A Small Open Economy Model with a Nontradable Sector, Foreign Currency Denominated Debt and a Financial Accelerator Mechanism: a Steady State Analysis

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Abstract

We develop a two-sectors small open economy model that considers imperfect competition, one-period nominal price rigidity and a financial accelerator mechanism. The later is modelled assuming an asymmetric information problem between lenders and borrowers. An important finding, not often highlighted in the literature, is that the model's equilibrium with the financial accelerator implies that credit is not rationed in the sense of Stiglitz and Weiss (1981). It is also shown that the imperfection in the credit market provides an upward sloping supply curve of capital. The model is fully solved for the steady state. It is demonstrated that the structural parameters affecting credit market variables are only the subjective discount factor, monitoring costs and the fraction of profits that capital producers (i.e., entrepreneurs) devote to consumption. It is also shown how the model with the financial accelerator nests a fairly standard RBC model; where the later is obtained by setting monitoring costs to zero (i.e., the model without asymmetric information).

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1 Introduction

The Mundell-Fleming textbook model indicates that a currency depreciation has an expansionary effect on output through expenditure-switching effects. With the recent experiences of financial distress in emerging markets, however, an important strand of the literature has focused on the relevance of financial channels. Calvo and Reinhart (1999) suggest the following explanation. When liabilities are denominated in the foreign currency while assets are denominated in the domestic currency, an exchange rate depreciation increases the domestic-value of liabilities. If the domestic-value of assets does not increase with the exchange rate, indebted agents face negative net worth effects. Putting it another way, Calvo and Reinhart introduce in a small open economy framework the negative net worth effects generated by debt burdens as stressed in Fisher (1933).

To motivate the discussion, Table 1 shows, for selected countries recently affected by currency crises, the negative relation between currency depreciations and the GDP growth rate.

Table 1. Selected macroeconomic indicators

Country	Year	Nominal depreciation	Δ real GDP
		(Dec-y_t/Dec-y_{t-1}) -in %-	
Argentina	2002	67.4	-10.9
Indonesia	1998	70.9	-13.1
Korea	1998	32.1	-6.9
Malaysia	1998	28.3	-7.4
Philippines	1998	27.9	-0.6
Thailand	1998	24.2	-10.5

Source: International Financial Statistics, IMF.

Krugman (1999) has firstly formalised this argument in the so-called balance-sheet approach. He introduces a combination of currency mismatches, imperfections in credit markets and sudden changes in expectations. A step forward is made in Aghion et al (2001), where they develop a model with a higher degree of formalisation. Although these models highlight important features of these crises, they also face important drawbacks: i) are highly stylised, thus ignoring microfoundations and ii) do not consider the role of nontradable goods. In particular, they assume that the economy produces a tradable good that faces nominal price rigidities in the period of the currency depreciation. As liabilities are denominated in the foreign currency, an exchange rate depreciation generates negative net worth effects reducing investment and output (i.e., balance-sheet effects). Their assumption that the tradable good faces nominal rigidities, however, is difficult to accept empirically. For instance, Burstein et al (2005 p.743) highlight that the source of nominal rigidity seems to be mostly in the nontradable sector.

A number of researchers has moved towards the microfoundations of these models. Cespedes et al (2004), Cook (2004) and Devereux et al (2006), among others, have developed well microfounded dynamic general equilibrium models that incorporate the abovementioned features.

Essentially, these models build on variants of the financial accelerator mechanism developed in Carlstrom and Fuerst (1997) and Bernanke et al (1999). The source of credit market imperfection is an asymmetric information problem between lenders and borrowers. Among other specificities, the asymmetric information problem yields a premium between the rate of return of capital and the risk-free interest rate; it also provides a spread between the non-default interest rate and the risk-free interest rate.

Since the works of Cook (2004) and Devereux et al (2006) are quantitative they are not interested in obtaining an analytical solution. In that aspect, there are relevant results and interactions in the model that are hidden in the "black box" typically associated with numerical analysis. Cespedes et al (2004), although they solve the model analytically, they take and adapt to the small open economy framework some of the main results obtained in Bernanke et al (1999). In particular, they incorporate one of the key equations that define the financial accelerator mechanism in a reduced-form way (Eq. 11 in their paper); therefore, ignoring important links between the endogenous variables associated to the credit market and the structural parameters of the model.

This paper intends to fill this gap. Formally, the model extends the small open economy model developed in Obstfeld and Rogoff (1996, Ch. 10.2) in two ways: i) incorporating capital and ii) incorporating credit market imperfections. The model is multisectorial, with two final goods sectors in the economy: one tradable (exogenous) and one nontradable. There is also one intermediate nontradable sector that demands capital and labour. To motivate that output could be demand-determined in the short run, it is assumed that this sector faces monopolistic competition as in Blanchard and Kiyotaki (1987). In allowing for a non-trivial role of monetary policy, we assume that this sector is exposed to an unexpected shock in period $t = 1$, giving place to a one-period nominal price rigidity.

The production of capital is a key element in the model, being modelled as in Carlstrom and Fuerst (1997). Each entrepreneur produces capital with only one input, that belongs to the final nontradable sector. The production function has a stochastic and idiosyncratic component. To determine the amount of investment placed in production, each entrepreneur uses his or her net worth in conjunction with external funding. This funding, however, is subject to frictions (i.e., there is an endogenous risk premium for obtaining external funding). It is assumed, moreover, that all borrowing is denominated in foreign currency.

Interestingly, it is shown that the variables associated to the financial accelerator mechanism do not depend on other structural parameters of the model but the subjective discount factor, monitoring costs and the fraction of expected profits that entrepreneurs devote to consumption. Moreover, the perfect information case, when monitoring costs are equal to zero, converges to a fairly standard RBC model. It is also shown that monetary policy, conceived as a permanent and unanticipated change in the level of the money supply, affects the short run ratio of intermediate output to capital but not the ratio of final output to capital.

The remainder of the paper is organised as follows. Section 2 develops the main elements of the model, with the exception of capital goods production. Section 3 incorporates capital producers and develops how credit constraints are introduced in the model. Section 4 deals with aggregation and defines equilibrium conditions. Section 5 solves the model for the steady state as well as for the short-run equilibrium. Finally, section 6 presents concluding remarks.

2 The model

We consider a small open economy model with two sectors: one tradable and one nontradable. To simplify the analysis, output in the tradable sector is assumed to be exogenously given in each period t. The economy is composed of firms, households, the government and entrepreneurs that mutually interact within a monetary framework.

2.1 Firms

2.1.1 Tradable sector

There is a single homogeneous tradable good whose supply is exogenously given each period t and is denoted by $\overline{Y}_{T,t}$. This output, in turn, becomes each period household's endowment.

2.1.2 Nontradable sector

The nontradable sector is composed of a continuum of intermediate firms that produce differentiated inputs and a perfectly competitive producer of the nontradable final good. There are a large number of firms indexed by i in the intermediate sector, where each one specialises in producing a particular input. Each firm, therefore, has some degree of monopoly power over its production. The imperfect competition in the production of nontradable inputs combined with nominal price rigidities in setting their prices (as explained below), provide an economic framework in which to rationalise that output could be demand-determined in the short-run.

The intermediate output of firm i at period t is produced by combining capital and labour services with a Cobb-Douglas production function as follows,

$$Z_{i,t} = A_t K_{i,t}^{\alpha} L_{i,t}^{1-\alpha}, i \in [0,1], \tag{1}$$

where $Z_{i,t}$ indicates the production of input i, A_t is a technology parameter assumed to be common to all firms, $K_{i,t}$ is the stock of capital rented to entrepreneurs at the beginning of period t, $L_{i,t}$ indicates labour services obtained from households and α is the share of capital in the nontradable intermediate input (which is assumed to be the same for all firms).

The producer of the final nontradable good combines the inputs provided by intermediate firms and a tradable input with a Cobb-Douglas-type production function. This output is afterwards sold to domestic agents for consumption or to entrepreneurs for using it as an input in the production of the capital good. The production function is defined as follows,

$$Y_t = \{[\int_0^1 (Z_{i,t})^{\frac{\theta-1}{\theta}} di]^{\frac{\theta}{\theta-1}}\}^{\gamma}\{X_{T,t}\}^{1-\gamma}, \ \theta > 1, \ 0 < \gamma < 1, \tag{2}$$

where Y_t is the final nontradable good, θ is the elasticity of substitution between different nontradable inputs, γ is the share of nontradable components in the final nontradable good and $X_{T,t}$ is the tradable input that is used in producing the final good. Each intermediate firm in the nontradable sector, therefore, faces the following downward sloping demand curve[1],

$$Z_{i,t} = Y_t\gamma^{\theta}[\frac{(1-\gamma)}{P_{T,t}}]^{\frac{(1-\gamma)(\theta-1)}{\gamma}}(P_{i,t})^{-\theta}(P_t)^{\frac{\theta-1+\gamma}{\gamma}}, \tag{3}$$

where $P_{T,t}$, $P_{i,t}$ and P_t are the prices of the tradable good, the intermediate good and the final good, respectively. It is worth noting that the marginal cost of the final producer firm is defined as $MC_t = \gamma^{-\gamma}(1-\gamma)^{(\gamma-1)}(P_{T,t})^{1-\gamma}[\int_0^1 P_{i,t}^{1-\theta}di]^{\frac{\gamma}{1-\theta}}$[2].

Let us define $P_{N,t} = [\int_0^1 P_{i,t}^{1-\theta}di]^{\frac{1}{1-\theta}} = [P_t\gamma^{\gamma}(1-\gamma)^{(1-\gamma)}(P_{T,t})^{\gamma-1}]^{\frac{1}{\gamma}}$. We can therefore rewrite the demand curve that each intermediate firm faces as,

$$Z_{i,t} = Y_t(1-\gamma)^{\frac{\gamma-1}{\gamma}}(\frac{P_t}{P_{T,t}})^{\frac{\gamma-1}{\gamma}}(\frac{P_{i,t}}{P_{N,t}})^{-\theta}. \tag{4}$$

Note that when $\gamma = 1$ Eq. 4 takes the conventional form $Z_{i,t} = Y_t(\frac{P_{i,t}}{P_t})^{-\theta}$ while the final producer marginal cost becomes $MC_t = [\int_0^1 P_{i,t}^{1-\theta}di]^{\frac{1}{1-\theta}}$.

It will be considered that the law of one price (LOOP) holds for tradable goods at all t, implying that,

$$P_{T,t} = S_t,$$

where S_t denotes the nominal exchange rate measured as the domestic price of foreign exchange. Note that the foreign price of the tradable good was normalised to one.

[1]The final good producer solves the following cost minimisation problem: $\min \int_0^1 Z_{i,t}P_{i,t}di + P_{T,t}X_{T,t}$ s.t. $Y_t = \{[\int_0^1 (Z_{i,t})^{\frac{\theta-1}{\theta}} di]^{\frac{\theta}{\theta-1}}\}^{\gamma}\{X_{T,t}\}^{1-\gamma}$, giving the inverse demand function stated in Eq. 3.

[2]Notice that in equilibrium, the marginal cost of the final producer firm will be equal to the price of the final good P_t.

2.1.3 Demand for factors by intermediate firms

Intermediate firms determine their demand for factors by solving the following cost minimisation problem (taking the output level $Z_{i,t}$ as given):

$$\min_{\{K_{i,t},L_{i,t}\}} R_t^k K_{i,t} + W_t L_{i,t} \text{ s.t. } Z_{i,t} = A_t K_{i,t}^{\alpha} L_{i,t}^{1-\alpha} \tag{5}$$

where R_t^k indicates the nominal rental price of capital and W_t denotes the nominal wage. It is worth highlighting that $K_{i,t}$ is a homogeneous capital good demanded by intermediate firms and supplied by a large number of entrepreneurs. This capital completely depreciates within the period. $L_{i,t}$, on the other hand, is a homogeneous type of labour demanded by intermediate firms and supplied by a large number of households. Since both inputs are homogeneous, supplied by a large number of agents and demanded by a large number of firms, at the individual level each firm takes the nominal rental price of capital R_t^k and the nominal wage W_t as given.

The first order conditions associated with this problem are,

$$K_{i,t}^{*} = (\frac{1-\alpha}{\alpha})^{\alpha-1} \frac{Z_{i,t}}{A_t} (\frac{W_t}{R_t^k})^{1-\alpha} \tag{6}$$

and

$$L_{i,t}^{*} = (\frac{1-\alpha}{\alpha})^{\alpha} \frac{Z_{i,t}}{A_t} (\frac{W_t}{R_t^k})^{-\alpha}. \tag{7}$$

Note that the cost function evaluated at $K_{i,t}^{*}$ and $L_{i,t}^{*}$ takes the form,

$$C_{i,t}(Z_{i,t}, R_t^k, W_t) = C_{i,t}^{*} = \alpha^{-\alpha}(1-\alpha)^{\alpha-1} A_t^{-1} Z_{i,t} W_t^{1-\alpha} (R_t^k)^{\alpha}. \tag{8}$$

2.1.4 Profit maximisation problem of intermediate firms

Intermediate firms determine the price level $P_{i,t}$ and output $Z_{i,t}$ that maximise profits subject to the cost function obtained in Eq. 8 and the inverse demand function stated in Eq. 4,

$$\max_{\{P_{i,t}\}} \pi_{i,t} = P_{i,t} Z_{i,t} - C_{i,t}^{*} \text{ s.t. } Z_{i,t} = Y_t (1-\gamma)^{\frac{\gamma-1}{\gamma}} (\frac{P_t}{P_{T,t}})^{\frac{\gamma-1}{\gamma}} (\frac{P_{i,t}}{P_{N,t}})^{-\theta}$$

The solution of this problem gives the following price setting equation,

$$P_{i,t} = \frac{\theta}{\theta-1} \alpha^{-\alpha} (1-\alpha)^{\alpha-1} A_t^{-1} W_t^{1-\alpha} (R_t^k)^{\alpha}, \tag{9}$$

where $\frac{\theta}{\theta-1}$ is a markup over marginal costs[3].

