Estimating Time-Varying Policy Neutral Rate in Real Time

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ABSTRACT

This paper examines policy neutral rate in real time for the Czech Republic in 2001:1-2006:09 estimating various specifications of simple Taylor-type monetary policy rules. First, we estimate it using GMM. Second, we apply a structural time-varying parameter model with endogenous regressors to evaluate the fluctuations of policy neutral rate over time. The results suggest that there is substantial interest rate smoothing and central bank primarily responds to inflation (forecast) developments. The estimated parameters seem to sustain the equilibrium determinacy. We find that the policy neutral rate gradually decreased over sample period to the levels comparable to those of in the euro area reflecting capital accumulation, smaller risk premium and successful disinflation in the Czech economy.

JEL Classification: E43, E52, E58.

Keywords: policy neutral rate, Taylor rule, time-varying parameter model with endogenous regressors.

1. Introduction

Inflation targeting regimes are increasingly popular around the world. For example, regarding the Central and Eastern Europe while the first two countries adopted explicit inflation targeting regime in 1998, there are already seven countries operating inflation targeting in 2006 and others are contemplating to do so (International Monetary Fund, 2006).[1] A characteristic feature of inflation targeting is that central banks set short-term interest rate in the way to get inflation and output at their targeted levels. The level of interest rates that should provide this objective is typically labeled as policy neutral rate.[2]

In this regard, Woodford (2003) notes that central banks should track the policy neutral rate to stabilize the economy. In a similar fashion, Taylor (1999) emphasizes that the measurement of policy neutral rate is one of key issues for countries targeting inflation. In this respect, it is of paramount importance for central banks to identify as precisely as possible the policy neutral rate. This is quite intricate exercise, as the policy neutral rate is unobservable, however the mis-measurement of policy neutral rate is high-priced, as it likely results in over- or undershooting the inflation target.

In this light, it is quite striking that remarkably little evidence is available for Central and Eastern European Countries (CEECs) on the estimation of policy neutral rate. While there are dozens of studies on *equilibrium exchange rates* in the EU new members, there is surprisingly very little evidence on *equilibrium interest rates* (Brzoza-Brzezina, 2006, seem to the only exception with evidence on Poland). This imbalance is rather striking, as nearly half of EU

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[1] Czech Republic and Poland adopted inflation targeting in 1998, followed by Hungary in 2001, Romania and Slovakia in 2005 and Armenia and Serbia in 2006 (note this is an updated list of Table 1 in IMF, 2006). Ukraine is likely to adopt inflation targeting in near future (IMF, 2006).

[2] Note that we use policy neutral rate, natural rate of interest and equilibrium interest rate in the following text interchangeably. There is however tendency to understand policy neutral rate as the short-term interest rate over which central bank has substantial control and thus, natural rate of interest and equilibrium interest rate may be understood as a bit more general concept.

new members target inflation (Czech Republic, Hungary, Poland and Slovakia), for which the concept as well as measurement of policy neutral rate is of primary importance for the conduct of monetary policy.[3] Consequently, this paper tries to bridge this gap.

This paper addresses the issue of policy neutral rate estimation in one of EU new member states, the Czech Republic, based on various specifications of simple Taylor-type monetary policy rules. Former transition country provides an interesting case to evaluate policy neutral interest rate, as one can expect certain pattern in the path of nominal and real equilibrium interest rates over longer time horizon. Lipschitz *et al.* (2006) points out that at the outset of transition the capital/labor ratios were much lower than those in Western Europe and therefore the marginal product of capital and for that reason real equilibrium interest rate was rather high. Given the capital accumulation over the course of transition, there should be tendency for the real equilibrium rates to decrease. From open economy perspective, EU new members exhibited a fall of exchange rate risk premium during their transition process to market economy, which also puts a downward pressure on real equilibrium interest rates (Archibald and Hunter, 2001). Additionally, the path of nominal equilibrium interest rates should reflect not only the decrease of real equilibrium rates, but also successful disinflation in transition countries (see Korhonen and Wachtel, 2006). All in all, aforementioned arguments provide rationale to model policy neutral rate as time-varying.

In this paper we provide first the estimation of monetary policy rules using GMM, from which we can obtain *time-invariant* estimate of policy neutral interest rate (defined as short-term real equilibrium interest rate plus expected inflation). Second, monetary policy rules with time-varying intercept are used to assess the fluctuations of equilibrium interest rates over time. The novelty of our approach is estimation of *time-varying* policy neutral rate by the

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[3] See Coats *et al.* (2003) and Kotlan and Navratil (2003) on Czech monetary policy.

time-varying parameter model with endogenous regressors (Kim, 2006).[4] Unlike conventional time-varying parameter model, this approach is robust to endogeneity of explanatory variables, which, as we find, is indeed relevant when estimating the monetary policy rules. Additional feature of this paper is that in line with Orphanides (2001) we utilize ex-post as well as real-time based data, namely real-time Czech National Bank's (CNB) output gap and inflation forecast to estimate the monetary policy rules.

The paper is organized as follows. Section 2 discusses various approaches to measure equilibrium interest rates. Section 3 describes our data and empirical methodology. Section 4 gives the results on the estimation of intercept-invariant monetary policy rules as well as time-varying estimates of equilibrium interest rates. Section 5 concludes. Appendix with additional results follows.

2. Methods for Natural Rate of Interest Estimation

Generally, there are several main methods to estimate the natural rate of interest (see e.g. Giammarioli and Valla, 2004, for survey and the benefits and costs of various methods).[5] The simplest is to assume that the equilibrium is captured reasonably well by the trend. The trend can be estimated by some univariate filter such as HP filter. Nevertheless, a number of papers document that the estimates based on these filters can be often misleading (Clark and Kozicki, 2005). In general, the limitations of the univariate methods have been pointed out by many authors (e.g. Canova, 1998).

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[4] Note that in previous version of Kim (2006), this is also labeled as augmented Kalman filter.
[5] Note that this is not the exhaustive list of methods for equilibrium interest rate estimation that is presented here, e.g. Brzoza-Brzezina (2006) proposes structural vector autoregression model in this regard. In general, In this regard, the role of equilibrium interest rate for monetary policy conduct is discussed extensively by e.g. Taylor (1993), Woodford (2003) or Amato (2005).

Another method to derive equilibrium interest rates is based on the estimation of simple monetary policy rule of central bank (Taylor, 1993). The reaction function typically associates short-term interest rates to its lagged value, a difference between inflation (forecast) and its target, and output gap. The intercept of the estimated reaction function can be interpreted as the equilibrium interest rate (this is, the interest rate that would prevail when inflation is at the targeted value and output on its potential). This method has been applied to estimate the equilibrium interest rates by e.g. Clarida *et al.* (1998, 2000) and Orphanides (2001) for the United States and Germany, Adam *et al.* (2005) for the United Kingdom and Gerdesmeier and Roffia (2004, 2005) for the euro area. Nevertheless, the assumption of constant equilibrium interest rates is often found too restrictive. It is possible to model the equilibrium interest rate as time-varying within this approach using Kalman filter (see Plantier and Scrimgeour, 2002, and Elkhoury, 2006). Typically, these studies find rationale to model the monetary policy rule as time-varying, given that the equilibrium interest rate sometimes fluctuates considerably over longer time horizons. Generally, the monetary policy rules approach measures the behavior of central bank and assumes that central bank measures equilibrium interest rates correctly. In case of central bank's systematic mis-measurement of equilibrium rates, it is likely that equilibrium rates retrieved from the estimation of reaction function are mis-measured as well.

