ for its product in order to maximize the expected discounted value of its profits. Nominal price rigidities are introduced in the present framework by assuming á la Calvo (1983) staggered price setting.

According to Calvo (1983), each firm resets the price $P_t(j)$ whenever receives a random signal with probability $1-\theta$, which is independent from the time elapsed since the last adjustment and the pricing decisions of other firms. As a result, during every period t, a fraction $1-\theta$ of intermediate firms receives the random signal and resets its price, while the remaining fraction of firms $\theta\epsilon(0,1)$ charges a price equal to the general price index of the previous period, that is $P_t(j) = \pi P_{t-1}$ (static indexation formulation). The representative intermediate good firm chooses a price $P_t(j)$ for its product so as to maximize the expected sum of its future profits, discounted by the pricing kernel and the probability that its optimal price will remain fixed in the future. Thus,

$$\max_{\{P_t(j)\}} E_t \sum_{i=0}^{\infty} \theta^i \Lambda_{t+i} \left[D_{t+i}(j)\right]$$

where

$$D_t(j) \equiv D_{d,t}(j) = \left[\frac{P_t(j)}{P_t}\right] Y_t(j) - \left[MC_t(j)^*\right] Y_t(j)$$

determines real profits of period t, in aggregate terms. As long as natives are the exclusive owners of domestic firms, in effect, real profits are earned only by natives, and as a consequence the subjective discount factor of real profits, given by the term Λ_{t+i}, is the marginal utility value to representative domestic household of an additional unit of profits. Thus, the pricing kernel is given by

$$\Lambda_{t+i} = \beta^i \left(\frac{\alpha_{d,t+i}}{C_{d,t+i}}\right)$$

In per capita terms, representative firm $j\epsilon[0,1]$ maximizes

$$\max_{\{P_t(j)\}} E_t \sum_{i=0}^{\infty} \theta^i \beta^i \left(\frac{\alpha_{d,t+i}}{C_{d,t+i}}\right) N_{d,t+i} \left(\frac{1}{N_{d,t+i}}\right) \left[\left(\frac{P_t(j)}{P_{t+i}}\right) Y_{t+i}(j) - MC^*_{t+i}(j) Y_{t+i}(j)\right]$$

or

$$\max_{\{P_t(j)\}} E_t \sum_{i=0}^{\infty} \theta^i \beta^i \left(\frac{\alpha_{d,t+i}}{c_{d,t+i}}\right) \left(\frac{1}{\gamma_{t+i}}\right) \left[\left(\frac{P_t(j)}{P_{t+i}}\right) y_{t+i}(j) - mc^*_{t+i}(j) y_{t+i}(j)\right]$$

subject to the demand for product $j\epsilon[0,1]$ by the final good firm, given by condition (2.25). The first order condition of the above problem

$$\frac{P_t^*}{P_t} = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \frac{E_t \sum_{i=0}^{\infty} \theta^i \beta^i \left[\alpha_{d,t+i} c_{d,t+i}^{-1} mc_{t+i} \left(\frac{P_{t+i}}{\pi^i P_t}\right)^{\varepsilon} y_{t+i} \gamma_{t+i}^{-1}\right]}{E_t \sum_{i=0}^{\infty} \theta^i \beta^i \left[\alpha_{d,t+i} c_{d,t+i}^{-1} \left(\frac{P_{t+i}}{\pi^i P_t}\right)^{\varepsilon - 1} y_{t+i} \gamma_{t+i}^{-1}\right]} \tag{2.34}$$

along with the condition of aggregate price level dynamics

$$P_t^{1-\varepsilon} = \theta \left(\pi P_{t-1}\right)^{1-\varepsilon} + (1-\theta) \left(P_t^*\right)^{1-\varepsilon} \tag{2.35}$$

determine the equilibrium conditions of the supply side of the economy.

In case there were no nominal price rigidities, the above pricing decision condition would have reduced to

$$\frac{P_t(j)}{P_t} = mc_t^* \left(\frac{\varepsilon}{\varepsilon - 1}\right)$$

which states that the price $P_t(j)$ of an individual firm $j\epsilon[0,1]$ relatively to the aggregate price index P_t is a mark-up ($m_t = \varepsilon/(\varepsilon - 1) > 1$) on its marginal cost of production. This comes as a consequence of the monopolistically competitive nature of the intermediate goods markets. The elasticity of substitution among intermediate goods ($\varepsilon > 1$) determines negatively the mark-up, which means that the higher the elasticity ε becomes, the lower value the mark-up of prices on marginal cost takes.

2.4 Aggregation

Given that total population is composed by native-born and foreign-born households, each per capita aggregate variable is a weighted average of the corresponding disaggregated magnitudes. For example, total consumption is the sum of natives and immigrants' one,

$$C_t = C_{d,t} + C_{im,t}$$

which in per capita terms is given by

$$c_t = \gamma_t \cdot c_{d,t} + (1 - \gamma_t) c_{im,t}$$

Taking into account that immigrants do not invest in capital holdings, aggregate per capita capital stock (k_t) and investment (x_t) are defined as follows:

$$k_t = \gamma_t \cdot k_{d,t} + (1 - \gamma_t) k_{im,t} \Rightarrow k_t = \gamma_t \cdot k_{d,t} \quad \text{since} \quad k_{im,t} = 0$$

$$x_t = \gamma_t \cdot x_{d,t} + (1 - \gamma_t) x_{im,t} \Rightarrow x_t = \gamma_t \cdot x_{d,t} \quad \text{since} \quad x_{im,t} = 0$$

In addition, aggregate profits of intermediate good-producing firms are distributed only among domestic households. In other words, there is no portion of total profits that goes to immigrants; thus, $D_{im,t} = 0$. Therefore, we may write

$$D_t = D_{d,t} + D_{im,t} \quad \Rightarrow \quad D_t = D_{d,t}$$

which in per capita terms becomes

$$\frac{D_t}{N_t} \frac{N_t}{N_{d,t}} = \frac{D_{d,t}}{N_{d,t}} \quad \Rightarrow \quad d_t \left(\frac{1}{\gamma_t} \right) = d_{d,t}$$

where d_t defines the dividend share in case all agents had access to profits allocation, and $d_{d,t}$ determines the dividend share earned by each native agent.

Accordingly, given that immigrants do not have access to contingent nominal asset markets, total bonds in the host country equal bond holdings of domestic households. Thus,

$$B_t = B_{d,t} \qquad \text{or} \qquad b_t = \gamma_t b_{d,t}$$

2.5 Monetary policy

We consider an independent central banker who conducts monetary policy by simply following a modified Taylor rule of the form

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1 - \rho_r) \phi_\pi \hat{\pi}_t + (1 - \rho_r) \phi_y \hat{y}_t + v_t \tag{2.36}$$

where the exogenous variable $v_t = \varepsilon_{r,t}$ is a white noise with $\varepsilon_{r,t} \sim iid(0, \sigma_r^2)$, which represents the unexpected increase of gross nominal interest rate by the central bank (unexpected monetary policy). Coefficients $\phi_\pi > 0$ and $\phi_y > 0$ satisfy the Taylor principle, and an interest rate smoothing component ($0 < \rho_r < 1$) is included. Following Rabanal (2007) we take $\rho_r = 0.8$, $\phi_\pi = 1.5$, and $\phi_y = 0.1$. Note that for the calibrated values of the above Taylor rule coefficients, the existence of a unique equilibrium for the present rational expectations model is guaranteed (Gali et al. 2004).

2.6 Market Clearing conditions

In a symmetric equilibrium, all agents take identical decisions and all markets clear. As a result, $h^s_{d,t}(j) = h^s_{d,t}$, $h^u_{d,t}(j) = h^u_{d,t}$, $h^s_{im,t}(j) = h^s_{im,t}$, $h^u_{im,t}(j) = h^u_{im,t}$, $h^e_t(j) = h^e_t$, $k_t(j) = k_t$, $mc^*_t(j) = mc^*_t$, $y_t(j) = y_t$, $d_{d,t}(j) = d_{d,t}$, $P_t(j) = P_t$, for all $j\epsilon[0,1]$ and $t = 0, 1, 2, \ldots$

Clearing in bond market requires $B_t = B_{t-1} = 0$ or accordingly $b_{d,t} = b_{d,t-1}(1/g^d) = 0$. Taking into account the above, the market clearing condition (aggregate resource constraint) becomes

$$y_t = \gamma_t c_{d,t} + (1-\gamma_t)c_{im,t} + \gamma_t x_{d,t}$$

3 Solving the model

3.1 Symmetric equilibrium

In a symmetric equilibrium all markets clear and all agents take identical decisions. Thus, the equilibrium conditions of the model are summarized as follows:

Domestic Households:

$$c_{d,t} + x_{d,t} = w^s_{d,t}\phi h^s_{d,t} + w^u_{d,t}(1-\phi)h^u_{d,t} + r^k_t k_{d,t} + d_{d,t} \tag{3.1}$$

$$g^d \cdot k_{d,t+1} = (1-\delta)k_{d,t} + x_{d,t} \tag{3.2}$$

$$\frac{\alpha_{d,t}}{c_{d,t}} = \left(\frac{\beta}{g^d}\right) r_t E_t\left[\frac{\alpha_{d,t+1}}{c_{d,t+1}}\left(\frac{1}{\pi_{t+1}}\right)\right] \quad \text{with} \quad \pi_{t+1} = \frac{P_{t+1}}{P_t} \quad \text{and} \quad r_t = 1 + i_t \tag{3.3}$$

$$\frac{\alpha_{d,t}}{c_{d,t}} = \frac{\beta}{g^d}E_t\left[\left(\frac{\alpha_{d,t+1}}{c_{d,t+1}}\right)R_{t+1}\right] \quad \text{with} \quad R_{t+1} = r^k_{t+1} + 1 - \delta \tag{3.4}$$

$$\left(\frac{1}{1-h_{d,t}}\right)\left(\frac{\phi h^s_{d,t}}{h_{d,t}}\right)^{\frac{1}{\sigma}} = \left(\frac{\alpha_{d,t}}{c_{d,t}}\right) w^s_{d,t} \tag{3.5}$$

$$\left(\frac{1}{1-h_{d,t}}\right)\left(\frac{(1-\phi)h^u_{d,t}}{h_{d,t}}\right)^{\frac{1}{\sigma}} = \left(\frac{\alpha_{d,t}}{c_{d,t}}\right) w^u_{d,t} \tag{3.6}$$

$$h_{d,t} = \left[\left(\phi \cdot h^s_{d,t}\right)^{\frac{\sigma+1}{\sigma}} + \left\{(1-\phi)h^u_{d,t}\right\}^{\frac{\sigma+1}{\sigma}}\right]^{\frac{\sigma}{\sigma+1}} \tag{3.7}$$