This equation defines how intermediate firms optimally set the price level of their output $P_{i,t}$. It is worth highlighting that firms decide the price level that will prevail at period t at the end of period $t-1$. To be more precise, we can think of Eq. 9 as implicitly given by the following expression,

$$P_{i,t} = E_{i,t-1}\{\frac{\theta}{\theta-1}\alpha^{-\alpha}(1-\alpha)^{\alpha-1}A_t^{-1}W_t^{1-\alpha}(R^k)^{\alpha}\},$$

where $E_{i,t-1}$ indicates the expectation hold by agent i at the end of period $t-1$ given the information available at that time. This model assumes "perfect foresight". Therefore, the above expression will be identical to Eq. 9 for all periods but $t=1$, when an unexpected shock hits the economy. During that period, the price $P_{i,0}$ differs from what firm i would have optimally chosen had the firm known the shock in advance. It is in this context that we can consider that the price level of the intermediate firm i is "given" at period $t=1$.

2.2 Households

The representative household obtains utility from consumption of the final good C_s, real money balances $\frac{M_s}{P_s}$ and leisure (given by the disutility associated with working in the production of the nontradable input $-\frac{\kappa}{2}(L_s)^2$). Therefore, lifetime utility of the representative agent takes the form,

$$U_t = \sum_{s=t}^{\infty}\beta^{s-t}[\log C_s + \chi\log(\frac{M_s}{P_s}) - \frac{\kappa}{2}(L_s)^2]. \tag{10}$$

The budget constraint that the household faces when maximising utility, expressed in nominal terms, is defined by,

$$P_tC_t + M_t + S_tD_{t+1} = P_{T,t}\overline{Y}_{T,t} + W_tL_t + \pi_t + S_tR_t^*D_t + M_{t-1} + P_tT_t. \tag{11}$$

Household's sources of funding are given by the endowment of the tradable good $P_{T,t}\overline{Y}_{T,t}$, wage earnings for working in the nontradable intermediate sector W_tL_t, dividends from owning intermediate firms π_t, nominal gross return from previous-period foreign currency denominated deposits $S_tR_t^*D_t$[4], holdings of previous period nominal money balances M_{t-1} and

[3] Note that in the perfect competition case, when $\theta \to \infty$, the price of the intermediate firm is equal to the marginal cost.

[4] Since one of the main objectives of the model is highlighting problems associated with currency mismatches, it is assumed that households only hold deposits denominated in the foreign currency.

lump-sum government transfers P_tT_t[5]. These resources may be used to purchase consumption goods P_tC_t, to accumulate nominal money balances M_t or to acquire new interest-bearing deposits S_tD_{t+1}.

First order conditions associated to this problem are obtained by maximising Eq. 10 with respect to D_{t+1}, M_t, and L_t subject to the budget constraint stated in Eq. 11. We therefore have,

$$C_{t+1} = \beta R^*_{t+1} \frac{S_{t+1}}{S_t} \frac{P_t}{P_{t+1}} C_t \tag{12}$$

$$\frac{M_t}{P_t} = \chi C_t \frac{R_{t+1}}{R_{t+1} - 1} \tag{13}$$

$$\frac{1}{\kappa} \frac{1}{C_t} \frac{W_t}{P_t} = L_t. \tag{14}$$

As conventional, Eq. 12 is a Euler equation indicating that the marginal rate of substitution of consumption in two subsequent periods must be equal to the real interest rate. Note that the UIP condition takes the form $R_{t+1} = R^*_{t+1}\frac{S_{t+1}}{S_t}$, where R_{t+1} and R^*_{t+1} indicate the gross nominal risk-free domestic and foreign interest rates, respectively.

The demand for real balances stated in Eq. 13 is positively associated with consumption and the weight in the utility function of having an extra-unit of real balances; and negatively related to the gross nominal interest rate. Finally, the labour supply equation shown in Eq. 14 increases in the real wage, while decreases in consumption and in the weight that the household gives to the disutility of working.

2.3 Government

It is assumed that government spending affects only the final nontradable good. In this simple setting, the only source of funding for the government's current spending and the lump-sum transfer that the government makes towards households, is real seigniorage. Observe that the interpretation of T_t is twofold: whenever it takes a positive value it refers to a lump-sum transfer from the government to households, while if it takes a negative value it implies a lump-sum tax paid from households to the government. The government's budget constraint can therefore be expressed as,

$$G_t + T_t = \frac{(M_t - M_{t-1})}{P_t}, \tag{15}$$

[5] In facilitating the analysis it is assumed that government's transfers are only made in the final nontradable good.

where G_t indicates government's expenditure on the final good and $\frac{(M_t - M_{t-1})}{P_t}$ is the real seigniorage that the government is obtaining for issuing money between t and $t-1$. In facilitating the analysis, unless otherwise stated, it will be assumed that $G_t \equiv 0$ and therefore any revenue due to seigniorage is immediately rebated to households in a lump-sum way.

3 Entrepreneurs

Entrepreneurs will play a central role in the model. They will produce the capital good that is afterwards rented to firms. In producing the capital good, however, they must obtain external funding, which is denominated in foreign currency and subject to frictions. The present section provides a detailed analysis of the entrepreneurs' behaviour and their interactions with the credit market.

3.1 Partial equilibrium contracting problem

The analysis of the debt contracting problem under asymmetric information developed in this section closely follows Carlstrom and Fuerst (1997). It is assumed a continuous number of entrepreneurs indexed by j in the interval $[0,1]$ producing a homogeneous capital good. Each entrepreneur has the following stochastic linear technology,

$$K_{j,t+1} = \omega_{j,t} i_{j,t}, \tag{16}$$

where $K_{j,t+1}$ indicates the capital good produced by entrepreneur j at period t, that will be incorporated in the production process of firms in period $t+1$; $i_{j,t}$ denotes the input utilised by entrepreneur j to produce the capital good, which is part of the final good produced in the economy; $\omega_{j,t}$ is a *iid* random variable with a common distribution across j, where the cumulative and density functions have positive supports and are denoted by $\Phi(\cdot)$ and $\phi(\cdot)$, respectively. To simplify the analysis it is assumed that $E(\omega) = 1$.

When the entrepreneur decides how much to invest at period t, he or she faces the following budget constraint,

$$S_t B^*_{j,t+1} = P_t (i_{j,t} - n_{j,t}), \tag{17}$$

where $S_t B^*_{j,t+1}$ indicates the domestic value of the foreign currency denominated debt[6] contracted at period t to be repaid at period $t+1$ and $n_{j,t}$ is the net worth of entrepreneur

[6] The fact that the entrepreneur can only obtain foreign currency denominated debt is taken as given in the model. It can be thought that the reason behind this situation is the so-called "original sin problem" (see Hausmann, 1999).

j at the beginning of period t. This constraint simply indicates that the entrepreneur can purchase inputs beyond his or her net worth only by contracting foreign currency denominated debt.

Following Townsend (1979) and Gale and Hellwig (1985) among others, the model assumes a costly state verification problem. In this context, the optimal contract between the borrower and the lender will take the form of a standard non-contingent debt contract. To simplify the model it will be assumed that there is enough anonymity in the credit market, so as to avoid issues related to how past records of interactions between entrepreneurs and lenders may affect the characteristics of the financial contract.

The contract specifies a fixed payment to the lender in all states where the project generates a nominal gross return above the fixed nominal value of the debt repayment. In contrast, when this condition is not satisfied, the entrepreneur defaults on the debt and the lender recoups as much as he or she can from the project, after paying a fixed monitoring cost.

The random variable $\omega_{j,t}$, which can be thought of as a productivity parameter, is neither observed by the entrepreneur nor by the lender ex-ante. For the entrepreneur, however, it is costless to observe the ex-post value of $\omega_{j,t}$. The lender, in contrast, must incur in a monitoring cost to observe the true value of $\omega_{j,t}$.

The monitoring cost is given by the payment of $\mu i_{j,t}$ units of the final capital good, where $0 \leq \mu \leq 1$[7]. The payment to observe $\omega_{j,t}$, however, is only made case the entrepreneur defaults on the debt. It is clear now where the costly state verification problem arises in the model: in order to observe the true realisation of $\omega_{j,t}$, the lender must incur in a deterministic pecuniary cost.

Let $\overline{\omega}_{j,t}$ denote the minimum value of $\omega_{j,t}$ at which default does not occur and let $R^{nd}_{j,t+1}$ indicate the non-default gross nominal interest rate charged on entrepreneur j when contracting the debt at period t. $R^{nd}_{j,t+1}$ and $\overline{\omega}_{j,t}$ therefore satisfy,

$$R^k_{t+1}\overline{\omega}_{j,t}i_{j,t} = R^{nd}_{j,t+1}S_tB^*_{j,t+1} = R^{nd}_{j,t+1}P_t(i_{j,t} - n_{j,t}). \tag{18}$$

Eq. 18 indicates that entrepreneur j, with the associated value for the productivity parameter given by $\overline{\omega}_{j,t}$, produces $\overline{\omega}_{j,t}i_{j,t}$ units of the capital good that are afterwards rented to firms at the nominal rental price R^k_{t+1}. The term $R^k_{t+1}\overline{\omega}_{j,t}i_{j,t}$, therefore, represents the

[7] The assumption regarding the form of the monitoring cost implies that there is a fixed cost $\mu i_{j,t}$, known ex-ante by the lender, for observing the true realisation of the project. Note that this cost depends on the scale of the investment $i_{j,t}$, but it is independent of the ex-post realisation of $\omega_{j,t}$. A slightly different approach is taken in Bernanke et al (1999), where the monitoring cost is a fraction of the ex-post realisation of the project. It is worth observing, however, that the main results of the model remain the same, independently of the form in which monitoring costs are defined.

minimum nominal gross return of the produced capital required to repay the principal and interests on the debt, $R^{nd}_{j,t+1} S_t B^*_{j,t+1}$.

Note that Eq. 18 can be rewritten as follows,

$$R^{nd}_{j,t+1} = \frac{R^k_{t+1} \overline{\omega}_{j,t}}{P_t(1 - \frac{n_{j,t}}{i_{j,t}})}. \tag{19}$$

Eq. 19 gives a simple relation between $R^{nd}_{j,t+1}$ and $\overline{\omega}_{j,t}$. It is worth highlighting that R^k_{t+1} is a market price, and as such will be determined by the equilibrium conditions between aggregate supply and aggregate demand for capital. The general price index, P_t, is also a market price determined by equilibrium conditions in the market for goods. Therefore, from the entrepreneur's viewpoint, these variable are taken as given.

Also note that taking the net worth of entrepreneur j as given, the contractual problem between the lender and the entrepreneur is fully specified either in terms of $R^{nd}_{j,t+1}$ and $i_{j,t}$, or $\overline{\omega}_{j,t}$ and $i_{j,t}$ (see Eq. 18). Since the contract in terms $\overline{\omega}_{j,t}$ and $i_{j,t}$ is slightly easier to study, in the remainder of the section the optimal contractual problem is analysed only in terms of these two variables.

3.2 Expected profits

In determining the optimal contract it is assumed that both the entrepreneur and the lender are risk neutral. The net expected profit of the entrepreneur in nominal terms can be expressed as follows,

$$R^k_{t+1} \int_{\overline{\omega}_{j,t}}^{\infty} i_{j,t} \omega \phi(\omega) d\omega - [1 - \Phi(\overline{\omega}_{j,t})] R^{nd}_{j,t+1} P_t (i_{j,t} - n_{j,t}),$$

where the first term indicates the expected gross income for producing the capital good whenever $\omega_{j,t} > \overline{\omega}_{j,t}$, while the second term shows the expected cost of the debt repayment in case the entrepreneur repays the debt as established in the contract (i.e., whenever $\omega_{j,t} > \overline{\omega}_{j,t}$). The term $[1 - \Phi(\overline{\omega}_{j,t})]$ thus indicates the probability that the entrepreneur does not default on the debt. Observe that in case of default, or whenever $\omega_{j,t} < \overline{\omega}_{j,t}$, the entrepreneur receives nothing, and any remaining value of the project is completely seized by the lender.

Using Eq. 18 it is possible to rewrite the above expression as follows,

$$R^k_{t+1} i_{j,t} f(\overline{\omega}_{j,t}) = R^k_{t+1} i_{j,t} \{ \int_{\overline{\omega}_{j,t}}^{\infty} \omega \phi(\omega) d\omega - [1 - \Phi(\overline{\omega}_{j,t})] \overline{\omega}_{j,t} \}$$

where $f(\overline{\omega}_{j,t}) = \{\int_{\overline{\omega}_{j,t}}^{\infty} \omega\phi(\omega)d\omega - [1-\Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}\}$ indicates the expected share of the investment that the entrepreneur keeps when undertaking a successful project.

Following a similar way of reasoning, the net expected profit of the lender can be expressed as follows,

$$R_{t+1}^{k}\int_{0}^{\overline{\omega}_{j,t}} i_{j,t}\omega\phi(\omega)d\omega - R_{t+1}^{k}\mu i_{j,t}\Phi(\overline{\omega}_{j,t}) + [1-\Phi(\overline{\omega}_{j,t})]R_{j,t+1}^{nd}P_t(i_{j,t}-n_{j,t}).$$

In this case $R_{t+1}^{k}\int_{0}^{\overline{\omega}_{j,t}} i_{j,t}\omega\phi(\omega)d\omega$ indicates the expected gross income generated by the project that is seized by the lender whenever $\omega_{j,t} < \overline{\omega}_{j,t}$ and $R_{t+1}^{k}\mu i_{j,t}\Phi(\overline{\omega}_{j,t})$ denotes the expected payment of the monitoring cost[8]. Note that $\Phi(\overline{\omega}_{j,t})$ indicates the probability that entrepreneur j defaults on the debt. In the case in which $\omega_{j,t} > \overline{\omega}_{j,t}$, on the other hand, the entrepreneur repays the loan as established in the contract, and thus the lender expects to receive $[1-\Phi(\overline{\omega}_{j,t})]R_{j,t+1}^{nd}P_t(i_{j,t}-n_{j,t})$.