Structural time series models represent another common method to measure equilibrium interest rates as well. The primary contribution in this area is Laubach and Williams (2003), who formulate a simple empirical model containing IS curve, Phillips curve and an equation linking equilibrium interest rate to trend growth, and model equilibrium interest rates and potential output as unobserved components. Their method has gained popularity recently and has been applied by Manrique and Marques (2004) for the U.S. and Germany, Mesonnier and

Renne (2004) for the euro area and Wintr, Guarda and Rouabah (2005) for the euro area[6] and Luxembourg as well. In principle, the joint estimation of equilibrium interest rates and output gap is an advantage of this approach; however it also reduces the degrees of freedom, which may be an issue for transition countries with rather short time series.

Equilibrium interest rates can also be estimated within stochastic dynamic general equilibrium models. The advantage of this type of literature is that it specifies the structure of economy and thus in principle allows an identification of variety of shocks hitting the economy. On the other hand, Levin *et al.* (1999) find that more complex models seem to be less robust to model uncertainty (see also Giammarioli and Valla, 2004). Consequently, these model outcomes may be quite sensitive to model assumptions. The recent examples of this approach to estimate equilibrium interest rates include Giammarioli and Valla (2003), Neiss and Nelson (2003) and Smets and Wouters (2003).

The last major stream of literature estimates equilibrium interest rates from the yield curve and asset pricing models. Bomfim (2001) uses inflation linked bonds in order to eliminate the distortions from inflation expectations and retrieves equilibrium interest rates from the realized yields on U.S. Treasury inflation-indexed securities. In this regard, Giammarioli and Valla (2004) discuss equilibrium interest rate estimates in relation to consumption capital asset pricing models. In general, this stream of literature hinges on a notion of liquid financial markets and thus this approach is viable especially for countries with developed financial markets.

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[6] See Crespo-Cuaresma *et al.* (2004) on related estimates on Euro area using somewhat different methodology.

3. Data and Empirical Methodology

In this part, we discuss the methodology we employ to evaluate the equilibrium interest rate in the Czech Republic in order to estimate a variety of backward or forward looking monetary policy rules with time-invariant or time-varying policy neutral rate.

3.1 Monetary Policy Rules

A starting point for a formal derivation of monetary policy rule is a reasonable assumption that central bank targets to set nominal interest rate in line with the state of economy (see Clarida, Gali and Gertler, 1998, 2000), as postulated in Eq. (1):

$$r_t^* = \bar{r} + \alpha \left(E\left\{ \pi_{t+i} | \Omega_t \right\} - \pi_{t+i}^* \right) + \beta E\left\{ x_t | \Omega_t \right\} \tag{1}$$

r_t^* denotes the targeted interest rate, $\bar{r}$ is the equilibrium nominal interest rate (policy neutral rate), π_{t+i} stands for the central bank forecast of yearly inflation rate i periods ahead (hereinafter, we set i either equal to 12 months to reflect the CNB's actual targeting horizon[7] or alternatively to 0, i.e. using the current inflation for sensitivity analysis), π_{t+i}^* is the central bank's inflation target. x_t represents a measure of output gap. $E(.)$ is the expectation operator and Ω_t is the information set available at the time when interest rates are set. Therefore, Eq. (1) links targeted nominal interest rates to a constant (i.e. interest rate – policy neutral rate – that would prevail, when expected inflation is at the target and output gap is null), the deviation of expected inflation from target and the output gap.

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[7] This in line with the CNB main forecasting model – Quarterly Prediction Model, see Coats *et al.*, 2003. The actual targeting horizon is 12-18 months, but due to data limitations we prefer to work with 12 months. In general, see Batini and Nelson, 1999, for contributions on optimal targeting horizon. Note also that policy neutral rate is defined as the real rate plus the expected inflation in period $t+k$, where k is given by the maturity of interbank rate (in our case $k=3$). k is thus different from forecasting horizon i. As argued by Clarida *et al.* (2000), this is not very relevant in practice, as the short-term interbank rates at various maturities are strongly linked together. Indeed, the correlation of 3M PRIBOR and 12M PRIBOR – to reflect that $i=12$ – stands at 0.991 in our sample.

Nevertheless, Eq. (1) is often argued to be too restrictive, as it does not account for important empirical regularity - interest rate smoothing of central banks.[8] Typically, central bank adjusts the interest rate sluggishly to the targeted value. This is so for a number of reasons such as the concerns over the stability of financial markets (Goodfriend, 1991) or uncertainty about the effects of interest rate changes on the economy (Sack, 1997). Instead of explicit listing of various factors behind the interest rate smoothing, it is assumed for simplicity that actual policy interest rate is a combination of its lagged value and the targeted policy rate as in Eq. (2).

$$r_t = \rho r_{t-1} + (1 - \rho) r_t^* + v_t \qquad (2),[9]$$

where $\rho \in [0,1]$. In line with Clarida *et al.* (1998), substituting Eq. (2) into Eq. (1) and eliminating unobserved forecast variables results in Eq. (3):

$$r_t = (1 - \rho)\left[\bar{r} + \alpha\left(\pi_{t+i} - \pi_{t+i}^*\right) + \beta x_t\right] + \rho r_{t-1} + \varepsilon_t \qquad (3)$$

Note that disturbance term ε_t is a combination of forecast errors and is thus orthogonal to all information available in time t (Ω_t). $\bar{r}$ denotes the estimate of policy neutral rate.

First, Eq. (3) is estimated by GMM. Next, we estimate time-varying short-term real equilibrium interest rate and neutral policy rate and for this reason, we apply structural time-varying coefficient model with endogenous regressors. Kim (2006) shows that conventional time-varying parameter model delivers inconsistent estimates, when explanatory variables are correlated with the disturbance term, which is relevant, when estimating policy rules. Note that π_{t+i} and x_t in (3) are correlated with ε_t, which is typically not taken into account in

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[8] We refer a reader to Rudebusch (2006) for discussion on the extent of monetary policy inertia.
[9] We have estimated the monetary policy rules including higher lags of interest rates, but failed to find it significant.

