Consumption Risk over the Frequency Domain

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Abstract

In this paper we extend the concept of ultimate consumption risk analyzed by Parker and Julliard (*Journal of Political Economy*, 2005), and we evaluate the Consumption Capital Asset Pricing Model by employing as an explanatory variable consumption risk over the frequency domain. We find that at lower frequencies consumption risk explains up to 98% of the cross-sectional variation of expected returns and the equity premium puzzle is resolved. Our evidence is consistent with a coefficient of risk aversion between 1 and 4 in the very long run.

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1. Introduction

A persistent puzzle in the macroeconomics and finance literature has been the failure of the Consumption Capital Asset Pricing model (C-CAPM), which measures risk by consumption beta, first to explain empirically the differences in expected stock returns by the variation in the covariance of consumption and returns, and second to provide plausible levels of risk aversion.[1] In this paper we re-evaluate the validity of the C-CAPM and provide additional insights into the relationship between returns and long-term consumption dynamics, as well as its implications for risk aversion, by assessing the explanatory power of consumption risk over the frequency domain. Our findings indicate that as lower frequencies of consumption risk are taken into account and thus the horizon of consumption growth increases (eventually reaching infinity), consumption risk explains almost entirely the cross-sectional variation of expected returns and, moreover, is consistent with reasonable and statistically significant values of the coefficient of risk aversion.

The idea of measuring the risk of a portfolio by its covariance with consumption over longer time horizons is not novel. Brainard et al. (1991) have shown that the performance of the C-CAPM improves as the horizon increases. Breeden et al. (1989) argue that at short horizons consumption should be replaced with a portfolio that exhibits higher correlations with long-run movements in consumption. Daniel and Marshall (1997) find that aggregate returns and consumption growth are more correlated at lower frequencies and that the behavior of the equity premium becomes less puzzling. More recently, Bansal and Yaron (2004), Bansal et al. (2005), and Hansen et al. (2005) show that when consumption risk is measured by the covariance between long-run cashflows from holding a security and long-run consumption growth in the economy, the differences in consumption risk provide useful information about the expected return differentials across assets.

The papers closest in spirit to ours are Parker (2001, 2003) and, in particular, Parker and Julliard (2005). These studies focus on the ultimate risk to consumption, which is defined as the covariance between an asset's return during a quarter and consumption growth over the quarter of the return and several following quarters. According to the empirical evidence, ultimate consumption risk explains the cross-sectional variation in returns surprisingly well, but the equity premium puzzle

[1] See, among others, Mankiw and Shapiro (1986) and Breeden et al. (1989). Mehra (2003) and Cochrane (2005) provide extensive surveys of the relevant literature.

persists and high levels of risk aversion are required to line up the model with the data. While similar in spirit, our approach allows for long-term consumption dynamics by performing a dynamic analysis of consumption risk with the C-CAPM at several frequencies rather than over the time domain. As pointed out by Granger and Hatanaka back in 1964, according to the spectral representation theorem a time series can be seen as the sum of waves of different periodicity and, hence, there is no reason to believe that economic variables should present the same lead/lag cross-correlation at all frequencies. We incorporate this rationale into the context of the single-factor C-CAPM by using well-developed techniques to estimate the coherency (the analog of the correlation coefficient in the time domain) and the gain (the analog of the regression coefficient) between returns and consumption risk over the frequency domain.[2]

The advantage of measuring the portfolio risk of consumption over the whole frequency domain is that it enables us to separate different layers of dynamic behavior within the standard C-CAPM by distinguishing between the short run (fluctuations of 2 to 6 quarters), the medium run or business cycle (lasting from 8 to 32 quarters), and the long run (oscillations of duration above 32 quarters). If consumption risk is a more persistent process than suggested by the conventional analysis, identifying the impact of lower frequencies of consumption risk can alter the implied long-run riskiness in ways that are empirically important and cannot be addressed by standard time-domain techniques, which aggregate over the entire frequency band and are not robust when frequency variations are large.[3] Moreover, our approach can circumvent several caveats associated with unmodeled frictions, time aggregation or measurement error in the consumption data, which are often found to account for the short-run predictability of the pricing errors.[4]

In this respect, cross-spectral analysis provides a powerful tool for the exploration of unknown relationships between two series where the correlation structure may vary over the time horizon considered. To our knowledge, the spectral estimation of the C-CAPM has only been previously considered by Berkowitz (2001), who provides a framework for estimating parameters of a wide

[2] See Hamilton (1994) for a general overview of spectral analysis.

[3] For example, employing a standard VAR model between 2 variables and k lags requires the solution of a $2k$ eigenproblem for both eigenvalues and eigenvectors to assess the relative importance of each cyclical component. More importantly, the limiting covariance structure as the horizon tends to infinity cannot be estimated.

[4] See Grossman et al. (1987) and Wheatley (1988).

class of dynamic rational expectations models in the frequency domain. The author applies his Generalized Spectral estimation technique to the C-CAPM under the assumption of constant relative risk aversion and finds that when the focus is oriented towards lower frequencies both risk aversion and the discount factor attain more plausible values. These findings indicate that the empirical failure of the C-CAPM is due to high-frequency noise that the model is not capable nor intended to match in the time domain. Going a step further in this direction, the approach adopted here allows us to examine the (range of) frequencies along which C-CAPM performs poorly or well rather than simply rejecting or accepting its empirical fit.

We use the 25 Fama-French size and book-to-market portfolios to explore the implications of spectral analysis for the C-CAPM. We find that at lower frequencies consumption risk explains up to 98% of the cross-sectional variation of expected returns and the equity premium puzzle is eliminated. In addition, we show that the value of risk aversion implied by the use of long-run consumption risk of stockholders becomes as low as 3.3 to 4.3. This range of values stems mainly from the increased variability of long-run consumption, which is inversely related to risk aversion, and is far below the level of 10 considered as reasonable by Mehra and Prescott (1985). Our findings are robust to the definitions of the variables, the sample span and the set of portfolios utilized. We also relate the use of consumption risk over the frequency domain with the ultimate consumption risk approach by Parker and Julliard (2005), and we are able to find a significant reduction for the estimates of risk aversion. Finally, given the importance of long-run consumption risk for the dynamics of the C-CAPM, we address the impact of long-term risk-free rates within the spectral approach. We find that the model preserves its significance and continues to yield plausible values of risk aversion for low frequencies of consumption risk, which lie between 3.5 and 1 for risk-free rates with longer maturity. Thus, we confirm that long-term consumption risk can provide useful information for the variation of excess returns in the context of the single-factor C-CAPM by reconciling through our spectral approach the increased importance of consumption dynamics over the very long run with plausible values of constant relative risk aversion.

The rest of the paper is organized as follows. Section 2 presents long-term consumption risk within the C-CAPM and its modified version in the context of spectral analysis. Section 3 describes the estimation method and the data. Section 4 presents the empirical results for consumption risk over the frequency domain. Section 5 presents some robustness tests and section 6 provides a comparison with ultimate consumption risk. Section 7 investigates the impact of long-term risk-free rates and, finally, section 8 concludes the paper.

2. Expected returns and the risk to consumption over the frequency domain

The standard C-CAPM assumes that the representative household maximizes the expected present discounted value of utility flows from consumption by allocating wealth to consumption and different investment opportunities. At the optimal allocation a marginal investment at time t in any asset should yield the same expected marginal increase in utility at $t+1$, which for the constant relative risk aversion utility function implies that:

$$E_t[C_{t+1}^{-\gamma} R_{j,t+1}] = E_t[C_{t+1}^{-\gamma}] R_{t,t+1}^f \tag{1}$$

where C_{t+1} is consumption at $t+1$, $R_{j,t+1}$ is the gross real return on portfolio j of stocks unknown at t and known at $t+1$, $R_{t,t+1}^f$ is the gross real return on a risk-free asset between t and $t+1$, and γ is the representative household's constant coefficient of relative risk aversion. Equation (1) can be written as a model of average cross sectional returns by manipulating it to a beta representation or factor model, in which the expectation of the equity premium, $E[R_{j,t+1}^e] = E[R_{j,t+1} - R_{t,t+1}^f]$, is given in terms of covariances by:

$$E[R_{j,t+1}^e] = \alpha_0 + \beta_{j,0}\lambda_0 \tag{2}$$

where $\alpha_0 = 0$, $\beta_{j,0} = \frac{Cov[\Delta \ln C_{t+1}, R_{j,t+1}^e]}{Var[\Delta \ln C_{t+1}]}$, $\lambda_0 = \frac{\gamma Var[\Delta \ln C_{t+1}]}{E[1-\gamma \Delta \ln C_{t+1}]}$. Equation (2) provides an external test of the structure embodied in the model with consumption growth, $\Delta \ln C_{t+1}$, being the stochastic discount factor that prices returns. The estimated α_0 should be equal to zero and the expected excess return on a portfolio is equal to the scaled consumption risk of the portfolio, $\beta_{j,0}\lambda_0$. The estimated λ_0 and moments of consumption growth imply a level of the risk aversion for the representative investor according to:

$$\gamma = \frac{\lambda_0}{E[\Delta \ln C_{t+1}]\lambda_0 + Var[\Delta \ln C_{t+1}]} \tag{3}$$

Equations (1) to (2) evaluate the risk of a portfolio based solely on its covariance with contemporaneous consumption growth. They maintain the assumption that the intertemporal allocation of consumption is optimal from the perspective of the textbook model of consumption smoothing, so that any change in marginal utility is reflected instantly and completely in consumption.

Now, departing from the time domain to the frequency domain, we can rewrite equations (2) and (3) for each frequency. After dropping the time subscript for notational simplicity, we get that the beta-form representation is given by the response of excess returns to consumption risk over the whole band of frequencies, ω, where ω is a real variable in the range $0 \preceq \omega \preceq \pi$:[5]

$$E[R_j^e] = \alpha_\omega + \beta_{j,\omega}\lambda_\omega \tag{4}$$

where

$$\alpha_\omega = 0, \beta_{j,\omega} = G_{R_j^e,\Delta \ln C}(\omega), \lambda_\omega = \frac{\gamma f_{\Delta \ln C,\Delta \ln C}(\omega)}{E[1-\gamma\Delta \ln C, \omega]} \tag{5}$$

The cross-spectrum between any two variables is complex-valued, therefore it can be decomposed into its real and imaginary components, which are given here by:

$$f_{R_j^e,\Delta \ln C}(\omega) = C_{R_j^e,\Delta \ln C}(\omega) - iQ_{R_j^e,\Delta \ln C}(\omega), \tag{6}$$

where $C_{R_j^e,\Delta \ln C}(\omega)$ is the *co-spectrum* and $Q_{R_j^e,\Delta \ln C}(\omega)$ is the *quadrature spectrum.* The measure of comovement between returns and consumption risk over the frequency domain is the well-known squared *coherency*, $c^2_{R_j^e,\Delta \ln C}(\omega)$, defined as:

$$c^2_{R_j^e,\Delta \ln C}(\omega) \equiv \frac{\left|f_{R_j^e,\Delta \ln C}(\omega)\right|^2}{f_{\Delta \ln C,\Delta \ln C}(\omega) f_{R_j^e,R_j^e}(\omega)} = \frac{C^2_{R_j^e,\Delta \ln C} + Q^2_{R_j^e,\Delta \ln C}}{f_{\Delta \ln C,\Delta \ln C}(\omega) f_{R_j^e,R_j^e}(\omega)} \tag{7}$$