Immigrants:

$$c_{im,t} = w^s_{im,t}\lambda_t h^s_{im,t} + w^u_{im,t}(1-\lambda_t)h^u_{im,t} \tag{3.8}$$

$$\left(\frac{1}{1-h_{im,t}}\right)\left(\frac{\lambda_t h^s_{im,t}}{h_{im,t}}\right)^{\frac{1}{\sigma}} = w^s_{im,t}\left(\frac{\alpha_{im,t}}{c_{im,t}}\right) \tag{3.9}$$

$$\left(\frac{1}{1-h_{im,t}}\right)\left(\frac{(1-\lambda_t)h^u_{im,t}}{h_{im,t}}\right)^{\frac{1}{\sigma}} = w^u_{im,t}\left(\frac{\alpha_{im,t}}{c_{im,t}}\right) \tag{3.10}$$

$$h_{im,t} = \left[\left(\lambda_t h^s_{im,t}\right)^{\frac{\sigma+1}{\sigma}} + \left\{(1-\lambda_t)h^u_{im,t}\right\}^{\frac{\sigma+1}{\sigma}}\right]^{\frac{\sigma}{\sigma+1}} \tag{3.11}$$

Population Dynamics:

$$\gamma_t = \left(\frac{g^d}{g^T_t}\right)\gamma_{t-1} \tag{3.12}$$

$$g^T_t = g^d\gamma_{t-1} + g^{im}_t(1-\gamma_{t-1}) \tag{3.13}$$

$$g^{im}_t = g^s_{im,t}\lambda_{t-1} + g^u_{im,t}\left(1-\lambda_{t-1}\right) \tag{3.14}$$

$$\lambda_t = \left(\frac{g^s_{im,t}}{g^{im}_t}\right)\lambda_{t-1} \tag{3.15}$$

Supply Side:

$$\frac{w^s_{d,t}}{w^s_{im,t}} = \omega_s\left[\frac{(1-\gamma_t)\lambda_t h^s_{im,t}}{\gamma_t\phi h^s_{d,t}}\right]^{\rho_1} \tag{3.16}$$

$$\frac{w^s_{d,t}}{w^u_{d,t}} = \zeta\left(\frac{\omega_s}{\omega_u}\right)h^{\rho_1-\rho}_{s,t}h^{\rho-\rho_2}_{u,t}\gamma^{\rho_2-\rho_1}_t\left[\phi h^s_{d,t}\right]^{-\rho_1}\left[(1-\phi)h^u_{d,t}\right]^{\rho_2} \tag{3.17}$$

$$\frac{w^s_{d,t}}{w^u_{im,t}} = \zeta\omega_s h^{\rho_1-\rho}_{s,t}h^{\rho-\rho_2}_{u,t}\left[\gamma_t\phi h^s_{d,t}\right]^{-\rho_1}\left[(1-\gamma_t)(1-\lambda_t)h^u_{im,t}\right]^{\rho_2} \tag{3.18}$$

$$\frac{w^s_{d,t}}{r^k_t} = \zeta\omega_s\left(\frac{a}{1-a}\right)(h^e_t)^{\rho-1}k_t h^{\rho_1-\rho}_{s,t}\left[\gamma_t\phi h^s_{d,t}\right]^{-\rho_1} \tag{3.19}$$

$$y_t = Z_t\left[h^e_t\right]^a k^{1-a}_t \tag{3.20}$$

$$h^e_t = \left[\zeta h^{1-\rho}_{s,t} + h^{1-\rho}_{u,t}\right]^{\frac{1}{1-\rho}} \tag{3.21}$$

$$h_{s,t} = \left[\omega_s\left\{\gamma_t\phi h^s_{d,t}\right\}^{1-\rho_1} + \left\{(1-\gamma_t)\lambda_t h^s_{im,t}\right\}^{1-\rho_1}\right]^{\frac{1}{1-\rho_1}} \tag{3.22}$$

$$h_{u,t} = \left[\omega_u \left\{\gamma_t(1-\phi)h^u_{d,t}\right\}^{1-\rho_2} + \left\{(1-\gamma_t)(1-\lambda_t)h^u_{im,t}\right\}^{1-\rho_2}\right]^{\frac{1}{1-\rho_2}} \tag{3.23}$$

$$\frac{P^*_t}{P_t} = \left(\frac{\varepsilon}{\varepsilon-1}\right) \frac{E_t \sum_{i=0}^{\infty} \theta^i \beta^i \left[\alpha_{d,t+i} c^{-1}_{d,t+i} mc_{t+i} \left(\frac{P_{t+i}}{\pi^i P_t}\right)^{\varepsilon} y_{t+i} \gamma^{-1}_{t+i}\right]}{E_t \sum_{i=0}^{\infty} \theta^i \beta^i \left[\alpha_{d,t+i} c^{-1}_{d,t+i} \left(\frac{P_{t+i}}{\pi^i P_t}\right)^{\varepsilon-1} y_{t+i} \gamma^{-1}_{t+i}\right]} \tag{3.24}$$

$$P^{1-\varepsilon}_t = \theta\left(\pi P_{t-1}\right)^{1-\varepsilon} + (1-\theta)\left(P^*_t\right)^{1-\varepsilon} \tag{3.25}$$

$$mc_t = \frac{r^k_t k_t}{(1-a)\, y_t} \tag{3.26}$$

$$d_{d,t} = \frac{1}{\gamma_t}\left[y_t - mc_t \cdot y_t\right] \tag{3.27}$$

Taylor Rule:

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1-\rho_r)\phi_\pi \hat{\pi}_t + (1-\rho_r)\phi_y \hat{y}_t + \varepsilon_{r,t} \tag{3.28}$$

Since the model cannot be solved for analytically, we log-linearize the above system of equilibrium conditions around the steady-state, and, then, we solve it according to the complex generalized Schur decomposition method presented by Klein (2000).

3.2 Baseline Calibration

We calibrate the model to a quarterly frequency by setting reasonable values for the "deep" parameters. With regard to preference parameters, we set the subjective discount factor β equal to 0.99, which implies a value of the steady-state annual real interest rate equal to 4%.

With respect to labor market, we assume that each type of worker spends one third of her total time endowment to labor activities (Ghez and Becker 1975, Hansen 1985) that is $\overline{h}^s_d = \overline{h}^u_d = \overline{h}^s_{im} = \overline{h}^u_{im} = 1/3$. We also take $\sigma = 1$ (Bouakez et al., 2005), the coefficient of productivity difference between skilled and unskilled labor equal to $\zeta = 1.3$ (Canova - Ravn, 1997) [5], the coefficients of productivity difference between native-born and foreign-born skilled and unskilled workers equal to $\omega_s = \omega_u = 1$. In other words, we adopt the standard assumption that natives and immigrants within the same skill-educational group have the same labor efficiency (Ottaviano - Peri, 2005, 2006).

With respect to labor substitutability parameters, we take $\rho = 0.67$, which implies an imperfect substitutability between skilled and unskilled labor (the elasticity of substitution between skilled and unskilled labor is given by $\sigma = 1/\rho$) [6], and also $\rho_1 = 0.2$, $\rho_2 = 0.1$. The calibrated values of ρ_1 and ρ_2 denote an

[5] Canova - Ravn (2000) take $\zeta = 2$, which is considered a reasonable upper bound for this parameter according to Kydland and Rios-Rull (Canova-Ravn 2000: 440).

[6] Ottaviano - Peri (2005) estimate the elasticity of substitution between group of workers which is found on the interval [1.5, 2]. Such values imply an interval [0.5, 0.67] for parameter ρ. Also, Canova - Ravn (1997) take two alternative values for labor substitutability: $\rho = 0$, where the two types of labor are perfect substitutes, and $\rho = 0.25$, where there is a moderate degree of substitutability. Finally, Canova - Ravn (2000) set either $\rho = 0$ or $\rho = 0.5$.

elasticity of substitution between natives and immigrants for skilled and unskilled labor equal to $\sigma_k = 5$ and $\sigma_k = 10$, respectively (Ottaviano - Peri, 2005), which means that immigrants within a skill-educational group are not perfect substitutes of native-born agents, but between the two groups unskilled workers are more substitutable.

The elasticity of substitution among intermediate goods is set equal to $\varepsilon = 6$ which, in a flexible price equilibrium regime, implies a constant mark-up of prices on marginal cost equal to 20% ($\overline{m} = 1.2$)[7]. Also, the average duration of optimally charged prices is one year; this is translated to a fraction of firms that retain constant prices equal to $\theta = 0.75$.

About population dynamics, the steady-state fraction of optimizing domestic households on total population is set $\overline{\gamma} = 0.75$ (Canova-Ravn, 1997), that is 75% of the total population of the host country consists of optimizing domestic households while the remaining 25% are rule-of-thumb immigrants. The growth rate of domestic population is considered constant and equal to unity; namely, $g_t^d \equiv g^d = 1$ which implies a stationary domestic population: $N_{d,t} = N_{d,t-1}, \quad \forall \quad t = 0, 1, 2, \ldots$ The portion of highly skilled natives on domestic population is assumed 40%, and a half of foreign-born agents are unskilled workers.

Finally, the depreciation rate of capital is set $\delta = 0.025$, which means an annual 10% depreciation, and the share of labor on total output is assumed to be $a = 0.64$. According to Ireland (2004), the steady-state values of aggregate productivity and inflation rate leave the dynamics of the model unaffected. Thus, we may set $\overline{Z} = 1.0048$, and $\overline{\pi} = 1.0086$ which are the mean values of the corresponding magnitudes, using quarterly data of the US economy for the period 1948:1 - 2003:1.

For $\overline{\gamma} \neq 0$, condition $\overline{\gamma} = \left(\frac{g^d}{\overline{g}^T}\right)\overline{\gamma}$ yields $\overline{g}^T = g^d = 1$. Then, substituting $\overline{g}^T = g^d = 1$ in the definition $\overline{g}^T = g^d\overline{\gamma} + \overline{g}^{im}(1-\overline{\gamma})$ we take for $\overline{\gamma} \neq 1$ that $\overline{g}^{im} = \overline{g}^d = 1$. From the ratio of skilled workers on foreign - born population and for $\overline{\lambda} \neq 0$ we derive $\overline{g}_{im}^s = \overline{g}^{im} = 1$. Accordingly, from the definition of immigrants' growth rate population we take for $\overline{\lambda} \neq 1$ that $\overline{g}_{im}^u = \overline{g}_{im}^s = \overline{g}^{im} = 1$.

Then, the Euler equation for bonds gives $1 = (\beta\overline{r})/(g^d\overline{\pi})$ and the Euler equation for capital $1 = \beta\overline{R}/g^d$. Therefore, we obtain the steady-state values of gross nominal interest rate ($\overline{r}$), of the gross rental rate of capital net of depreciations ($\overline{R}$) and of the rental price of capital ($\overline{r}^k = \overline{R} - 1 + \delta$).