the previous literature on time-varying monetary policy rules. Kim (2006) derives a consistent estimator of time-varying parameter model, when regressors are endogenous. In line with Kim (2006), we estimate the following empirical model:

$$r_t = (1 - \rho)\left[\bar{r}_t + \alpha\left(\pi_{t+i} - \pi_{t+i}^*\right) + \beta x_t \right] + \rho r_{t-1} + \varepsilon_t \tag{4}$$

$$\bar{r}_t = \bar{r}_{t-1} + \vartheta_t, \quad \vartheta_t \sim i.i.d.N\left(0, \sigma_\vartheta^2\right) \tag{5}$$

$$\pi_{t+i} = Z_{t-j}'\xi + \sigma_\varphi \varphi_t, \quad \varphi_t \sim i.i.d.N(0,1) \tag{6}$$

$$x_t = Z_{t-j}'\psi + \sigma_v v_t, \quad v_t \sim i.i.d.N(0,1) \tag{7}$$

The measurement equation (4) is Taylor rule with policy neutral rate, $\bar{r}_t$, as outlined above (analogously for short-term real equilibrium interest rate). However, we relax here the assumption of constant policy neutral rate and let it vary over time, $\bar{r}_t$, as specified in the transition equation (5). We assume that $\bar{r}_t$ follows random walk without drift.[10] Given the data limitations and interest in examining fluctuations of equilibrium interest rates, we do not allow α, β and ρ being time-varying.[11] The "first-stage" equations (6) and (7) lay out the relationship between endogenous regressors (π_{t+i} and x_t) and its instruments, Z_t. The list of instruments, Z_{t-j}, is as follows: π_{t-1}, π_{t-2}, x_{t-1}, x_{t-2} and r_{t-1}. We assume that the parameters in the equations (6) and (7) are time-invariant. Next, it is also assumed that the correlation between the standardized residuals φ_t and v_t with ε_t is $\kappa_{\varphi,\varepsilon}$ and $\kappa_{v,\varepsilon}$, respectively (note that σ_φ and σ_v are standard errors of φ_t and v_t, respectively). The consistent estimates of coefficients in the equation (4) are then obtained in two steps. In the first step, we estimate the equations (6) and (7) and save the standardized residuals φ_t and v_t. In the second step, we

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[10] We also experimented with AR(1) structure in the equation (5), but it just marginally reduced the likelihood and the estimated AR parameter has been close to one, anyway.

[11] Kim and Nelson (2006) estimate monetary policy rule for the U.S. and allow all these parameters to be time-varying.

estimate Eq. (8) along with Eq. (5) using maximum likelihood via the Kalman filter. Note that (8) now includes bias correction terms, (standardized) residuals from Eqs. (6) and (7), to address the aforementioned endogeneity of regressors. Consequently, the estimated parameters in Eq. (8) are consistent.

$$r_t = (1 - \rho)\left[\bar{r}_t + \alpha\left(\pi_{t+i} - \pi^*_{t+i}\right) + \beta x_t\right] + \rho r_{t-1} + \kappa_{v,\varepsilon}\sigma_{\varepsilon,t}v_t + \kappa_{\varphi,\varepsilon}\sigma_{\varepsilon,t}\varphi_t + \iota_t,$$

$$\iota_t \sim N\left(0, (1 - \kappa^2_{v,\varepsilon} - \kappa^2_{\varphi,\varepsilon})\sigma^2_{\varepsilon,t}\right) \tag{8}$$

It is common in literature (see for example Gerdesmeier and Roffia, 2004) to include additional economic variables in Eq. (3) trying to capture the state of economy in a fuller manner. Nevertheless, it is important to emphasize that this is typically done in an *ad hoc* manner. Literature typically assumes that interests rates depends only on inflation and output and there is no role for other variables such as (real) exchange rate to enter Taylor rule directly, as they influence inflation and output and therefore affect the interest rate setting *indirectly*. As Taylor (2001, p. 266) puts it: *"Although the policy rule ... may not appear to involve interest rate reaction to exchange rate, it implies such a reaction. What might appear to be a closed economy policy rule is actually just as much as open economy rule as if the exchange rate appeared directly."*

In a book describing the forecasting and policy analysis process in the CNB, Coats *et al.* (2003) report that no other variables than inflation and output gap enter into the monetary policy rule in the CNB Quarterly Projection Model. Therefore, our prior is that it is rather unlikely to find any other variables significant when estimating policy rule for the CNB. Regarding money aggregates, one can expect that inflation targeting central bank in general views it as supplementary information about the degree of economic activity and/or

inflationary pressures. However, it has to be noted that in case of exchange rate fluctuations, the picture is less clear-cut. While the CNB has stated several times that it does not directly react to exchange rate fluctuations, it acknowledged that exchange rate plays important role for inflation developments in small open economies and that it might react indirectly to exchange rate fluctuations, if they jeopardize the inflation developments (Kotlán and Navrátil, 2005). In addition, CNB used to intervene in the foreign exchange market at the beginning of our sample period (2001-September 2002), despite the intervention activity has rather supported the inflation targeting in the sense that the interventions did not go against future fulfillment of the inflation targets (Geršl and Holub, 2006). As a result, there might be some potential in analyzing the role of exchange rate in shaping the CNB interest rate setting process. Therefore, for the sensitivity of results we examine our baseline specification in Eq. (3) including real effective exchange rate as well as money growth in the policy rule.

Originally, the design of Taylor rule lacked forward-looking element characteristic for the modern monetary policy conduct. Additional way to address the sensitivity of our results is estimation of both backward- and forward-looking Taylor-type rules. Therefore, we formulate the monetary policy rule in the Eq. (3) in case of backward-looking policy rule such that we set $i = 0$, i.e. we use current inflation rate instead of its forecasted value (which is utilized for the forward-looking policy rule). Another important point has been raised about timeliness of information in the monetary policy conduct (Orphanides, 2001). Output data are typically revised at later stage, but monetary policy is conducted based on information available at the time. Therefore, we collect real-time based CNB output gap estimates (note that inflation is not revised at later stage by the Czech Statistical Office) and re-estimate the monetary policy rule with real-time output gap. Analogously, we use one year ahead CNB's real-time inflation forecasts in estimating monetary policy rule.

There are further modeling issues stemming from the fact that policy interest rates are not changed in a continuous fashion. For instance, the CNB Bank Board meets on a monthly basis to discuss the policy interest rate settings. Besides, the policy rate change itself is not continuous. Typically, if the rates are changed, the respective magnitude is 0.25 percentage points (or eventually multiple of 0.25), despite the change maximizing economic stability according to model-based forecast might be of (slightly) different magnitude. In consequence, policy rate is not only discrete, but also censored.

Given the inherent censoring of policy interest rates, majority of authors such as Clarida *et al.* (1998, 2000) or Adam *et al.* (2005) rely on using 3 months interbank rate as the approximation of the censored policy rate. How accurate the approximation is depends on the ability of market participants to predict the policy interest rate changes. If the central bank monetary policy actions were largely predictable, short-term interbank rate is likely to serve as a useful approximation of the policy rate. Perez-Quiros and Sicilia (2002) argue that market has predicted the monetary policy decisions of the European Central Bank and the Federal Reserve relatively well. In the case of the CNB, Kotlán and Navrátil (2005) find that about 75% of monetary policy decisions have been priced in by the market participants in 2000-2004.

Other authors such as Choi (1999) or Carstensen (2006) therefore put forward modeling censoring in policy rate directly by employing e.g. ordered probit model. The advantage of this approach is that it models interest rate setting more realistically and does not have to make a rather simplifying assumption by utilizing the short-term interbank rate. On the other hand, this stream of literature so far models only censoring in the policy rate, but there it has

been stressed that there is also censoring in the policy rate change (Podpiera, 2006). In addition, censored models are known to be less efficient and interestingly, the results based on them seem to be relatively close to those of using short-term interbank rate (for example, consider the extent of interest rate smoothing). Additional drawback of this approach in our case is that our main estimation technique is time-varying parameter model with endogenous regressors (Kim, 2006) in order to estimate the fluctuations of equilibrium interest rates over time and to our knowledge, the time-varying parameter model with endogenous regressors and with censored dependent variable is simply not available. Having all pros and cons of these two approaches – using short-term interbank rate vs. policy interest rate – in mind, in consequence we opt for using short-term interbank rate in estimation of the monetary policy rules.