Using Eq. 18 it is possible to define the expected profit for the lender as,

$$R_{t+1}^{k}i_{j,t}g(\overline{\omega}_{j,t}) = R_{t+1}^{k}i_{j,t}\{\int_{0}^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega - \mu\Phi(\overline{\omega}_{j,t}) + [1-\Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}\},$$

where $g(\overline{\omega}_{j,t}) = \int_{0}^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega - \mu\Phi(\overline{\omega}_{j,t}) + [1-\Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}$ indicates the expected share of the investment that the lender keeps from the project.

Considering the definitions of $f(\overline{\omega}_{j,t})$ and $g(\overline{\omega}_{j,t})$ it is possible to show that $f(\overline{\omega}_{j,t}) + g(\overline{\omega}_{j,t}) = 1-\mu\Phi(\overline{\omega}_{j,t})$[9]. This fact implies that a fraction $\mu\Phi(\overline{\omega}_{j,t})$ of the total investment made by entrepreneur j is expected to be lost due to the presence of monitoring costs.

[8] Recall that when the entrepreneur defaults, the lender must pay $\mu i_{j,t}$ units of the capital good, which must therefore be priced at the rental price of capital R_{t+1}^{k}.

[9] To obtain this result, notice that $f(\overline{\omega}_{j,t})+g(\overline{\omega}_{j,t})$ can be written as $\int_{0}^{\infty} \omega\phi(\omega)d\omega - \mu\Phi(\overline{\omega}_{j,t})$. Recalling that $E(\omega) = \int_{0}^{\infty} \omega\phi(\omega)d\omega = 1$ gives $f(\overline{\omega}_{j,t}) + g(\overline{\omega}_{j,t}) = 1-\mu\Phi(\overline{\omega}_{j,t})$.

3.2.1 A note on the behaviour of $f(\overline{\omega}_{j,t})$ and $g(\overline{\omega}_{j,t})$

Let us consider again the fraction of the investment that the entrepreneur and the lender keep from the project $f(\overline{\omega}_{j,t})$ and $g(\overline{\omega}_{j,t})$, respectively. In Appendix B it is shown that $f'(\overline{\omega}_{j,t}) = -[1-\Phi(\overline{\omega}_{j,t})]$ and $f''(\overline{\omega}_{j,t}) = \phi(\overline{\omega}_{j,t})$, implying that $f(\overline{\omega}_{j,t})$ is a convex function of $\overline{\omega}_{j,t}$ (notice that monitoring costs, μ, do not affect $f(\overline{\omega}_{j,t})$) . In particular, note that $f'(\overline{\omega}_{j,t})$ will always be negative, unless $\overline{\omega}_{j,t}$ takes the highest value for which ω is defined, in which case $f'(\overline{\omega}_{j,t}) = 0$. Therefore, for a given level of investment $i_{j,t}$, entrepreneur's expected profits, $R^k_{t+1} i_{j,t} f(\overline{\omega}_{j,t})$, do not increase in $\overline{\omega}_{j,t}$.

Regarding $g(\overline{\omega}_{j,t})$, in this Appendix it is also shown that $g'(\overline{\omega}_{j,t}) = -\mu\phi(\overline{\omega}_{j,t}) + [1-\Phi(\overline{\omega}_{j,t})]$ and that $g''(\overline{\omega}_{j,t}) = -[\mu \frac{\partial \phi(\overline{\omega}_{j,t})}{\partial \overline{\omega}_{j,t}} + \phi(\overline{\omega}_{j,t})]$. Note that, without monitoring costs (i.e., whenever $\mu = 0$), $g(\overline{\omega}_{j,t})$ is concave in $\overline{\omega}_{j,t}$ and $g'(\overline{\omega}_{j,t}) \geq 0$[10]. When monitoring costs are introduced in the model, there is an additional effect on $g(\overline{\omega}_{j,t})$. It can be shown that $g''(\overline{\omega}_{j,t}) < 0$ but, for sufficiently high values of $\overline{\omega}_{j,t}$, $g'(\overline{\omega}_{j,t}) < 0$ (i.e., whenever $\mu\phi(\overline{\omega}_{j,t}) > [1-\Phi(\overline{\omega}_{j,t})]$). Therefore, with monitoring costs $g(\overline{\omega}_{j,t})$ becomes a hump shaped concave function, with a maximum at the value of $\overline{\omega}_{j,t}$ for which $\mu\phi(\overline{\omega}_{j,t}) = [1-\Phi(\overline{\omega}_{j,t})]$, call it $\overline{\omega}^*_{j,t}$.

Observe that for a given $i_{j,t}$, the behaviour of the lender's expected profits, $R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t})$, depends on the the behaviour of $g(\overline{\omega}_{j,t})$. In particular, whenever $\overline{\omega}_{j,t} < \overline{\omega}^*_{j,t}$ a small rise in $\overline{\omega}_{j,t}$ must increase lender's expected profits. To gain intuition on this result, note that a small rise in $\overline{\omega}_{j,t}$ has three effects on the lender's expected profits: i. Increases the expected gross revenue of what the lender would recoup when the entrepreneur defaults on the debt, ii. Increases the expected monitoring costs and iii. Reduces the expected nominal value of the lender's debt repayment. Therefore, it must be true that the first effect overcomes the second and third effects when $g'(\overline{\omega}_{j,t}) > 0$, so as to have that the lender's expected profits increase in $\overline{\omega}_{j,t}$ when $\overline{\omega}_{j,t} < \overline{\omega}^*_{j,t}$.

3.3 Determining the optimal contract

The optimal debt contract will be determined by the pair of values for $i_{j,t}$ and $\overline{\omega}_{j,t}$ that maximises the entrepreneur's expected profits, subject to the lender receiving at least the opportunity cost of the loan.

In what follows, it is assumed that the entrepreneur's participation constraint, given by $R^k_{t+1} i_{j,t} f(\overline{\omega}_{j,t}) > R_{t+1} P_t n_{j,t}$, holds. This condition indicates that the gross nominal rate of return that the entrepreneur expects to obtain for undertaking the project, $\frac{R^k_{t+1} i_{j,t} f(\overline{\omega}_{j,t})}{P_t n_{j,t}}$, must

[10] $g'(\overline{\omega}_{j,t})$ will always be positive unless $\overline{\omega}_{j,t}$ takes the highest value for which ω is defined, in which case $g'(\overline{\omega}_{j,t}) = 0$.

be greater than the gross nominal interest, R_{t+1}.

The lender's participation constraint, in turn, is given by $R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t}) \geq R_{t+1} P_t (i_{j,t} - n_{j,t})$, indicating that the lender's expected gross nominal rate of return, $\frac{R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t})}{P_t (i_{j,t} - n_{j,t})}$, must be at least the opportunity cost of the loan, R_{t+1}. Assuming that there are a large number of lenders in this economy, we can invoke arbitrage conditions so as to guarantee that the lender's participation constraint binds.

The optimisation problem that the entrepreneur faces, therefore, can be stated as follows,

$$\max_{\{i_{j,t}, \overline{\omega}_{j,t}\}} R^k_{t+1} i_{j,t} f(\overline{\omega}_{j,t}) \text{ s.t. } R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t}) = R_{t+1} P_t (i_{j,t} - n_{j,t}).$$

From the first order conditions it is possible to obtain,[11]

$$R^k_{t+1}\{g(\overline{\omega}_{j,t}) - \frac{f(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})} g'(\overline{\omega}_{j,t})\} = R_{t+1} P_t, \tag{20}$$

and

$$i_{j,t} = \frac{n_{j,t}}{1 - \frac{R^k_{t+1}}{R_{t+1} P_t} g(\overline{\omega}_{j,t})}. \tag{21}$$

It is worth observing that Eqs. 19, 20 and 21 constitute a system of three equations in three unknowns ($\overline{\omega}_{j,t}$, $R^{nd}_{j,t+1}$ and $i_{j,t}$), since $n_{j,t}$, P_t, R^k_{t+1}, and R_{t+1} are taken as given. In solving this system, notice that Eq. 20 gives an implicit function of the form,[12]

$$\overline{\omega}_{j,t} = F(\frac{R^k_{t+1}}{R_{t+1} P_t}) = \overline{\omega}_t, \text{ where } \frac{\partial \overline{\omega}_t}{\partial R^k_{t+1}} = \frac{F'(\cdot)}{R_{t+1} P_t} > 0. \tag{22}$$

Observe that Eq. 22 implies that, in equilibrium, the value of $\overline{\omega}_{j,t}$ is the same for all entrepreneurs (and thus it is denoted by $\overline{\omega}_t$). Using Eqs. 21 and 22 it is possible to rewrite the demand function for the input $i_{j,t}$ as,

$$i_{j,t} = \frac{n_{j,t}}{1 - \frac{R^k_{t+1}}{R_{t+1} P_t} g(\overline{\omega}_t)}. \tag{23}$$

Taking as given the net worth of entrepreneur j, Eq. 23 gives a positive relation between the rental price of capital, R^k_{t+1}, and the investment demand, $i_{j,t}$. Formally, differentiating Eq. 23 with respect to R^k_{t+1} it is possible to obtain,

[11] See Appendix A for details.

[12] See Appendix B for details.

$$\frac{\partial i_{j,t}}{\partial R^k_{t+1}} = \frac{1}{R_{t+1}P_t}\frac{i^2_{j,t}}{n_{j,t}}[g(\overline{\omega}_t) + R^k_{t+1}g'(\overline{\omega}_t)\frac{\partial \overline{\omega}_t}{\partial R^k_{t+1}}] > 0^{13}, ^{14}. \tag{24}$$

It is important to highlight that Eq. 23 indicates that the demand function for the input $i_{j,t}$ linearly depends on the net worth of agent j, fact that facilitates aggregation. Moreover, from this equation it can be seen that $i_{j,t}$ only depends on $n_{j,t}$, $\overline{\omega}_t$ and market prices.

Remark 1 *The optimal leverage ratio $i_{j,t}/n_{j,t}$ is independent of any idiosyncratic variable of entrepreneur j and, therefore, in equilibrium, will be the same for all entrepreneurs.*

Combining Eqs. 19, 22 and 23 we can compute the solution for $R^{nd}_{j,t+1}$,

$$R^{nd}_{j,t+1} = \frac{R^k_{t+1}\overline{\omega}_{j,t}}{P_t(1 - \frac{n_{j,t}}{i_{j,t}})} = R_{t+1}\overline{\omega}_t g(\overline{\omega}_t)^{-1} = R^{nd}_{t+1}. \tag{25}$$

This equation indicates that the non-default interest rate will be the same for all entrepreneurs, since does not depend on any variable of entrepreneur j.

Remark 2 *Since in equilibrium the probability of default is the same for all entrepreneurs, all of them are equally risky and therefore they face the same non-default interest rate. Hence, the risk premium on each loan, defined as the difference between the non-default interest rate and the risk-free interest rate, must be the same for all entrepreneurs.*

Proof. *In equilibrium, the probability of default of each entrepreneur j is the same and is given by $\Phi(\overline{\omega}_t)$. Therefore, it must be true that all entrepreneurs are equally risky. Let $\rho_{j,t}$ indicate the risk premium that entrepreneur j faces when contracting the loan (i.e., $R^{nd}_{t+1} - R_{t+1}$). In equilibrium this risk premium takes the form: $\rho_{j,t} = R_{t+1}\{\overline{\omega}_t g(\overline{\omega}_t)^{-1} - 1\} = \rho_t$, which does not depend on any idiosyncratic variable of entrepreneur j.* ■

[13] A sufficient condition for having $\frac{\partial i_{j,t}}{\partial R^k_{t+1}} > 0$ is that $g'(\overline{\omega}_t) > 0$. In Appendix D it is shown that this condition must hold in order to satisfy the second order condition of the entrepreneur's maximisation problem when ω is uniformly distributed in the interval $[0, 2]$. Moreover, the fact that in equilibrium $g'(\overline{\omega}_t) > 0$ can also be determined by analysing the maximand and the constraint of the entrepreneur's optimisation problem. To see this: let $g'(\overline{\omega}_t) < 0$. From the constraint, this fact implies that $\frac{\partial \overline{\omega}_t}{\partial i_t} < 0$. Using this result, we can see from the maximand that the entrepreneur can always increase the expected profits by increasing investment (since $f'(\overline{\omega}_t) \leq 0$), fact that must not be true in equilibrium. We therefore have that, in equilibrium, it must be true that $g'(\overline{\omega}_t) > 0$.

[14] It is worth emphasizing the close link between the entrepreneur's optimal contracting problem and the modern literature on credit rationing. In particular, the fact that in equilibrium it must be true that $g'(\overline{\omega}_t) > 0$, suggests that this model does not show "equilibrium credit rationing" in the sense of Stiglitz and Weiss (1981). Therefore, on the upward sloping part of $g(\overline{\omega}_t)$, the lender may provide any extra funding required by the entrepreneur at a higher interest rate R^{nd}_{t+1}, since lender's expected profits increase in that region. In contrast, whenever $g'(\overline{\omega}_t) \leq 0$, any further increase in the interest rate, which is associated with a higher probability of default of the entrepreneur, reduces lender's expected profits thus giving place to a situation where credit rationing holds.

4 Aggregation and equilibrium conditions

4.1 Aggregate net worth and aggregate investment of the entrepreneurial sector

A key variable of the model is given by the entrepreneur's net worth. In what follows, it will be assumed that entrepreneurs have an infinite horizon and that each period they devote a fraction $\upsilon \in (0,1)$ of their aggregate net profits to the consumption of the final good.

