Economic Development, Exchange Rates, and the Structure of Trade

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Abstract

The paper builds a two-country open economy model of incomplete exchange rate pass-through. The paper contributes to the existing literature in two ways. First, incomplete pass-through is the result of price discrimination, and not any assumption about price rigidities. The flexible-price model is capable of delivering empirically plausible magnitudes of pass-through, as long as the exchange rate shock is temporary and not very persistent. Second, the model is also used to shed light on the empirically observed differences in exchange rate pass-through between developing and developed countries. In particular, the discrepancy is explained by the different composition of consumption and trade patterns of rich and poor countries - an assumption to which some empirical support is also presented.

1 Introduction

The extent of exchange rate pass-through is an important question both for economic research and for policymakers. For the latter, the size of short-

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and long-run pass-through is a key input into monetary policy decisions. For academic economists, many puzzling facts have emerged that challenge researchers to try and explain them.

This paper tackles some stylized facts related to exchange rate pass-through. First, empirical research has shown[1] that price discrimination is a very important determinant of the reaction in exchange rate changes, but dynamic general equilibrium models that incorporate oligopolistic behavior are still lacking. Thus one purpose of the paper is to provide a tractable, but rich framework where price discrimination is the primary force behind incomplete exchange rate pass-through.

Second, the extent of exchange rate pass-through seems to be systematically related to the level of economic development. In particular, various articles document that pass-through is faster in less-developed countries.[2] The arguments advanced in earlier work to explain this phenomenon rely on features of the macroeconomic environment that may be different between developed and developing economies. Campa and Goldberg (2005), however, provide evidence that macroeconomic differences explain little of the variability of pass-through. Instead they find that most of the heterogeneity is accounted for by differences in industrial structure. The model presented here is consistent with this observation, and relies on differences in trade and consumption patterns to explain differences in pass-through.

The framework I use is a two-sector, two-country open economy model where money in the utility provides a nominal side. The model relies on a modified version of the Helpman-Krugman model of international trade (Helpman and Krugman 1985, Chapter 6-8), which combines insights from

[1] For a recent contribution, see Hellerstein (2005).

[2] See, for example, Calvo and Reinhart (2002), Choudhri and Hakura (2001) and Devereux et al. (2005)

models based on comparative advantage with those from models based on increasing returns to scale. One sector (food) produces a homogeneous good, where firms are price takers. The other sector (manufacturing) features differentiated products and market power.

To introduce price discrimination, I follow the strategy pioneered by Neary (2003). Thus I assume that there are a continuum of industries, hence each industry forms a negligible part of the economy. On the other hand, firms are large in their industries, and are able to set prices/output. In particular, I assume that in each industry a foreign and a domestic firm compete in Cournot fashion, a setting that was introduced by Krugman and Brander (1983).

The key mechanism is the following. As countries develop, they switch production from homogeneous products produced by competitive firms towards differentiated goods produced by companies with market power. Since in the former sector firms are price takers, pass-through is expected to be fast and complete. In the latter case, however, as firms make profits and are price setters, they can accommodate some of the exchange rate change in the short run. Thus for differentiated products pass-through should be incomplete and gradual.

Since the structure of production is systematically related to the level of development, and this structure has implications for pricing, exchange rate pass-through is linked to the level of development through this mechanism. Consistent with the Helpman-Krugman model, as countries become richer, they not only produce more differentiated products, but a larger share of their total trade is composed of such goods, which leads to an overall lower pass-through, consistent with the available evidence(some of which is presented below).

A final important assumption that is needed to explain pass-through differences into the consumer price index (CPI) concerns the non-homotheticity of consumption expenditure. While empirically strongly supported, non-homothetic preferences do not usually feature in open economy macro models. A simple assumption that leads to a declining share of food in consumption is that preferences are quasi-linear in food, which guarantees that - absent of price changes - food consumption does not change with the level of economic development.

The rest of the paper is structured as follows. Section 2 presents some evidence on the relation between trade patterns and the level of development. Section 3 describes the theoretical model, while Section 4 presents the equilibrium conditions. Section 5 shows the main results through simulations. Finally, Section 6 concludes.

2 Some evidence

An important element of the argument presented in the Introduction is that the structure of trade is systematically different between rich and poor countries. In this section I present some evidence that supports this assumption.