3.2 Data

Our sample contains monthly data over the period 2001:1-2006:09 on yearly CPI inflation ($\pi_t = p_t - p_{t-12}$, where p_t is the log of price level at time t), yearly net inflation (price indexes of regulated goods excluded from the price index, thus $\pi_t^{net} = p_t^{net} - p_{t-12}^{net}$, where p_t^{net} is the log of net price level at time t), output gap (x_t, a difference between actual and potential GDP growth, defined as below), short-term interbank rate (3M PRIBOR), real effective exchange rate, ($reer_t$), and the yearly growth rate of monetary aggregate M2 ($\Delta m_t = m_t - m_{t-12}$, where m_t is the log of money level). We also use the real-time CNB internal forecasts of CPI $\left(\pi_{t+12}^f\right)$ and net inflation $\left(\pi_{t+12}^{net,f}\right)$ and output gap. Consequently, we use three different estimates of output gap: a) estimate using HP filter[12], b) ex-post revised output gap from CNB's QPM (main forecasting model) as of their October 2006 forecast

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[12] Standard smoothing parameter of 14440 has been used. Different smoothing parameter, as the one suggested by Ravn and Uhlig (2002), had very little impact on the resulting estimates of policy neutral rate.

round and c) real-time based output gap collected from CNB's QPM. The source of our data is the CNB (note that inflation forecasts and real-time and ex-post output gap is not available within their public database system ARAD).

All our variables are available on the monthly basis, except the output gap. Following Adam *et al.* (2005), we linearly interpolate quarterly estimates of output gap to monthly values. We use the mid-points of CNB inflation target. The choice of 2001-2006 period is motivated to have as long sample period as possible, while not rejecting stationarity of all variables at 5% significance level (using KPSS test). More importantly, real-time output gap and inflation forecast are not available before 2001. As a robustness check, we also estimate the monetary policy rules with net inflation (regulated prices are excluded from the consumer basket) instead of CPI inflation.

4. Results

In this section we first report the GMM estimates of monetary policy rules. Next, we relax the restriction of constant policy neutral rate, so the estimates of time-varying policy neutral rate follow. This is done using time-varying parameter models with endogenous regressors (Kim, 2006). Appendix presents some additional results.

4.1 GMM Results

As a prelude to the GMM estimation, we test the stationarity of the variables and do not reject the stationarity of the variables at 5% level using KPSS test (see Table A.1 in the Appendix). In this context, we also find OLS estimates inconsistent based on the results of Hausman test.

The GMM estimates of backward-looking monetary policy rules based on various specifications are reported in Table 1. The results indicate that policy neutral rate lies somewhere between 2-3%. Notably, there is evidence for substantial interest rate smoothing (note that we use monthly data, the value of smoothing parameter corresponding to the quarterly frequency is around 0.75). As expected, greater inflation tends to increase the nominal interest rates. We use three alternative estimates of output gap: HP filter, real-time and ex-post gap from the CNB's QPM (see Chart B.2 in the Appendix for a comparison of output gap estimates), but fail to find them significant. Given the extent of interest rate smoothing, it should not come as surprise that interest rate predicted based on Taylor rule describes well the actual path of interest rates (the Chart available upon request).

Table 1 – Backward looking Policy Rules

$$r_t = (1 - \rho)\left[\bar{r} + \alpha\left(\pi_t - \pi_t^*\right) + \beta x_t \right] + \rho r_{t-1} + \varepsilon_t$$

$\bar{r}$	ρ	α	β	Sargan test	Notes:
3.27***	0.95***	0.69**	0.88	0.3	GDP HP-filtered gap, CPI current
(0.67)	(0.02)	(0.33)	(0.73)	(0.58)	inflation
2.4***	0.91***	0.32	-0.22	2.04	CNB GDP gap (ex-post), CPI
(0.58)	(0.02)	(0.32)	(0.6)	(0.56)	current inflation
		Real-time estimates			
2.02***	0.89***	0.65*	-1.03	2.63	CNB GDP gap (real-time), CPI
(0.53)	(0.03)	(0.34)	(0.68)	(0.11)	current inflation

Note: $\bar{r}$ stands for the policy neutral rate. Inflation is not revised ex-post. Therefore, inflation series is identical for all specifications in this Table. HAC robust standard errors in brackets. Sargan test is test for overidentifying restrictions (p-value in the brackets). ***, **, and * - denotes significance at 1 percent, 5 percent, and 10 percent, respectively.

Next, we evaluate the equilibrium determinacy ("equilibrium with stable prices") of the estimated monetary policy rules. This is important in order to assure that economy is not subject to sunspot equilibria, when there is infinite number of paths for the real variables leading back to equilibrium. In this context, Woodford (2003) in a basic model for analysis of

monetary policy effects shows that in case of backward-looking Taylor rule with interest rate smoothing it is necessary for the determinacy of equilibrium that $\alpha \succ 1 - \rho - (1 - \psi/q)\beta$, where α, β, ρ are defined as above, ψ is discount factor and q is a parameter linking output gap to inflation in the standard New Keynesian Phillips curve. Note that it is assumed that $\alpha, \beta, \rho \geq 0$. The results in Table 1 indicate that α is around 0.7 and ρ is around 0.9. Assuming realistically that ψ is close to 1 and $q = 0.5$ (see Coats *et al.*, 2003 for a calibration of Phillips curve for the Czech Republic), the inequality above is fulfilled, when $\beta \prec 0.6$, which is the case in Table 1, as β is not significantly different from zero at 5% significance level. Although the results must be interpreted with caution, as there is uncertainty in the estimated parameter values, CNB monetary policy does not seem to lead to equilibrium indeterminacy, where the fluctuations of output and inflation would result from self-fulfilling expectations.

Table 2 reports estimation of the forward-looking monetary policy rule. The results suggest that policy neutral rate, $\bar{r}$, is now more precisely estimated around the value of 2.5%. The estimated parameters yield now their expected signs with reasonable values, but standard errors of the estimates in Table 2 remain rather large.

Table 2 – Forward looking Policy Rules

$$r_t = (1 - \rho)\left[\bar{r} + \alpha\left(\pi_{t+12} - \pi_{t+12}^*\right) + \beta x_t \right] + \rho r_{t-1} + \varepsilon_t$$

$\bar{r}$	ρ	α	β	Sargan test	Notes:
2.54***	0.92***	0.85	0.01	4.8*	CNB GDP HP-filtered gap, CPI
(0.45)	(0.03)	(0.61)	(0.66)	(0.09)	inflation forecast (real-time)
2.43***	0.92***	0.58	0.08	2.16	CNB GDP gap (ex-post), CPI
(0.7)	(0.03)	(0.72)	(0.39)	(0.54)	inflation forecast (real-time)
Real-time estimates					
2.43***	0.93***	0.62*	0.05	2.1	CNB GDP gap (real-time), CPI
(0.77)	(0.03)	(0.21)	(0.41)	(0.35)	inflation forecast (real-time)

Note: See Table 1 for mnemonics.