Recall that $R^k_{t+1} f(\overline{\omega}_{j,t}) i_{j,t}$ denotes the expected net profits of entrepreneur j at period t. Using the fact that in equilibrium $\overline{\omega}_{j,t} = \overline{\omega}_t$ and summing over j, we can define the net expected profits of the entrepreneurial sector as $R^k_{t+1} f(\overline{\omega}_t) i_t$, where $i_t = \int_0^1 i_{j,t} dj$ denotes aggregate investment (defined below). Recalling that $f(\overline{\omega}_t) = 1 - \mu\Phi(\overline{\omega}_t) - g(\overline{\omega}_t)$, nominal aggregate net worth at the beginning of period $t+1$ can be defined as,

$$P_{t+1} n_{t+1} = (1-\upsilon) R^k_{t+1} (1 - \mu\Phi(\overline{\omega}_t) - g(\overline{\omega}_t)) i_t.$$

Notice that the lender's constraint in the entrepreneur's maximisation problem can be written in the aggregate as $R^k_{t+1} i_t g(\overline{\omega}_t) = R_{t+1} P_t (i_t - n_t)$[15]. Using this expression and the budget constraint of the entrepreneurial sector, $S_t B^*_{t+1} = P_t(i_t - n_t)$, we can rewrite the above equation as follows,

$$P_{t+1} n_{t+1} = (1-\upsilon)\{R^k_{t+1}(1 - \mu\Phi(\overline{\omega}_t)) i_t - R_{t+1} S_t B^*_{t+1}\}. \tag{26}$$

Aggregate consumption of the entrepreneurial sector at period $t+1$, C^e_{t+1}, is hence defined as,

$$P_{t+1} C^e_{t+1} = \upsilon\{R^k_{t+1}(1 - \mu\Phi(\overline{\omega}_t)) i_t - R_{t+1} S_t B^*_{t+1}\}. \tag{27}$$

Considering Eqs. 26 and 27 lagged one period, we can rewrite the budget constraint of the entrepreneurial sector at period t (*i.e.*, $S_t B^*_{t+1} = P_t(i_t - n_t)$) as,

$$P_t i_t + P_t C^e_t + R_t S_{t-1} B^*_t = R^k_t (1 - \mu\Phi(\overline{\omega}_{t-1})) i_{t-1} + S_t B^*_{t+1}. \tag{28}$$

Each period t, the entrepreneurial sector invests $P_t i_t$ to produce the capital good, consumes $P_t C^e_t$ of the final produced good and repays capital and interests of the debt contracted at

[15] Recall that the constraint that entrepreneur j faces when maximising is given by $R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t}) = R_{t+1} P_t (i_{j,t} - n_{j,t})$. Since in equilibrium $\overline{\omega}_{j,t} = \overline{\omega}_t$, we can sum over j thus obtaining $R^k_{t+1} i_t g(\overline{\omega}_t) = R_{t+1} P_t (i_t - n_t)$, where $i_t = \int_0^1 i_{j,t} dj$ denotes aggregate investment and $n_t = \int_0^1 n_{j,t} dj$ denotes aggregate net worth.

period $t-1$, $R_t S_{t-1} B_t^{*}$[16]. These expenditures are financed with the aggregate income obtained for renting the produced capital good to firms, $R_t^k(1-\mu\Phi(\overline{\omega}_{t-1}))i_{t-1}$[17], and by issuing new debt $S_t B_{t+1}^{*}$.

Aggregate investment of the entrepreneurial sector can be obtained by summing over j the demand function for the input $i_{j,t}$ stated in Eq. 23, thus yielding,

$$i_t = \int_0^1 i_{j,t}dj = (1 - \frac{R_{t+1}^k}{R_{t+1}P_t} g(\overline{\omega}_t))^{-1} n_t = (1 - \frac{f'(\overline{\omega}_t)g(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)}) n_t, \tag{29}$$

where the last term in this expression is obtained using Eq. 20[18]. Eq. 29 shows that i_t linearly depends on n_t, the aggregate net worth available at the beginning of period t. It also indicates that, in equilibrium, aggregate investment at period t is determined by the aggregate net worth in the same period scaled by the factor $(1 - \frac{f'(\overline{\omega}_t)g(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)})$, which can be thought of as a measure of the leverage ratio of the entrepreneurial sector as a whole.

Introducing Eqs. 20 and 29 into Eq. 26 it is possible to obtain,

$$P_{t+1}n_{t+1} = (1-\upsilon)R_{t+1}\{-\frac{f'(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)}(1-\mu\Phi(\overline{\omega}_t))P_t n_t - S_t B_{t+1}^{*}\}. \tag{30}$$

Similarly, entrepreneur's consumption can be expressed as,

$$P_{t+1}C_{t+1}^e = \upsilon R_{t+1}\{-\frac{f'(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)}(1-\mu\Phi(\overline{\omega}_t))P_t n_t - S_t B_{t+1}^{*}\}. \tag{31}$$

Eq. 30, becoming one of the main equations of the model, defines the evolution of aggregate net worth as a first order non-autonomous difference equation. It indicates that entrepreneurs obtain in the aggregate the gross nominal return $-\frac{f'(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)}(1-\mu\Phi(\overline{\omega}_t))R_{t+1}$ for investing their aggregate net worth $P_t n_t$ to produce the capital good. They utilise this return to repay the amount $R_{t+1}S_t B_{t+1}^{*}$ for the debt contracted at period t. The difference between these two flows multiplied by $(1-\upsilon)$, the fraction of entrepreneurs' net profits not consumed, defines aggregate net worth at the beginning of period $t+1$.

[16] It is worth noting that, although each individual entrepreneur has to repay $R_{j,t}^{nd} S_{t-1} B_{j,t}^{*}$ to lenders (whenever the debt is repaid as established in the contract), at the aggregate level lenders receive the opportunity cost of the loan, $R_t S_{t-1} B_t^{*}$. Indeed, by charging a risk-premium on each entrepreneur when lending and by seizing the project of those entrepreneurs that defaulted on the debt, lenders guarantee that at the aggregate level they obtain $R_t S_{t-1} B_t^{*}$.

[17] The fact that the term $(1-\mu\Phi(\overline{\omega}_{t-1}))i_{t-1}$ is equal to the supply of capital at period t is discussed in detail in the next subsection.

[18] Recall that Eq. 20 determines the relation between R_{t+1}^k and R_{t+1}. Since in equilibrium $\overline{\omega}_{j,t} = \overline{\omega}_t$ we have that the rental price of capital satisfies : $R_{t+1}^k = \{g(\overline{\omega}_t) - \frac{f(\overline{\omega}_t)}{f'(\overline{\omega}_t)}g'(\overline{\omega}_t)\}^{-1} R_{t+1}P_t$.

Finally, note that from the budget constraint of the entrepreneurial sector $S_tB^*_{t+1} = P_t(i_t - n_t)$ and Eq. 29 it is possible to obtain the demand for credit at the aggregate level,

$$S_tB^*_{t+1} = -P_t\frac{f'(\overline{\omega}_t)g(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)}n_t. \tag{32}$$

4.2 Aggregate supply of the capital good

From the previous section we know that a fraction $\mu\Phi(\overline{\omega}_t)$ of the total investment made by entrepreneur j at period t is expected to be lost due to the presence of monitoring costs. The aggregate supply of the capital good is hence defined as,

$$K^s_{t+1} = i_t(1 - \mu\Phi(\overline{\omega}_t)). \tag{33}$$

The existence of asymmetric information problems between lenders and entrepreneurs implies that the aggregate supply of capital, K^s_{t+1}, is a fraction $(1 - \mu\Phi(\overline{\omega}_t))$ of what would be supplied under perfect information (i.e., whenever $\mu = 0$).

Notice that, as R^k_{t+1} increases K^s_{t+1} is affected by two effects: i. Aggregate investment, i_t, increases, positively affecting K^s_{t+1} and ii. Expected monitoring costs, $\mu\Phi(\overline{\omega}_t)$, rises, negatively affecting K^s_{t+1}. Formally, we have that,

$$\frac{\partial K^s_{t+1}}{\partial R^k_{t+1}} = \frac{\partial i_t}{\partial R^k_{t+1}}(1 - \mu\Phi(\overline{\omega}_t)) - i_t\mu\phi(\overline{\omega}_t)\frac{\partial \overline{\omega}_t}{\partial R^k_{t+1}}. \tag{34}$$

Determining analytically the sign of this partial derivative is not easy. In Appendix D, however, this derivative is numerically evaluated for the case in which ω is uniformly distributed in the interval $[0, 2]$, showing that it is always positive provided that $\mu + \overline{\omega} < 2$ (i.e., the required condition for having $g'(\overline{\omega}_t) > 0$) and that $\mu \in (0, 1]$. It is also shown that as $\mu \rightarrow 0$ the partial derivative $\frac{\partial K^s_{t+1}}{\partial R^k_{t+1}} \rightarrow \infty$, whenever $\overline{\omega} \in [0, 2)$. This fact implies that the existence of monitoring costs provides an upward sloping supply curve of capital in the (R^k_{t+1}, K_{t+1}) space, rather than a horizontal supply curve as would be the case without asymmetric information problems and a constant returns to scale technology. Using Eq. 29 we can express the aggregate supply of capital as,

$$K^s_{t+1} = (1 - \frac{f'(\overline{\omega}_t)g(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)})(1 - \mu\Phi(\overline{\omega}_t))n_t. \tag{35}$$

It is worth noting that, in equilibrium, K^s_{t+1} only depends on $\overline{\omega}_t$, μ and n_t. Therefore, changes in R^k_{t+1} will affect K^s_{t+1} only indirectly through changes in $\overline{\omega}_t$.

4.3 Aggregate demand for the capital good

In this model only intermediate firms demand the capital produced by entrepreneurs. Using the fact that in a symmetric equilibrium each firm i sets the same price for the produced intermediate good (i.e., $P_{i,t} = P_{N,t}$), Eq. 4 thus implies that $Z_{i,t} = Z_t$ for all i. From Eq. 6, the aggregate demand for the capital good in period $t+1$ takes the form,

$$K_{t+1}^d = (\frac{1-\alpha}{\alpha})^{\alpha-1} \frac{Z_{t+1}}{A_{t+1}} (\frac{W_{t+1}}{R_{t+1}^k})^{1-\alpha}. \tag{36}$$

It can be easily seen that K_{t+1}^d negatively depends on R_{t+1}^k, as we would expect[19].

4.4 Equilibrium conditions

To define the equilibrium of the model it is still necessary to specify: i. Money market equilibrium, ii. Goods market equilibrium, iii. Capital good market equilibrium, iv. Labour market equilibrium, v. Intertemporal balance of trade equilibrium and vi. Credit market equilibrium.

4.4.1 Money market equilibrium

Money market equilibrium is given by Eq. 13 under the assumption that aggregate supply (which is assumed to be exogenous) equates aggregate demand for money.

4.4.2 Goods market equilibrium

To determine the equilibrium conditions in the goods market it is worth recalling that there are two sectors in this economy: one tradable and one nontradable. Noting that the only source of absorption of tradable output is given by the demand for tradable inputs by the final producer firm, we can define the clearing market condition in the tradable sector as follows,

$$P_{T,t}(\overline{Y}_{T,t} - X_{T,t}) = TB_t, \tag{37}$$

where TB_t denotes the trade balance at period t measured in terms of tradable goods.

As previously pointed out, in a symmetric equilibrium we have that $P_{i,t} = P_{N,t}$ and that $Z_{i,t} = Z_t$ for all i. Therefore, the production function of the nontradable intermediate firm becomes $Z_t = A_t K_t^{\alpha} L_t^{1-\alpha}$. Due to the existence of imperfect competition in this sector, it must

[19] Formally, the negative effect of R_{t+1}^k on K_{t+1}^d is given by the following partial derivative, $\frac{\partial K_{t+1}^d}{\partial R_{t+1}^k} = -(1-\alpha)^{\alpha}\alpha^{1-\alpha}\frac{Z_{t+1}}{A_{t+1}}W_{t+1}^{1-\alpha}(R_{t+1}^k)^{\alpha-2} < 0$.

be true that the aggregate income of intermediate firms equates the payment of the two factors of production plus any remaining profits at the aggregate level or: $P_{N,t}Z_t = R_t^k K_t + W_t L_t + \pi_t$.

Regarding the final producer firm, noting that in equilibrium the marginal cost of the final producer firm equates the price level, we can obtain the following expression for the domestic general price level P_t,

$$P_t = \gamma^{-\gamma}(1-\gamma)^{(\gamma-1)} {P_{T,t}}^{1-\gamma} P_{N,t}^{\gamma}, \tag{38}$$

where $P_{N,t}$ is given by,

$$P_{N,t} = \frac{\theta}{\theta - 1}\alpha^{-\alpha}(1-\alpha)^{\alpha-1}A_t^{-1}W_t^{1-\alpha}(R_t^k)^{\alpha}. \tag{39}$$

In equilibrium, moreover, the production function of the final firm takes the simpler form $Y_t = Z_t^{\gamma} X_{T,t}^{1-\gamma}$. Cost minimisation hence implies $P_t Y_t = P_{N,t}Z_t + P_{T,t}X_{T,t}$, and thus the demand functions for the nontradable and tradable inputs are given by $P_{N,t}Z_t = \gamma P_t Y_t$ and $P_{T,t}X_{T,t} = (1-\gamma)P_t Y_t$, respectively.

Finally, the clearing market condition for the final nontradable good implies,

$$Y_t = C_t + C_t^e + i_t. \tag{40}$$

4.4.3 Capital good market equilibrium

In equilibrium, the rental price of capital will adjust so as to equate aggregate supply and aggregate demand for the capital good. From Eqs. 35, 36 and 39 we have that $K_t^s = K_t^d = K_t$ thus implying,

$$R_t^k = \alpha \frac{\theta - 1}{\theta} \frac{P_{N,t}Z_t}{K_t}, \tag{41}$$

where $K_t = (1 - \frac{f'(\overline{\omega}_{t-1})g(\overline{\omega}_{t-1})}{f(\overline{\omega}_{t-1})g'(\overline{\omega}_{t-1})})(1 - \mu\Phi(\overline{\omega}_{t-1}))n_{t-1}$.