The important question concerns the relationship between the level of development and the nature of the product composition of trade. To measure this, one needs data on the extent of product differentiation among traded products. The dataset I use is described in Rauch (1999). Rauch organizes traded goods into three categories: (1) products that are traded at organized exchanges, (2) products that have a reference price, and (3) products that do not have a reference price.

I merge the Rauch categorization with export-import data from the World Bank database on international trade and GDP data from the Penn

	Imports	Exports
Poor countries	3.12	2.15
Rich countries	6.67	17.56

Tab. 1: Importance of differentiated products in trade (country averages)

World Tables. The dataset contains 110 countries between 1980-1988.

The hypothesis I test is that a more developed country's trade pattern is tilted towards category (3). The main variables I use to measure specialization is the ratio of category (3) imports (exports) to category (1) imports (exports). Given the panel nature of the data, I estimate a fixed effects specification where the explanatory variables include per capita GDP, population and general openness as measured by the share of exports plus imports in GDP.

Table 1 presents mean values for the relative import and export measures for two country groups: poor nations with a per capita GDP below \$5,000, and rich nations with a per capita GDP above \$15,000. The table clearly shows that in both imports and exports rich countries have a much larger category (3) share. While already large for imports, the differences are very dramatic for exports.

Table 2 shows the estimation results for imports and exports. The same pattern emerges as in Table 1. GDP has a large and significant impact on the share of category (3) imports and exports relative to category (1) imports and exports. Thus I take that the evidence strongly supports this key assumption of the model.

3 The model

I focus on two open economies (Home and Foreign) that produce two goods, food and manufactures. Food is homogeneous, and it is produced by com-

	Imports	Exports
Log(GDP)	5.03	8.32
	(0.595)**	(1.786)**
Log(Population)	6.05	-10.46
	(1.538)**	(4.612)*
Openness	0.01	-0.01
	-0.01	-0.03
Constant	-92.17	33.38
	(12.281)**	-36.83
Observations	985	985
Number of ccode	110	110
R-squared	0.16	0.03

Standard errors in parentheses
* significant at 5%; ** significant at 1%

Tab. 2: Importance of differentiated goods in trade (fixed effects)

petitive firms who use only land, which is available in a fixed amount to household.

Manufactures require labor (human capital), supplied by household through their labor/leisure decisions. There are a continuum of varieties in the manufacturing sector, which are completely symmetric, but imperfect substitutes in consumption.

The key assumption of the model is that manufacturing is oligopolistic, with two firms (a domestic and a foreign) competing in Cournot fashion in the market of each variety. Thus companies choose output, given their competitor's decision and the inverse demand curve for their product.

Manufacturing firms produce both for the domestic and foreign markets, which are segmented so that there may be different prices offered on the two markets. One reason for the lack of arbitrage is that exporting is subject to a transportation cost. Food, on the other hand, is costlessly tradable, so there is full and immediate pass-through of the exchange rate into food prices.

The modelling horizon is the short-run, so I assume that the export and import sectors require specialized labor. In particular, households supply labor for both sectors, but the labor amounts are not perfect substitutes. The importance of this assumption is that it prevents implausibly large sectoral reallocations of labor in response to exchange rate movements.

3.1 Households

Households solve the following problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left[C_t + \alpha \log Y_t + \mu \log \frac{M_t}{P_t} - \frac{\chi}{1+\varphi} L_{1t}^{1+\varphi}(j) - \frac{\chi}{1+\varphi} L_{2t}^{1+\varphi}(j) \right]$$

$$\text{s.t.} \quad M_t - M_{t-1} = V_t T + W_{1t} L_{1t}(j) + W_{2t} L_{2t}(j) + \Pi_{1t} + \Pi_{2t} - S_t Y_t - P_t C_t$$

where C is a composite good of varieties (manufacturing), Y is a homogeneous good (food), M is money demand by households, $L_i(j)$ is the labor supply of household j in either the export (1) or the import (2) industry, W_i is the nominal wage in sector i, V is the rental rate of land, T is the stock of land available for food production, and S is the nominal exchange rate. I assume that the Law of One Price holds for food, so after normalizing its Foreign price to unity, the Home price of food is S.