[5] In general, the spectrum of a process, say x_t, can be written as $f_{xx}(\omega) = \rho_0 + 2\sum_{k=1}^{\infty} \rho_k \cos(k\omega)$, where ρ_k is the k-order autocovariance function of the series. In turn, we can consider the multivariate spectrum, $F_{yx}(\omega)$, for a bivariate zero mean covariance stationary process $Z_t = [y_t, x_t]^\top$ with covariance matrix $\Gamma(\cdot)$, which is the frequency domain analog of the autocovariance matrix. The diagonal elements of $F_{yx}(\omega)$ are the spectra of the individual processes, $f_{yy}(\omega)$ and $f_{xx}(\omega)$, while the off-diagonal ones refer to the cross-spectrum or cross spectral density matrix of y_t and x_t. In detail, $F_{yx}(\omega) = \frac{1}{2\pi}\sum_{k=-\infty}^{\infty}\Gamma(k)e^{-ik\omega} = \begin{bmatrix} f_{xx}(\omega) & f_{yx}(\omega) \\ f_{xy}(\omega) & f_{yy}(\omega)\end{bmatrix}$, where $F_{yx}(\omega)$ is an Hermitian, non-negative definite matrix, i.e. $F_{yx}(\omega) = F^*_{yx}(\omega)$, with $*$ denoting the complex conjugate transpose since $f_{yx}(\omega) = \overline{f_{xy}(\omega)}$.

where $0 \leq c_{R_j^e,\Delta \ln C}(\omega) \leq 1$. Intuitively, coherency provides a measure of the correlation between the two series at each frequency and can be interpreted as the frequency domain analog of the correlation coefficient. We can then define the *gain* as:

$$G_{R_j^e,\Delta \ln C}(\omega) \equiv \frac{\left| f_{R_j^e,\Delta \ln C}(\omega) \right|}{f_{\Delta \ln C,\Delta \ln C}(\omega)} \tag{8}$$

which provides a scalar measure of the amplitude of the relationship between the components at hand at each frequency. The *gain* can be interpreted here as the beta coefficient of the $\omega-$frequency component of R_j^e on the corresponding component of $\Delta \ln C$.

Once the price of risk, λ_ω, is estimated from a cross-section regression, the implied relative risk aversion of the representative agent at each frequency can be retrieved by:

$$\gamma_\omega = \frac{\lambda_\omega}{E[\Delta \ln C, \omega]\lambda_\omega + f_{\Delta \ln C,\Delta \ln C}(\omega)]} \tag{9}$$

which is the analog of (3) in the frequency domain.

3. Estimation methodology and data

In this section we first outline the estimation methodology of the C-CAPM in the context of the spectral analysis developed above and then we present the dataset and briefly discuss the spectral properties of the data.

3.1. Estimation methodology

Estimation of (2) is typically performed in the literature within a two-step approach. The first step involves a time series regression of the return of the j portfolio onto a constant and consumption growth, $\Delta \ln C_{t+1}$, in order to obtain an estimate of the slope coefficient $\beta_{j,0}$. As a second step, the estimated coefficients are employed in the cross-section regression (2) in order to get the estimate of the price of risk, λ_0.[6] By employing excess returns, we can test whether our model contains an equity premium by simply testing the significance of the constant. The adjusted R^2 of this equation measures the fraction of the cross-sectional variation explained by the data.

[6]See Fama and French (1992). Alternatively, the Fama and MacBeth (1973) methodology can be employed.

Furthermore, inference regarding the risk aversion of the representative investor can be conducted taking as given the mean and variance of consumption growth by employing (3) with the standard errors of γ calculated by the delta method.

Our methodology differs in the way betas are obtained by calculating from (8) the gain between each portfolio excess return and consumption growth for every frequency. Specifically, the spectra and co-spectra of a vector of time-series for a sample of T observations can be estimated for a set of frequencies $\omega_n = 2\pi n/T$, $n = 1, 2, ..., T/2$. The relevant quantities are estimated through the periodogram, which is based on a representation of the observed time-series as a superposition of sinusoidal waves of various frequencies; a frequency of π corresponds to a time period of two quarters, while a zero frequency corresponds to infinity. However, the estimated periodogram is an unbiased but inconsistent estimator of the spectrum because the number of parameters estimated increases at the same rate as the sample size. Consistent estimates of the spectral matrix can be obtained by either smoothing the periodogram, or by employing a lag window approach that both weighs and limits the autocovariances and cross-covariances used.[7] We use here the Bartlett window that assigns linearly decreasing weights to the autocovariances and cross-covariances in the neighborhood of the frequencies considered and zero weight thereafter.[8]

3.2. Data

For our portfolios and returns series we use quarterly returns on the 25 Fama and French portfolios, which are the intersections of 5 portfolios formed on size (market equity, ME) and 5 portfolios formed on the ratio of book equity to market equity (B/M). B/M used during a fiscal year is based on the book equity for the previous fiscal year divided by ME for December of the previous year. The B/M breakpoints are the NYSE quintiles. The portfolios include all NYSE, AMEX, and NASDAQ stocks for which there is market equity data for December and June of the previous fiscal year, and (positive) book equity data for the previous fiscal year. The series are available on a monthly basis and excess returns are constructed by subtracting the three-month

[7]For example, the spectrum of x_t is estimated by $f_{xx}(\omega) = \frac{1}{2\pi}\sum_{k=-(T-1)}^{T-1} w(k)\widehat{\rho_k}e^{-ik\omega}$, where the kernel, $w(k)$, is a series of lag windows.

[8]The lag, k, is set using the rule $k = 2\sqrt{T}$, as suggested by Chatfield (1989).

Treasury Bill rate, which proxies the risk-free rate. To match consumption data we use a quarterly frequency and set our timing convention so that $R_{j,t+1}$ represents the return on portfolio j during the quarter $t+1$. We measure consumption as personal consumption expenditures on nondurable goods from the National Income and Product Accounts. We make the 'end-of-period' timing assumption that consumption during quarter t takes place at the end of the quarter. The data are made real using a chain weighted price deflator, spliced across periods, produced by the Bureau of Economic Analysis. These series determine the sample, which covers the second quarter of 1947 to the last quarter of 2001, and the frequency (quarterly) utilized.[9]

3.3. Spectral properties of the data

Before moving on with the estimation results, we report some evidence on the comovement between returns and consumption growth in the frequency domain. Figures 1A and 1B depict the spectra of the series under scrutiny (along with 95% confidence intervals) and can be interpreted as the variance decompositions over various frequency bands (stated as a fraction of π).[10] As can be readily observed, the variability of returns does not exhibit substantial changes over the frequency domain. On the other hand, the variability of non-durables consumption is muted for 2 to 32 quarters; however, for horizons exceeding 32 quarters a steep increase is prevalent. As t approaches infinity, the variance of consumption is seven times greater than its 32-quarter value and 52 times greater than its short-run value. The concentration of variance in low frequencies is an indication of short-term correlation in consumption growth, such as an AR(1) with a positive coefficient, rather than an indication of non-stationarity of the process, which can be ruled out for the series at hand.[11] This finding has direct implications for the subsequent analysis, especially when the coefficient of risk aversion is calculated from the estimates of our model. Since the variance of consumption growth is inversely related to the coefficient of risk aversion by (3), we expect that as

[9]We obtained the Fama and French portfolio data from Kenneth French's web page (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). The rest of the data were obtained from Jonathan Parker's web page (http://www.princeton.edu/~jparker/research/crisk.html); see Parker and Julliard (2005) for a more detailed description of the dataset.

[10]Confidence intervals were derived based on a normal approximation of the spectra of the series; see Priestley (1981) for a detailed description.

[11]See Campbell (2003, section 3.2) and the references cited therein for some evidence the properties of US consumption growth.

the lower frequencies are taken into account, risk aversion will decrease.

Figure 1C presents the coherency (along with 95% confidence intervals) between the market excess return and non-durables consumption growth for all frequencies. This analysis has been undertaken for every portfolio but to save space we report only the results for the aggregate market return. Overall our estimates suggest that the correlation (measured by coherency) between returns and consumption growth exhibits an upward trend as we move from high to low frequencies. Specifically, as regards the short-run correlation for frequencies between π and $7\pi/8$ corresponding to around 2 quarters, coherency fluctuates around 20%. Then it plunges to around 5% and steadily increases to reach a local peak of 60% at frequencies corresponding to 3-4 quarters. Two more cycles are observable with peaks at 6 and 16 quarters. The maximum is reached at zero frequency, i.e. for an infinite horizon. In this case, the coherency between the series at hand is estimated at 79%. On the whole, the short-run correlation between returns and consumption growth is low, the business-cycle correlation amounts on average to roughly 50%, while the long-run correlation exceeds 70%.

4. Empirical findings

This section asks whether consumption risk explains the cross-sectional variation in expected returns for various frequencies. In particular, the questions we seek to answer are the following. First, does consumption risk at various frequencies explain a large share of variation of average returns? Second, is the price of risk, λ_ω, statistically significant? Third, does the estimate of α_ω corroborate the existence of an equity premium? Last, what is the estimate of the risk aversion coefficient, γ_ω?

To allow for comparisons with the rest of the literature, in this section we take the standard route and we estimate the model by employing non-durables consumption and gross excess returns from the Fama-French 25 portfolios. As a first step, we estimate model (4) by imposing the coefficient restriction $\alpha_\omega = 0$. Table 1 (Panel A) reports the estimation results for a range of frequencies corresponding from 2 quarters to infinity. The first row reports the results for the highest frequency considered (which corresponds to two quarters in the time domain). Our results suggest that at this frequency consumption risk does not explain variation in returns and is associated with a significant and positive price of risk (given by the estimate of λ_ω). Moreover, the coefficient of risk aversion is estimated at 71 and is found to be significant. These findings are in line with those typically reported in the literature on the C-CAPM. As we move to lower frequencies (and consequently increase the time horizon) consumption risk still fails to explain a larger share of the cross-sectional variation; however, the implied risk aversion declines almost monotonically and reaches 8.8 for the 16-quarter horizon. When even lower frequencies are taken into account the performance of the model improves substantially. For the 32-quarter horizon, consumption risk is positive and significant and explains 66% of the cross-sectional variation of the returns. More importantly, the coefficient of risk aversion is significant and reduced to 4.6. The performance of the C-CAPM is further improved at zero frequency (infinite horizon). The model succeeds in explaining 98.1% of the cross-sectional variation of returns. The associated price of risk is significant and estimated at 0.007, almost three times greater than, for example the one at 2-quarters. More importantly, risk aversion is estimated at just 4.3 and remains significant.

Next, we assess model (4) by estimating α_ω rather than imposing $\alpha_\omega = 0$. In this respect, we separately evaluate the ability of the model to explain the equity premium and the cross section of expected stock returns, and we are able to measure the extent to which the model addresses the equity premium puzzle. Panel B of Table 1 reports the estimation results. The evidence suggests that at a high frequency consumption risk does not explain variation in returns and is associated with a significant equity premium of the magnitude of 2.3% per quarter. Moreover, the coefficient of risk aversion is estimated at 42.5 and found to be insignificant. This poor performance of contemporaneous consumption risk is also depicted in the left upper panel of Figure 2, which plots the consumption betas (gains) and the average realized returns along with the second-stage regression line associated with this frequency. The overall picture indicates an almost flat relationship between consumption risk and returns at this frequency. Figure 3 plots in turn the predicted and average returns of the portfolios. The horizontal distance between a portfolio and the 45-degree line is the extent to which the expected return based on fitted consumption risk (on the vertical axis) differs from the observed average return (on the horizontal axis). As expected, at the 2-quarter horizon there is almost no relation between predicted and realized returns.