4.4.4 Labour market equilibrium

Recalling that only intermediate firms demand labour, we can obtain the demand function for labour at the aggregate level in a symmetric equilibrium from Eq. 7: $L_t^d = (\frac{1-\alpha}{\alpha})^{\alpha}\frac{Z_t}{A_t}(\frac{W_t}{R_t^k})^{-\alpha}$. Aggregate labour supply, on the other hand, is given by Eq. 14. Using Eq. 39 and the equilibrium condition $L_t^s = L_t^d = L_t$ gives,

$$W_t = (1-\alpha)\frac{\theta - 1}{\theta}\frac{P_{N,t}Z_t}{L_t},$$

where $L_t = \frac{1}{\kappa}\frac{1}{C_t}\frac{W_t}{P_t}$.

4.4.5 Intertemporal balance of trade equilibrium

By adding the budget constraints of households, government and entrepreneurs we can obtain the budget constraint of the economy as a whole (i.e., the balance of payment) as follows,

$$\begin{aligned} & P_t C_t + P_t C_t^e + P_t i_t + S_t R_t^* B_t^* + S_t D_{t+1} \\ = \; & P_{T,t}\overline{Y}_{T,t} + S_t B_{t+1}^* + R_t^k (1 - \Phi(\overline{\omega}_{t-1})) i_{t-1} + W_t L_t + \pi_t + S_t R_t^* D_t. \end{aligned}$$

Using the fact that $R_t^k(1 - \mu\Phi(\overline{\omega}_{t-1}))i_{t-1} = R_t^k K_t$; $P_{N,t}Z_t = R_t^k K_t + W_t L_t + \pi_t$ and $P_t Y_t = P_{N,t}Z_t + P_{T,t}X_{T,t}$ as well as the clearing market conditions for the tradable and nontradable sectors gives,

$$S_t(D_{t+1} - B_{t+1}^*) = TB_t + S_t R_t^*(D_t - B_t^*).$$

Let $F_t = D_t - B_t^*$ and $F_{t+1} = D_{t+1} - B_{t+1}^*$ denote the net foreign assets accumulated by households at period t and $t+1$, respectively, denominated in foreign currency. Observe that F_t and F_{t+1} will also indicate the accumulation of net foreign assets by the economy as a whole. The intertemporal national budget constraint in foreign currency can thus be written as,

$$F_{t+1} = S_t^{-1} TB_t + R_t^* F_t. \tag{42}$$

Lagging one period Eq. 42, rearranging and iterating forward yields the conventional expression,

$$-F_{-1}R_{-1}^* = \sum_{t=0}^{\infty} S_{t-1}^{-1} TB_{t-1}(1.(R_0^*)^{-1}...(R_{t-1}^*)^{-1}) + \lim_{T\to\infty}(-F_T)(1.(R_0^*)^{-1}...(R_{T-1}^*)^{-1}).$$

Imposing the "no-Ponzi-game-condition", implying that $\lim_{T\to\infty}(-F_T)(1.(R_0^*)^{-1}...(R_{T-1}^*)^{-1}) = 0$, gives,

$$-F_{-1}R_{-1}^* = \sum_{t=0}^{\infty} S_{t-1}^{-1} TB_{t-1}(1.(R_0^*)^{-1}...(R_{t-1}^*)^{-1}). \tag{43}$$

As usual, Eq. 43 simply states that any initial net foreign-currency indebtedness must be equal to the present value of future trade balance surpluses, guaranteeing that the economy is solvent from an intertemporal perspective.

4.4.6 Credit market equilibrium

From the intertemporal national budget constraint we have,

$$S_t B^*_{t+1} = S_t(D_{t+1} - F_{t+1}),$$

which indeed defines the equilibrium condition in the credit market. Notice that the aggregate demand for credit is defined by $S_t B^*_{t+1} = -P_t \frac{f'(\overline{\omega}_t)g(\overline{\omega}_t)}{f(\overline{\omega}_t)g'(\overline{\omega}_t)} n_t$ as stated in Eq. 32; while the aggregate supply is given by $D_{t+1} - F_{t+1}$. We can see, therefore, that the overall amount of credit that is available in the economy is provided by domestic households as deposits D_{t+1}, and by foreigners in the form of net foreign liabilities, $-F_{t+1}$.

5 Solving the model

5.1 Solving the monetary sector

Fender and Rankin (2003) point out that in a model where households' preferences are logarithmic over consumption and real balances, the monetary sector can be solved independently of the real sector. We will use this property in solving the model. Notice that using Eq. 12 and the UIP condition it is possible to rewrite Eq. 13 as follows,

$$\frac{M_t}{P_t C_t} = \chi + \beta \frac{M_t}{M_{t+1}} \frac{M_{t+1}}{P_{t+1} C_{t+1}},$$

where $\frac{M_t}{P_t C_t}$ can be interpreted as money demand per unit of consumption. $\frac{M_t}{M_{t+1}}$, on the other hand, indicates the inverse of the money supply growth rate. Let us define $X_t = \frac{M_t}{P_t C_t}$ and $H_t = \frac{M_t}{M_{t+1}}$. We can therefore rewrite the above equation as,

$$X_t = \chi + \beta H X_{t+1}, \tag{44}$$

under the assumption of a constant money supply growth rate. Notice that Eq. 44 represents a first order autonomous linear difference equation in the forward-looking variable X_t. Since $\beta < 1$ and, under the assumption of a non-negative money supply growth rate, $H \leq 1$ the solution of X_t is unstable in the forward dynamics. Therefore, the condition for saddlepoint stability requires X_t being equal to the steady state value $\frac{\chi}{1-\beta H}$.

The implications of this fact are relevant in terms of the dynamics of the model. For instance, if the economy is initially at the steady state, an unanticipated and permanent change in the level of the money supply at period t will not affect X. Since X remains at the steady state level $\frac{\chi}{1-\beta H}$, it must be true that $P_t C_t$ rises proportionately to M_t. From

Eq. 13 we can see that R_{t+1} must also remain unchanged. Observing the UIP condition, it is also clear that S_{t+1}/S_t must be unaffected, implying that the exchange rate immediately jumps to its new steady state value. Hence, the rate of change of the nominal exchange rate is determined, but not the level of the nominal exchange rate. This model, therefore, will not show non-trivial exchange rate dynamics such as the overshooting process described in Dornbusch (1976). To understand what happens in the rest of the economy, on the other hand, it is necessary to firstly analyse the steady state equilibrium of the model.

5.2 Describing the symmetric steady state

As a first step towards solving the complete model it is worth analysing how the economy behaves in a symmetric steady state. In this regard, it is useful to start with the aggregate net worth of the entrepreneurial sector. From Eq. 30 it is possible to obtain for the steady state,

$$n[f(\overline{\omega})g'(\overline{\omega}) + (1-\upsilon)Rf'(\overline{\omega})(f(\overline{\omega}) + g(\overline{\omega}))] = -b(1-\upsilon)Rf(\overline{\omega})g'(\overline{\omega}), \tag{45}$$

where $b = \frac{SB^*}{P}$ and the fact that $(1-\mu\Phi(\overline{\omega})) = f(\overline{\omega}) + g(\overline{\omega})$ has been introduced in the above expression. The aggregate demand for credit defined in Eq. 32 yields, in the steady state,

$$b = -\frac{f'(\overline{\omega})g(\overline{\omega})}{f(\overline{\omega})g'(\overline{\omega})}n. \tag{46}$$

Observe that Eqs. 45 and 46 bring the following relation,

$$-\frac{g'(\overline{\omega})}{f'(\overline{\omega})} = (1-\upsilon)R.$$

Using the above expression and Eq. 46 we can also obtain,

$$C^e = \frac{\upsilon}{1-\upsilon}n.$$

To solve for R note that Eq. 12 evaluated at the zero-inflation steady state gives,

$$R = r \equiv \beta^{-1}, \tag{47}$$

where the Fisher relation $R_{t+1}\frac{P_t}{P_{t+1}} = r_{t+1}$, with r_{t+1} denoting the domestic risk-free gross real interest rate at period $t+1$, has been introduced.

Hence, we have,

$$-\frac{g'(\overline{\omega})}{f'(\overline{\omega})} = (1-\upsilon)\beta^{-1}, \tag{48}$$

expression from which the steady state value of $\overline{\omega}$ can be obtained. This equation will be extensively used in solving for the other endogenous variables of the system.

Let us rewrite the nominal variables in real terms as follows: $r^k = \frac{R^k}{P}$, $R^{nd} = r^{nd}$, $p_N = \frac{P_N}{P}$, $s = p_T = \frac{S}{P}$, $w = \frac{W}{P}$, $m = \frac{M}{P}$, $b = \frac{SB^*}{P}$, $tb = \frac{TB}{P}$, $f_{-1} = \frac{SF_{-1}}{P}$ and $d = \frac{SD}{P}$. To facilitate the exposition, the key endogenous variables of the model in the zero-inflation/zero-money growth rate steady state are listed below:

$$C = m\frac{(1-\beta)}{\chi}, \tag{49}$$

$$r^{nd} = \beta^{-1}\overline{\omega}g(\overline{\omega})^{-1} \tag{50}$$

$$r^k = \beta^{-1}(g(\overline{\omega}) - \frac{f(\overline{\omega})}{f'(\overline{\omega})}g'(\overline{\omega}))^{-1}, \tag{51}$$

$$r^k = \alpha p_N \frac{\theta-1}{\theta}\frac{Z}{K}, \tag{52}$$

$$K = (1 - \frac{f'(\overline{\omega})g(\overline{\omega})}{f(\overline{\omega})g'(\overline{\omega})})(f(\overline{\omega}) + g(\overline{\omega}))n, \tag{53}$$

$$w = (1-\alpha)p_N\frac{\theta-1}{\theta}\frac{Z}{L} \tag{54}$$

$$L = \frac{1}{\kappa}\frac{1}{C}w \tag{55}$$

$$\gamma Y = p_N Z, \tag{56}$$

$$(1-\gamma)Y = sX_T, \tag{57}$$

$$1 = \gamma^{-\gamma}(1-\gamma)^{(\gamma-1)}s^{1-\gamma}p_N^{\gamma}, \tag{58}$$

$$p_N = \frac{\theta}{\theta-1}\alpha^{-\alpha}(1-\alpha)^{\alpha-1}A^{-1}w^{1-\alpha}(r^k)^{\alpha}, \tag{59}$$

$$C^e = \frac{\upsilon}{1-\upsilon} n, \tag{60}$$

$$i = (1 - \frac{f'(\overline{\omega})g(\overline{\omega})}{f(\overline{\omega})g'(\overline{\omega})})n, \tag{61}$$

$$p_T(\overline{Y}_T - X_T) = tb, \tag{62}$$

$$Y = C + C^e + i, \tag{63}$$

$$b = -\frac{f'(\overline{\omega})g(\overline{\omega})}{f(\overline{\omega})g'(\overline{\omega})} n, \tag{64}$$

$$-\frac{g'(\overline{\omega})}{f'(\overline{\omega})} = (1-\upsilon)\beta^{-1}, \tag{65}$$

$$f_{-1} = d - b, \tag{66}$$

$$-f_{-1}\beta^{-1} = (1-\beta)^{-1} tb. \tag{67}$$

To further simplify the model it is assumed that $f_{-1} \equiv 0$, implying that $X_T \equiv \overline{Y}_T$ and thus $s = Y(1-\gamma)\overline{Y}_T^{-1}$ from Eqs. 57, 62 and 67. On the other hand, since the production function of entrepreneurs is stochastic, to solve the model analytically it becomes necessary to assume a distribution function for the random variable ω. In the case in which ω is uniformly distributed in the interval $[0, 2]$ Eq. 65 yields,

$$\overline{\omega} = 2 + \frac{\mu\beta}{1-(\beta+\upsilon)} \equiv \overline{\omega}^*. \tag{68}$$

For ω to be within the interval $[0, 2]$ it is required that $\beta + \upsilon > 1$ and that $-2 \leq \frac{\mu\beta}{1-(\beta+\upsilon)} \leq 0$. From here onwards, we will assume that these conditions are satisfied. Notice that the parameters affecting $\overline{\omega}^*$ are only μ, β and υ; therefore, the steady state solution of $\overline{\omega}$ is not affected by those parameters related to production functions or preferences other that β. Also observe that for having $g'(\overline{\omega}) > 0$ it is required that $\upsilon < 1$, condition that is satisfied since $\upsilon \in (0, 1)$[20].

In the steady state, therefore, $\overline{\omega}$ decreases with monitoring costs μ (and thus, the probability of default decreases in μ), while increases in the subjective discount factor β and in the

[20] Indeed, notice that $g'(\overline{\omega}) = \frac{\mu}{2}\frac{1-\upsilon}{\beta+\upsilon-1}$ when ω is uniformly distributed.

fraction of expected profits devoted to consumption, υ. Observe that once the solution of $\overline{\omega}$ is obtained r^k can also be pinned-down. Eqs. 51 and 68 thus give,

$$r^k = (\beta(1 - \frac{1}{2}\mu - \frac{\mu\overline{\omega}}{4}))^{-1} = (\beta(1 - \mu - \frac{\mu^2}{4}\frac{\beta}{1-(\beta+\upsilon)}))^{-1} \equiv r^{k*}. \tag{69}$$

From Eq. 69 it can be shown that r^k increases in μ if $\beta > \frac{2(1-\upsilon)}{2-\mu}$. It is also possible to demonstrate that r^k increases in β if $\frac{r^k\beta^2\mu^2}{4} > \frac{[1-(\beta+\upsilon)]^2}{1-\upsilon}$. On the other hand, r^k will always increase in υ.