Let Λ_t be the Lagrangian multiplier assigned to the period budget constraint. The first-order conditions of the problem[3] associated with the consumption and money holding choices are:

$$\begin{aligned} \frac{1}{\Lambda_t} &= P_t \\ \frac{\alpha}{Y_t} &= \frac{S_t}{P_t} \\ \frac{1}{P_t} &= \beta \frac{\mu}{M_t} + \beta E_t \frac{1}{P_{t+1}} \end{aligned}$$

[3] In what follows, for simplicity I often omit the analogous conditions for Foreign.

Notice that since utility is quasilinear in the manufacturing good, rising incomes will have no effect on food consumption, unless they lead to changes in the relative price of food.

3.2 Producers

3.2.1 Food

Food is produced by constant returns-to-scale, perfectly competitive firms. The unit land requirement is normalized to 1 for simplicity, which guarantees that

$$V_t = S_t.$$

Land is supplied inelastically, so domestic production must equal the amount of available land, T.

3.2.2 Manufactures

The manufacturing aggregate is assembled from individual varieties by perfectly competitive firms, whose cost-minimization problem is as follows:

$$\min \int_0^1 P_t(i) X_t(i) di$$

$$\text{s.t.} \quad \left[\int_0^1 X_t(i)^{1-1/\sigma} di \right]^{\frac{\sigma}{\sigma-1}} = X_t.$$

The solution gives the well-known CES demand function for individual varieties as

$$\begin{aligned} X_t(i) &= \left[\frac{P_t(i)}{P_t} \right]^{-\sigma} X_t \\ P_t(i) &= P_t X_t^{1/\sigma} X_t(i)^{-1/\sigma}. \end{aligned} \tag{1}$$

The market for each variety is oligopolistic, and Home and Foreign markets are internationally segmented. This means that prices for the same product can differ across borders, and international arbitrage cannot equalize them. I assume that on each market, one Home and one Foreign firm compete in Cournot fashion. This could easily be generalized to a situation where there are n_k firms originating from country k. As long as costs structures are identical within countries, the results would remain intact.

Since markets operate the same way in the two countries, and there are no international cost linkages, I will only explicitly derive conditions for the Home market. Home producers produce with a marginal cost of W/A, where $A \leq 1$ is the productivity parameter which reflects the level of development of the Home economy. For simplicity, I assume that A is given exogenously, and it is constant for the time horizon of the monetary model. The second assumption is conceptually straightforward, but computationally tedious, to relax, so I opt for the simpler case. The marginal cost of foreign companies is τSW^*, where $\tau \geq 1$ represents transportation costs.

Equation (1) can be rearranged to get the inverse demand function for an individual variety,

$$P(i) = PX^{1/\sigma}X(i)^{-1/\sigma} = kX(i)^{-1/\sigma},$$

where k represents variables that cannot be influenced by individual firms. Let us ignore the index i for product varieties and let x_h and x_f indicate production by the Home and Foreign firm, respectively.

Home market profits for Home and Foreign companies can then be written as

$$\begin{aligned}
\pi &= \left[k(X_h + X_f)^{-1/\sigma} - W/A\right] X_h \\
\pi^f &= \left[k(X_h + X_f)^{-1/\sigma} - \tau S W^*\right] X_f.
\end{aligned}$$

After maximizing profits, finding the Nash equilibrium, and using the definition of k, we arrive at the following:

$$P = \frac{W/A + \tau S W^*}{2 - 1/\sigma} \tag{2}$$

$$X_h = \frac{\sigma X(P - W/A)}{P} \tag{3}$$

$$X_f = \frac{\sigma X(P - \tau S W^*)}{P} \tag{4}$$

The Foreign market operates the same way, and the analogous conditions are written as

$$P_t^* = \frac{\tau W_t/AS_t + W_t^*}{2 - 1/\sigma} \tag{5}$$

$$X_h^* = \frac{\sigma X^*(P^* - \tau W/AS)}{P^*} \tag{6}$$

$$X_f^* = \frac{\sigma X^*(P^* - W^*)}{P^*}. \tag{7}$$

Finally, the total supply for each variety in a country is the sum of domestic and imported production,

$$\begin{aligned}
X &= X_h + X_f \\
X^* &= X_h^* + X_f^*.
\end{aligned}$$

3.3 The labor market

As mentioned above, individual households supply specialized labor for both the import and export sectors. Manufacturing firms use a labor aggregate, given by

$$L_i = \left[\int_0^1 L_i(j)^{1-1/\theta} dj\right]^{\frac{1}{1-\theta}}.$$

Thus demand for an individual household's services is given by

$$\begin{aligned} L_i^D(j) &= \left[\frac{W_i(j)}{W_i}\right]^{-\theta} L_i^D \\ W_i &= \left[\int_0^1 W_i(j)^{1-\theta} dj\right]^{\frac{1}{1-\theta}}. \end{aligned}$$