When we move to lower frequencies consumption risk explains a larger share of the cross-sectional variation, reaching 12% for the 8-quarter horizon. However, the implied premium remains large and significant, whereas the price of risk turns out insignificant and negative. This general picture is also depicted in the regression line in the upper right part of Figure 3. Furthermore, a significant and high risk aversion is estimated at this frequency reaching 20.9. Similar findings pertain with respect to the 16-quarter frequency with risk aversion now declining and reaching 6.8, but with a large standard error.

As lower frequencies are further considered the performance of the model improves substantially. For the 32-quarter horizon, consumption risk is positive and significant, and explains 66% of the cross-sectional variation of the returns. These findings are depicted in left lower panel of Figures 2 and 3. The regression line is positive, quite steep and suggests a strong relationship between betas and returns. As expected, the deviation between fitted and realized returns is sufficiently reduced. More importantly, the coefficient of risk aversion becomes significant and is now reduced to 4.6. Associated with this horizon is a negligible and insignificant equity premium of -0.3%. The performance of the C-CAPM is further improved at zero frequency (infinite horizon). The model succeeds in explaining 98.6% of the cross-sectional variation of returns coupled with an insignificant pricing error. The associated price of risk is significant and estimated at 0.007; however, at this frequency our model overpredicts average returns by just 0.2%, which is marginally significant. More importantly, risk aversion is estimated at 4.3 and is significant. These features are also illustrated in the lower right part of Figures 2 and 3, in which the average realized and fitted returns are almost perfectly aligned on the regression line and the 45-degree line, respectively.

To sum up, we find that when higher frequencies of consumption risk are considered the results replicate the typical findings of the literature, i.e. the C-CAPM fails to explain the differences in expected stock returns by the variation in the covariance of consumption and returns, and to provide plausible levels of risk aversion. In contrast, as lower frequencies of consumption risk are taken into account, consumption risk explains almost entirely the cross-sectional variation of expected returns and the equity premium puzzle is eliminated. Moreover, the coefficient of risk aversion implied by the cross-sectional reward for long-run consumption risk is found to be approximately 4.3 and is statistically significant.

5. Robustness tests

In this section we present some sensitivity tests on the relationship between consumption risk and the expected returns over the frequency domain. We first consider the impact of alternative specifications by using a smaller sample size as well as alternative definitions of returns and consumption, and subsequently we examine the impact of alternative portfolios on our results.

5.1. Alternative specifications

Some studies (including, among others, Fama and French, 1992, 1993, and Lettau and Ludvigson, 2001) have used a shorter time period than the one analyzed in our baseline results. To allow for comparisons, Panel A of Table 2 shows the results of estimating our model on a sample of returns that starts in the third quarter of 1963. In this sub-period, the pattern of coefficients and the fit tell a similar story, except that low-frequency consumption risk does even better at explaining expected returns. Around 67% and almost 100% of the variation in expected returns is explained by consumption risk over the 32-quarter and infinite horizons with the level of risk aversion again found to be slightly larger than 4 (reaching 4.6 and 4.3, respectively). Similar to the baseline specification, the fitted model understates the average return on all portfolios by 0.5% and 0.2%. The fit of the model for the infinite horizon is depicted on the upper part of Figure 4.

Second, we measure consumption risk using total consumption instead of non-durables consumption. Ait-Sahalia et al. (2004) argue that the consumption risk of equity is understated by NIPA nondurable goods because it contains many necessities and few luxury goods. As pointed out by the authors, consumers have more discretion over their consumption of luxury goods than essential goods, and consumption of the former is found to covary more strongly with stock returns.[12] Panel B of Table 2 shows that using total consumption risk in place of nondurable consumption risk leads to a slightly different picture. Long-run total consumption risk fits the cross-section of expected returns somewhat better than non-durables consumption and, interestingly, lowers the

[12] See also Parker (2001). The usual concern when total consumption is used is that it contains the flow of expenditures on durable goods instead of the -theoretically desired- stock of durable goods. However, expenditures and stocks are cointegrated and, hence, the long-term movement in expenditures following an innovation to equity returns also measures the long-term movement in consumption flows.

level of risk aversion relative to the previous specifications at 3.3. This finding accords well with nonseparability over time (or habits) in the utility function, which is expected to be stronger for durable consumption goods, which are now included in consumption. Past consumption levels are expected to affect more negatively the marginal utility of consumption for durable goods when longer horizons are considered, which drives down the estimates of risk aversion. The bottom part of Figure 4 plots the performance of the specification with total consumption.

Finally, we use consumption risk over the frequency domain to price long-horizon returns. Long-horizon returns are calculated as cumulative returns over the next 11 quarters.[13] Panel C of Table 2 shows some improvements of our model for shorter horizons compared to the baseline specification. Specifically, for an horizon of 8 quarters, the model succeeds in explaining almost half the cross-sectional variation of returns; however, the price of risk is negative and the associated risk aversion is found to be quite high, estimated at 18.8. As we move to lower frequencies, and specifically to the 32-quarter horizon the explanatory power of the model is lower than the baseline specification (42.2% as opposed to 65.5%), but the remaining attributes of the model are in line with the theoretical one. The price of risk is positive and significant, the equity premium is insignificant and the estimated risk aversion decreases to 4.7. This specification yields similar findings to the baseline specification for the infinite horizon and its performance is depicted at the bottom part of Figure 4.

5.2. Other portfolios

The C-CAPM as any asset pricing model should be able to explain expected returns on any set of portfolios. So far, the portfolios considered are the double-sorted 25 Fama-French B/M and ME value-weighted portfolios, which basically aim at capturing the value and size premia. We consider here alternative portfolios sorted on both firm characteristics and overall economic factors or systematic risk factors in order to check whether consumption risk over the frequency domain succeeds in explaining risk premia generated by these portfolios.

As a first step, we consider a slightly different set of returns, namely the equal-weighted Fama

[13]For comparison purposes, the choice of the horizon is the one that corresponds to the selected model of Parker and Juliard (2005).

and French 25 portfolios that is also examined by Parker and Julliard (2005). In line with these authors, low-frequency consumption risk does an even better job of explaining the cross-sectional pattern of expected returns for these portfolios (see Panel A of Table 3). A slightly increased proportion of the variation in expected returns is explained along with low coefficients of risk aversion (reaching 4.3), whereas the equity risk premium is found to be insignificant. The fit of the model for the infinite horizon is depicted on the upper right part of Figure 5.

Second, we consider a set of single sorted portfolios, namely the 10 size (ME), 10 book to market (B/M) and 10 dividend yield (D/P) portfolios of Fama and French. These portfolios sort firms on the basis of their characteristics that lead to cross-sectional dispersion in measured risk premia and are behind the factor models of Fama and French (1993).[14] This set of portfolios aims at disentangling the value and size premia. To the extent that the C-CAPM holds, we expect to find growth firms to have less exposure to consumption risk than value firms and smaller firms to be exposed to higher consumption risk when compared to larger firms.[15] Our results (reported in Panel B of Table 3) are in line with those of our baseline specification. At a high frequency, the C-CAPM explains 13% of the cross-sectional variation in expected returns associated with a significant risk premium and a high coefficient of risk aversion estimated at 57.7. The estimate of risk aversion decreases with the frequency decline, whereas the fit of the model improves. At the 32-quarter horizon, half of the variation is explained and risk aversion declines to 4.6, while at an infinite horizon, the respective figures are 93.9% and 4.3. The upper part of Figure 5 plots the actual and the predicted returns for this set of portfolios.

Third, we use the 20 risk-sorted portfolios employed by Campbell and Vuolteenaho (2004).[16] The authors follow Daniel and Titman's (1997) point that sorting only on firm characteristics

[14] The 10 size value-weighted portfolios are formed on the basis of market capitalization and include all NYSE, AMEX, and NASDAQ stocks in the CRSP database which are ranked at the end of June of each year using NYSE capitalization breakpoints. The 10 B/M portfolios are formed at the end of each June using NYSE breakpoints. The BE used in June of year t is the book equity for the last fiscal year ending in t-1 and ME is price times shares outstanding at the end of December of t-1. The 10 D/P portfolios include all NYSE, AMEX, and NASDAQ stocks for which ME for June of year t, and at least 7 monthly returns (to compute the dividend yield) from July of t-1 to June of t are available. Portfolios are formed on D/P at the end of each June using NYSE breakpoints. The dividend yield used to form portfolios in June of year t is the total dividends paid from July of t-1 to June of t per dollar of equity in June of t. The returns on these portfolios are taken from Kenneth French's web site, where more details on their construction can be found.

[15] See also Jagganathan and Wang (2005), and Cochrane (2005).

[16] These portfolios are available at http://post.economics.harvard.edu/faculty/vuolteenaho/papers.html.

could generate a spurious link between premia and risk measures, and sort common stocks into 20 portfolios according to their past loadings with state variables that are useful in predicting the aggregate market return.[17] The purpose of their strategy is to generate portfolios with a large spread in these loadings and thus overcome Daniel and Titman's (1997) problem. Panel C of Table 3 reports our results for this set of portfolios. Interestingly, the C-CAPM fails in at least one of its aspects for all the frequencies under consideration with the exception of the infinite horizon. For this horizon, 82% of the cross-sectional variation of the returns is explained and risk aversion is estimated at 4.3. Figure 5 (bottom part), which plots realized returns versus predicted returns, shows that the spread in returns across portfolios is lower than the one generated by the portfolios considered so far explaining the somewhat worse performance of this model.

Fourth, we consider 34 industry-sorted portfolios, which have posed a particularly challenging feature from the perspective of systematic risk measurement (see Fama and French, 1997). Value-weighted industry portfolios are formed by sorting all NYSE, AMEX, and NASDAQ stocks by their CRSP four-digit SIC Code at the end of June of each year.[18] Similar to the previous set of portfolios, our findings suggest that systematic industry-specific risk is priced only for the infinite horizon (see Panel D of Table 3). The risk aversion for the 32-quarter and the infinite horizon is estimated at 4.2, a value that is very close to the one attained by every specification and portfolio considered when non-durables consumption is employed.

6. A connection with ultimate consumption risk

As discussed earlier on, in a series of papers Parker (2001, 2003) and Parker and Julliard (2005) have allowed for the slow response of consumption to market returns and have evaluated the risk/return trade-off among portfolios of stocks by focusing on the ultimate consumption risk measured by the covariance of the return at $t+1$ and the change in consumption from t to $t+1+s$,

[17] These state variables include the excess log return on the market, the term yield spread (computed as the difference between ten-year and short-term bonds) and the small stock-value spread (computed as the difference between the log(B/M) of the small high B/M portfolio and the small low B/M portfolio). More details can be found at the Appendix of Campbell and Vuolteenaho (2004), which is available at http://kuznets.fas.harvard.edu/~campbell/papers.html.