Turning to supply side equations, given that each type of worker spends one third of its total time endowment to labor activities, we find the hours of total domestic labor ($\overline{h}_d$), the hours of foreign-born labor ($\overline{h}_{im}$), the hours of total skilled ($\overline{h}_s$), total unskilled ($\overline{h}_u$), and efficient labor ($\overline{h}^e$).

The optimal pricing decision condition in steady-state defines that marginal cost is equal to the inverse of the mark-up which depends on the elasticity of intermediate product (ε); that is,

$$\overline{mc} = \frac{\varepsilon - 1}{\varepsilon}$$

We use the above condition and the definition of real marginal cost so as to derive the steady-state value of per capita capital ($\overline{k}$) and capital per native agent ($\overline{k}_d$). Specifically,

[7] Both $\varepsilon = 6$ and $\varepsilon = 10$ are standard calibrated values for this elasticity found in business cycle literature. For example, Dotsey (1999), Chari - Kehoe - McGrattan (2000) and Wang - Wen (2006) set $\varepsilon = 10$, while Yun (1996), Ireland (1997, 2001, 2002, 2004), Rabanal (2007), Gali-Salido-Vallés (2003, 2004, 2007) take $\varepsilon = 6$

$$\overline{k}=\left[\frac{\overline{Z}\,\overline{mc}(1-a)}{\overline{r}^{k}}\right]^{\frac{1}{a}}\overline{h}^{e}\quad\text{and}\quad\overline{k}_d=\left(\frac{1}{\overline{\gamma}}\right)\overline{k}$$

Having found capital per native we directly compute investment spending per native which is given by $\overline{x}_d=\delta\overline{k}_d$. Then, using the production function and the definition of real dividends per native-born agent we find the steady-state value of per capita output ($\overline{y}$) and of real dividend shares ($\overline{d}_d$). Skilled and unskilled real wages for natives ($\overline{w}_d^s$, $\overline{w}_d^u$) and immigrants ($\overline{w}_{im}^s$, $\overline{w}_{im}^u$) are derived from the first order conditions of the cost minimization problem:

$$\overline{w}_d^s=(\overline{mc})\,a\overline{Z}\left[\overline{h}^{e}\right]^{a-1+\rho}\left(\overline{k}\right)^{1-a}\zeta\left(\overline{h}_s\right)^{\rho_1-\rho}\omega_s\left[\overline{\gamma}\phi\overline{h}_d^{s}\right]^{-\rho_1}$$

$$\overline{w}_{im}^s=(\overline{mc})\,a\overline{Z}\left[\overline{h}^{e}\right]^{a-1+\rho}\left(\overline{k}\right)^{1-a}\zeta\left(\overline{h}_s\right)^{\rho_1-\rho}\left[\left(1-\overline{\gamma}\right)\overline{\lambda}\overline{h}_{im}^{s}\right]^{-\rho_1}$$

$$\overline{w}_d^u=(\overline{mc})\,a\overline{Z}\left[\overline{h}^{e}\right]^{a-1+\rho}\left(\overline{k}\right)^{1-a}\left(\overline{h}_u\right)^{\rho_2-\rho}\omega_u\left[\overline{\gamma}\left(1-\phi\right)\overline{h}_d^{u}\right]^{-\rho_2}$$

$$\overline{w}_{im}^u=(\overline{mc})\,a\overline{Z}\left[\overline{h}^{e}\right]^{a-1+\rho}\left(\overline{k}\right)^{1-a}\left(\overline{h}_u\right)^{\rho_2-\rho}\left[\left(1-\overline{\gamma}\right)\left(1-\overline{\lambda}\right)\overline{h}_{im}^{u}\right]^{-\rho_2}$$

Finally, from each representative agent optimality conditions, we obtain consumption spending and the preference weight on consumption for each type of agent (natives and immigrants). Specifically,

$$\overline{c}_d=\phi\overline{w}_d^s\overline{h}_d^s+(1-\phi)\overline{w}_d^u\overline{h}_d^u+\left(\overline{r}^k-\delta\right)\overline{k}_d+\overline{d}_d$$

$$\overline{c}_{im}=\overline{\lambda}\overline{w}_{im}^s\overline{h}_{im}^s+\left(1-\overline{\lambda}\right)\overline{w}_{im}^u\overline{h}_{im}^u$$

$$\overline{\alpha}_d=\frac{\overline{c}_d}{\overline{l}_d\overline{w}_d^s}\left(\frac{\phi\overline{h}_d^s}{\overline{h}_d}\right)^{\frac{1}{\sigma}}\quad\text{and}\quad\overline{\alpha}_{im}=\frac{\overline{c}_{im}}{\overline{l}_{im}\overline{w}_{im}^s}\left(\frac{\overline{\lambda}\overline{h}_{im}^s}{\overline{h}_{im}}\right)^{\frac{1}{\sigma}}\quad\text{where}\quad\overline{l}_j=1-\overline{h}_j,\quad\text{for}\,j=d,im$$

4 Impulse Response Analysis

In the present model, we have introduced five distinct shocks: two preference shocks, each for every type of representative agent ($\hat{\alpha}_{d,t}$ and $\hat{\alpha}_{im,t}$ for natives and immigrants, respectively), a technology shock ($\hat{Z}_t$), a policy shock ($v_t=\varepsilon_{r,t}$), and finally an immigration shock ($\hat{\mu}_t$). As long as preference shocks are included among the potential sources of business cycle dynamics (Ireland, 2004), and given the difference in optimizing behavior between native and foreign-born agents, we distinguish immigrants' preference shock from that of natives so as to investigate whether this heterogeneity creates different implications. In addition, we describe both cases of modeling immigration shocks: inflows that alter the composition of total populations temporarily or permanently. We examine the impulse responses of the model to the aforementioned shocks, by assuming that each exogenous term follows a stationary first order autoregressive process with high serial correlation. Thus, we set $\rho_{\alpha_d} = 0.9048$, $\rho_{\alpha_{im}} = 0.9048$, $\rho_z = 0.94$ (Ireland, 2004), and $\rho_\mu = 0.6$ (Canova-Ravn, 1997).

The main purpose of this paper is to examine the potential effects of skilled and unskilled, temporary or permanent immigration inflows in a host economy, represented by a standard New Keynesian model. For that purpose, we place special attention to immigration inflows shocks as well as to immigrants' preference behavior shock. Regarding the rest of the unexpected disturbances that may hit the theoretical economy, the present analysis verifies existing results of similar DSGE studies.

4.1 *Immigrants' preference shock*

The preference shock on consumption ($\hat{\alpha}_{im,t}$) affects the marginal rate of substitution between consumption and leisure; specifically, it causes an increase of the marginal utility of consumption. Rule-of-thumb immigrants work more hours in order to increase their labor income, and thus their consumption. Both skilled ($\hat{h}^s_{im,t}$) and unskilled hours ($\hat{h}^u_{im,t}$) of work by immigrants show a hump-shaped behavior. Total labor supply of foreign-born population ($\hat{h}_{im,t}$) follows a similar pattern. The upward movement of immigrants' labor supply causes a downward pressure on their skilled and unskilled wage earnings ($\hat{w}^s_{im,t}$, $\hat{w}^u_{im,t}$). Firms substitute domestic labor and capital with foreign-born labor, which is now cheaper. Demand for skilled and unskilled domestic labor ($\hat{h}^s_{d,t}$ and $\hat{h}^u_{d,t}$) declines, and thus wages paid to native workers go down as well. Nevertheless, due to imperfect substitutability between native-born and foreign-born labor the negative movement of domestic wages ($\hat{w}^s_{d,t}$, $\hat{w}^u_{d,t}$) is not as intense as the decline of immigrants' wages. Also, impulse responses show that the expanded labor supply of immigrants ($\hat{h}_{im,t}$) overcomes the negative response of domestic labor ($\hat{h}_{d,t}$), which means a hump-shaped behavior of total skilled ($\hat{h}_{s,t}$), total unskilled ($\hat{h}_{u,t}$) and efficient labor ($\hat{h}^e_t$).

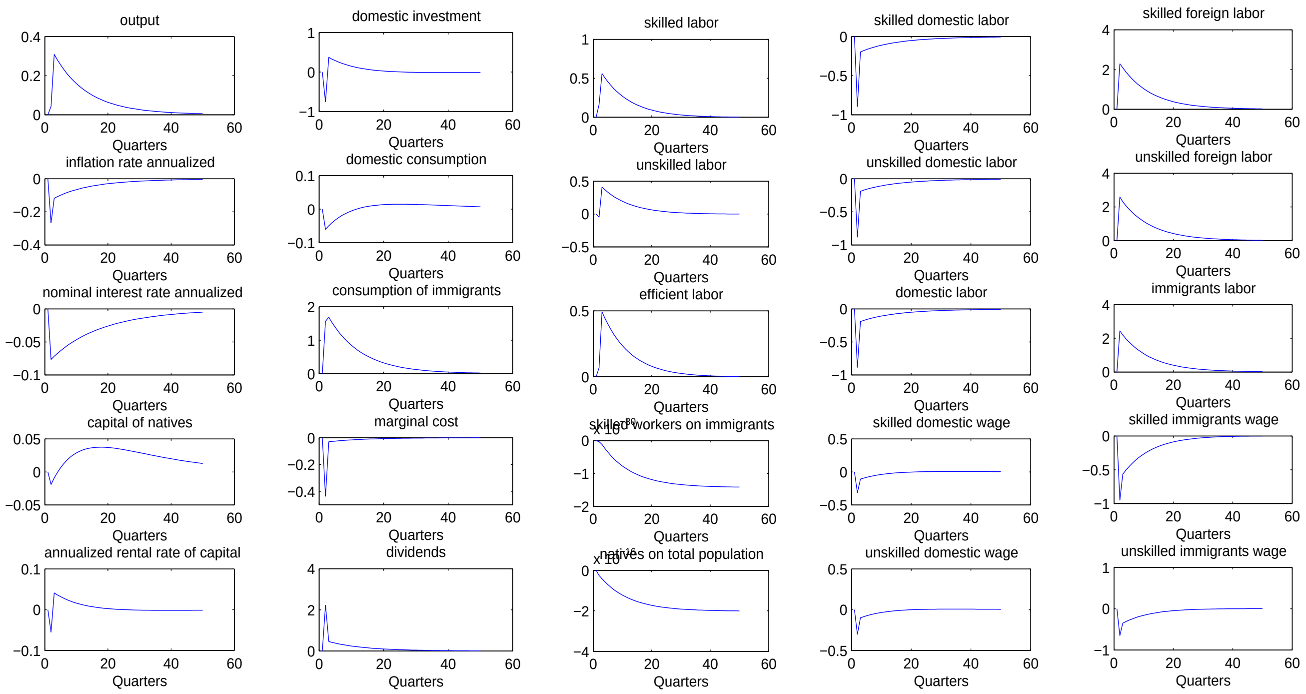
Accordingly, in the short-run, the substitution of capital with foreign-born labor lowers the demand for capital and pushes downwards its rental price; however, as long as firms expand their production capabilities and, thus, their demand for capital, its rental price starts going upwards.