Households maximize their utility from supplying labor and earning a wage. Since there are no intertemporal linkages, we can focus on the static problem:

$$\max \frac{W_{it}(j)}{P_t} L_{it}(j) - \frac{\chi}{1+\varphi} L_{It}(j)^{1+\varphi}$$

The solution is given by the following labor supply function:

$$\frac{W_{it}}{P_t} = \frac{\theta\chi}{\theta-1} L_{it}^{\varphi}, \tag{8}$$

where I utilized the assumption that all households are identical.

Finally, labor demand comes from manufacturing firms, and is given as

$$L_{1t} = \frac{X_{h,t}}{A} \tag{9}$$

$$L_{2t} = \frac{\tau X_{ft}}{A} \tag{10}$$

Again, Foreign equations are completely analogous.

4 Equilibrium

4.1 The dynamic conditions

The dynamic system that determines the evolution of the remaining endogenous variables consists of three equations with four variables: X, X^*, M and M^*. All other variables have been expressed as functions of these through (2), (3), (4), (5), (6), (7), (8), (9), (10) and the Foreign counterparts for the latter three. The dynamic equations then follow from the budget constraints and the money demand equations:

$$
\begin{aligned}
M_t - M_{t-1} &= S_t T - \alpha P_t + S_t P_t^* X_{f,t}^* - P_t X_{f,t} && (11)\\
M_t^* - M_{t-1}^* &= T^* - \alpha P_t^* + \frac{P_t X_{f,t}}{S_t} - P_t^* X_{f,t}^* && (12)\\
\frac{1}{P_t} &= \beta E_t \frac{1}{P_{t+1}} + \beta \frac{\mu}{M_t} && (13)\\
\frac{1}{P_t^*} &= \beta E_t \frac{1}{P_{t+1}^*} + \beta \frac{\mu}{M_t^*}. && (14)
\end{aligned}
$$

Finally, we have to characterize the evolution of the nominal exchange rate to close the system. I will examine two alternative scenarios. First, I assume that the exchange rate is completely exogenous, and follows a first-order autoregressive process. The advantage of this assumption is that we can analyze the impact of a "pure" (temporary) exchange rate shock. If the exchange rate is endogenous, exchange rate pass-through depends to a large extent on the nature of the underlying shock that causes the exchange rate to react.[4]

Second, I will examine the case of a permanent and credible devaluation when the exchange rate is fixed both before and after the policy change. As I will show later, this provides a nice contrast with the previous case,

[4] For a very informative discussion on this issue, see Bouakez and Rebei (2005).

and highlights the different reactions of firms to temporary and permanent shocks.

Third, returning to a temporary shock, I incorporate nominal rigidities into the model for both the manufacturing price and wage. The main message of this excercise is that in this setting nominal rigidities do not play an important role in explaining exchange rate pass-through, but lead to somewhat different implications for other variables.

4.2 Choosing parameter values

The choice of some of the parameters is not obvious, since the model is non-standard. While I believe the values are meaningful, the results should be viewed as illustration for the qualitative conclusions, rather than quantitative predictions.

That said, I use the following values in the baseline simulations:

- $T = T^* = 1$: a normalization of the land endowment
- $\sigma = \theta = 4$: a value that is common in the literature for the extent of market power
- $\alpha = 0.2$: the relative importance of food in consumption
- $\beta = 0.95$: the (yearly) discount rate
- $\mu = 0.02$: this implies that households' yearly money holding equals about 4 months' consumption
- $\chi = 1$: the relative disutility of work
- $\varphi = 1$: the elasticity of labor supply
- $\tau = 1.1$: trade costs are moderate