The non-default real interest rate, r^{nd}, can also be pinned-down using Eq. 68 yielding,

$$r^{nd} = (\beta(1 - \frac{1}{2}\mu - \frac{\overline{\omega}}{4}))^{-1} = (\beta(\frac{1}{2} - \frac{1}{2}\mu - \frac{1}{4}\mu\frac{\beta}{1-(\beta+\upsilon)}))^{-1} \equiv r^{nd*}. \tag{70}$$

Notice the important similarity between Eq. 69 and Eq. 70. It is possible to verify that r^{nd} increases in μ if $\beta > 2(1-\upsilon)$ and that it increases in β if $\frac{r^{nd}\beta^2\mu}{4} > \frac{[1-(\beta+\upsilon)]^2}{1-\upsilon}$. As in the case of the rental rate of capital, r^{nd} univocally rises in υ. Table 2, below, summarises these results.

Table 2. Summary of static comparative analysis

Effect on	Change in		
	μ	β	υ
$\overline{\omega}^*$	(-)	(+)	(+)
r^{k*}	(+) if $\beta > \frac{2(1-\upsilon)}{2-\mu}$	(+) if $r^k > \frac{4}{\beta^2\mu^2}\frac{(1-(\beta+\upsilon))^2}{1-\upsilon}$	(+)
r^{nd*}	(+) if $\beta > 2(1-\upsilon)$	(+) if $r^{nd} > \frac{4}{\beta^2\mu^2}\frac{(1-(\beta+\upsilon))^2}{1-\upsilon}$	(+)

To bring forward a more precise answer to the question of the effects of monitoring costs μ and the share of entrepreneurs' expected profits devoted to consumption υ on $\overline{\omega}^*$, r^{k*} and r^{nd*} a small scale calibration-type analysis is followed. There are three parameters affecting $\overline{\omega}^*$, r^{k*} and r^{nd*} : μ, υ and β. Following Carlstrom and Fuerst (1997) we will set $\beta = 0.99$. We will consider two cases regarding monitoring costs: $\mu = 0$ (i.e., the perfect information case) and $\mu = 0.25$ (i.e., the Carlstrom and Fuerst case). The parameter υ will be chosen so as to satisfy the restrictions $\beta + \upsilon > 1$ and $-2 \leq \frac{\mu\beta}{1-(\beta+\upsilon)} \leq 0$. In particular, notice that the second restriction implies $\upsilon > \frac{2+\beta(\mu-2)}{2}$. Hence, two cases are analysed: $\upsilon = 0.15$ and $\upsilon = 0.25$.

Table 3. A numerical example

	$\mu = 0, \upsilon = \frac{3}{20}$	$\mu = \frac{1}{4}, \upsilon = \frac{3}{20}$	$\mu = 0, \upsilon = \frac{1}{4}$	$\mu = \frac{1}{4}, \upsilon = \frac{1}{4}$
$\overline{\omega}^*$	2	0.23	2	1.58
r	1.01	1.01	1.01	1.01
r^{k*}	1.01	1.17	1.01	1.24
r^{nd*}	2.02	1.24	2.02	1.60

From Table 3, it is worth observing that when $\mu = 0$ the model collapses to a fairly standard RBC model. The fact that $\overline{\omega}^*$ takes the highest value of the domain of the distribution, implying that the steady state probability of default is 1 (i.e., $\Phi(\overline{\omega}^*) = 1$) indicates that there is no credit relation between the entrepreneur and the lender anymore: the lender and the entrepreneur are the same agent. Therefore, $r^{nd*}/\mu = 0$ is just an indicative steady state gross interest rate. As expected, the gross rate of return of capital, r^{k*}, converges to the risk-free interest rate, r. Also notice that with $\mu = 0$ a change in υ does not affect the steady state values of $\overline{\omega}^*$, r, r^{k*} and r^{nd*}.

As the asymmetric information problem takes place, (i.e., $\mu > 0$), it is possible to observe that r^{k*} also rises. Therefore, the steady state difference $r^{k*} - r$ increases in μ as suggested in Carlstrom and Fuerst (1997) and Bernanke et al (1999). In this case, the rise in the entrepreneur fraction of consumption increases both, the gross rate of return of capital and the gross non default interest rate (see Table 3).

5.3 Steady State Solution with flexible prices

To start solving for the other endogenous variables of the model under flexible prices, notice that since $s = Y(1-\gamma)\overline{Y}_T^{-1}$, Eq. 58 gives,

$$Y = \overline{Y}_T(\frac{p_N}{\gamma})^{\frac{\gamma}{\gamma-1}}.$$

This equation, however, does not pin-down Y since p_N is endogenous. To solve for p_N note that the equilibrium condition in the labour market, described by Eqs. 54 and 55, in conjunction with Eq. 56 bring the relation,

$$w = [\gamma(1-\alpha)\frac{\theta-1}{\theta}Y\kappa C]^{\frac{1}{2}}. \tag{71}$$

Hence, the price index for the nontradable good given in Eq. 59 can be expressed as,

$$p_N = A^{-1}(\frac{r^{k*}}{\alpha})^{\alpha}\frac{\theta}{\theta-1}^{\frac{1+\alpha}{2}}[\frac{\gamma\kappa}{(1-\alpha)}YC]^{\frac{1-\alpha}{2}}.$$

Combining the above equation with the expression for Y obtained previously and solving for C gives,

$$C = Y^{\frac{\gamma(1+\alpha)-2}{(1-\alpha)\gamma}}[A^{-1}(\frac{r^{k*}}{\alpha})^{\alpha}(\frac{\theta}{(\theta-1)\gamma})^{\frac{1+\alpha}{2}}(\frac{\kappa}{(1-\alpha)})^{\frac{1-\alpha}{2}}\overline{Y}_T^{\frac{\gamma-1}{\gamma}}]^{\frac{2}{\alpha-1}}.$$

This equation relates two endogenous variables: C and Y. The clearing market condition for the nontradable good, Eq. 63, in combination with Eqs. 60, 61 and 65 also give,

$$n = (1-\upsilon)(1+\beta\frac{g(\overline{\omega}^*)}{f(\overline{\omega}^*)})^{-1}[Y - Y^{\frac{\gamma(1+\alpha)-2}{(1-\alpha)\gamma}}[A^{-1}(\frac{r^{k*}}{\alpha})^{\alpha}\frac{\theta}{(\theta-1)\gamma}^{\frac{1+\alpha}{2}}(\frac{\kappa}{(1-\alpha)})^{\frac{1-\alpha}{2}}\overline{Y}_T^{\frac{\gamma-1}{\gamma}}]^{\frac{2}{\alpha-1}}], \quad (72)$$

after substituting for C. Observe that Eq. 72 brings a relation between the two endogenous variables n and Y. We can obtain a second expression in these two variables as follows. The production function of intermediate firms, $Z = AK^{\alpha}L^{1-\alpha}$,[21] and Eq. 56 give,

$$Y = A\gamma^{-1}K^{\alpha}L^{1-\alpha}p_N.$$

Notice that $L^{1-\alpha}p_N = A^{-1}(\frac{\theta}{\theta-1}\frac{r^{k*}}{\alpha})^{\alpha}(\gamma Y)^{1-\alpha}$ (from Eqs. 55, 59 and 71) and that $r^{k*}K = (1-\upsilon)^{-1}(1+\frac{g(\overline{\omega}^*)}{f(\overline{\omega}^*)})n$ (from Eqs. 51, 53 and 65). Therefore, it is possible to obtain,

$$n = Y\alpha\gamma(1-\upsilon)\frac{\theta-1}{\theta}(1+\frac{g(\overline{\omega}^*)}{f(\overline{\omega}^*)})^{-1}. \quad (73)$$

Substituting Eq. 73 into 72 and rearranging gives the solution for Y,

$$\begin{aligned} Y &= [1-\alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}]^{\frac{(1-\alpha)\gamma}{2(\alpha\gamma-1)}} \\ &\quad [A^{-1}\overline{Y}_T^{\frac{\gamma-1}{\gamma}}(\frac{r^{k*}}{\alpha})^{\alpha}(\gamma\frac{\theta-1}{\theta})^{-\frac{1+\alpha}{2}}\frac{\kappa}{1-\alpha}^{\frac{1-\alpha}{2}}]^{\frac{\gamma}{\alpha\gamma-1}}, \end{aligned} \quad (74)$$

where Eqs. 51 and 65 have been used in obtaining Y. Net worth is thus given by,

$$\begin{aligned} n &= (1-\upsilon)(1+\frac{g(\overline{\omega}^*)}{f(\overline{\omega}^*)})^{-1}[1-\alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}]^{\frac{(1-\alpha)\gamma}{2(\alpha\gamma-1)}} \\ &\quad [\alpha^{-1}A^{-\gamma}\overline{Y}_T^{\gamma-1}(\gamma\frac{\theta-1}{\theta})^{\frac{\gamma(\alpha-1)-2}{2}}(r^{k*})^{\alpha\gamma}\frac{\kappa}{1-\alpha}^{\frac{\gamma(1-\alpha)}{2}}]^{\frac{1}{\alpha\gamma-1}}. \end{aligned} \quad (75)$$

Having solved for Y and n we can now recover all the other endogenous variables. Rewriting the production function of the final firm as[22], $Z = \overline{Y}_T^{\frac{\gamma-1}{\gamma}}Y^{\frac{1}{\gamma}}$, gives,

$$\begin{aligned} Z &= \overline{Y}_T^{\frac{\alpha(\gamma-1)}{\alpha\gamma-1}}[1-\alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}]^{\frac{(1-\alpha)}{2(\alpha\gamma-1)}} \\ &\quad [A^{-1}(\frac{r^{k*}}{\alpha})^{\alpha}(\frac{\theta}{\theta-1}\frac{1}{\gamma})^{\frac{1+\alpha}{2}}\frac{\kappa}{1-\alpha}^{\frac{1-\alpha}{2}}]^{\frac{1}{\alpha\gamma-1}}. \end{aligned}$$

[21] The production function of Intermediate firms can be recovered by substituting Eqs. 52 and 54 into Eq. 59.

[22] The production function of the final firm, $Y = Z^{\gamma}X_T^{1-\gamma}$, can be obtained by substituting Eqs. 56 and 57 into Eq. 58.

Following a similar procedure as before, considering Eqs. 53, 61, 64, 65 and 75, we can solve for K, i and b yielding:

$$\begin{aligned} K &= (f(\overline{\omega}^*)(1-\upsilon)+\beta g(\overline{\omega}^*))[1-\alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}]^{\frac{(1-\alpha)\gamma}{2(\alpha\gamma-1)}} \\ & [\alpha^{-1}A^{-\gamma}\overline{Y}_T^{\gamma-1}(\gamma\frac{\theta-1}{\theta})^{\frac{\gamma(\alpha-1)-2}{2}}(r^{k*})^{\alpha\gamma}\frac{\kappa}{1-\alpha}^{\frac{\gamma(1-\alpha)}{2}}]^{\frac{1}{\alpha\gamma-1}}, \end{aligned} \tag{76}$$

$$\begin{aligned} i &= \frac{f(\overline{\omega}^*)(1-\upsilon)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}[1-\alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}]^{\frac{(1-\alpha)\gamma}{2(\alpha\gamma-1)}} \\ & [\alpha^{-1}A^{-\gamma}\overline{Y}_T^{\gamma-1}(\gamma\frac{\theta-1}{\theta})^{\frac{\gamma(\alpha-1)-2}{2}}(r^{k*})^{\alpha\gamma}\frac{\kappa}{1-\alpha}^{\frac{\gamma(1-\alpha)}{2}}]^{\frac{1}{\alpha\gamma-1}}, \end{aligned} \tag{77}$$

$$\begin{aligned} b &= \beta\frac{g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}[1-\alpha\gamma\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}]^{\frac{(1-\alpha)\gamma}{2(\alpha\gamma-1)}} \\ & [\alpha^{-1}A^{-\gamma}\overline{Y}_T^{\gamma-1}(\gamma\frac{\theta-1}{\theta})^{\frac{\gamma(\alpha-1)-2}{2}}(r^{k*})^{\alpha\gamma}\frac{\kappa}{1-\alpha}^{\frac{\gamma(1-\alpha)}{2}}]^{\frac{1}{\alpha\gamma-1}}. \end{aligned} \tag{78}$$

5.4 Short-run equilibrium with pre-set prices

As already discussed, in period $t = 1$ intermediate firms do not adjust their prices; being the price level the same as in period $t = 0$ and is thus denoted $\overline{P}_{N0}$. It is worth highlighting that Eq. 59 does not hold in this case. Also recall that intermediate output will be demand-determined. Since only the final firm demands this output, we have from Eq. 56 that $Z_0 = \gamma\frac{P_0}{\overline{P}_{N0}}Y_0$, where the subindex 0 denotes steady state values at period $t = 0$. Using the production function of the final firm and the definition of Z_0 it is possible to obtain,

$$Y_0 = \overline{Y}_T(\gamma\frac{P_0}{\overline{P}_{N0}})^{\frac{\gamma}{1-\gamma}} = \overline{Y}_T(\gamma\frac{M}{\overline{P}_{N0}}\frac{(1-\beta)}{\chi})^{\frac{\gamma}{1-\gamma}}C_0^{\frac{\gamma}{\gamma-1}},$$

where $P_0 = \frac{M}{C_0}\frac{(1-\beta)}{\chi}$ is obtained from Eq. 49. Notice that the expression above gives a relation between two endogenous variables: Y_0 and C_0.

From Eqs. 60, 61, 63 and 65, and substituting for C_0 according to the previous expression it is possible to obtain,

$$n_0 = (1-\upsilon)(1+\beta\frac{g(\overline{\omega}^*)}{f(\overline{\omega}^*)})^{-1}[Y_0 - \gamma\frac{M}{\overline{P}_{N0}}\frac{(1-\beta)}{\chi}(\frac{Y_0}{\overline{Y}_T})^{\frac{\gamma-1}{\gamma}}] \tag{79}$$

This equation, derived from the clearing market condition for nontradable goods, relates the two endogenous variables n_0 and Y_0.