Similarly to aforementioned backward-looking Taylor rule, Woodford (2003) demonstrates that for forward-looking Taylor rule with interest rate smoothing it is necessary for the determinacy of equilibrium that both $\alpha \prec 1 + \rho + \left([1 + \psi]/q\right)\left(\beta + 8\sigma^{-1}(1 + \rho)\right)$ and $\alpha \succ 1 - \rho - \beta(1 - \psi)/q$ are satisfied. $\alpha, \beta, \rho, \psi, q$ are defined as above, and σ is intertemporal elasticity of substitution of aggregate expenditures. Regarding the former inequality note that $\left([1 + \psi]/q\right)\left(\beta + 8\sigma^{-1}(1 + \rho)\right)$ is expected to be positive (if $\beta \geq 0$, it is always positive) and thus it is enough, if $\alpha \prec 1 + \rho$, which is the case for all estimated specifications reported in Table 2. Analogously for the latter inequality, it is vital to note that if $\beta \geq 0$, then the equilibrium is determinate, if $\alpha \succ 1 - \rho$, which is again the case for all the specifications reported in Table 2.[13]

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[13] It is worth emphasizing that we also estimated the monetary policy rules including either real effective exchange rate or the growth rate of monetary aggregate M2 into the baseline specification as in Eq. (3). Both these terms have been included either as the current value or lagged by one period. However, neither of these additional variables is found to be significant in any specification. The results using net inflation are generally quite close to those of CPI inflation. This is not surprising, as the correlation of net inflation and CPI inflation is 0.94 in our sample. These results are available upon request.

4.2 Time-varying equilibrium interest rates

Generally, we find that the policy neutral rate decreases over time. This is in line with Lipschitz *et al*. (2006) noting that real equilibrium interest rate should be initially at high levels in transition countries, as capital accumulation was typically low and thus marginal product of capital high. Consequently, as capital accumulates over the course of transition, real equilibrium interest rates should decrease over longer time horizons. In addition, a decrease in risk premium inevitably provides a downward pressure on the real equilibrium rates as well (Archibald and Hunter, 2001). This corresponds to Beneš and N'Diaye (2004), who find that risk premium decreased during transition of Czech economy. The respective decrease of policy neutral rate also reflects successful disinflation of Czech economy and well-anchored inflation expectations (see chart B.1 in the Appendix on inflation developments in 2001-2006).

The estimated path of policy neutral rate is in line with the aforementioned fundamental factors, as the results presented in Charts 1-2 indicate. Policy neutral rate gradually decreased from some 5% to the values around 2% at the end of 2005 and subsequently slightly increased to some 2.5% over the course of 2006. This supports substantial interest rate convergence to the levels comparable to the euro area countries. For example, Messonier and Renne (2004) estimate euro area real equilibrium interest rate around 1% at the end of their sample (i.e. year 2002) and Wintr *et al.* (2005) find it a bit below 1% in 2004. If we add to these estimates 2% for the expected inflation – to reflect the European Central Bank definition of price stability –, we receive the estimate of policy neutral rate of about 3% for the euro area. For Charts 1-2, we report two generic specifications, the first one, backward-looking policy rule with the output gap estimated by HP filter and on the other hand, the second one, forward looking rule

with the real-time output gap. All other policy neutral rate estimates are available in the Appendix C.

Additionally, the results support the usefulness of applying time-varying parameter model with endogenous regressors. The bias correction terms, φ_t and ν_t, in Eq. (8), are typically significant and the log likelihood improves after their inclusion. Comparing the estimated policy neutral rate with those implied by conventional time-varying parameter model (not accounting for endogeneity of regressors), we find that the resulting difference between these two varies according to the specification policy rule as well as over time. While the median difference is 0.05 p.p. in the absolute terms, the maximum difference, that the inclusion of bias correction terms amounts to, is 1.8 p.p.

The Appendix D presents a comparison of policy neutral rate based on identical specification of policy rule, but estimated either by the time-varying parameter model with endogenous regressors or by the conventional time-varying parameter model. Denoting the policy neutral rate estimated by the former method, $\bar{r}_{t,er}$, and , $\bar{r}_{t,c}$, by the latter, the Charts report a difference between these two. Obviously, if $\bar{r}_{t,er} - \bar{r}_{t,c} = 0$, the bias correction terms do not matter. However, we can see from the results that albeit the two methods yield in general rather similar estimates of policy neutral rate, there are periods, when the bias correction terms matter considerably, i.e. when the estimates by the conventional time-varying parameter model do not even lie inside the confidence interval of the policy neutral rate estimated by the time-varying parameter model with endogenous regressors.

**Chart 1 – Real-Time Time-Varying Policy Neutral Rate, Backward Looking**

$$r_t = (1 - \rho)\left[\bar{r}_t + \alpha\pi_t + \beta x_t\right] + \rho r_{t-1} + \varepsilon_t \ , \ \bar{r}_t = \bar{r}_{t-1} + v_t$$
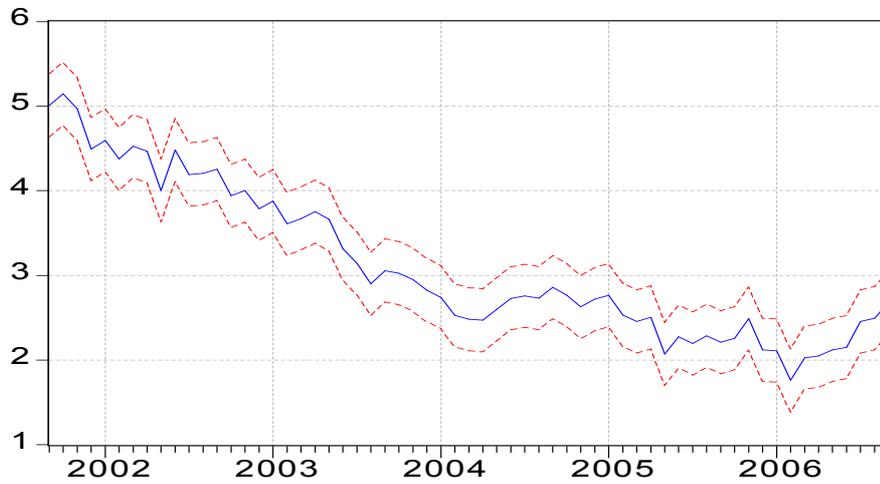
Note: Smoothed parameter estimate based on time-varying coefficient model with endogenous regressors and ±2 standard errors. Output gap estimate, x_t, based on HP filter.