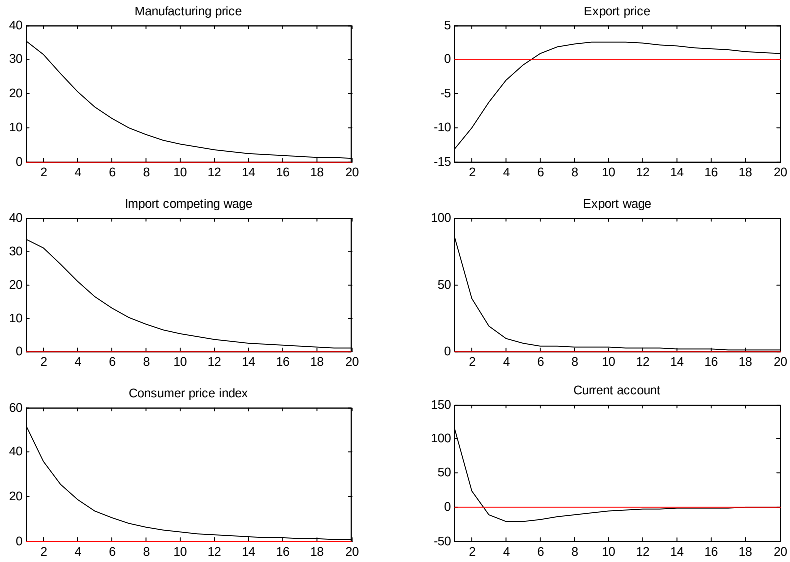

Fig. 1: A temporary devaluation in a rich country, prices and wages

5 Results

5.1 A temporary exchange rate shock

As I indicated above, the first scenario I examine is a temporary exchange rate depreciation, which is treated as an exogenous shock. Thus I assume that the (log of the) exchange rate follows an AR(1) process:[5]

$$s_t = \phi s_{t-1} + \epsilon_t$$

In what follows, I examine the impulse responses of the endogenous variables to a shock in the innovation ϵ_t. Since this section looks at a temporary devaluation, I choose $\phi = 0.5$, so that the shock is not very persistent.

Figure 1 and figure 2 show the results for the case when Home is a

[5] I use lowercase variables to indicate logarithms.

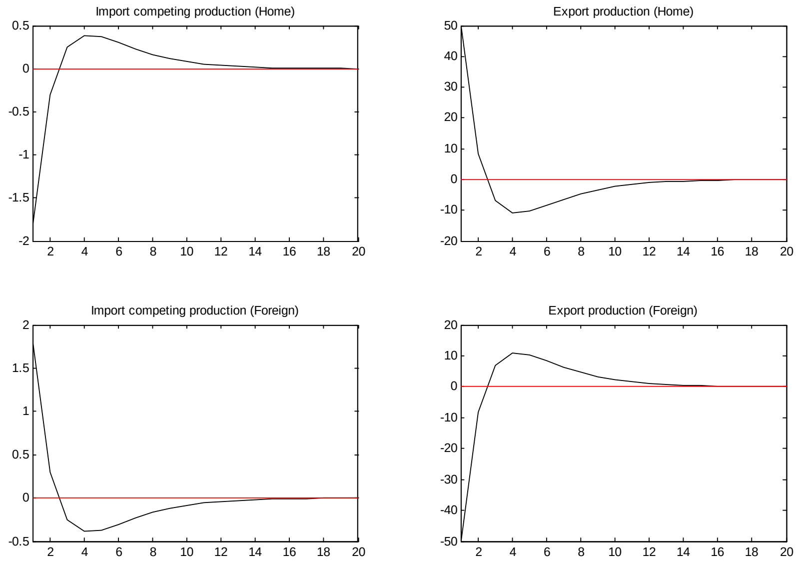

Fig. 2: A temporary devaluation in a rich country, production

developed country ($A = 1$). The responses are measured in percentages of the shock. The most important result to note is that the manufacturing price increases on impact by only about 35%. Given that there are no price rigidities built into the model (but see below), this number - which is in line with estimates of exchange rate pass-through into wholesale prices - is remarkable. Since by assumption the Law of One Price holds for food, the implied CPI pass-through is about 51%.

Consistent with the Home price response, export prices - measured in the Foreign currency - decline substantially on impact. Home manufacturing production expands (not shown) modestly, which is also in line with expectations. This expansion, however, is export biased. Home exports increase by 46%, while domestic sales of home firms contract by about 2%. Foreign production, on the other hand, contracts modestly - but it is mostly a consequence of foreign companies' reduced export performance.