A second expression in these two variables can be obtained from Eq. 52, which takes the form $r^{k*} = \alpha \frac{\theta-1}{\theta} \frac{\overline{P}_{N0}}{P_0} \frac{Z_0}{K_0}$. Notice that from Eqs. 51 and 53 we have $r^{k*} K_0 = (1-\upsilon)^{-1}(1+\frac{g(\overline{\omega}^*)}{f(\overline{\omega}^*)})n_0$. Therefore, using the definition of Z_0 stated previously gives,

$$n_0 = \alpha\gamma(1-\upsilon)\frac{\theta-1}{\theta}(1+\frac{g(\overline{\omega}^*)}{f(\overline{\omega}^*)})^{-1} Y_0. \tag{80}$$

Eqs. 79 and 80 determine the solutions for n_0 and Y_0. Substituting Eq. 80 into Eq. 79 and rearranging brings the solution for Y_0,

$$Y_0 = \{\frac{M}{\overline{P}_{N0}} \frac{(1-\beta)}{\chi}[\gamma^{-1} - \alpha\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}]^{-1}\}^{\gamma} \overline{Y}_T^{1-\gamma}. \tag{81}$$

In the above expression monitoring costs will affect output through the term $\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)} \leq 1$, which must be close to one as far as β is close to one (as should be the case in order to have a reasonable level for the steady state net real interest rate). Moreover, since $\frac{\theta-1}{\theta} \leq 1$, the term $[\gamma^{-1} - \alpha\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}]$ will always be positive[23]. From Eq. 81 it can be easily seen that Z_0 must take the form,

$$Z_0 = \frac{M}{\overline{P}_{N0}} \frac{(1-\beta)}{\chi}[\gamma^{-1} - \alpha\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}]^{-1}. \tag{82}$$

The steady state solution for net worth is therefore given by,

$$n_0 = \alpha\gamma(1-\upsilon)\frac{\theta-1}{\theta}(1+\frac{g(\overline{\omega}^*)}{f(\overline{\omega}^*)})^{-1}\{\frac{M}{\overline{P}_{N0}} \frac{(1-\beta)}{\chi}[\gamma^{-1} - \alpha\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}]^{-1}\}^{\gamma} \overline{Y}_T^{1-\gamma}. \tag{83}$$

Similarly, using Eqs. 53, 61, 64, 65 and 83 it is possible to obtain the solutions for K_0, i_0 and b_0,

$$K_0 = \alpha\gamma\frac{\theta-1}{\theta}(\beta g(\overline{\omega}^*) + (1-\upsilon)f(\overline{\omega}^*))\{\frac{M}{\overline{P}_{N0}} \frac{(1-\beta)}{\chi}[\gamma^{-1} - \alpha\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}]^{-1}\}^{\gamma} \overline{Y}_T^{1-\gamma}.$$

$$i_0 = \alpha\gamma\frac{\theta-1}{\theta}\frac{\beta g(\overline{\omega}^*) + (1-\upsilon)f(\overline{\omega}^*)}{g(\overline{\omega}^*)+f(\overline{\omega}^*)}\{\frac{M}{\overline{P}_{N0}} \frac{(1-\beta)}{\chi}[\gamma^{-1} - \alpha\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}]^{-1}\}^{\gamma} \overline{Y}_T^{1-\gamma}.$$

[23] Unless the extreme case $\gamma = 1$, $\beta = 1$, $\alpha = 1$ and $\theta \to \infty$ takes place, which is ruled out by assumption (since in this situation $[\gamma^{-1} - \alpha\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}]^{-1}$ is not defined).

$$b_0 = \alpha\gamma\beta\frac{\theta-1}{\theta}\frac{g(\overline{\omega}^*)}{g(\overline{\omega}^*)+f(\overline{\omega}^*)}\{\frac{M}{\overline{P}_{N0}}\frac{(1-\beta)}{\chi}[\gamma^{-1}-\alpha\frac{\theta-1}{\theta}\frac{f(\overline{\omega}^*)+\beta g(\overline{\omega}^*)}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}]^{-1}\}^{\gamma}\overline{Y}_T^{1-\gamma}.$$

Continuing in this way we can obtain the solutions of all the variables included in the model. For concreteness, the steady state solutions discussed previously can also be represented relative to K, as shown in Table 4.

Table 4. Steady State Solutions with $\mu \neq 0$

Variable	Value
$\frac{Y}{K}=\frac{Y_0}{K_0}$	$\frac{1}{f(\overline{\omega}^*)(1-\upsilon)+\beta g(\overline{\omega}^*)}\frac{1}{\alpha\gamma}\frac{\theta}{\theta-1}$
$\frac{i}{K}=\frac{i_0}{K_0}$	$\frac{1}{f(\overline{\omega}^*)+g(\overline{\omega}^*)}$
$\frac{b}{K}=\frac{b_0}{K_0}$	$\frac{Y}{K}\frac{i}{K}\frac{\theta-1}{\theta}\alpha\gamma\beta g(\overline{\omega}^*)$
$\frac{n}{K}=\frac{n_0}{K_0}$	$\frac{Y}{K}\frac{i}{K}\frac{\theta-1}{\theta}\alpha\gamma(1-\upsilon)f(\overline{\omega}^*)$
$\frac{Z}{K}$	$\frac{Y}{K}(A(\frac{r^{k*}}{\alpha})^{-\alpha}(\gamma\frac{\theta-1}{\theta})^{\frac{1+\alpha}{2}}(\overline{Y}_T(\frac{\kappa}{1-\alpha})^{\frac{1}{2}})^{\alpha-1})^{\frac{1-\gamma}{1-\alpha\gamma}}$
$\frac{Z_0}{K_0}$	$\frac{Y}{K}(\frac{Z_0}{\overline{Y}_T})^{1-\gamma}$

From Table 4 it is possible to observe that these ratios are the same for the steady state and for the short-run equilibrium with only one exception: intermediate nontradable output relative to capital (i.e., $\frac{Z}{K}$ and $\frac{Z_0}{K_0}$). Therefore, monetary policy viewed as a change in the nominal money supply will only have short-run effects on this ratio. As previously mentioned, the case in which monitoring costs are zero (i.e., $\mu = 0$) brings a fairly standard RBC model. Table 5 summarises this case.

Table 5. Steady State Solutions with $\mu = 0$ (RBC case)

Variable	Value
$\frac{Y}{K}=\frac{Y_0}{K_0}$	$\frac{1}{\beta\alpha\gamma}\frac{\theta}{\theta-1}$
$\frac{i}{K}=\frac{i_0}{K_0}$	1
$\frac{b}{K}=\frac{b_0}{K_0}$	1
$\frac{n}{K}=\frac{n_0}{K_0}$	0
$\frac{Z}{K}$	$(A^{1-\gamma}[(\overline{Y}_T(\frac{\kappa}{1-\alpha})^{\frac{1}{2}})^{1-\gamma}(\frac{\theta-1}{\theta}\gamma)^{\frac{1+\gamma}{2}}\alpha\beta]^{\alpha-1})^{\frac{1}{1-\alpha\gamma}}$
$\frac{Z_0}{K_0}$	$\frac{1}{\beta\alpha\gamma}\frac{\theta}{\theta-1}(\frac{\frac{M}{\overline{P}_{N0}}\frac{(1-\beta)}{\chi}(\gamma^{-1}-\alpha\beta\frac{\theta-1}{\theta})^{-1}}{\overline{Y}_T})^{1-\gamma}$

Since the model assumes that capital fully depreciates within the period, when $\mu = 0$ we have that investment i and the capital stock K coincide. As before, the short run equilibrium ratios will be the same as the steady state ratios, with the only exception of intermediate nontradable output relative to capital. Again, monetary policy will only have a short-run effect on $\frac{Z_0}{K_0}$. Also notice that when $\mu = 0$ the real amount of debt b and the stock of capital

K are equivalent (since the entrepreneur and the lender coincide, as previously pointed out). We also have that the steady state ratio $\frac{n}{K} = \frac{n_0}{K_0} = 0$. When $\mu = 0$ entrepreneur's net worth and consumption are zero.

6 Concluding Remarks

This paper develops a fully microfounded two sectors small open economy model with imperfect competition in the intermediate nontradable sector, infinitely lived agents, currency mismatches in the denomination of assets and liabilities and imperfect capital markets. Although similar models have been introduced in the literature (e.g., Cespedes et al (2004), Cook (2004) and Devereux et al (2006)) the approach taken here has been slightly different. It provides a model that is solvable analytically from primitive assumptions, rather than a model that is either solved numerically or relying on ad-hoc reduced-form relations for introducing the financial accelerator. The gains of taking this approach are not only a better understanding of the mechanisms by which monetary policy may affect the economy, but also a transparent and a neat way of studying how structural parameters affect the steady state solutions of the model.

Along the paper a number of results have been emphasized. Due to the assumption of Cobb-Douglas households' preferences over consumption and real balances, it was shown that the nominal exchange rate does not show non-trivial dynamics such as in Dornbusch (1976) well known overshooting model. Of particular relevance are the results associated to credit market imperfections. These findings can be separated in two strands: the characteristics of the form in which credit market imperfections are introduced in the model and the steady state solutions in the general equilibrium analysis.

On the first strand, it has been shown that the solution of the entrepreneur's maximisation problem, which ultimately determines the amount of capital supplied in the economy, implies the absence of equilibrium credit rationing in the sense of Stiglitz and Weiss (1981). It was also demonstrated that the asymmetric information problem, defined generically as the case where monitoring costs are greater than zero, brings a positively sloped supply curve of capital. Opposed is the perfect information case, where the supply of capital becomes horizontal (as expected with a constant returns to scale technology).

On the second strand, it has been demonstrated that those variables associated with the financial accelerator mechanism only depend on the following structural parameters: the subjective discount factor, monitoring costs and the share of expected profits that entrepreneurs devote to consumption.

Solving for the remaining steady state endogenous variables of the model brought the fact that monetary policy, viewed as an unanticipated and permanent change in the level of the nominal money supply, does not affect the ratio of final output to capital but the ratio of intermediate output to capital. By eliminating the asymmetric information problem from the model, setting monitoring costs to zero, it is shown that the model converges to a fairly standard RBC model.

Up to now, it has been obtained the steady state solution of the model. The necessary next step in the analysis should be the study of the dynamic behaviour of the model. Due to the high degree of non-linearities, a log-linear version of the model seems to be the obvious place to start with the analysis. Once the order of the system is determined, and its stability properties understood, it will be possible to study how the endogenous variables of the model converges to the steady state after different monetary policy shocks. Moreover, it would be desirable to further study the dynamics of the model under different exchange rate regimes. Since the model is derived from utility maximisation principles, welfare comparisons can easily be introduced into the analysis.

Appendix A

The maximisation problem of the entrepreneur can be written in terms of the following Lagrangean,

$$\max_{\{i_{j,t},\overline{\omega}_{j,t},\lambda\}} L = R^k_{t+1} i_{j,t} f(\overline{\omega}_{j,t}) + \lambda[R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t}) - R_{t+1} P_t (i_{j,t} - n_{j,t})], \tag{AA1}$$

The associated first order conditions are,

$$\frac{\partial L}{\partial i_{j,t}} = R^k_{t+1} f(\overline{\omega}_{j,t}) + \lambda[R^k_{t+1} g(\overline{\omega}_{j,t}) - R_{t+1} P_t] = 0, \tag{AA2}$$

$$\frac{\partial L}{\partial \overline{\omega}_{j,t}} = R^k_{t+1} i_{j,t} f'(\overline{\omega}_{j,t}) + \lambda[R^k_{t+1} i_{j,t} g'(\overline{\omega}_{j,t})] = 0, \tag{AA3}$$

and

$$\frac{\partial L}{\partial \lambda} = R^k_{t+1} i_{j,t} g(\overline{\omega}_{j,t}) - R_{t+1} P_t (i_{j,t} - n_{j,t}) = 0. \tag{AA4}$$

Note that Eq. AA3 implies $\lambda = -\frac{f'(\overline{\omega}_{j,t})}{g'(\overline{\omega}_{j,t})}$. Replacing this expression in Eq. AA2 and rearranging gives Eq. 20. Solving Eq. AA4 for $i_{j,t}$ gives Eq. 21.

Appendix B

From the main text we have: $f(\overline{\omega}_{j,t}) = \int_{\overline{\omega}_{j,t}}^{\infty} \omega\phi(\omega)d\omega - [1 - \Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}$. Observe that $f(\overline{\omega}_{j,t})$ can be written as $f(\overline{\omega}_{j,t}) = \int_0^{\infty} \omega\phi(\omega)d\omega - \int_0^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega - [1 - \Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}$. Recalling that $E(\omega) = \int_0^{\infty} \omega\phi(\omega)d\omega = 1$, we can obtain,

$$f(\overline{\omega}_{j,t}) = 1 - \int_0^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega - [1 - \Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}.$$

Taking derivatives with respect to $\overline{\omega}_{j,t}$ gives,

$$f'(\overline{\omega}_{j,t}) = -[1 - \Phi(\overline{\omega}_{j,t})], \tag{AB1}$$

and

$$f''(\overline{\omega}_{j,t}) = \phi(\overline{\omega}_{j,t}), \tag{AB2}$$

implying that $f(\overline{\omega}_{j,t})$ is a convex function of $\overline{\omega}_{j,t}$.