Chart 2 – Real-Time Time-Varying Policy Neutral Rate, Forward Looking

$$r_t = (1 - \rho)\left[\bar{r}_t + \alpha\pi_{t+12} + \beta x_t\right] + \rho r_{t-1} + \varepsilon_t \ , \ \bar{r}_t = \bar{r}_{t-1} + v_t$$
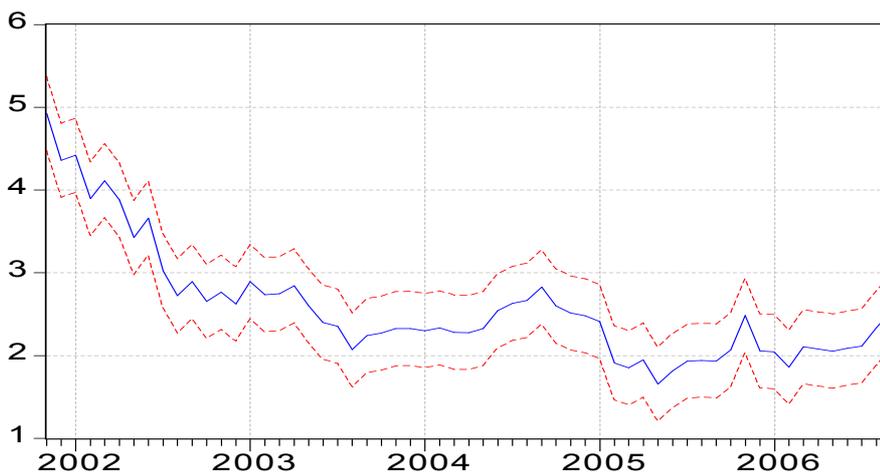
Note: Smoothed parameter estimate based on time-varying coefficient model with endogenous regressors and ±2 standard errors. Output gap estimate, x_t, is real-time CNB's output gap.

5. Conclusions

This paper analyzes the equilibrium interest rates in the Czech Republic. In order to do so, we estimate various specifications of simple monetary policy rules at the monthly frequency from 2001 to 2006. To address the sensitivity of results, the specifications differ based on whether we include real-time or ex-post revised data, employ backward or forward-looking monetary policy rules or vary the measure of output gap.

Generalized method of moments is first used to retrieve the estimates of policy neutral rate (i.e. nominal interest rate that arises, when inflation is at the target and output at its potential). According to our results, we find that Czech National Bank primarily responds to inflation (forecast) developments and interest rate setting process is relatively inertial. The estimated parameters seem to support equilibrium determinacy. Subject to various sensitivity tests, the results indicate that policy neutral rate lies around 2.5-3%.

We also estimate time-varying policy neutral rate from the simple Taylor-type monetary policy rule. For this purpose, we use time-varying parameter model with endogenous regressors (Kim, 2006). This approach is especially appealing, when estimating monetary policy rules, as it addresses the endogeneity of inflation and output gap. Indeed, the results support the usefulness of applying time-varying parameter model with endogenous regressors. The bias correction terms, accounting for the endogeneity of regressors, are typically significant, the log likelihood improves after their inclusion and the estimated paths of policy neutral rate is for certain periods considerably different.

The results indicate that policy neutral rate decreases gradually over the course of sample period from some 5% in 2001 to about 2.5% in 2006 showing a substantial interest rate

convergence to the levels comparable to the euro area. Over the longer time horizon, the decrease may be supported a number of factors such as capital accumulation, the decrease in risk premium as well as successful disinflation of Czech economy and well-anchored inflation expectations.

References

Adam, Christopher, Cobham, David, Girardin, Eric, 2005. Monetary Frameworks and Institutional Constraints: UK Monetary Policy Reaction Functions, 1985-2003. Oxford Bulletin of Economics and Statistics 67, 497-516.

Amato, Jeffery, 2005. The Role of Natural Rate of Interest in Monetary Policy. CESifo Economic Studies 51, 729-755.

Archibald, Joanne, Hunter, Leni, 2001. What is the Neutral Interest Rate, and How Can We Use It?. Reserve Bank of New Zealand Bulletin 64, 15-28.

Batini, Nicoletta, Nelson, Edward, 1999. Optimal Horizons for Inflation Targeting. Journal of Economic Dynamics and Control 25, 891-910.

Beneš, Jaromír, N'Diaye, Papa, 2004. A Multivariate Filter for Measuring Potential Output and the NAIRU: Application to the Czech Republic, IMF Working Paper, No. 04/45.

Bomfim, Antulio, N., 2001. Measuring Equilibrium Real Interest Rates: What Can We Learn from Yields on Indexed Bonds?. Board of Governors of the Federal Reserve System, mimeo.

Brzoza-Brzezina, Michal, 2006. The Information Content of Neutral Rate of Interest. Economics of Transition 14, 391-412.

Canova, Fabio, 1998. Detrending and business cycle facts. Journal of Monetary Economics 41, 475-512.

Carstensen, Kai, 2006. Estimating the ECB Policy Reaction Function, German Economic Review 7, 1-34.

Choi, Woon Gyu, 1999. Estimating the Discount Rate Policy Reaction Function of the Monetary Authority. Journal of Applied Econometrics 14, 379-401.

Clarida, Richard, Gali, Jordi, Gertler, Mark, 1998. Monetary Policy Rules in Practice: Some International Evidence. European Economic Review 42, 1033-1067.

Clarida, Richard, Gali, Jordi, Gertler, Mark, 2000. Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory. The Quarterly Journal of Economics 115, 147-180.

Clark, Todd, E., Kozicki, Sharon, 2005. Estimating Equilibrium Real Interest Rates in Real Time. North American Journal of Economics and Finance 16, 395-413.

Coats, Warren, Laxton, Douglas, Rose, David (Eds.), 2003. The Czech National Bank's Forecasting and Policy Analysis System. Czech National Bank (available at www.cnb.cz).

Crespo-Cuaresma, Jesús, Gnan, Ernest, Ritzberger-Gruenwald, Doris, 2004. Searching for the Natural Rate of Interest: A Euro Area Perspective. Empirica 31, 185-204.

Elkhoury, Marwan, 2006. Time-Varying Parameter Model of a Monetary Policy Rule in the Switzerland. HEI working paper, No.01, Geneva.

Gerdesmeier, Dieter, Roffia, Barbara, 2004. Empirical Estimates of Reaction Functions for the Euro Area. Swiss Journal of Economics and Statistics 140, 37-66.

Gerdesmeier, Dieter, Roffia, Barbara, 2005. The Relevance of Real-Time Data in Estimating Reaction Functions for the Euro Area. North American Journal of Economics and Finance 16, 293-307.

Geršl, Adam, Holub, Tomáš, 2006. Foreign Exchange Interventions under Inflation Targeting: The Czech Experience, Contemporary Economic Policy 24, 475-491.

Giammarioli, Nicola, Valla, Natacha, 2003. The Natural Real Rate of Interest in the Euro Area. European Central Bank Working Paper, No. 233.

Giammarioli, Nicola, Valla, Natacha, 2004. The Natural Real Interest Rate and Monetary Policy: A Review. Journal of Policy Modeling 26, 641-660.

Goodfriend, Marvin, 1991. Interest Rates and the Conduct of Monetary Policy. Carnegie-Rochester Conference Series on Public Policy 34, 7-30.

International Monetary Fund, 2006. Inflation Targeting and the IMF. IMF: Washington D.C.

Kim, Chang-Jin, 2006. Time-Varying Parameter Models with Endogenous Regressors. Economics Letters 91, 21-26.