[18] The industry definitions are available at Kenneth French's web site. We include in our analysis the portfolios for which we have returns for the whole sample period.

where s is the horizon over which the consumption response is studied:

$$Cov[\ln\left(\frac{C_{t+1+s}}{C_t}\right), R^e_{j,t+1}] \tag{10}$$

In beta representation we have:

$$E[R^e_{j,t+1}] = \alpha_s + \beta_{j,s}\lambda_s \tag{11}$$

where $\alpha_s = 0$, $\beta_{j,s} = \frac{Cov[\ln\left(\frac{C_{t+1+s}}{C_t}\right), R^e_{j,t+1}]}{Var[\ln\left(\frac{C_{t+1+s}}{C_t}\right)]}$, $\lambda_s = \frac{\gamma_s Var[\ln\left(\frac{C_{t+1+s}}{C_t}\right)]}{E[1-\gamma_s \ln\left(\frac{C_{t+1+s}}{C_t}\right)]}$. When $S = 0$, equation (11) yields the standard beta representation (2). For $S > 0$, the stochastic discount factor considered is one minus the long-horizon consumption growth times the risk aversion of the representative agent, γ_s. The estimated λ_s and moments of consumption growth imply then a level of relative risk aversion given by:

$$\gamma_s = \frac{\lambda_s}{E[\ln\left(\frac{C_{t+1+s}}{C_t}\right)]\lambda_s + Var[\ln\left(\frac{C_{t+1+s}}{C_t}\right)]} \tag{12}$$

Equations (11) and (12) show a modification of the standard C-CAPM developed in section 2 over the time domain. Clearly by varying the horizon, S, consumption risks take a range of values from the short-run to the long-run along with the corresponding asset pricing implications of these risks. In their empirical results, Parker and Julliard (2005) find a model improvement as the horizon increases accompanied by lower estimates of the risk-free rate and the coefficient of risk aversion. However, the authors do not report results beyond 15 quarters, as the trade-off between a larger horizon and optimal inference leads to a choice of 11 quarters as the preferred specification.

We relate the methodology of Parker and Julliard (2005), which is based on the employment of ultimate consumption risk in the time domain, to the current one conducted in the frequency domain. To study the implications of ultimate consumption risk for our methodology, we utilize the 11-quarter consumption growth rate that coincides with the preferred specification of Parker and Julliard (2005), and contrast it with our specification.[19] To gain some insight on the effects that this transformation of consumption growth has on the spectral estimates, Figure 6A depicts the log-spectrum of 1-quarter consumption growth rate versus the 11-quarter one.[20] Significant differences

[19]To save space the results reported here refer to the total market return. The results for the individual portfolios are qualitatively similar.

[20]We plot the spectral densities in a log scale to accentuate the cyclical properties of the data.

between the spectral densities of the two series are detected for frequencies lower than $5\pi/16$ (beyond 5.3 quarters). At zero frequency the 11-quarter consumption growth variance is 125 times greater than the 1-quarter ahead. This is expected since the transformation employed strengthens lower frequencies and attenuates the impacts of the higher ones. Defining $\ln(C_{t+1+s}/C_t) \equiv \Delta^s \ln C_t$, we can show that we can obtain $\Delta^s \ln C_t$ from $\Delta \ln C_t$ through the transformation $H(L) = (1 + L + L^2 + ... + L^s)$, where L is the usual lag operator. The spectrum of $\Delta^s \ln C_t$ is then linked with the one of $\Delta \ln C_t$ by $f_{\Delta^s \ln C_t} = H(e^{-i\omega})H(e^{i\omega})f_{\Delta \ln C_t}$.[21] For $\omega = 0$, the variance of $\Delta^2 \ln C_t$ is 4 times the variance of $\Delta \ln C_t$, while the respective variance for $\omega = \pi$ is eliminated.

Turning to Figure 6B that plots the estimated coherencies between the two measures of consumption growth and returns, we observe that the transformation over the following 11 quarters has increased the short-run comovement of consumption with returns, which is estimated at around 50% as opposed to around 20% for the 1-quarter ahead consumption growth. Given that our cross-section analysis is based on the estimated gains, the most important finding is the change imposed on the gain through this transformation of the data. Figure 6C plots the respective estimated gains over the frequency domain. The results suggest that the aggregation of consumption growth now leads to an escalation of the short-term gain combined with an attenuation of the long-run one. On the other hand, the gain corresponding to the 1-quarter consumption growth remains fairly stable over the whole frequency domain.

The preceding analysis on the spectral properties of ultimate consumption growth compared to the ones of typical consumption growth seems to stress the high frequency gains. If this result is combined with a similar responsiveness for the portfolios at hand, we expect to find increased empirical validity over the short-term as well. Our results (see Table 4) of the cross-section estimation corroborate to some extent such a conjecture. Specifically, for a short-run horizon (ranging from 2-3 quarters) our model explains half of the cross-sectional variation of expected returns, as opposed to 25% based on the 1-quarter consumption growth (see Table 1). However, the other features of the model, like the negative prices of risk, the significant pricing errors and the high values of risk aversion, point to the empirical failure of the C-CAPM; this is expected since the risk aversion

[21]For example, for $s = 2$, $f_{\Delta^s \ln C_t} = (2 + 2\cos\omega)f_{\Delta \ln C_t}$.

is calculated on the basis of the relative contribution of the variance over the specific wavelength and not over the whole domain. As the frequency approaches zero, the empirical validity of the C-CAPM is restored and the model explains 64% and 98% of the cross-sectional variation of expected returns. Moreover, in accordance with our general findings, lower frequencies are associated with significant decreases of the equity premium, leading to an even lower risk aversion estimated at 2.3 generated by the increased low-frequency variance.

7. Long-run risk-free rates and consumption risk over the frequency domain

The previous sections have established that the low frequencies of consumption risk, which are associated with the long-run pattern of C-CAPM, improve the empirical fit of the model and provide plausible values of risk aversion. An extension of this approach envisages the impact of risk-free rates of longer maturity, which are likely to embed useful information when the horizon of consumption risk widens. Intuitively, if long-term interest rates are negatively related to consumption growth, then they provide a hedge against bad states and individuals will sell short-term bonds and buy long-term bonds to receive payoffs when their consumption level is expected to be lower, thus resulting in a falling or negative term structure. On the flip side, if long-run rates earn a low return when consumption growth is negative, holding long-term bonds exacerbates consumption risk resulting in a rising term premium.[22]

To assess the impact of long-run risk-free rates and consumption risk over the frequency domain, we develop in the next section a variant of the model presented in section 2 that incorporates risk-free rates of longer maturity in the C-CAPM and then we present some empirical results.

7.1. A beta representation of long-run risk-free rates and consumption risk

The solution of the investor optimization problem implies that:

$$E_t[\frac{1}{1+\rho}(1+R^f_{t+1,t+1+s})\frac{u'(C_{t+1+s})}{u'(C_t)}] = 1 \tag{13}$$

[22]Estrella and Mishkin (1996) have found that inverted yield curves can be leading indicators of recessions and hence of reduced consumption growth rates. The empirical implications of long-run risk-free rates (and the associated term structure) for the C-CAPM have been investigated by several studies including, among others, Harvey (1988, 1989, 1991 and 1993), Estrella and Hardouvelis (1991), Plosser and Rouwenhorst (1994), Kamara (1997), Roma and Torous (1997), and Hamilton and Kim (2002). The general empirical contention from these studies is that the slope of the term spread is positively associated with future economic activity.

where ρ is the rate of time preference and $R^f_{t+1,t+1+s}$ is the risk-free rate with s-periods ahead maturity. In turn, we can re-write the Euler equation (1) as:

$$E_t[R^f_{t+1,t+1+s}\frac{u'(C_{t+1+s})}{u'(C_t)}R_{j,t+1}] = E_t[R^f_{t+1,t+1+s}\frac{u'(C_{t+1+s})}{u'(C_t)}]R^f_{t,t+1} \tag{14}$$

Assuming that $R^f_{t+1,t+1+s}$ is orthogonal to $R^f_{t,t+1}$, we can get the following beta representation for the excess return of portfolio j:

$$E[R^e_{j,t+1}] = \alpha^s + \beta^s_j\lambda^s \tag{15}$$

where

$$\alpha^s = \frac{Cov[R^f_{t+1,t+1+s}\frac{u'(C_{t+1+s})}{u'(C_t)}, R^f_{t,t+1}]}{E[R^f_{t+1,t+1+s}\frac{u'(C_{t+1+s})}{u'(C_t)}]},\ \beta^s_j = \frac{Cov[R^f_{t+1,t+1+s}\frac{u'(C_{t+1+s})}{u'(C_t)}, R^e_{j,t+1}]}{Var[R^f_{t+1,t+1+s}\frac{u'(C_{t+1+s})}{u'(C_t)}]},$$

$$\text{and } \lambda^s = \frac{\gamma^s Var[R^f_{t,t+1+s}\frac{u'(C_{t+1+s})}{u'(C_t)}]}{E[R^f_{t,t+1+s}\frac{u'(C_{t+n})}{u'(C_t)}]}$$

Equation (15) renders an alternative specification to (2) and shows how risk-free rates of longer maturity affect the single factor C-CAPM with the interaction of the long-term risk-free rate scaling consumption growth over the corresponding period and affecting risk aversion. In turn, defining $R^f_{t+1,t+1+s} \equiv R^f_{s,t}$ for notational simplicity and adopting the standard constant relative risk aversion parametrization, which implies that $\frac{u'(C_{t+1+s})}{u'(C_t)} \simeq 1 - \gamma\Delta^s \ln C_t$, we get that:

$$\alpha^s = \frac{Cov[R^f_{s,t}(1-\gamma\Delta^s \ln C_t), R^f_{t,t+1}]}{E[R^f_{s,t}(1-\gamma\Delta^s \ln C_t)]},\ \beta^s_j = \frac{Cov[R^f_{s,t}\Delta^s \ln C_t, R^e_{j,t+1}]}{Var[R^f_{s,t}\Delta^s \ln C_t]},$$

$$\text{and } \lambda^s = \frac{\gamma_s Var[R^f_{s,t}\Delta^s \ln C_t]}{E[R^f_{s,t}(1-\gamma^s\Delta^s \ln C_t)]}$$

The implied risk aversion can be given in terms of the long-term price of risk, λ^s, and equals:

$$\gamma^s = \frac{\lambda^s R^f_{s,t}}{E[R^f_{s,t}\Delta^s \ln C_t]\lambda^s + Var[R^f_{s,t}\Delta^s \ln C_t]} \tag{16}$$

which can be larger or smaller than the one implied by (3) depending upon the magnitude of the long-run risk-free rate and the expected value and variance of scaled consumption growth.

Following the spectral approach adopted in section 2, equation (15) can be estimated over the frequency domain as:

$$E[R^e_{j,t+1}] = \alpha^s_\omega + \beta^s_{j,\omega}\lambda^s_\omega \tag{17}$$

where the components, after dropping the time subscript, are given by:

$$\alpha^s_\omega = \frac{G_{R^f_s(1-\gamma\Delta^s \ln C),R^f}(\omega)}{E[R^f_s(1-\gamma\Delta^s \ln C)]},\ \beta^s_{j,\omega} = G_{R^e_j, R^f_s\Delta^s \ln C}(\omega),\ \lambda^s_\omega = \frac{\gamma^s_\omega f_{R^f_s\Delta^s \ln C, R^f_s\Delta^s \ln C}(\omega)}{E[R^f_s(1-\gamma^s_\omega\Delta^s \ln C),\omega]} \tag{18}$$

and the coefficient of risk aversion is given by:

$$\gamma^s_\omega = \frac{\lambda^s_\omega E[R^f_s,\omega]}{E[\Delta^s \ln C,\omega]\lambda^s_\omega + f_{R^f_s\Delta^s \ln C, R^f_s\Delta^s \ln C}(\omega)]} \tag{19}$$

which is the analog of (9) when long-term risk-free rates are taken into account.