The positive response of immigrants' labor supply, induced by the preference shock, overcomes the decline of skilled and unskilled wages, and as a result immigrants' total labor income and consumption are positively sloping. On the contrary, consumption of natives shows an inverse hump-shaped behavior. There are mainly two opposing effects on domestic households: on the one hand, a negative effect, caused by the decline of skilled and unskilled wage earnings; on the other hand, a positive effect, due to the upward movement of dividend shares. Real profits go up in the short-run, because wages and the rental price of capital push downwards the marginal cost. The decline of labor income overcomes the increase of dividend shares and the final result on domestic consumption is negative but modest.

A preference shock of immigrants operates as a positive supply shock. Labor supply increases in the short-run, wages decline, and total output expands. Clearing in good market implies that firms shall satisfy the higher consumption by producing more output. Also, as long as wages and the rental price of capital go downwards in the short-run, marginal cost is inverse hump-shaped. Since marginal cost is the accurate determinant of inflation (Gali and Gertler, 1999), inflation rate follows a similar monotone shaped path.

Interestingly, immigrants' preference shock creates an decrease rather than an increase of inflation rate,

Figure 1: Immigrants' preference shock ($\hat{\alpha}_{im,t}$) - Baseline Calibration



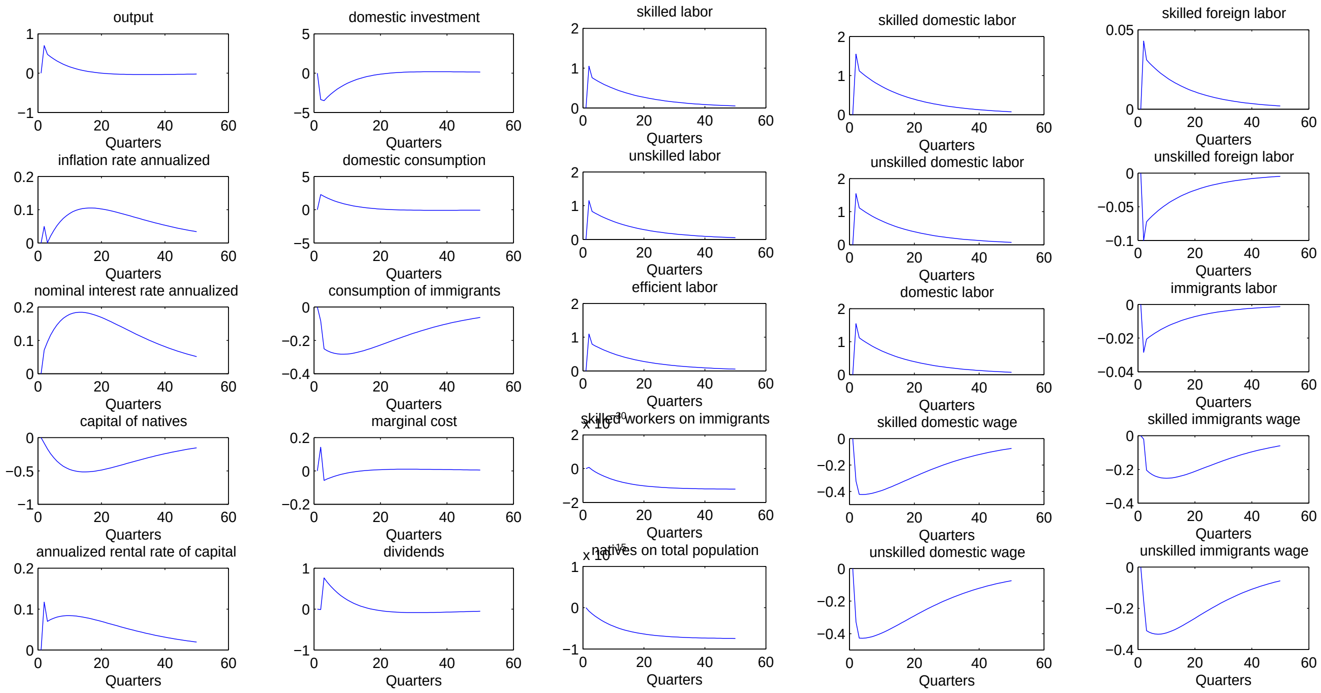
with the latter observed in case of domestic preference shock. Nominal interest rate, being more sensitive to inflation fluctuations rather than to output behavior, follows an inverse hump-shaped path. Setting a higher persistence coefficient or standard deviation to immigrants' preference shock, we take a larger decline of both inflation and interest rate, as well as a stronger increase of total output. However, output response is sensitive to the price rigidities parameter (the probability $0 \leq \theta \leq 1$). The model predicts a short-run decrease of total output under a more staggered price setting, but still inflation rate and nominal interest rate are monotone shaped. Overall, immigrants' preference shock seems to be a positive supply shock for the host country as the responses of aggregate magnitudes show.

Immigrants' preference shock captures successfully the general attitude found on immigration literature that immigrants work more intensively, and appear to be more industrious than natives. This position is justified by immigrants' strong motive for succeeding in the host country where they migrated so as to improve their standards of living. In the present context, immigrants' preference shock models their willingness to work more intensively so as to increase their labor income. Immigrants' behavior allow domestic firms to extend their production with cheaper labor force; output increases and inflation declines. The above impulse responses show that immigrants' preference shock may partly justify why immigration has been considered by the Federal Reserve Bank (FED) as a disinflation force for the host economy.

4.2 Immigration shock

In population dynamics section above, we considered two kinds of immigration inflows: on the one hand, we modeled immigration shock as an unexpected influx of foreign-born agents, either skilled or unskilled,

Figure 2: Natives' preference shock ($\hat{\alpha}_{d,t}$) - Baseline Calibration



that alters the composition of the total population of the host country between natives and immigrants temporarily; on the other hand, we modeled permanent immigration inflows that change the growth rates of immigration population, either skilled or unskilled. In the latter case, immigration shocks alter the fraction of natives to the total population of the host country permanently. We describe below the impulse responses in both of these cases.

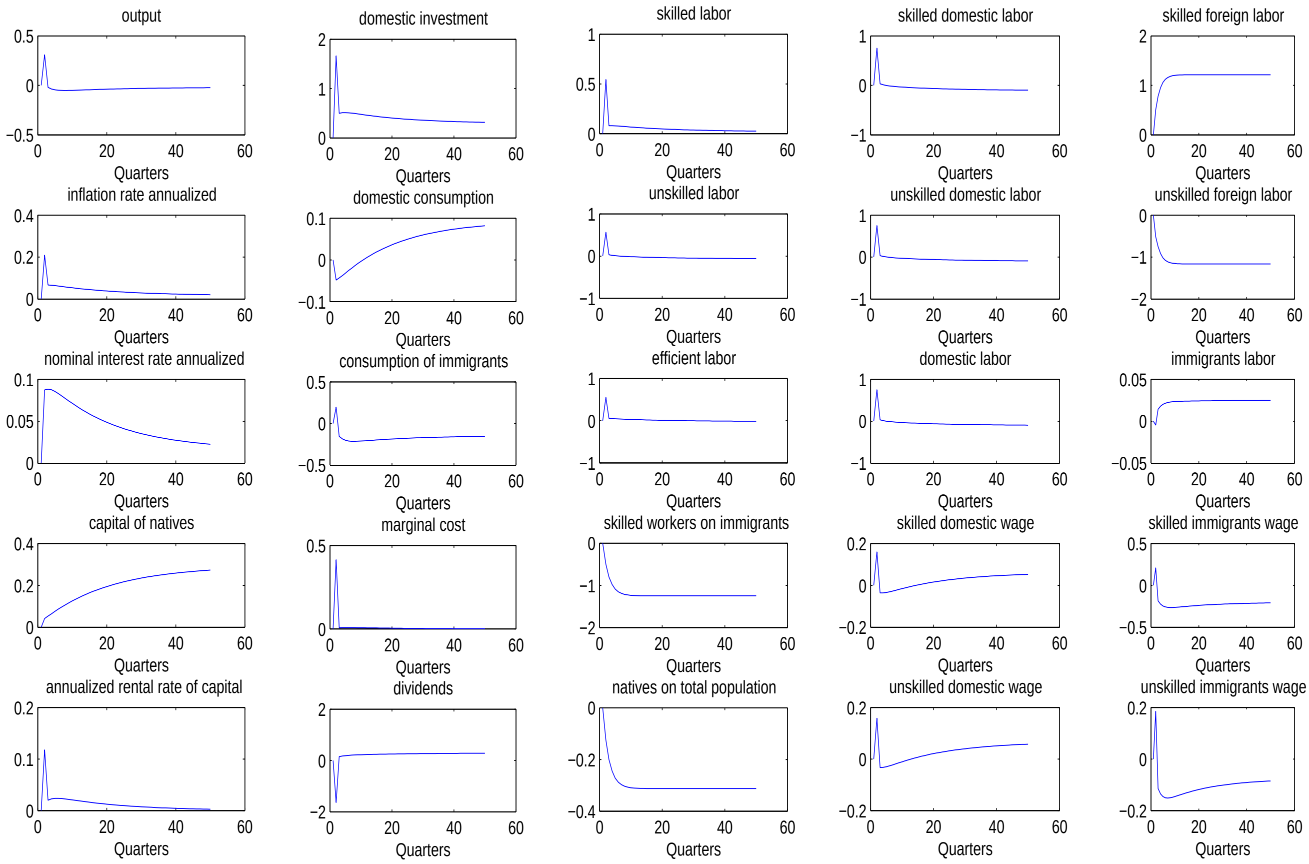
4.2.1 Permanent Immigration

Unskilled immigration inflow First, an immigration shock on the growth rate of low-skilled immigrants reduces the fraction of skilled immigrants on the foreign-born population (λ_t) and the ratio of domestic households to total population (γ_t).

With imperfect substitutability between skilled and unskilled labor ($\rho = 0.67$) and the baseline calibrated values for the rest of the model's parameters, unskilled immigration shock seems to be beneficial for domestic households, because it creates an income redistribution from immigrants to natives. A permanent immigration shock of unskilled agents increases skilled and unskilled wages paid to natives, because of the imperfect substitutability between native-born and foreign born population, and because of the lower fraction of natives on total population. Skilled and unskilled domestic labor follow a hump-shaped time path, but natives' wage earnings increase permanently.

Accordingly, an immigration inflow reduces the fraction of skilled immigrants and increases the portion of low-skilled foreigners. Skilled immigrants work in response more hours, but unskilled foreigners tend to work less. Both skilled and unskilled wages paid to immigrants increase in the short-run but decline after some quarters below steady-state. Impulse responses verify that newcomers compete with foreign-

Figure 3: Permanent unskilled immigration shock



born population for similar job opportunities and affect the wage earnings of pre-existing immigrants downwards. Immigrants' consumption mimics the permanent reduction of their wage earnings.