Kim, Chang-Jin, Nelson, Charles, R. 2006. Estimation of a Forward-Looking Monetary Policy Rule: A Time-Varying Parameter Model Using Ex-post Data, Journal of Monetary Economics 53, 1949-1966.

Korhonen, Iika, Wachtel, Paul, 2006. Observations on Disinflation in Transition Economies. In: Altig, David, Nosal, Ed, (Eds.), Monetary Policy in Low Inflation Economies. Cambridge University Press: Cambridge.

Kotlán, Viktor, Navrátil, David, 2003. Inflation Targeting as a Stabilization Tool: Its Design and Performance in the Czech Republic. Finance a uver – Czech Journal of Economics and Finance 53, 220-242.

Kotlán, Viktor, Navrátil, David, 2005. The CNB's Policy Decisions: Are They Priced in by the Markets. Czech National Bank Research and Policy Note, No.1/2005.

Kwiatkowski, Denis, Phillips, Peter, C.B., Schmidt, Peter, Shin, Yongcheol, 1992. Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root?. Journal of Econometrics 54, 159-178.

Laubach, Thomas, Williams, John, C., 2003. Measuring the Natural Rate of Interest. The Review of Economics and Statistics 85, 1063–1070.

Laxton, Douglas, Pesenti, Paolo, 2003. Monetary Policy Rules for Small, Open, Emerging Economies. Journal of Monetary Economics 50, 1109-1146.

Levin, Andrew, Wieland, Volker, Williams, John, C., 1999. Robustness of Simple Monetary Policy Rules under Model Uncertainty. In: J. B. Taylor (Ed), Monetary Policy Rules. Chicago: University of Chicago.

Lipshitz, Leslie, Lane, Timothy, Moumouras, Alex, 2006. Capital Flows to Transition Countries: Master or Servant. Finance a úvěr – Czech Journal of Economics and Finance 56, 202-222.

Manrique, Marta, Marques José, M., 2004. An Empirical Approximation of the Natural Rate of Interest and Potential Growth. Banco de Espana Working Paper, No. 0416.

Mesonnier, Jean-Stephane, Renne, Jean-Paule, 2004. Time Varying "Natural" Rate of Interest in the Euro Area. Bank of France Working Paper, No. 115.

Neiss, Katherine, S., Nelson, Edward, 2003. The Real Interest Rate Gap as an Inflation Indicator. Macroeconomic Dynamics 7, 239-262.

Orphanides, Athanasios 2001. Monetary Policy Rules Based on Real-Time Data. American Economic Review 91, 964-985.

Plantier, L. Christopher, Scrimgeour, Dean, 2002. Estimating a Taylor Rule for New Zealand with a Time-Varying Neutral Real Rate. Reserve Bank of New Zealand Discussion Paper Series, 06/2002.

Perez-Quiros, Gabriel, Sicilia, Jorge, 2002. Is the European Central Bank (and the United States Federal Reserve) Predictable?. European Central Bank Working Paper Series, No. 192/2002.

Podpiera, Jiří, 2006. Distinguishing Policy Rate Censoring from Smoothing: Identification Issues and Policy Implications. Czech National Bank, mimeo.

Ravn, Morten, O., Uhlig, Harald, 2002. On Adjusting the Hodrick-Prescott Filter for the Frequency of Observations. Review of Economics and Statistics 84, 371-375.

Rudebusch, Glenn, 2006. Monetary Policy Inertia: Fact or Fiction?. International Journal of Central Banking 2(4), 85-136.

Sack, Brian, 1997. Uncertainty, Learning and Gradual Monetary Policy, Board of Governors of the Federal Reserve System. Finance and Economics Discussion Series, 1997-34.

Smets, Frank, Wouters, Raf, 2003. An Estimated Dynamics Stochastic General Equilibrium Model of the Euro Area. Journal of the European Economics Association 1, 1123-1175.

Taylor, John, B., 1993. Discretion versus Policy Rules in Practice. Carnegie-Rochester Conference Series on Public Policy 39, 195-214.

Taylor, John, B. (Ed.), 1999. Monetary Policy Rules. The University of Chicago Press: Chicago.

Taylor, John, B., 2001. The Role of Exchange Rate in Monetary-Policy Rules, American Economic Review 91, 263-267.

Wintr, Ladislav, Guarda, Paolo, Rouabah, Abdelaziz, 2005. Estimating the Natural Interest Rate for the Euro Area and Luxembourg. Central Bank of Luxembourg Working Paper, No. 15.

Woodford, Michael, 2003. Interest and Prices: Foundations of a Theory of Monetary Policy. Princeton University Press, Princeton: New Jersey.

APPENDIX

A. Stationarity tests

Table A.1 – KPSS Test

Series	Test statistic
PRIBOR 3M	0.355*
CPI Inflation	0.165
CPI Inflation forecast (t+12)	0.163
Net Inflation	0.147
Output gap – HP filtered	0.106
Output gap – Real-time	0.293
Output gap – Ex-post	0.214
M2 growth	0.168
Real effective exchange rate	0.447*

Given the size of our time series, we opt for KPSS test (Kwiatkowski *et al.* 1992). The null hypothesis is that the series is level stationary. Critical values for the null hypothesis: 10% - 0.347, 5% - 0.463, 1% - 0.739. Sample period: 2001:1-2006:09. *, **, *** denotes significance at the 10, 5 and 1 percent level, respectively.

B. Additional Charts

Chart B.1 – Policy Rate, Output Gap and Inflation
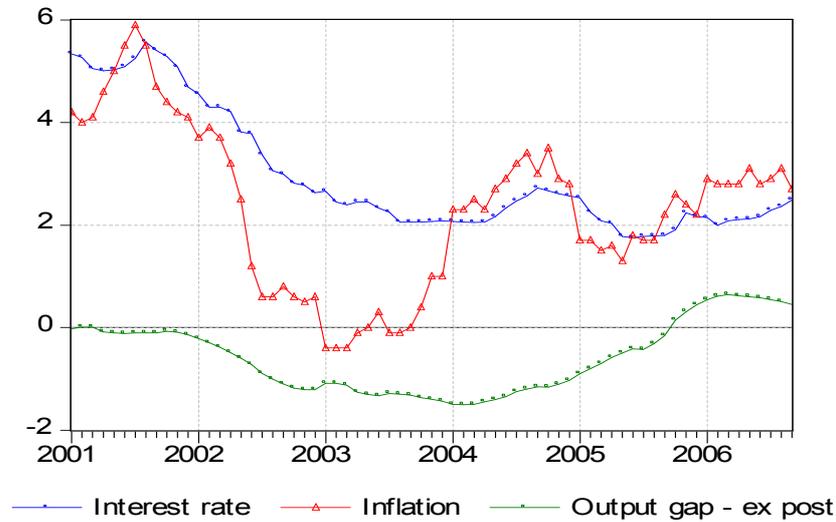
Note: This chart presents current inflation, short-term interbank interest rate (3M PRIBOR) and CNB output gap as of October 2006 forecast round