7.2. Empirical results with long-term risk-free rates

To estimate equation (17) we use data on long-term risk-free interest rates. Since data for each maturity, s, are not readily available to match our consumption and return series, we employ risk-free interest rates with maturities of 1, 3, 5 and 10 years starting in 1953:Q2.[23] Risk-free interest rates are made real by employing as a measure of inflation the q-o-q change in the chain weighted price deflator, spliced across periods, produced by the Bureau of Economic Analysis. In this respect, we proxy expected interest rates and expected inflation with their realized counterparts over the holding period of the corresponding risk-free asset.

First, we briefly discuss the spectral properties of the data. The first row of Figure 7 presents the log-spectra of 1, 3, 5, and 10-year consumption growth for all frequencies (stated as a fraction of π).[24] As expected, the volatility of consumption growth at any horizon increases sharply for lower frequencies and, given the propeties of consumption growth over the time domain, the low-

[23]The series codes are GS1, GS3,GS5 and GS10 and are available from the Board of Governors of the Federal Reserve System (http://www.research.stlouisfred.org/fred2/).

[24]Notice that the consumption data for the 3-year horizon are close to the 11-quarter horizon framework advocated by Parker and Julliard (2005) and examined in section 6.

frequency variability of consumption growth is amplified when the time horizon increases (see also Figure 6A). Again, the relative concentration of fluctuations in low frequencies is an indication of short-term correlation in consumption growth, which drives the estimates for the coefficients of risk aversion. The second row of Figure 7 plots the estimated coherencies between the long-term returns and the corresponding measures of consumption growth and shows that the relationship remains fairly stable over the whole frequency domain for all four horizons considered. The third row of Figure 7 plots the respective estimated gains over the frequency domain and, as can be readily seen, as the horizon of returns and the corresponding consumption growth rates increases the gains for higher frequencies are substantially lower.

Moving on to the main empirical results, Table 5 presents the estimates for the four maturities considered. The evidence from the 1-year interest rates (Panel A) replicates the usual failure of the C-CAPM; at a high frequency consumption risk explains only a small fraction of the variation in returns and is associated with a significant equity premium of the magnitude of 3.4% per quarter, whereas the coefficient of risk aversion is found to be 79.7 and is significant. For the 16-quarter horizon the coefficient of risk equity premium falls to 2%, but the model is overall unable to explain the cross-section of returns and the coefficient of risk aversion is statistically equal to zero. As we move to lower frequencies, the picture changes starkly. For the 32-quarter horizon, the performance of the model improves dramatically, the equity premium is negligible, and moreover the coefficient of risk aversion is estimated at 4.5 with a small standard error. The picture is further improved at the zero frequency, where the model explains 96.4% of the variability in returns with a zero equity premium and a significant coefficient of risk aversion found to be as low as 3.4.

A similar picture emerges from other risk-free rates of long-term maturities (Panels B to D of Table 5). In all cases, the model fails at high frequencies, but its performance is consistent with the C-CAPM at lower frequencies. The coefficient of risk aversion attains plausible values for horizons above 16-quarters for the 3 and 5-year interest rates, whereas the specification with the 10-year risk free rate produces plausible results with the 16-quarter horizon as well. Notably, when the 10-year rate is used the model yields a coefficient of risk aversion for the zero frequency (infinite horizon) that is in the vicinity of unity, thus providing evidence in favor of a log-utility specification for the long-run investment problem. These patterns are corroborated by Figure 8, in which the average realized and fitted returns from the various risk-free rates are found to be closely aligned.

8. Conclusions

In this paper we re-evaluated the C-CAPM by adopting a spectral approach to measure the covariance of an asset's return with consumption growth and its impact on expected stock returns over the frequency domain. We established that when lower frequencies of consumption risk are considered the validity of the C-CAPM is restored. For low frequencies the C-CAPM can explain almost entirely the cross-sectional variation of expected returns accompanied by a decrease in the equity premium, whereas the implied coefficient of risk aversion is found to lie between 1 to 4 and is statistically significant.

The paper is part of the upcoming literature that aims at capturing the behavior of aggregate and cross-sectional stock returns via the long-term dynamics of consumption. The approach adopted here remains, however, agnostic about the driving force of these dynamics. For instance, our findings are consistent with the general class of models that relax the assumption of costless adjustment in consumption plans by including the time spent to calculate and implement a new consumption-savings decision, or constraints in information and search costs that lead investors in making infrequent consumption and portfolio allocation decisions at discrete points in time. The impact of consumption risk measured over the frequency domain can also be consistent with models that entail monitoring costs and heterogeneous agents, in which only a fraction of households adjusts its consumption over discrete intervals.[25]

Recently, there have been some attempts to bring together longer-term consumption dynamics with theoretical explanations. Panageas and Yu (2005) claim that over the short run, consumption growth is dominated by small frequent shocks, while unpredicted and large technological innovations, which are embodied in the capital stock, prevail in the long run. The authors then show that this framework implies that consumption growth over the long run can reveal information about the degree to which the economy has absorbed a major technological shock. Malloy et al. (2005) show

[25] See, for instance, Grossman and Laroque (1990), Lynch (1996), Marshall and Parekh (1999), Gabaix and Laibson (2001) and Jagganathan and Wang (2005).

that in a model with recursive preferences the covariance of returns with long-run consumption growth of households who bear stock market risk captures the cross-sectional variation of average stock returns better than the covariance of returns with long-run aggregate or non-stockholder consumption growth. Thus, the question on why consumption takes so long to adjust to news in stock returns and what the underlying shocks driving stock returns and consumption are remains open and offers a promising route for further research.

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Table 1. Expected excess returns and consumption risk frequencies

Panel A

Frequency	*(Quarters)*	*R-sq(adj)*	λ_ω	*standard error*	*Relative risk aversion*	*standard error*
1	(2.000)	-2.728	0.003	0.000	71.002	0.581
15/16	(2.133)	-4.460	0.008	0.001	69.735	0.888
7/8	(2.286)	-1.931	0.010	0.001	68.572	0.594
13/16	(2.462)	-3.428	0.008	0.001	46.676	0.362
3/4	(2.667)	-5.218	0.011	0.002	48.650	0.229
5/8	(3.200)	-1.325	0.003	0.000	33.314	0.344
1/2	(4.000)	-0.298	0.004	0.000	36.222	0.057
3/8	(5.333)	-2.076	0.003	0.000	27.510	0.186
1/4	(8.000)	-3.160	0.004	0.000	17.857	0.073
3/16	(10.667)	-1.401	0.004	0.000	14.095	0.061
1/8	(16.000)	-0.443	0.003	0.000	8.822	0.031
1/16	(32.000)	0.660	0.007	0.000	4.621	0.002
0	(inf)	0.981	0.007	0.000	4.297	0.002

Panel B

Frequency	*(Quarters)*	*R-sq(adj)*	*Equity premium*	*standard error*	λ_ω	*standard error*	*Relative risk aversion*	*standard error*
1	(2.000)	-0.026	0.023	0.003	0.000	0.000	42.463	36.590
15/16	(2.133)	0.257	0.032	0.003	-0.003	0.001	96.259	8.239
7/8	(2.286)	0.006	0.029	0.003	-0.002	0.002	154.260	131.933
13/16	(2.462)	0.042	0.029	0.004	-0.002	0.001	80.082	40.057
3/4	(2.667)	-0.030	0.026	0.002	-0.001	0.001	101.154	161.341
5/8	(3.200)	-0.043	0.025	0.006	0.000	0.001	-6.818	103.596
1/2	(4.000)	-0.003	0.019	0.005	0.001	0.001	33.174	2.724
3/8	(5.333)	0.242	0.037	0.006	-0.002	0.001	35.221	2.675
1/4	(8.000)	0.121	0.031	0.004	-0.001	0.001	20.935	1.526
3/16	(10.667)	0.022	0.031	0.006	-0.001	0.001	17.991	3.572
1/8	(16.000)	-0.032	0.022	0.008	0.000	0.001	6.807	4.592
1/16	(32.000)	0.655	-0.003	0.003	0.008	0.001	4.629	0.007
0	(inf)	0.986	-0.002	0.001	0.007	0.000	4.323	0.010

Notes:
1) Frequency is expressed as a fraction of π.
2) See the text for the definition of λ_ω.
3) Newey-West heteroskedasticity and autocorrelation corrected standard errors.

Table 2.

Expected excess returns and consumption risk frequencies: Robustness tests

Frequency	*(Quarters)*	*R-sq(adj)*	*Equity premium*	*standard error*	λ_ω	*standard error*	*Relative risk aversion*	*standard error*
A. Original Fama-French start date (1963:03)								
1	(2)	0.296	0.017	0.002	0.001	0.000	60.739	3.902
1/2	(4)	-0.008	0.028	0.009	-0.001	0.001	45.227	12.923
1/4	(8)	0.314	0.033	0.003	-0.002	0.000	19.945	0.317
1/8	(16)	-0.016	0.017	0.007	0.001	0.001	7.468	2.107
1/16	(32)	0.671	-0.005	0.002	0.009	0.001	4.612	0.006
0	(inf)	0.995	-0.002	0.000	0.007	0.000	4.269	0.007
B. Total consumption								
1	(2)	0.016	0.018	0.006	0.001	0.001	23.540	4.031
1/2	(4)	0.000	0.020	0.005	0.000	0.000	23.674	4.005
1/4	(8)	0.098	0.012	0.006	0.002	0.001	9.721	0.287
1/8	(16)	-0.033	0.022	0.007	0.000	0.001	5.367	3.551
1/16	(32)	0.712	0.003	0.003	0.008	0.001	3.545	0.006
0	(inf)	0.989	0.000	0.001	0.009	0.000	3.311	0.007
C. Long-horizon returns								
1	(2)	-0.006	0.506	0.078	-0.011	0.013	76.719	1.777
1/2	(4)	0.022	0.407	0.033	0.016	0.011	37.100	0.205
1/4	(8)	0.421	0.529	0.030	-0.009	0.002	18.768	0.071
1/8	(16)	0.272	0.205	0.071	0.010	0.003	9.112	0.044
1/16	(32)	0.422	0.013	0.093	0.012	0.003	4.653	0.008
0	(inf)	0.979	0.017	0.018	0.007	0.000	4.306	0.013

Notes: See Table 1.

Table 3.