Unskilled immigration shock increases the rental price of capital as long as immigrants' influx reduces the capital-labor ratio. Unskilled immigrants neither have access to domestic capital markets nor bring with them from their home-country any capital stock. As a result, the rise of foreign-born population decreases the total capital stock per capita ($\hat{k}_t$), and increases the capital holdings per domestic household ($\hat{k}_{d,t}$). The aggregate capital stock of the host country belongs to domestic households, and thus only to a fraction of total population. As long as the inflow of immigrants increases the ratio of foreign-born population and decreases accordingly the fraction of natives on total population, total capital stock corresponds to a lower portion of population. Domestic households are motivated by the higher price of capital to invest more in real assets. Domestic investment shows a hump-shaped behavior, capital holdings of natives increase steadily, but the total capital stock declines sharply in the short-run by the unexpected increase of population.

The short-term increase of wages and of rental price of capital raise the marginal cost and reduce in turn real dividends. Nevertheless, labor income and capital earnings of natives overcome the reduction of dividend shares, which means that the final result on natives' wealth is positive. It is worth noting that under this context, immigration inflow creates a second wealth effect in natives' favor: firms' profits are allocated to a smaller fraction of total population, which means in effect that for every domestic agent dividend shares increase. After some quarters, dividends recover from the short-run decline (caused by the positive response of marginal cost) and increase above their steady-state value permanently. Domestic consumption falls in the short-run, because households spend more income in capital investment, but after some quarters consumption recovers and increases steadily above its steady-state value.

In aggregate terms, unskilled immigration shock seems to push upwards in the short-run both output and inflation, which return gradually back to steady-state values after some quarters. Thus, the demand side effect of immigration inflow overcomes the supply side effect, namely the increase of labor supply. For the central bank, the positive movements of output and inflation cause a hump-shaped response of interest rate.

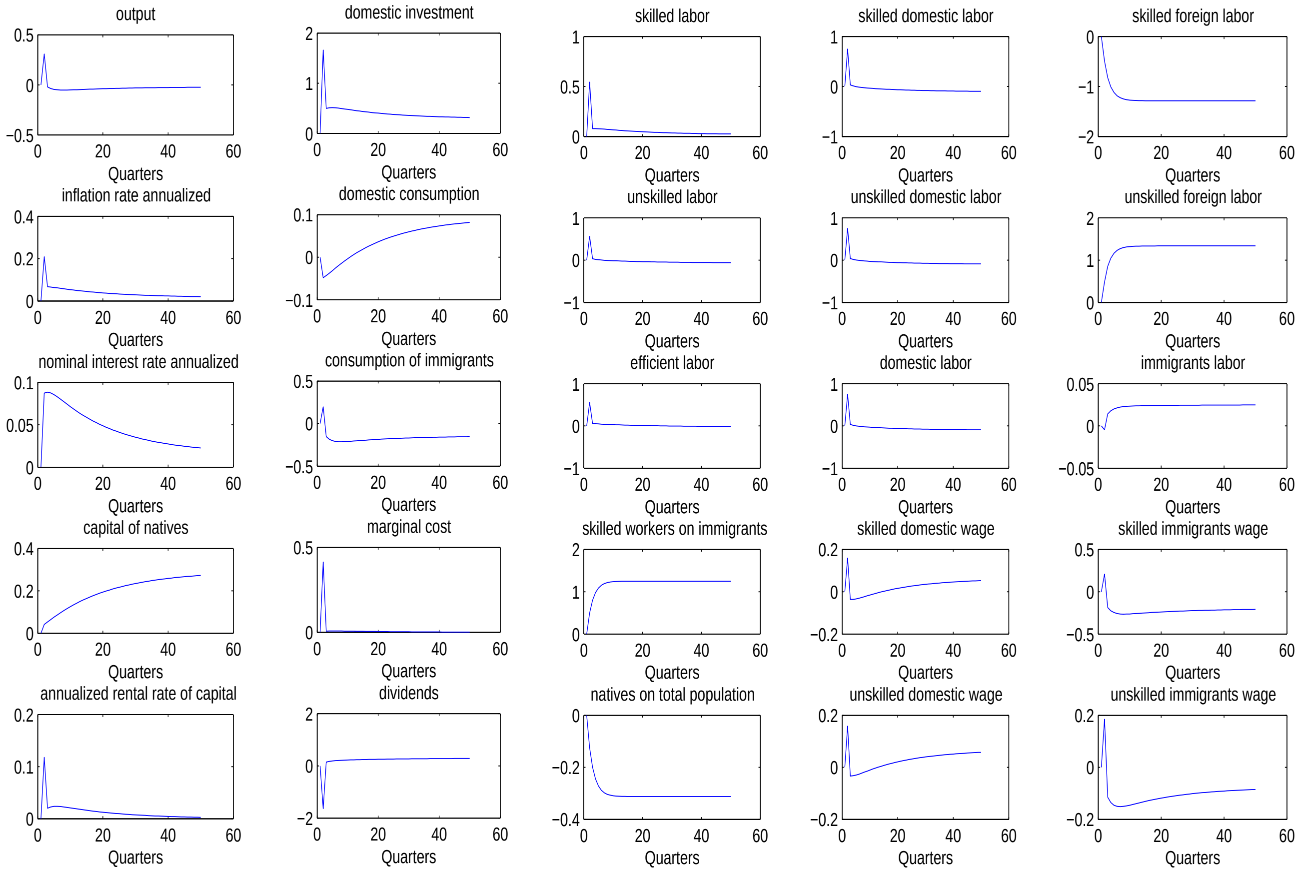
Overall, immigration shock can generate in a standard New Keynesian framework a positive "output effect". Under monopolistically competitive markets with staggered price setting, an immigration shock pushes in the short-run total output above steady-state. The inflow of unskilled immigrants increases in the short-run the total population, boosts total consumption and thus aggregate demand. Market clearing implies that firms will generate more output to satisfy the excess demand. Total output increases in the short-run as long as firms cannot re-adjust upwards their prices instantly, but with some time lags. As a result, in the short-run there is no any positive movement of price mark-ups over marginal costs which would generate contractionary pressures into aggregate demand. Output increase requires more inputs of production, both capital and labor; higher demand for labor and capital is translated into an upward short-term movement of real return on capital, of skilled and unskilled wages. After a short-run period of constant prices and of increased aggregate activity, firms re-adjust upwards their prices to re-establish their mark-ups. The rise of price level affects aggregate demand negatively and output declines gradually reaching its steady-state value in the long-run. Nominal interest rate shows a hump-shaped behavior with strong persistence. In sum, immigration shock destabilizes the host economy, but it seems to be welfare improving for natives.

Finally, coefficients of productivity difference between natives and immigrants, either for skilled or unskilled labor (ω_s and ω_u, respectively), determine the total labor in efficiency units and constitute crucial parameters for the long-run behavior of both inflation and output. Thus, a comparison of the standard assumption $\omega_s = \omega_u = 1$ with values that reflect productivity differences between natives and foreigners would create interesting implications.

Skilled immigration inflow A positive shock on the growth rate of skilled immigrants increases their fraction to the foreign-born population (λ_t) and reduces accordingly the ratio of domestic households to total population (γ_t).

Under the same baseline calibration, skilled immigration shock affects the aggregate magnitudes of the host country as unskilled immigration does. The sudden increase of the growth rate of total population, caused by immigration, pushes upwards in the short-run both output and inflation, which return gradually back to steady-state values after some quarters. Thus, even if we model an influx of skilled immigrants, who by definition are considered more productive than unskilled foreigners, the demand side effect of immigration inflow overcomes any supply side effect, caused by the expansion of labor supply, and the

Figure 4: Permanent skilled immigration shock



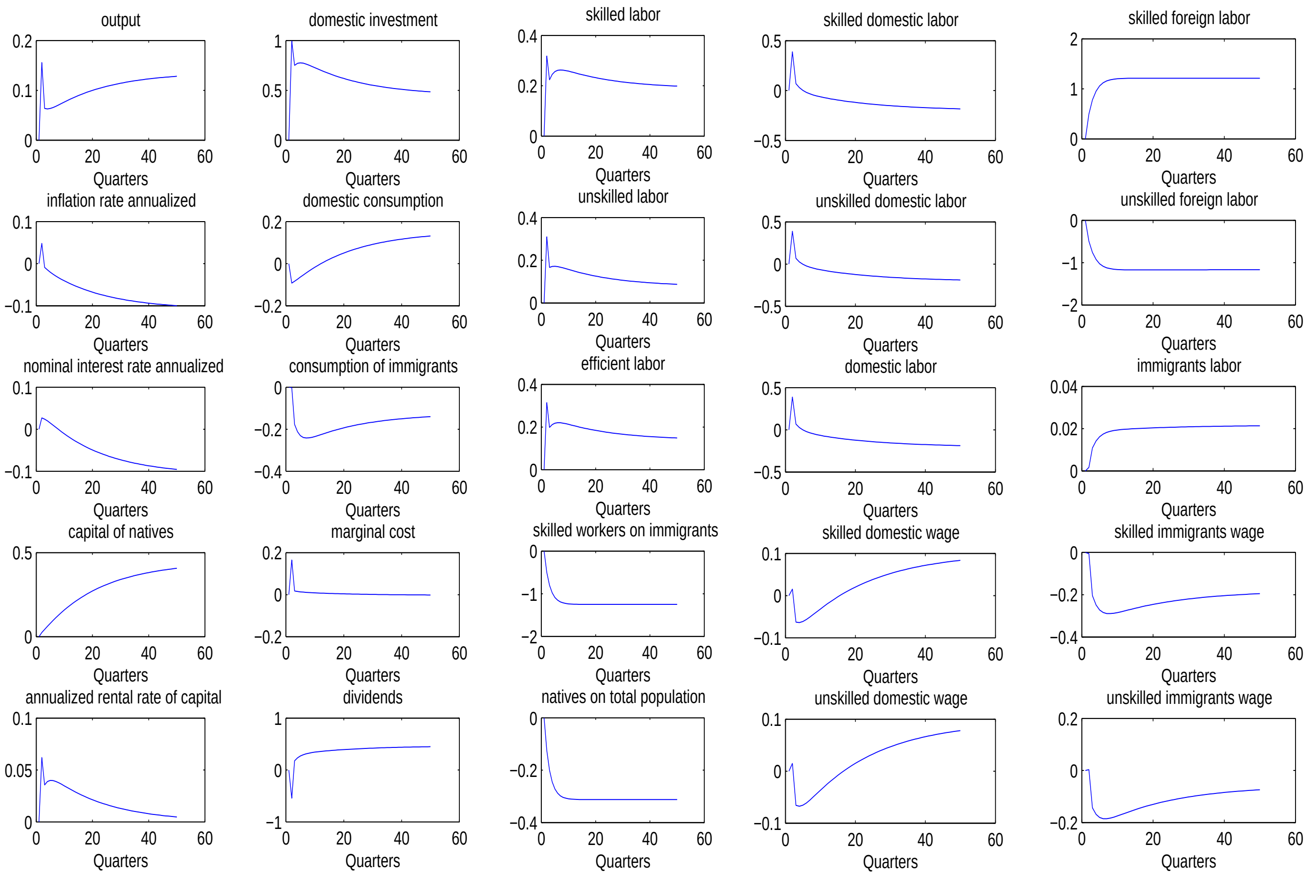
final result on inflation rate is still positive. For the policymaker, the upwards movements of output and inflation cause a hump-shaped response of nominal interest rate, which shows significant persistence.