Similarly, from the main text we have,

$$g(\overline{\omega}_{j,t}) = \int_0^{\overline{\omega}_{j,t}} \omega\phi(\omega)d\omega - \mu\Phi(\overline{\omega}_{j,t}) + [1-\Phi(\overline{\omega}_{j,t})]\overline{\omega}_{j,t}.$$

Taking derivatives with respect to $\overline{\omega}_{j,t}$ gives,

$$g'(\overline{\omega}_{j,t}) = -\mu\phi(\overline{\omega}_{j,t}) + [1-\Phi(\overline{\omega}_{j,t})], \tag{AB3}$$

and

$$g''(\overline{\omega}_{j,t}) = -[\mu\frac{\partial\phi(\overline{\omega}_{j,t})}{\partial\overline{\omega}_{j,t}} + \phi(\overline{\omega}_{j,t})]. \tag{AB4}$$

Let us now consider the first order condition stated in Eq. 20. After rearranging terms, this equation takes the form,

$$\frac{R^k_{t+1}}{R_{t+1}P_t} = \{g(\overline{\omega}_{j,t}) - \frac{f(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})}g'(\overline{\omega}_{j,t})\}^{-1}.$$

Let us define $G(\overline{\omega}_{j,t}) = \{g(\overline{\omega}_{j,t}) - \frac{f(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})}g'(\overline{\omega}_{j,t})\}^{-1}$. Therefore,

$$\frac{\partial G(\overline{\omega}_{j,t})}{\partial\overline{\omega}_{j,t}} = -\{g(\overline{\omega}_{j,t}) - \frac{f(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})}g'(\overline{\omega}_{j,t})\}^{-2}\frac{f(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})}\{-g''(\overline{\omega}_{j,t}) + g'(\overline{\omega}_{j,t})\frac{f''(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})}\}.$$

Introducing in this expression Eqs. AB1, AB2, AB3 and AB4 we can obtain,

$$\frac{\partial G(\overline{\omega}_{j,t})}{\partial\overline{\omega}_{j,t}} = \mu\{g(\overline{\omega}_{j,t}) - \frac{f(\overline{\omega}_{j,t})}{f'(\overline{\omega}_{j,t})}g'(\overline{\omega}_{j,t})\}^{-2}\frac{f(\overline{\omega}_{j,t})}{[1-\Phi(\overline{\omega}_{j,t})]}\{\frac{\partial\phi(\overline{\omega}_{j,t})}{\partial\overline{\omega}_{j,t}} + \frac{\phi(\overline{\omega}_{j,t})^2}{[1-\Phi(\overline{\omega}_{j,t})]}\}.$$

Note that $\frac{\partial G(\overline{\omega}_{j,t})}{\partial\overline{\omega}_{j,t}} > 0$ whenever $\{\frac{\partial\phi(\overline{\omega}_{j,t})}{\partial\overline{\omega}_{j,t}} + \frac{\phi(\overline{\omega}_{j,t})^2}{[1-\Phi(\overline{\omega}_{j,t})]}\} > 0$. In this case, therefore, we can write that $\overline{\omega}_{j,t} = F(\frac{R^k_{t+1}}{R_{t+1}P_t})$, with $\frac{\partial\overline{\omega}_{j,t}}{\partial R^k_{t+1}} = \frac{1}{R_{t+1}P_t}F'(\frac{R^k_{t+1}}{R_{t+1}P_t}) > 0$ as stated in the main text.

Appendix C

To obtain the second order conditions we also need the following partial derivatives of the Lagrangean analysed in Appendix A:

$$\frac{\partial^2 L}{\partial i^2_{j,t}} = 0,$$

$$\frac{\partial^2 L}{\partial i_{j,t}\partial\overline{\omega}_{j,t}} = \frac{\partial^2 L}{\partial\overline{\omega}_{j,t}\partial i_{j,t}} = R^k_{t+1}[f'(\overline{\omega}_{j,t}) + \lambda g'(\overline{\omega}_{j,t})] = 0^{24},$$

[24]To obtain this result, it is worth recalling that $\lambda = -\frac{f'(\overline{\omega}_{j,t})}{g'(\overline{\omega}_{j,t})}$.

$$\frac{\partial^2 L}{\partial \overline{\omega}_{j,t}^2} = R_{t+1}^k i_{j,t}[f''(\overline{\omega}_{j,t}) + \lambda g''(\overline{\omega}_{j,t})]$$

$$\frac{\partial}{\partial i_{j,t}}(\frac{\partial L}{\partial \lambda}) = R_{t+1}^k g(\overline{\omega}_{j,t}) - R_{t+1}P_t,$$

and

$$\frac{\partial}{\partial \overline{\omega}_{j,t}}(\frac{\partial L}{\partial \lambda}) = R_{t+1}^k i_{j,t} g'(\overline{\omega}_{j,t}).$$

We can now form the bordered Hessian,

$$H = \begin{bmatrix} 0 & R_{t+1}^k g(\overline{\omega}_{j,t}) - R_{t+1}P_t & R_{t+1}^k i_{j,t} g'(\overline{\omega}_{j,t}) \\ R_{t+1}^k g(\overline{\omega}_{j,t}) - R_{t+1}P_t & 0 & 0 \\ R_{t+1}^k i_{j,t} g'(\overline{\omega}_{j,t}) & 0 & R_{t+1}^k i_{j,t}[f''(\overline{\omega}_{j,t}) + \lambda g''(\overline{\omega}_{j,t})] \end{bmatrix}.$$

To satisfy the associated second order condition for a maximum, we need the determinant of the matrix H to be greater or equal to zero (see, for instance, Simon and Blume 1994, p. 461). This determinant takes the form,

$$|H| = -R_{t+1}^k i_{j,t}[R_{t+1}^k g(\overline{\omega}_{j,t}) - R_{t+1}P_t]^2[f''(\overline{\omega}_{j,t}) + \lambda g''(\overline{\omega}_{j,t})].$$

Since $[R_{t+1}^k g(\overline{\omega}_{j,t}) - R_{t+1}P_t]^2 \geq 0$, to satisfy the second order condition we need $[f''(\overline{\omega}_{j,t}) - \frac{f'(\overline{\omega}_{j,t})}{g'(\overline{\omega}_{j,t})} g''(\overline{\omega}_{j,t})] \leq 0$. This condition can be written as $f''(\overline{\omega}_{j,t}) \leq \frac{f'(\overline{\omega}_{j,t})}{g'(\overline{\omega}_{j,t})} g''(\overline{\omega}_{j,t})$.

Appendix D

In this Appendix, some of the key results obtained in Section 3 are analysed numerically. To avoid excessive notation, subscripts are avoided. For the sake of concreteness, let us assume that ω is uniformly distributed in the interval $[0, 2]$. In this case, we have that the mean of ω is 1, as stated in the main text. The following results immediately follows: $\phi(\overline{\omega}) = \frac{1}{2}$, $\frac{\partial \phi(\overline{\omega})}{\partial \overline{\omega}} = 0$, $\Phi(\overline{\omega}) = \frac{1}{2}\overline{\omega}$ and $1-$ $\Phi(\overline{\omega}) = 1 - \frac{1}{2}\overline{\omega}$. Therefore, using the results obtained in Appendix B, we can obtain:

$$g(\overline{\omega}) = -\frac{1}{4}\overline{\omega}^2 + \overline{\omega}(1 - \frac{\mu}{2}),$$

$$g'(\overline{\omega}) = 1 - \frac{1}{2}(\mu + \overline{\omega}),$$

$$g''(\overline{\omega}) = -\frac{1}{2},$$

$$f(\overline{\omega}) = \frac{1}{4}\overline{\omega}^2 - \overline{\omega} + 1,$$

$$f'(\overline{\omega}) = -(1 - \frac{1}{2}\overline{\omega}),$$

and

$$f''(\overline{\omega}) = \frac{1}{2}.$$

We can therefore conclude that the function $g(\overline{\omega})$ is concave while $f(\overline{\omega})$ is convex in $\overline{\omega}$. The second order condition of the maximisation problem derived in Appendix C is given by $\frac{1}{2} \leq \frac{\frac{1}{2}(1-\frac{1}{2}\overline{\omega})}{1-\frac{1}{2}(\overline{\omega}+\mu)}$. For this inequality to be satisfied, we need that $g'(\overline{\omega}) = 1 - \frac{1}{2}(\mu + \overline{\omega}) > 0$ (or $\mu < 2 - \overline{\omega}$). Note that Eq. 20 can be solved for $\frac{RP}{R^k}$, thus giving,

$$\frac{RP}{R^k} = 1 - \frac{1}{4}\overline{\omega}\mu - \frac{1}{2}\mu,$$

or,

$$\overline{\omega} = \frac{4}{\mu}[1 - \frac{PR}{R^k} - \frac{1}{2}\mu].$$

We can immediately see that,

$$\frac{\partial\overline{\omega}}{\partial R^k} = \frac{4}{\mu}\frac{PR}{(R^k)^2} > 0.$$

Now we want to determine the sign of the partial derivative $\frac{\partial K^s_{t+1}}{\partial R^k_{t+1}}$. Note that Eq. 33 can be written as,

$$K^s = i[f(\overline{\omega}) + g(\overline{\omega})].$$

Since to satisfy the second order condition of the maximisation problem it is required that $g'(\overline{\omega}) > 0$, in order to obtain $\frac{\partial K^s}{\partial R^k} > 0$ it is sufficient to demonstrate that,

$$\frac{\partial i f(\overline{\omega})}{\partial R^k} = \frac{\partial i}{\partial R^k} f(\overline{\omega}) + i f'(\overline{\omega})\frac{\partial\overline{\omega}}{\partial R^k} > 0.$$

Since $\frac{\partial i}{\partial R^k} f(\overline{\omega}) > 0$ while $i f'(\overline{\omega})\frac{\partial\overline{\omega}}{\partial R^k} < 0$ the sign of $\frac{\partial i f(\overline{\omega})}{\partial R^k}$, at first sight, is ambiguous. Using Eq. 24 we can rewrite this expression as,

$$\frac{\partial i f(\overline{\omega})}{\partial R^k} = \frac{i^2}{n}\frac{1}{PR}[g(\overline{\omega}) + R^k g'(\overline{\omega}_t)\frac{\partial\overline{\omega}}{\partial R^k}] f(\overline{\omega}) + i f'(\overline{\omega})\frac{\partial\overline{\omega}}{\partial R^k},$$

or, simplifying,

$$\frac{\partial if(\overline{\omega})}{\partial R^k} = \frac{i^2}{n}\{\frac{1}{PR}[g(\overline{\omega}) + R^k g'(\overline{\omega}_t)\frac{\partial \overline{\omega}}{\partial R^k}]f(\overline{\omega}) + \frac{n}{i}f'(\overline{\omega})\frac{\partial \overline{\omega}}{\partial R^k}\}.$$

From Eq. 21 we have that $\frac{n}{i} = 1 - \frac{R^k}{PR}g(\overline{\omega})$. Using this expression and the fact that $\frac{\partial \overline{\omega}}{\partial R^k} = \frac{4}{\mu}\frac{PR}{(R^k)^2}$ gives,

$$\frac{\partial if(\overline{\omega})}{\partial R^k} = \frac{i^2}{n}\frac{1}{(R^k)^2}\{\frac{(R^k)^2}{PR}g(\overline{\omega})f(\overline{\omega}) + \frac{4}{\mu}R^k g'(\overline{\omega}_t)f(\overline{\omega}) + \frac{4}{\mu}PRf'(\overline{\omega}) - \frac{4}{\mu}R^k g(\overline{\omega})f'(\overline{\omega})\}.$$

Since $R^k = RP(1 - \frac{1}{4}\overline{\omega}\mu - \frac{1}{2}\mu)^{-1}$, the above equation can be further expressed as

$$\begin{aligned} \frac{\partial if(\overline{\omega})}{\partial R^k} &= \frac{i^2}{n}\frac{PR}{(R^k)^2}\{(1 - \frac{1}{4}\overline{\omega}\mu - \frac{1}{2}\mu)^{-2}g(\overline{\omega})f(\overline{\omega}) + \frac{4}{\mu}(1 - \frac{1}{4}\overline{\omega}\mu - \frac{1}{2}\mu)^{-1}g'(\overline{\omega}_t)f(\overline{\omega}) \\ &\quad + \frac{4}{\mu}f'(\overline{\omega}) - \frac{4}{\mu}g(\overline{\omega})f'(\overline{\omega})(1 - \frac{1}{4}\overline{\omega}\mu - \frac{1}{2}\mu)^{-1}\}. \end{aligned}$$

Observe that the sign of $\frac{\partial if(\overline{\omega})}{\partial R^k}$ is determined by the term in curly brackets, which only depends on $\overline{\omega}$ and μ. Using the expressions for $g(\overline{\omega})$, $g'(\overline{\omega})$, $f(\overline{\omega})$ and $f'(\overline{\omega})$ obtained previously, after some algebra, it is possible to obtain,

$$\frac{\partial if(\overline{\omega})}{\partial R^k} = -\frac{i^2}{n}\frac{PR}{(R^k)^2}\{\overline{\omega}\frac{(\overline{\omega}-2)^2}{(2\mu + \mu\overline{\omega} - 4)^2}(2\mu + \overline{\omega} - 4)\} > 0,$$

since $(2\mu + \overline{\omega} - 4) < 0$ (recall that $g'(\overline{\omega}) > 0$ implies $\overline{\omega} + \mu < 2$).

In a sightlier more complicated way it is also possible to demonstrate that $\frac{\partial K^s}{\partial R^k} = \frac{\partial}{\partial R^k}(i(1 - \mu\Phi(\overline{\omega}))) > 0$. By following a similar procedure as the one applied above, it is possible to obtain the following expression,

$$\begin{aligned} \frac{\partial K^s}{\partial R^k} &= \frac{i^2}{n}\frac{PR}{(R^k)^2}\frac{2}{\mu\,(2\mu + \mu\overline{\omega} - 4)^2}\{\mu^3(\overline{\omega}^2 - 4\overline{\omega} - 4) + \mu^2(-6\overline{\omega}^2 + 8\overline{\omega} + 24) \\ &\quad + \mu(6\overline{\omega}^2 + 8\overline{\omega} - 48) - 16\overline{\omega} + 32\}, \end{aligned}$$

which is positive whenever $\mu + \overline{\omega} < 2$. Observe that as $\mu \to 0$ the partial derivative $\frac{\partial K^s_{t+1}}{\partial R^k_{t+1}} \to \infty$ whenever $\overline{\omega} \in [0, 2)$.

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