Expected excess returns and consumption risk frequencies: Alternative portfolios

Frequency	*(Quarters)*	*R-sq(adj)*	*Equity premium*	*standard error*	λ_{ω}	*standard error*	*Relative risk aversion*	*standard error*
A. Equally weighted portfolios								
1	(2)	0.092	0.021	0.003	0.001	0.000	58.129	8.248
1/2	(4)	-0.025	0.032	0.011	-0.001	0.001	43.895	13.017
1/4	(8)	0.034	0.032	0.004	-0.001	0.001	21.468	2.216
1/8	(16)	0.091	0.014	0.007	0.001	0.001	8.373	0.491
1/16	(32)	0.733	0.001	0.003	0.007	0.001	4.625	0.007
0	(inf)	0.992	-0.001	0.000	0.007	0.000	4.311	0.006
B. 10 size, 10 B/M and 10 D/P portfolios								
1	(2)	0.132	0.019	0.001	0.001	0.000	57.694	3.898
1/2	(4)	0.110	0.017	0.002	0.001	0.000	25.304	2.874
1/4	(8)	0.052	0.019	0.002	0.001	0.000	20.240	1.723
1/8	(16)	0.198	0.014	0.002	0.001	0.000	8.225	0.267
1/16	(32)	0.507	0.008	0.002	0.004	0.000	4.570	0.014
0	(inf)	0.939	-0.000	0.001	0.007	0.000	4.300	0.016
C. 20 risk-sorted portfolios								
1	(2)	-0.056	0.019	0.001	0.000	0.000	-1.509	60.938
1/2	(4)	0.024	0.022	0.002	-0.000	0.000	52.154	14.198
1/4	(8)	-0.051	0.019	0.001	-0.000	0.000	-24.615	168.524
1/8	(16)	-0.046	0.020	0.002	-0.000	0.000	-12.211	64.929
1/16	(32)	-0.054	0.019	0.002	0.000	0.000	1.908	5.983
0	(inf)	0.818	-0.003	0.003	0.008	0.001	4.333	0.040
D. 34 industry portfolios								
1	(2)	-0.019	0.019	0.002	0.000	0.000	27.162	36.261
1/2	(4)	0.003	0.022	0.002	0.000	0.000	52.657	43.332
1/4	(8)	0.062	0.016	0.002	0.001	0.000	20.054	3.040
1/8	(16)	-0.030	0.020	0.004	0.000	0.000	2.306	17.053
1/16	(32)	0.066	0.017	0.002	0.001	0.000	4.180	0.223
0	(inf)	0.853	0.002	0.001	0.006	0.001	4.240	0.036

Notes: See Table 1.

Table 4.

Expected excess returns and consumption risk frequencies: 11-quarter consumption growth

Frequency	*(Quarters)*	*R-sq(adj)*	*Equity premium*	*standard error*	λ_ω	*standard error*	*Relative risk aversion*	*standard error*
1	(2.000)	-0.006	0.022	0.004	0.001	0.001	50.752	25.363
15/16	(2.133)	0.278	0.030	0.002	-0.003	0.001	104.919	11.447
7/8	(2.286)	0.011	0.028	0.004	-0.002	0.003	93.301	25.406
13/16	(2.462)	-0.019	0.028	0.005	0.000	0.001	75.992	61.753
3/4	(2.667)	0.515	0.038	0.003	-0.004	0.001	54.989	1.068
5/8	(3.200)	-0.041	0.023	0.007	0.000	0.001	12.433	42.601
1/2	(4.000)	0.194	0.010	0.006	0.004	0.002	35.935	0.575
3/8	(5.333)	0.321	0.037	0.005	-0.003	0.001	35.656	2.140
1/4	(8.000)	0.223	0.035	0.006	-0.005	0.002	22.618	2.294
3/16	(10.667)	0.085	0.017	0.003	0.005	0.003	12.423	0.992
1/8	(16.000)	0.141	0.011	0.004	0.013	0.005	7.318	0.540
1/16	(32.000)	0.638	0.008	0.002	0.048	0.006	3.797	0.090
0	(inf)	0.984	-0.001	0.001	0.084	0.001	2.255	0.019

Notes: See Table 1.

Table 5.

Expected excess returns and consumption risk frequencies: Long-term interest rates

Frequency	*(Quarters)*	*R-sq(adj)*	*Equity premium*	*standard error*	λ_ω	*standard error*	*Relative risk aversion*	*standard error*
				A. 1-year interest rate				
1	(2)	0.143	0.034	0.006	0.000	0.000	79.742	1.324
1/2	(4)	0.156	0.013	0.006	0.000	0.000	37.199	0.579
1/4	(8)	0.103	0.032	0.005	-0.003	0.002	29.116	9.198
1/8	(16)	-0.017	0.020	0.006	0.002	0.003	4.711	3.308
1/16	(32)	0.730	-0.002	0.003	0.027	0.003	4.494	0.034
0	(inf)	0.964	0.000	0.001	0.029	0.001	3.450	0.043
				B. 3-year interest rate				
1	(2)	0.127	0.029	0.003	-0.001	0.001	112.306	22.363
1/2	(4)	0.059	0.016	0.006	0.003	0.003	35.623	3.243
1/4	(8)	0.499	0.035	0.003	-0.008	0.002	23.432	0.777
1/8	(16)	0.215	0.008	0.005	0.016	0.005	7.541	0.564
1/16	(32)	0.659	0.010	0.002	0.046	0.005	3.882	0.094
0	(inf)	0.968	0.002	0.001	0.082	0.003	2.177	0.047
				C. 5-year interest rate				
1	(2)	0.008	0.023	0.002	0.001	0.002	49.154	24.811
1/2	(4)	0.008	0.022	0.003	0.002	0.003	29.901	9.287
1/4	(8)	0.215	0.036	0.004	-0.010	0.003	25.273	1.514
1/8	(16)	0.266	0.001	0.008	0.028	0.010	8.252	0.550
1/16	(32)	0.760	0.010	0.002	0.102	0.012	3.978	0.101
0	(inf)	0.939	0.003	0.002	0.139	0.009	1.561	0.061
				D. 10-year interest rate				
1	(2)	0.021	0.023	0.002	0.002	0.003	20.723	15.347
1/2	(4)	0.011	0.021	0.003	0.003	0.003	13.238	8.227
1/4	(8)	0.295	0.034	0.003	-0.040	0.012	32.108	3.351
1/8	(16)	0.072	0.012	0.006	0.039	0.020	5.796	1.162
1/16	(32)	0.442	-0.003	0.003	0.450	0.054	4.021	0.117
0	(inf)	0.929	0.001	0.002	0.341	0.022	0.945	0.042

Notes: See Table 1.

Figure 1A. Spectrum of non-durables consumption growth

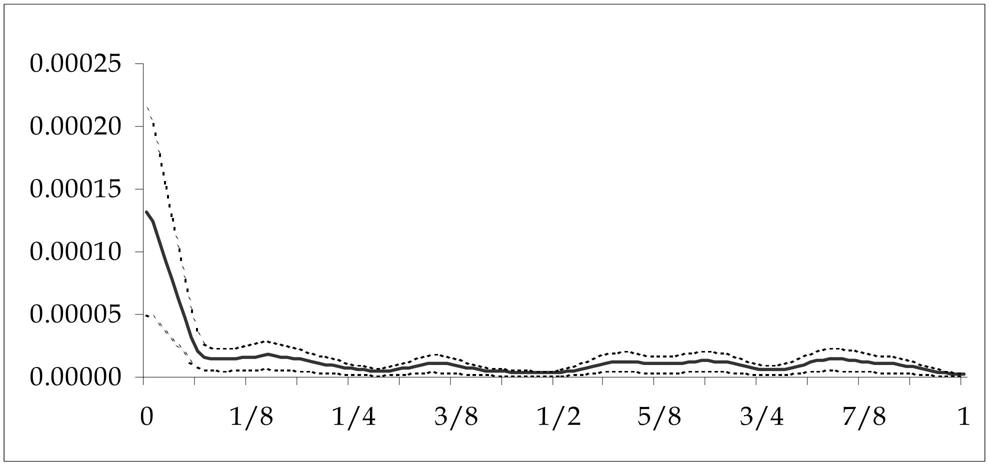


Figure 1B. Spectrum of excess returns

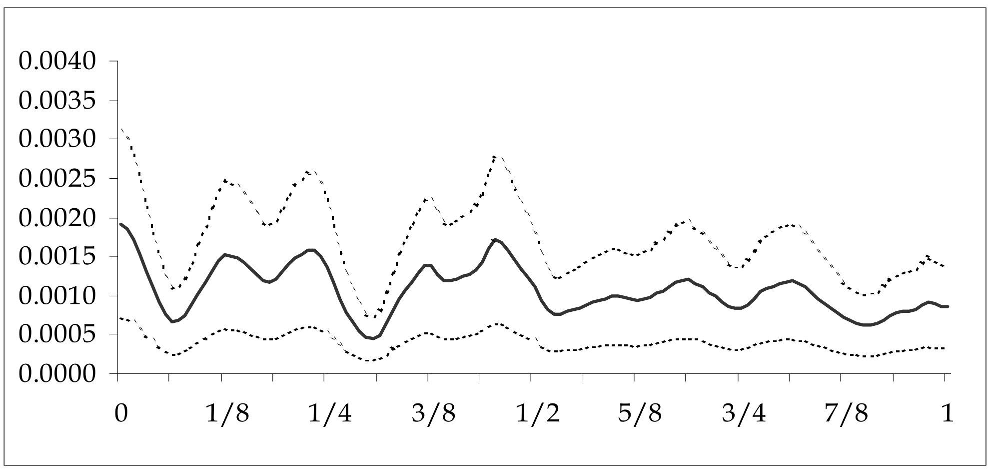
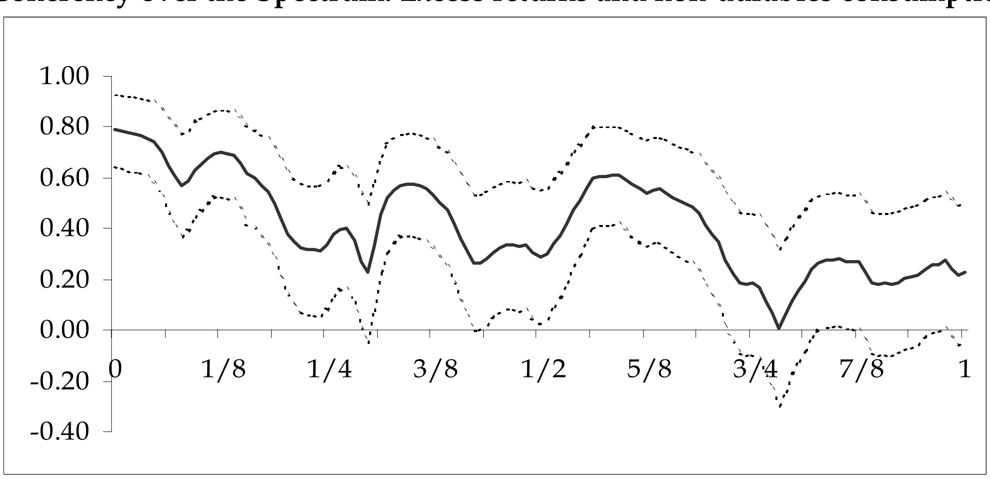


Figure 1C. Coherency over the Spectrum: Excess returns and non-durables consumption growth



Notes: 95% confidence intervals in dashed lines.