With respect to disaggregated variables, skilled immigration shock creates similarly an income redistribution from immigrants to natives. Wage earnings of natives are hump-shaped in the short-run, but increase permanently in the long-run. Dividend shares decline during the first quarters, due to the marginal cost behavior, but in the long-run remain above steady-state as long as the portion of natives on total population has declined. The combination of the increased capital stock per native agent with the upward movement of the rental price of capital implies that natives' capital earnings become higher. As a result, natives' wealth is positively affected by skilled immigration and domestic households are better off.

Under the present context, the only difference observed between skilled and unskilled immigration is the opposite change of immigrants' skilled and unskilled working hours. An inflow of skilled immigrants reduces the fraction of unskilled immigrants who work in turn more hours. Equivalently, skilled immigration increases the portion of skilled foreigners who tend to work less hours. Both skilled and unskilled wages paid to immigrants increase in the short-run, but decline after some quarters below steady state. Therefore, impulse responses verify that newcomers compete with foreign-born population for the same job positions; skilled immigration affects downwards wage earnings of pre-existing foreign-born population.

In sum, the way we have built the present model shows that there is no significant difference in the results obtained under each kind of immigration shock; the short-run increase of growth rate of the total population caused by immigration triggers an identical adjustment mechanism.

Figure 5: Immigration shock

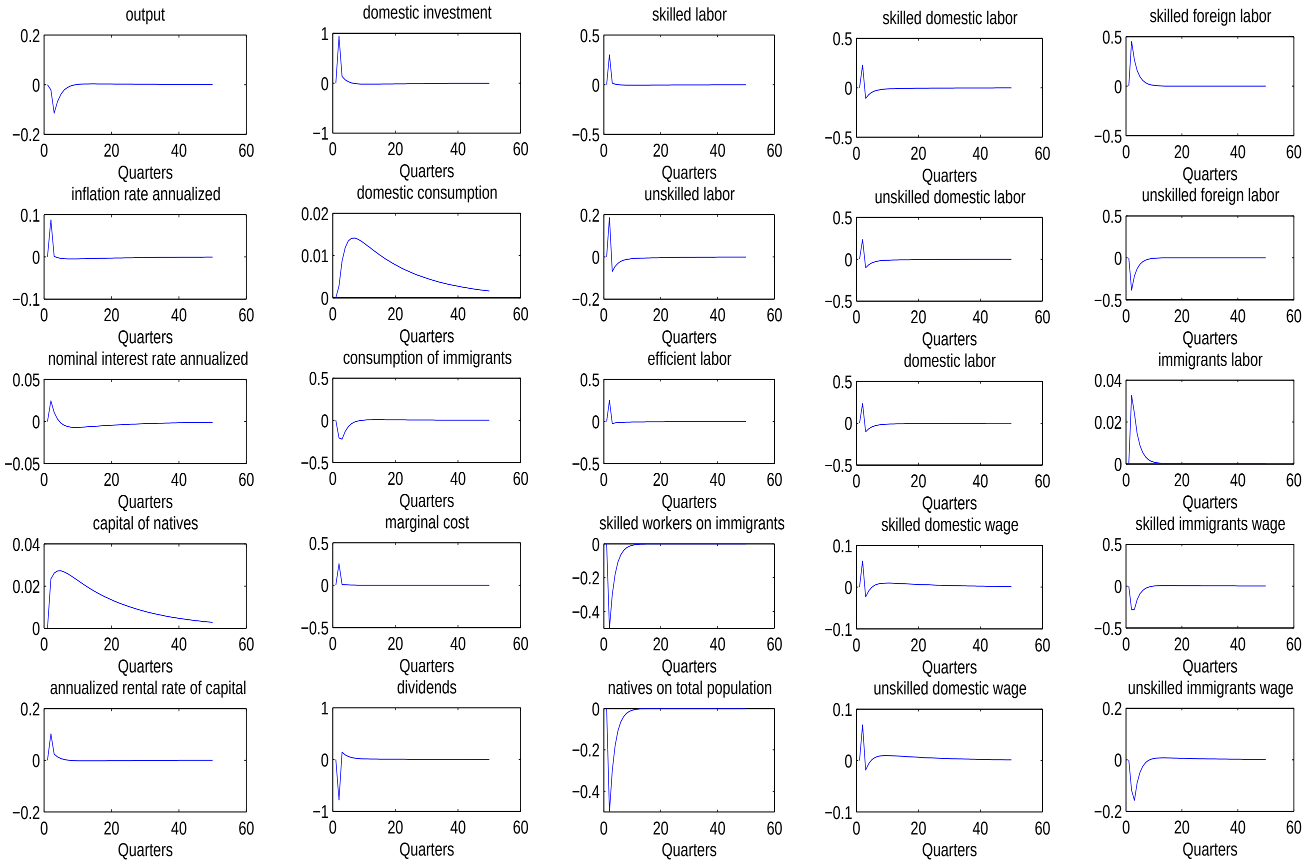


Finally, as long as productivity difference parameters (ω_s and ω_u) play a central role in determining the long-run adjustment of the host country to exogenous shocks, an estimation of their values would be beneficial. When the hypothesis of similar productivity between natives and immigrants is dropped out and immigrants are more industrious and productive than natives, then, after a short-run positive demand effect, immigration seems to operate as a positive supply side shock for the host country. In the long-run, immigrants' inflows may reduce inflation rate below steady-state and boost simultaneously aggregate output, as figure 5 depicts.

4.2.2 Temporary immigration shock

If we insert immigration shock in the number of foreigners that enter the host country rather than in the growth rate of the corresponding population, the fraction of the foreign-born agents to total population alters temporarily. As described above, the host country reacts identically to skilled or unskilled immigration shock as long as an influx of immigrants triggers the same adjustment mechanism. Other things equal, the difference between skilled and unskilled immigration inflows is still found on the symmetric response of labor offered by each kind of foreign-born agent: in case of skilled immigration, unskilled foreigners work more hours, skilled immigrants less hours, and vice versa. Hours of work by immigrants change temporarily as long as their fraction on total population deviates from its steady-state value.

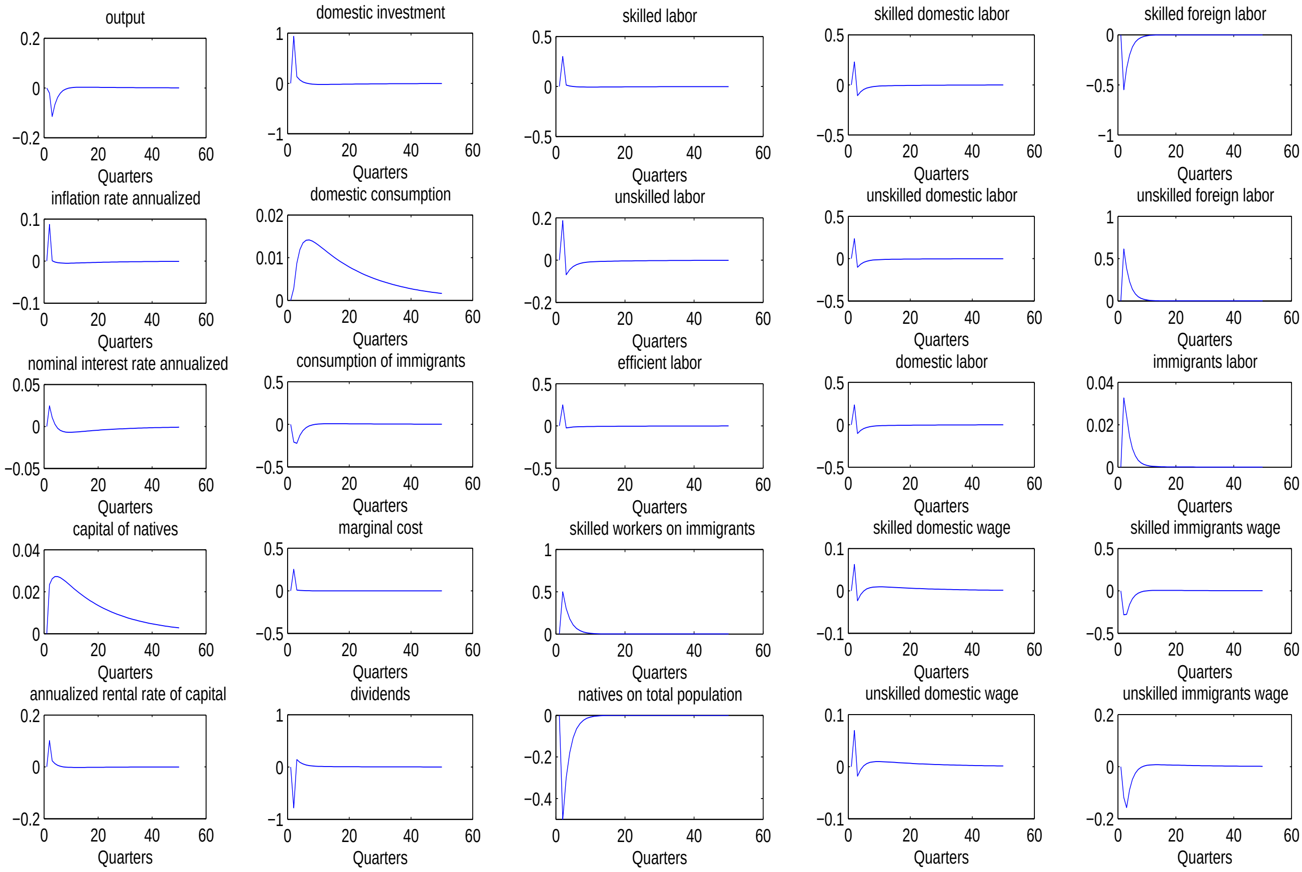
Figure 6: Temporary unskilled immigration shock



The supply side of the host economy shows that newcomers compete only with foreign-born residents. As a result, both skilled and unskilled immigration affect negatively wage earnings of foreigners. Due to imperfect substitutability, natives obtain higher wages, and simultaneously benefit from the reduction of capital-labor ratio which in turn increases the rental price of capital. The redistributive effects of each type of immigration, that benefit domestic households, are reflected on the hump-shaped behavior of domestic consumption, and on the monotone shaped time path of immigrants' consumption. When immigration shock changes the growth rate of foreign-born population, redistributive effects become more intensive and positive effects on domestic consumption are prolonged.