Chart B.2 – Comparison of Output Gap Estimates
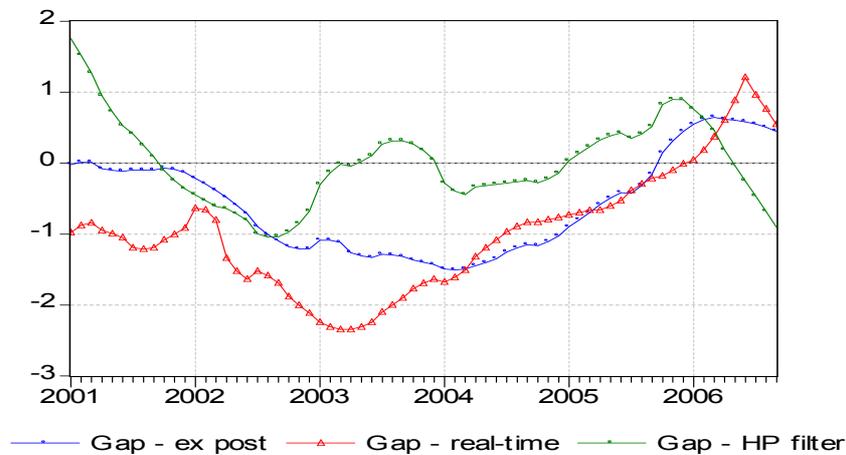
Note: This chart presents three measures of output gap used in the paper: Output gap estimated by the CNB as of their October 2006 forecast round (Gap - ex post), Real-time based output gap estimated by the CNB (Gap - real-time) and the output gap calculated using HP filter (Gap – HP filter) as the estimate of potential output.

C. Policy Neutral Rate Estimates

Time-varying coefficient model with endogenous regressors		
Backward-looking policy rule		

Current inflation, output gap – HP filter	Current inflation, output gap – ex-post	Current inflation, output gap – real-time

Forward-looking policy rule		

Inflation forecast – real-time, output gap – HP filter	Inflation forecast – real-time, output gap – ex-post	Inflation forecast – real-time, output gap – real-time

Conventional time-varying coefficient model		
Backward-looking policy rule		

Current inflation, output gap – HP filter	Current inflation, output gap – ex-post	Current inflation, output gap – real-time

Forward-looking policy rule		

Inflation forecast – real-time, output gap – HP filter	Inflation forecast – real-time, output gap – ex-post	Inflation forecast – real-time, output gap – real-time

Note: Policy neutral rate ±2 standard errors reported. Below the chart, it is reported, what measure of output gap and inflation is used for estimation of policy neutral rate.

D. Importance of Bias Correction Terms in Estimating Policy Rules

Backward-looking policy rule		
Current inflation, output gap – HP filter	Current inflation, output gap – ex-post	Current inflation, output gap – real-time

Forward-looking policy rule		
Inflation forecast – real-time, output gap – HP filter	Inflation forecast – real-time, output gap – ex-post	Inflation forecast – real-time, output gap – real-time
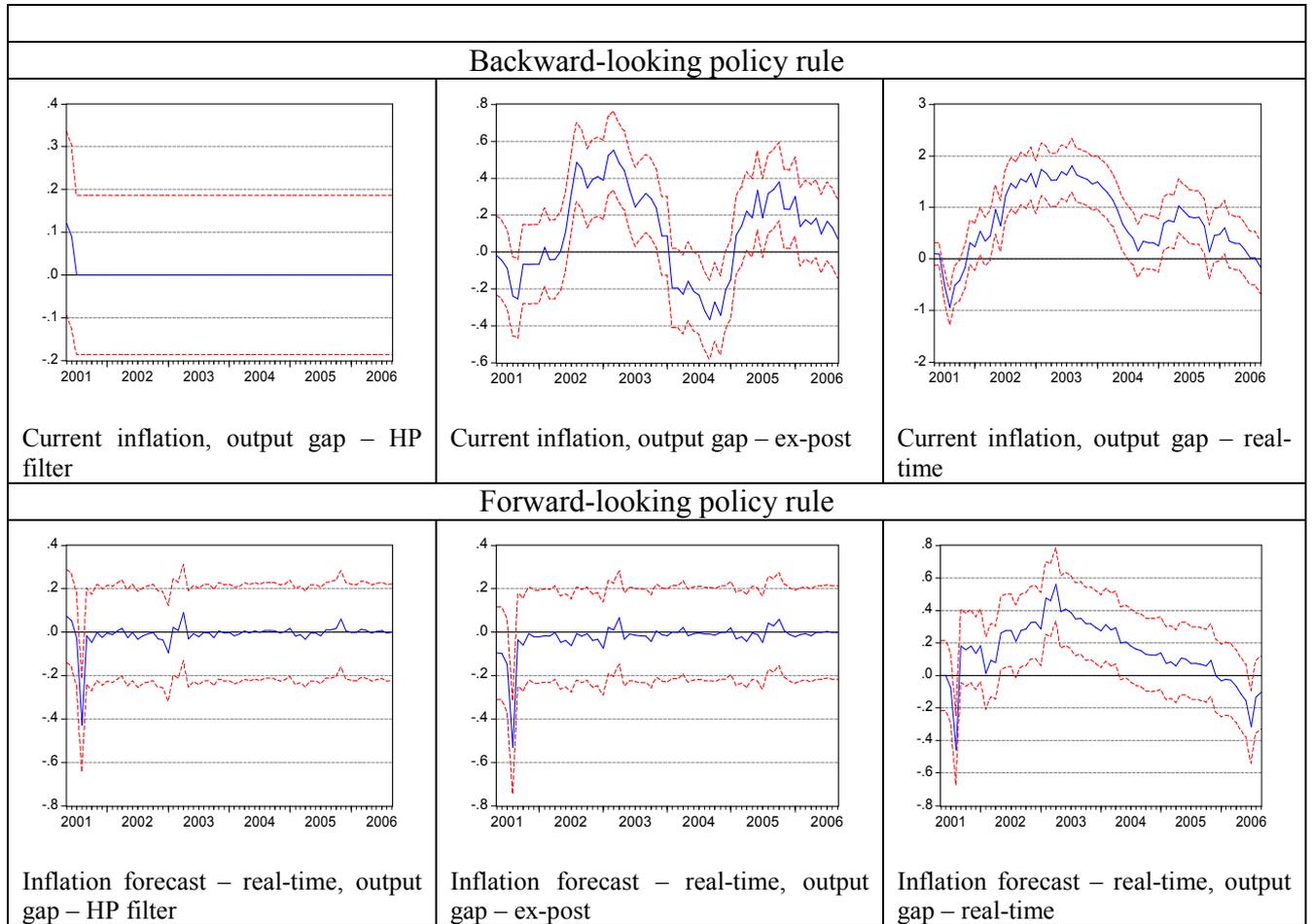
Note: The difference between the policy neutral rates estimated from the time-varying parameter model with endogenous regressors, $\bar{r}_{t,er}$, and from the conventional time-varying parameter model, $\bar{r}_{t,c}$, and ±2 standard errors is reported. ±2 Standard errors are computed as follows: $\bar{r}_{t,er} - \bar{r}_{t,c} \pm 2 s.e.\left(\bar{r}_{t,er}\right)$. Below the chart, it is reported, what measure of output gap and inflation is used for estimation of policy neutral rate.