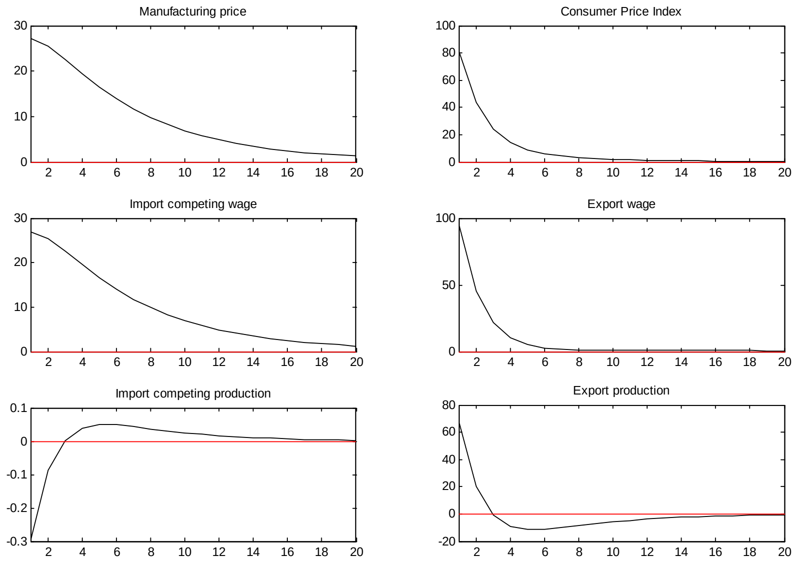

Fig. 3: A temporary devaluation in a poor country

It is interesting to note that the response of production is not monotonic, i.e. an initial export expansion is followed by a decline - an overshooting type of behavior. This is observable in all production sectors, in Home and Foreign. The reason for this behavior is a wealth effect: the exchange rate devaluation increases Home's competitiveness, and leads to higher wages and profits. But increased income leads to a preference for more leisure next period, which implies an overall reduction in production. Just as for the initial impact, it is asymmetric across sectors, and mostly shows up in export output.

Figure 3 shows a subset of the previous graphs for a developing country ($A = 1/3$). While the qualitative results are the same, the pass-through into the CPI is considerably faster, around 80%. Comparing with figure 1 reveals that the increase is completely a composition effect: the share of food in consumption is much larger for the developing country. This is, of course, due

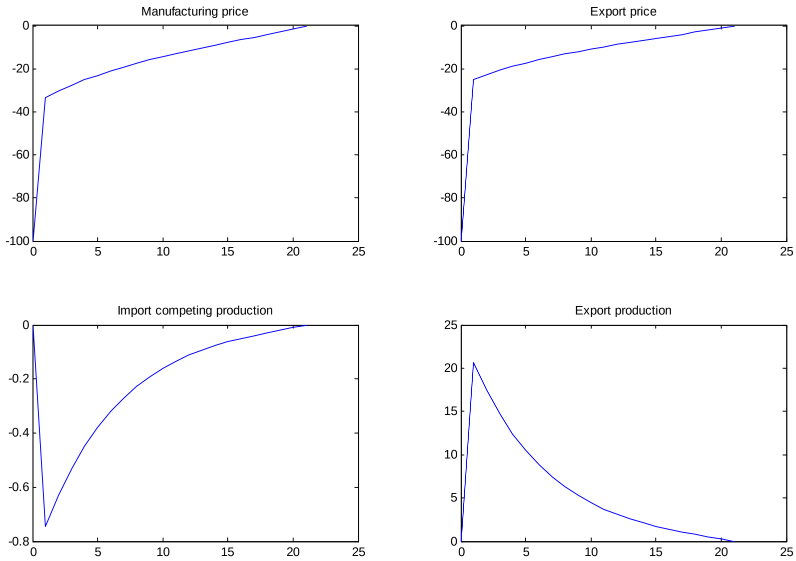

Fig. 4: A permanent devaluation, a rich country

to the assumption of quasi-linear preferences: a more productive, and hence richer, country consumes more manufactures. Interestingly, manufacturing pass-through is actually smaller - which means that structural differences at the industry level also play a role in determining the extent of pass-through.

5.2 A permanent devaluation

In this section I examine the impact of a permanent, 10% devaluation. I assume that agents believe the new exchange rate level will stay unchanged forever. The devaluation is unexpected, so it has an initial impact on the real wealth (money) of agents.