Although, impulse response analysis shows that any kind of immigration shock (permanent or temporary) causes income redistribution from immigrants to natives, immigration effects on aggregate magnitudes are not robust. When immigration alters permanently the composition of total population, output is positively affected in the short-run. In contrast, a temporary inflow of foreigners gives a monotone shaped time path for output which can be explained by the sudden decrease of total capital. The unexpected inflow

Figure 7: Temporary skilled immigration shock



of immigrants increases total workforce, and thus total labor in efficiency units, but the latter is unable to overcome the reduction of capital stock and the downward movement of total output.

We observe that, under this context, a temporary immigration shock acts as a negative supply shock. Output declines, but inflation increases in the short-run. The marginal cost of production (the driving force of inflation rate) increases, because the upward movement of the rental price of capital and of domestic wages overcome the decline of immigrants' wages. Although output responds negatively, nominal interest rate is mainly affected by the positive movement of inflation and follows a similar hump-shaped time path.

5 Conclusion

The present theoretical analysis offers a simple description of the potential effects of immigration inflows on the macroeconomic magnitudes of a large-developed economy. Mainly, we complement Canova-Ravn (2000) analysis by examining immigration shocks in a rather more realistic New Keynesian framework, which is widely being used by modern central banks. We adopt a simplified production function described by Ottaviano-Peri (2005, 2006) which has been used so far for the examination of immigrants' impact on domestic labor market outcomes only. We distinguish total workforce between skilled and unskilled agents, and within each class of workers we introduce foreign-born population. Following the current trend of immigration literature, in every skill-class population, immigrants and natives are considered imperfect substitutes. Also, the present set up allows one to undertake the alternative assumption of perfect substitutability between natives and immigrants, by assigning appropriate values to the corresponding coefficients.

We motivated to integrate immigration inflows into a standard New Keynesian model so as to test the bottom line that immigration can be placed among the factors that prevent or lessen inflationary pressures. The present study offers useful insights for central banks as long as it shows that immigration could be responsible for the business cycle behavior of developed countries which are destinations of immigration flows. The results show that immigration inflows destabilize the host economy temporarily or even permanently. In the short-run, an inflow of immigrants may create a positive output effect, but in the long-run the final result depends on the "deep" parameters of the model. Price stickiness, productivity differences between natives and immigrants, the degree of substitutability between skilled and unskilled workers or between natives and immigrants, as well as the Taylor rule coefficients should be of major concern by central banks in predicting the consequences of unexpected immigration inflows accurately. Nominal variables, namely inflation and interest rate, show a hump-shaped behavior. Therefore, the standard New Keynesian model does not verify as a whole that immigration is considered a deflationary force.

The standard New Keynesian model shows that in the short-run the demand side effect caused by immigration overcomes any supply side effect, and the final result is an upward movement of inflation. Policymakers should be interested in identifying whether immigration inflows alter the composition of the total population permanently or temporarily, because, in both cases, demand side pressures on inflation rate dominate, but permanent immigration shocks create a positive output effect too. We predict that a new version of New Keynesian model with unemployment rate would succeed in justifying the disinflation force of immigration. A fraction of immigration literature supports that newcomers do not compete with natives for the same job opportunities but fill up existing vacancies that native agents are unwilling to undertake. As a result, we expect that immigrants would reduce any domestic unemployment rate and increase total output without pushing inflation rate upwards.

Although the standard New Keynesian model predicts an upward movement of inflation in the short-run, due to demand side effect, there are still some cases that verify the deflationary impact of immigration. Immigrant's characteristics may lessen the inflation rate even in an economy that balances in full employment equilibrium. First, a shock on immigrants' preferences captures successfully the general attitude of recent immigration literature that foreigners work more intensively and appear to be more industrious than natives. That immigrants have strong motives to succeed in the host country justifies their behavior to spend more hours in labor market activities than natives. Indeed, their narrow financial abilities and their willingness to improve their standards of living establish their hard-working behavior. Immigrants allow domestic firms to extend their production capabilities with cheaper labor force. These potential shocks on immigrants' preferences (remittances shock) cause both an increase of total output and a decline of inflation rate. There is also the case that immigration may involve more productive agents than natives, who improve in the long-run the average productivity of the total workforce of the host country and push output and inflation above and below steady-state, respectively.

In general, the final results on the macro-economy depend on the calibrated parameters of the model and on whether immigration alters the composition of total workforce permanently or temporarily. In any case, however, with respect to disaggregated variables, immigration inflows remain welfare improving for domestic agents as they cause income redistribution from foreign-born to native-born households. Domestic consumption and labor market outcomes for natives are positively affected by immigration shocks, as long as newcomers compete with existing immigrants for similar job opportunities and affect the labor market of foreign-born agents negatively.

Finally, the present model shows that skilled and unskilled immigration influxes cause almost identical macroeconomic responses, as the adjustment mechanism of the host country remains unchanged between the two cases. Thus, the idea that policymakers should have preferences regarding the type of workers that immigration involves seems to collapse under the present context. The productivity difference between skilled and unskilled immigrants seems to be infinitesimal so as to make any difference in the host macroeconomy.

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A Appendix: The log-linearized equilibrium conditions

Having found steady-state conditions, we write the log-linearized conditions around the symmetric equilibrium. In general, the term $\hat{x}_t$ defines the percentage deviation of variable x_t around its steady - state value $\overline{x}$, that is $\hat{x}_t = \log(x_t) - \log(\overline{x})$. The equilibrium conditions of the model are summarized as follows:

Domestic households:

$$\hat{c}_{d,t} + \phi_1 \hat{x}_{d,t} = \phi_2 \hat{w}^s_{d,t} + \phi_2 \hat{h}^s_{d,t} + \phi_3 \hat{w}^u_{d,t} + \phi_3 \hat{h}^u_{d,t} + \phi_4 \hat{R}_t + \phi_5 \hat{k}_{d,t} + \phi_6 \hat{d}_{d,t} \tag{A.1}$$

$$\hat{k}_{d,t+1} = \phi_7 \hat{k}_{d,t} + \phi_8 \hat{x}_{d,t} \tag{A.2}$$

$$\phi_9 \hat{h}^s_{d,t} = \hat{a}_{d,t} - \hat{c}_{d,t} + \hat{w}^s_{d,t} + \phi_{10} \hat{h}_{d,t} \tag{A.3}$$

$$\phi_9 \hat{h}^u_{d,t} = \hat{a}_{d,t} - \hat{c}_{d,t} + \hat{w}^u_{d,t} + \phi_{10} \hat{h}_{d,t} \tag{A.4}$$

$$\hat{c}_{d,t} = E_t\left(\hat{c}_{d,t+1}\right) - \hat{r}_t + E_t\left(\hat{\pi}_{t+1}\right) + \phi_{11} \hat{\alpha}_{d,t} \tag{A.5}$$

$$\hat{c}_{d,t} = E_t\left(\hat{c}_{d,t+1}\right) - E_t\left(\hat{R}_{t+1}\right) + \phi_{11} \hat{\alpha}_{d,t} \tag{A.6}$$

$$\hat{h}_{d,t} = \phi_{12} \hat{h}^s_{d,t} + \phi_{13} \hat{h}^u_{d,t} \tag{A.7}$$

Immigrants:

$$\hat{c}_{im,t} = \phi_{14} \hat{w}^s_{im,t} + \phi_{14} \hat{h}^s_{im,t} + \phi_{15} \hat{w}^u_{im,t} + \phi_{15} \hat{h}^u_{im,t} + \phi_{16} \hat{\lambda}_t \tag{A.8}$$

$$\phi_9 \hat{h}^s_{im,t} + \phi_9 \hat{\lambda}_t = \hat{\alpha}_{im,t} - \hat{c}_{im,t} + \hat{w}^s_{im,t} + \phi_{17} \hat{h}_{im,t} \tag{A.9}$$

$$\phi_9 \hat{h}^u_{im,t} + \phi_{18} \hat{\lambda}_t = \hat{\alpha}_{im,t} - \hat{c}_{im,t} + \hat{w}^u_{im,t} + \phi_{17} \hat{h}_{im,t} \tag{A.10}$$

$$\hat{h}_{im,t} = \phi_{19} \hat{h}^s_{im,t} + \phi_{20} \hat{h}^u_{im,t} + \phi_{21} \hat{\lambda}_t \tag{A.11}$$

Immigration shock:

$$\hat{\lambda}_t = \hat{\lambda}_{t-1} + \phi_{22} \hat{\mu}_t \tag{A.12}$$

$$\hat{\gamma}_t = \hat{\gamma}_{t-1} - \phi_{23} \hat{\mu}_t \tag{A.13}$$

Supply Side:

$$\phi_{24}\hat{w}^{s}_{d,t} - \phi_{24}\hat{w}^{s}_{im,t} = \phi_{25}\hat{\gamma}_t + \hat{\lambda}_t + \hat{h}^{s}_{im,t} - \hat{h}^{s}_{d,t} \tag{A.14}$$

$$\hat{w}^{s}_{d,t} - \hat{w}^{u}_{d,t} = \phi_{26}\hat{h}_{s,t} + \phi_{27}\hat{h}_{u,t} + \phi_{28}\hat{\gamma}_t - \phi_{29}\hat{h}^{s}_{d,t} + \phi_{30}\hat{h}^{u}_{d,t} \tag{A.15}$$

$$\hat{w}^{s}_{d,t} - \hat{w}^{u}_{im,t} = \phi_{26}\hat{h}_{s,t} + \phi_{27}\hat{h}_{u,t} - \phi_{29}\hat{h}^{s}_{d,t} + \phi_{30}\hat{h}^{u}_{im,t} + \phi_{31}\hat{\lambda}_t - \phi_{32}\hat{\gamma}_t \tag{A.16}$$

$$\hat{w}^{s}_{d,t} - \phi_{33}\hat{R}_t = \phi_{34}\hat{h}^{e}_t + \phi_{26}\hat{h}_{s,t} - \phi_{29}\hat{h}^{s}_{d,t} + \hat{k}_{d,t} + \phi_{35}\hat{\gamma}_t \tag{A.17}$$

$$\hat{y}_t = \hat{Z}_t + \phi_{36}\hat{h}^{e}_t + \phi_{37}\hat{k}_{d,t} + \phi_{37}\hat{\gamma}_t \tag{A.18}$$

$$\hat{h}^{e}_t = \phi_{38}\hat{h}_{s,t} + \phi_{39}\hat{h}_{u,t} \tag{A.19}$$

$$\hat{h}_{s,t} = \phi_{40}\hat{h}^{s}_{d,t} + \phi_{41}\hat{h}^{s}_{im,t} + \phi_{41}\hat{\lambda}_t + \phi_{42}\hat{\gamma}_t \tag{A.20}$$