Figure 2. Average returns and betas

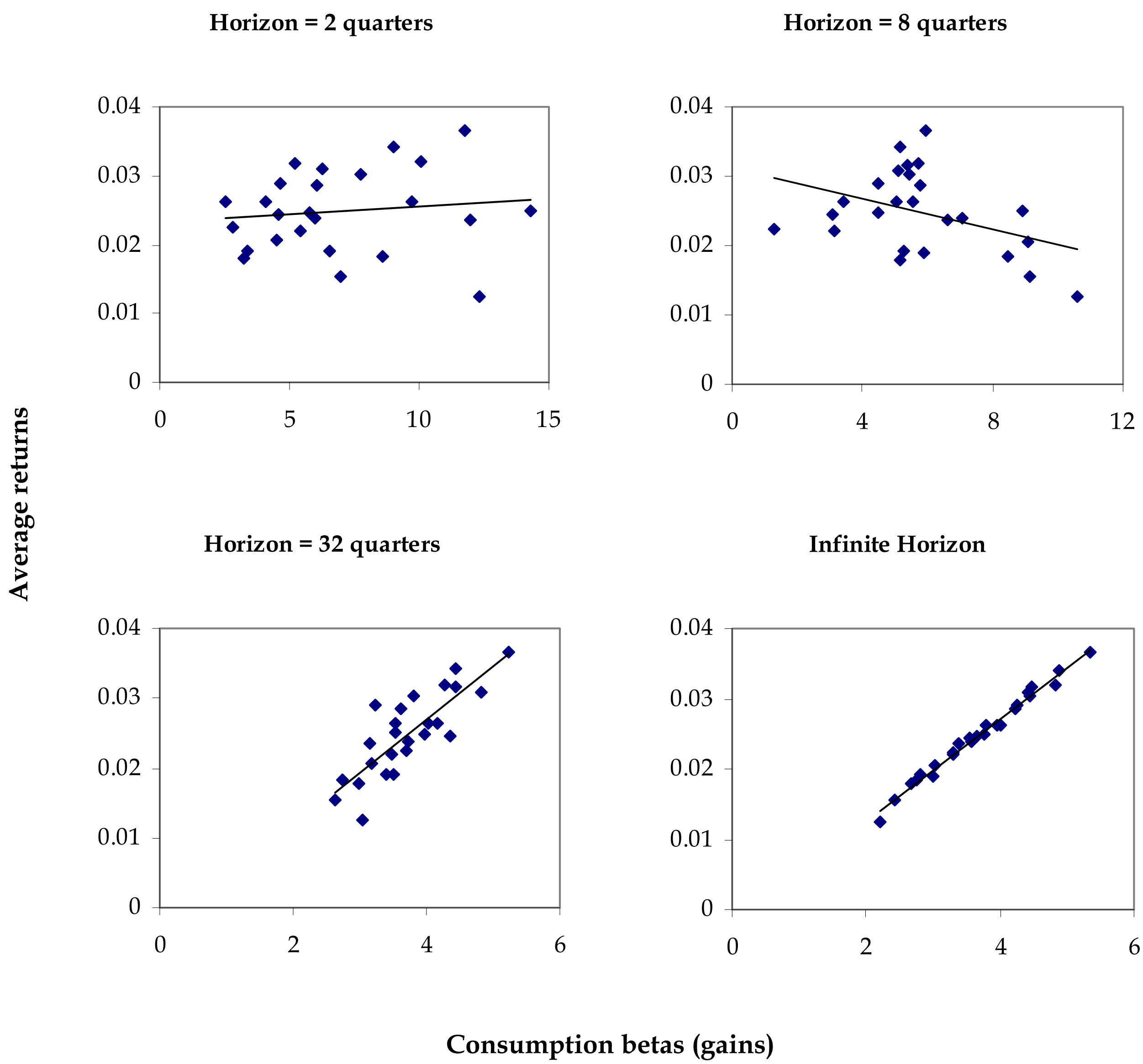


Figure 3. Fitted and average returns

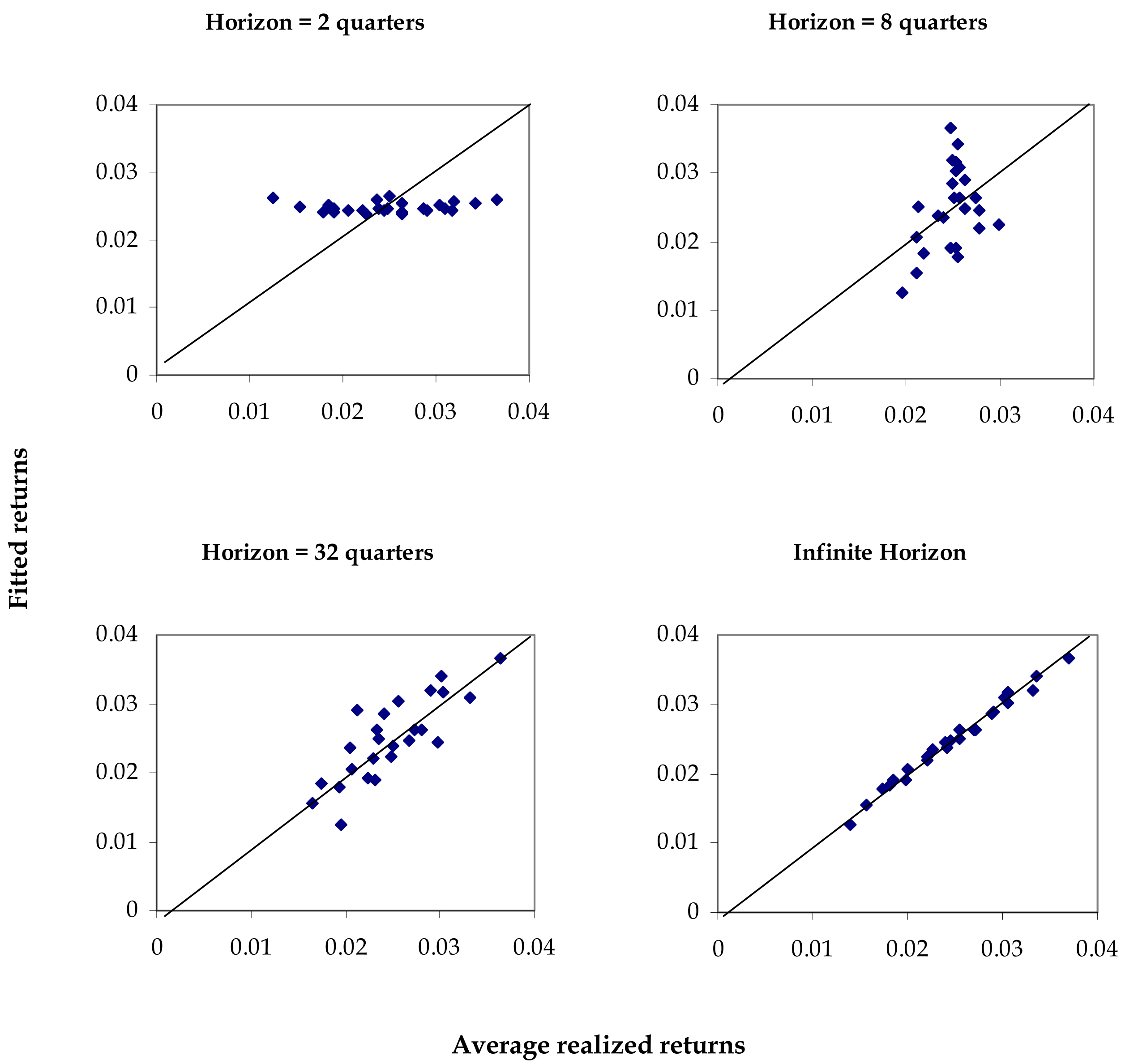


Figure 4. Fitted and average returns (alternative specifications, infinite horizon)

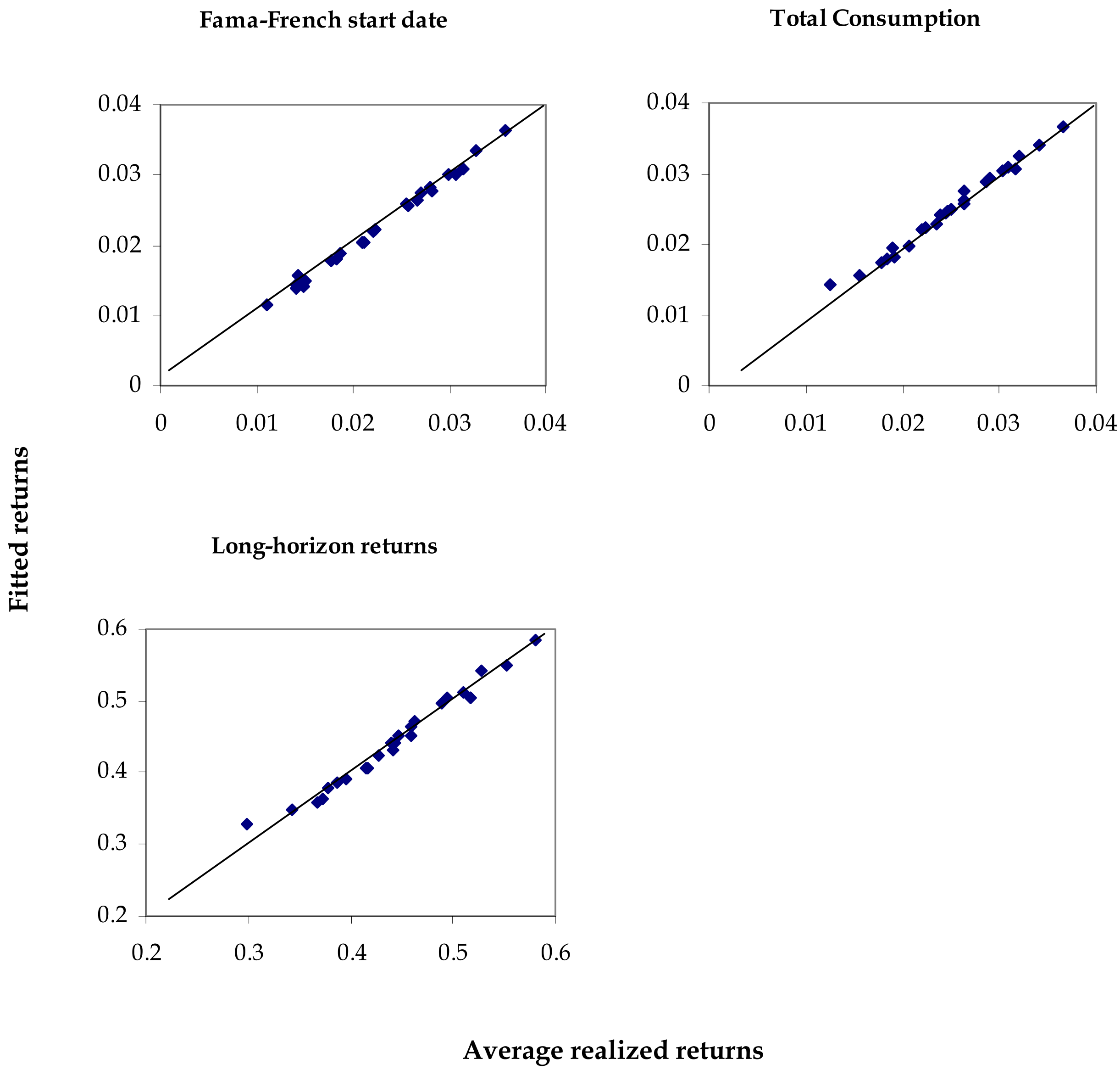


Figure 5. Fitted and average returns (alternative portfolios, infinite horizon)