$$\hat{h}_{u,t} = \phi_{43}\hat{h}^{u}_{d,t} + \phi_{44}\hat{h}^{u}_{im,t} + \phi_{45}\hat{\lambda}_t + \phi_{46}\hat{\gamma}_t \tag{A.21}$$

$$\hat{mc}_t = \phi_{33}\hat{R}_t + \hat{k}_{d,t} + \hat{\gamma}_t - \hat{y}_t \tag{A.22}$$

$$\hat{\pi}_t = \beta E_t\left(\hat{\pi}_{t+1}\right) + \phi_{47}\hat{mc}_t - \hat{e}_t \tag{A.23}$$

$$\hat{d}_{d,t} = \hat{y}_t - \phi_{48}\hat{mc}_t - \hat{\gamma}_t \tag{A.24}$$

$$\hat{r}_t = \phi_{49}\hat{r}_{t-1} + \phi_{50}\hat{\pi}_t + \phi_{51}\hat{y}_t + \hat{v}_t \tag{A.25}$$

and the stationary processes of the exogenous variables are given by

$$\hat{\alpha}_{d,t} = \rho_{\alpha_d}\hat{\alpha}_{d,t-1} + \varepsilon_{\alpha_d,t} \tag{A.26}$$

$$\hat{\alpha}_{im,t} = \rho_{\alpha_{im}}\hat{\alpha}_{im,t-1} + \varepsilon_{\alpha_{im},t} \tag{A.27}$$

$$\hat{e}_t = \rho_e\hat{e}_{t-1} + \varepsilon_{e,t} \tag{A.28}$$

$$\hat{Z}_t = \rho_z\hat{Z}_{t-1} + \varepsilon_{z,t} \tag{A.29}$$

$$\hat{\upsilon}_t = \varepsilon_{r,t} \tag{A.30}$$

$$\hat{\mu}_t = \rho_\mu \hat{\mu}_{t-1} + \varepsilon_{\mu,t} \tag{A.31}$$

Clearly, we have 25 equilibrium conditions along with 25 state variables: $\hat{c}_{d,t}$, $\hat{c}_{im,t}$, $\hat{h}^s_{d,t}$, $\hat{h}^u_{d,t}$, $\hat{h}_{d,t}$, $\hat{h}^s_{im,t}$, $\hat{h}^u_{im,t}$, $\hat{h}_{im,t}$, $\hat{h}_{s,t}$, $\hat{h}_{u,t}$, $\hat{h}^e_t$, $\hat{w}^s_{d,t}$, $\hat{w}^u_{d,t}$, $\hat{w}^s_{im,t}$, $\hat{w}^u_{im,t}$, $\hat{k}_{d,t}$, $\hat{x}_{d,t}$, $\hat{R}_t$, $\hat{r}_t$, $\hat{mc}_t$, $\hat{\pi}_t$, $\hat{y}_t$, $\hat{d}_{d,t}$, $\hat{\gamma}_t$, $\hat{\lambda}_t$.

The coefficients of the above log-linearized equilibrium conditions, which are functions of the calibrated parameters and the steady-state values of the model's variables, are the following:

$$\phi_1 = \frac{\overline{x}_d}{\overline{c}_d}, \qquad \phi_2 = \frac{\phi \overline{w}^s_d \overline{h}^s_d}{\overline{c}_d}, \qquad \phi_3 = \frac{(1-\phi)\overline{w}^u_d \overline{h}^u_d}{\overline{c}_d}, \qquad \phi_4 = \frac{\overline{R}\cdot\overline{k}_d}{\overline{c}_d}, \qquad \phi_5 = \frac{\overline{r}^k \overline{k}_d}{\overline{c}_d}, \qquad \phi_6 = \frac{\overline{d}_d}{\overline{c}_d}$$

$$\phi_7 = 1-\delta, \qquad \phi_8 = \delta, \qquad \phi_9 = \frac{1}{\sigma}, \qquad \phi_{10} = \frac{1}{\sigma} + \frac{\overline{h}_d}{\overline{h}_d - 1}, \qquad \phi_{11} = 1 - \rho_{\alpha_d}$$

$$\phi_{12} = \left(\frac{\phi\cdot\overline{h}^s_d}{\overline{h}_d}\right)^{\frac{\sigma+1}{\sigma}}, \qquad \phi_{13} = \left(\frac{(1-\phi)\overline{h}^u_d}{\overline{h}_d}\right)^{\frac{\sigma+1}{\sigma}}, \qquad \phi_{14} = \frac{\overline{w}^s_{im}\overline{\lambda}\cdot\overline{h}^s_{im}}{\overline{c}_{im}}, \qquad \phi_{15} = \frac{\overline{w}^u_{im}(1-\overline{\lambda})\overline{h}^u_{im}}{\overline{c}_{im}}$$

$$\phi_{16} = \phi_{14} - \left(\frac{\overline{\lambda}}{1-\overline{\lambda}}\right)\phi_{15}, \qquad \phi_{17} = \frac{1}{\sigma} + \left(\frac{\overline{h}_{im}}{\overline{h}_{im}-1}\right), \qquad \phi_{18} = \phi_9\left(\frac{\overline{\lambda}}{\overline{\lambda}-1}\right), \qquad \phi_{19} = \left(\frac{\overline{\lambda}\cdot\overline{h}^s_{im}}{\overline{h}_{im}}\right)^{\frac{\sigma+1}{\sigma}}$$

$$\phi_{20} = \left(\frac{(1-\overline{\lambda})\overline{h}^u_{im}}{\overline{h}_{im}}\right)^{\frac{\sigma+1}{\sigma}}, \qquad \phi_{21} = \phi_{19} - \left(\frac{\overline{\lambda}}{1-\overline{\lambda}}\right)\phi_{20}, \qquad \phi_{22} = 1-\overline{\lambda}, \qquad \phi_{23} = (1-\overline{\gamma})\overline{\lambda}$$

$$\phi_{24} = \frac{1}{\rho_1}, \qquad \phi_{25} = \frac{1}{\overline{\gamma}-1}, \qquad \phi_{26} = \rho_1 - \rho, \qquad \phi_{27} = \rho - \rho_2, \qquad \phi_{28} = \rho_2 - \rho_1$$

$$\phi_{29} = \rho_1, \qquad \phi_{30} = \rho_2, \qquad \phi_{31} = \rho_2\left(\frac{\overline{\lambda}}{\overline{\lambda}-1}\right), \qquad \phi_{32} = \rho_1 + \rho_2\left(\frac{\overline{\gamma}}{1-\overline{\gamma}}\right), \qquad \phi_{33} = \frac{\overline{R}}{\overline{r}^k}$$

$$\phi_{34} = \rho - 1, \qquad \phi_{35} = 1-\rho_1, \qquad \phi_{36} = a, \qquad \phi_{37} = 1-a, \qquad \phi_{38} = \zeta\left(\frac{\overline{h}_s}{\overline{h}^e}\right)^{1-\rho}$$

$$\phi_{39} = \left(\frac{\overline{h}_u}{\overline{h}^e}\right)^{1-\rho}, \qquad \phi_{40} = \omega_s\left(\frac{\overline{\gamma}\phi\overline{h}^s_d}{\overline{h}_s}\right)^{1-\rho_1}, \qquad \phi_{41} = \left(\frac{(1-\overline{\gamma})\overline{\lambda}\cdot\overline{h}^s_{im}}{\overline{h}_s}\right)^{1-\rho_1}$$

$$\phi_{42} = \phi_{40} + \left(\frac{\overline{\gamma}}{\overline{\gamma}-1}\right)\phi_{41}, \qquad \phi_{43} = \omega_u\left(\frac{\overline{\gamma}(1-\phi)\overline{h}^u_d}{\overline{h}_u}\right)^{1-\rho_2}, \qquad \phi_{44} = \left(\frac{(1-\overline{\gamma})(1-\overline{\lambda})\overline{h}^u_{im}}{\overline{h}_u}\right)^{1-\rho_2}$$

$$\phi_{45} = \left(\frac{\overline{\lambda}}{\overline{\lambda}-1}\right)\phi_{44}, \qquad \phi_{46} = \phi_{43} + \left(\frac{\overline{\gamma}}{\overline{\gamma}-1}\right)\phi_{44}, \qquad \phi_{47} = \frac{(1-\beta\cdot\theta)(1-\theta)}{\theta}$$

$$\phi_{48} = \frac{\overline{y} - \overline{\gamma}\overline{d}_d}{\overline{\gamma}\cdot\overline{d}_d}, \qquad \phi_{49} = \rho_r, \qquad \phi_{50} = (1-\rho_r)\phi_\pi, \qquad \phi_{51} = (1-\rho_r)\phi_y$$

B Klein's method

We solve the model by applying the generalized Schur decomposition described by Klein (2000). So, let s_t^0 be the (25x1) vector of state variables

$$s_t^0 = \Big[\hat{y}_{t-1}, \hat{c}_{im,t-1}, \hat{h}^s_{d,t-1}, \hat{h}^u_{d,t-1}, \hat{h}_{d,t-1}, \hat{h}^s_{im,t-1}, \hat{h}^u_{im,t-1}, \hat{h}_{im,t-1}, \hat{h}_{s,t-1}, \hat{h}_{u,t-1}, \hat{h}^e_{t-1},$$
$$\hat{k}_{d,t}, \hat{x}_{d,t-1}, \hat{w}^s_{d,t-1}, \hat{w}^u_{d,t-1}, \hat{w}^s_{im,t-1}, \hat{w}^u_{im,t-1}, \hat{r}_{t-1}, \hat{d}_{d,t-1}, \hat{\gamma}_{t-1}, \hat{\lambda}_{t-1}, \hat{mc}_{t-1}, \hat{R}_t, \hat{c}_{d,t}, \hat{\pi}_t\Big]'$$

and accordingly v_t the (6x1) vector of exogenous stochastic variables

$$v_t = [\hat{\alpha}_{d,t}, \hat{\alpha}_{u,t}, \hat{e}_t, \hat{z}_t, \hat{v}_t, \hat{\mu}_t]'$$

Following Klein (2000) we write the model in a matrix form as follows:

$$A \cdot E_t\left(s^0_{t+1}\right) = B \cdot s^0_t + C \cdot u_t$$

where A (25x25), B (25x25), and C (25x6) are the coefficient matrices.

The exogenous processes of preference, mark-up, technology, policy and immigration shock can be written in matrix form as

$$u_t = P \cdot u_{t-1} + \varepsilon_t$$

with

$$P = \begin{bmatrix} \rho_{\alpha_d} & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_{\alpha_{im}} & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_e & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_z & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_\mu \end{bmatrix}$$

and

$$\varepsilon_t = [\varepsilon_{\alpha_d,t}, \varepsilon_{\alpha_{im},t}, \varepsilon_{e,t}, \varepsilon_{z,t}, \varepsilon_{r,t}, \varepsilon_{\mu,t}]'$$