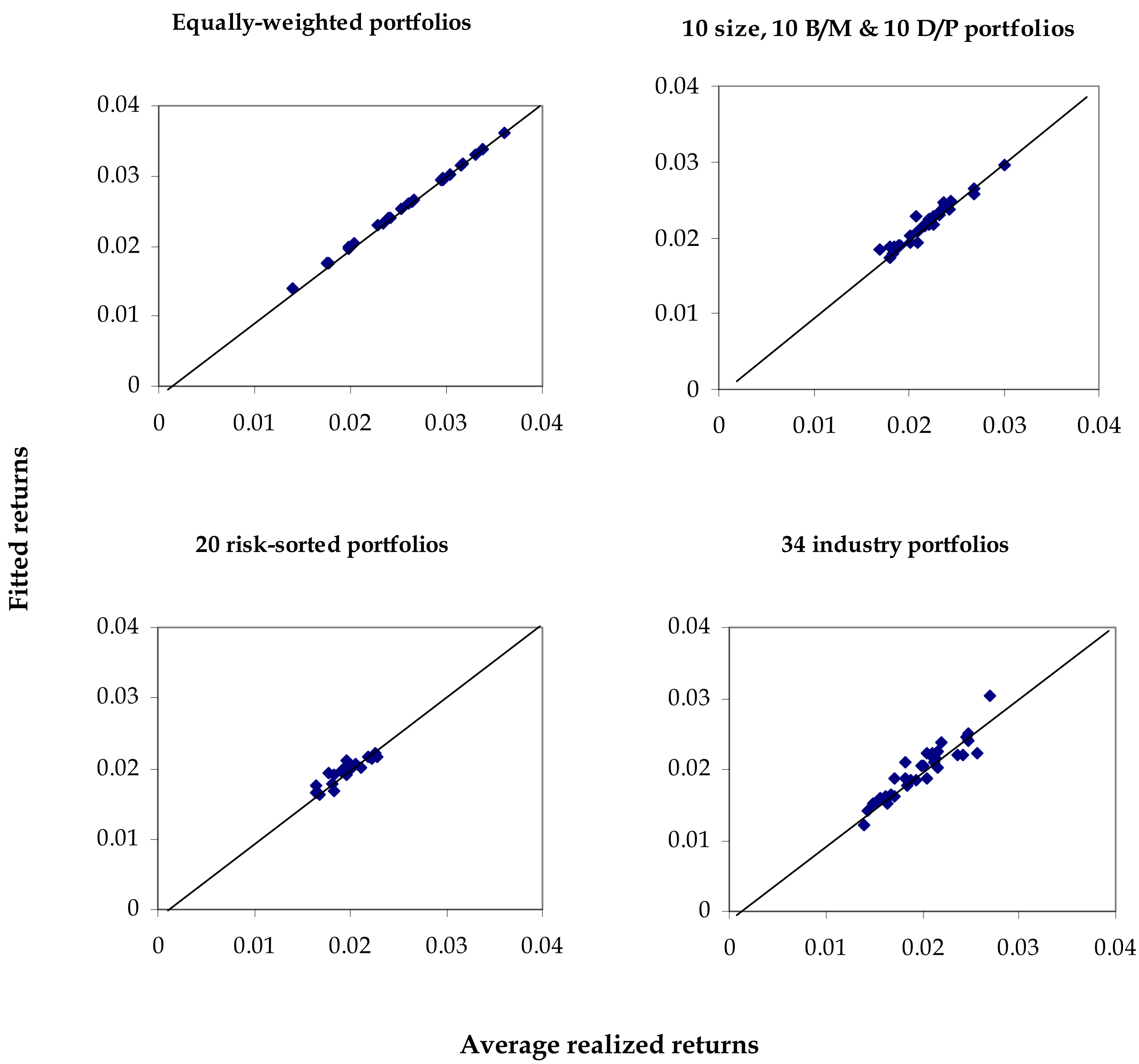


Figure 6A. Log-spectrum of consumption growth

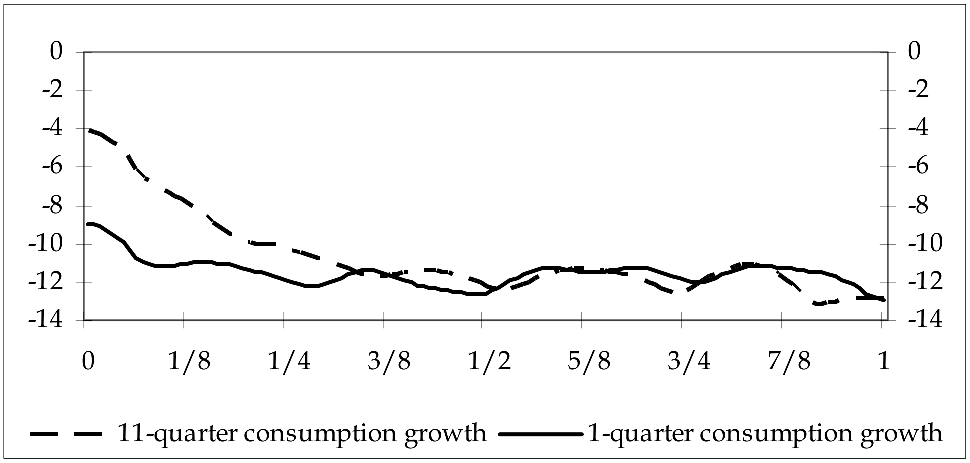


Figure 6B. Coherency of consumption growth and returns

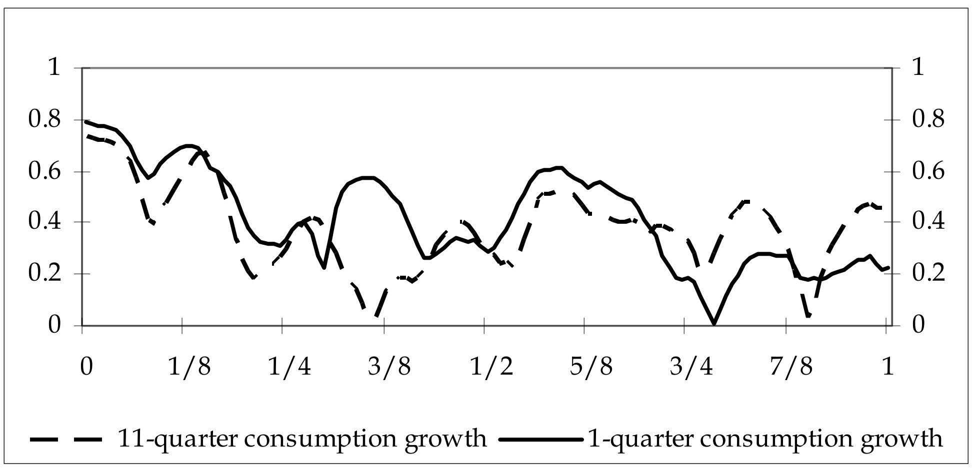


Figure 6C. Gain of consumption growth and returns

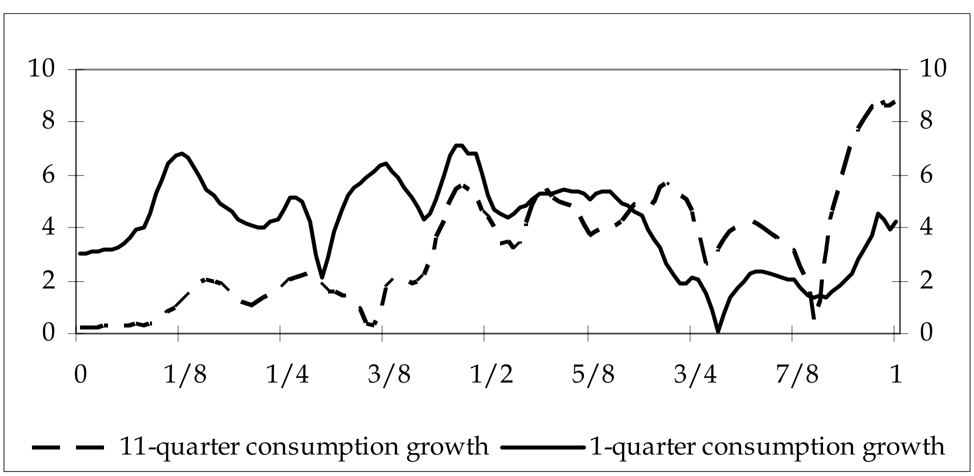


Figure 7. Spectral properties of C-CAPM and the term structure of interest rates

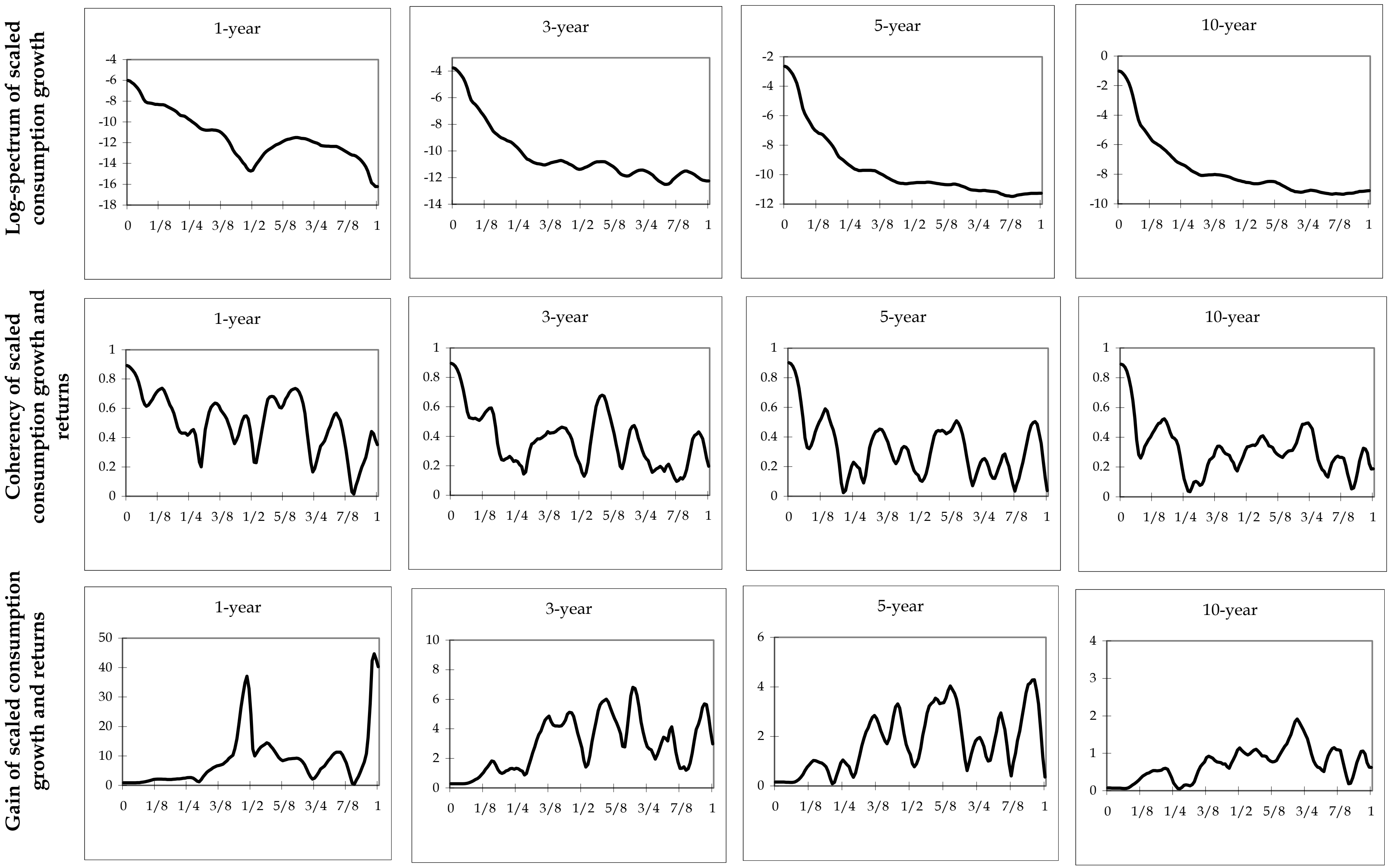


Figure 8. Fitted and average returns (term structure, infinite horizon)