Figure 4 shows the results for a rich country. As in the case of a temporary depreciation, exchange rate pass-through is incomplete and gradual, and the other variables react the same way as before. The most significant difference is that the initial effects are somewhat smaller. The manufactur-

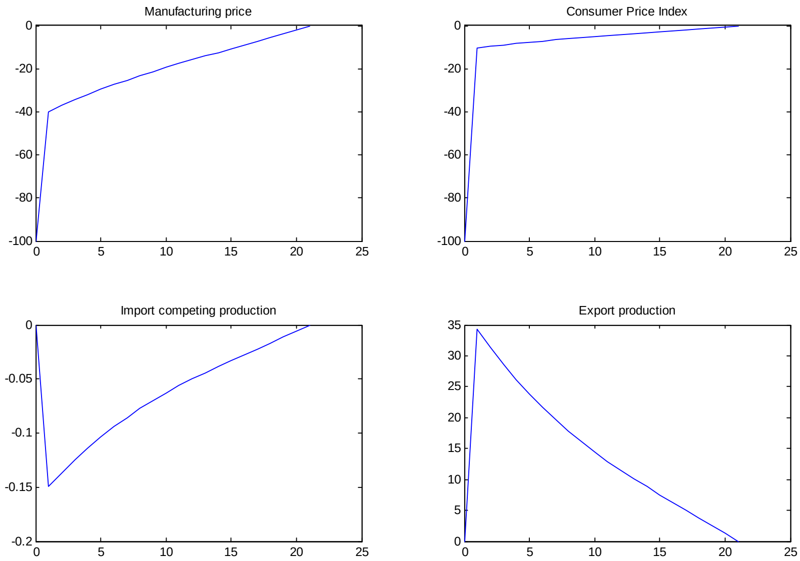

Fig. 5: A permanent devaluation, a poor country

ing price goes up by 6% on impact (so pass-through is 75%, given that the exchange rate shock is 10%), and then gradually converges to the new steady state. The CPI increases by 7% on impact. Production levels also change by less than for a temporary devaluation, reflecting the smaller change in competitiveness.

Figure 5 plots the same responses for a developing country. As expected, the qualitative results are the same as for a rich country. CPI pass-through, however, is again much faster: 8.5% in this case, compared to 7% for a rich country. The difference is almost entirely due to the composition effect, since manufacturing goods are much more important for rich countries.

Overall, while qualitatively similar to the temporary case, examining a permanent devaluation produces useful insights. One is the importance of the persistence of the shock: more persistence leads to higher pass-through, which makes perfect economic sense. Second, responses are now monotonic,

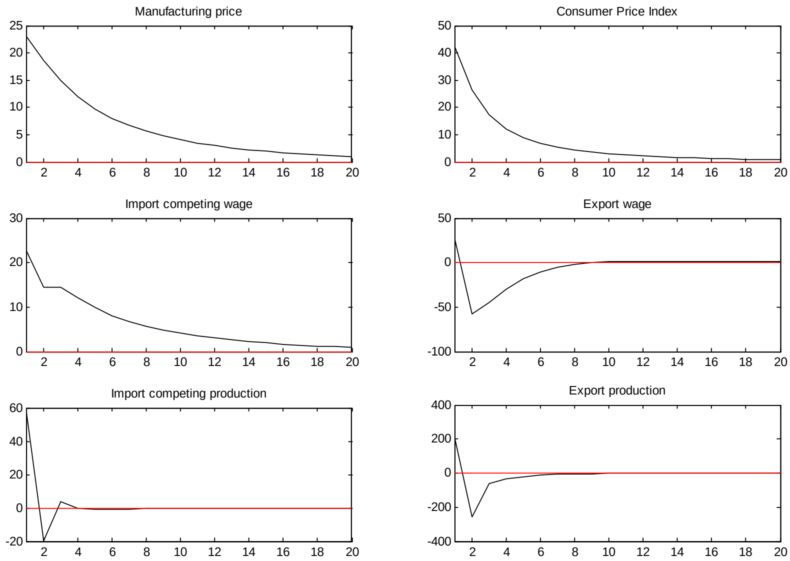

Fig. 6: Sticky wage sand proces

as opposed to overshooting in the temporary case for some variables.

5.3 The role of nominal rigidites

In this section I incorporate nominal rigidities into the model. I assume that firms and households are allowed to set prices and wages randomly, as in the Calvo model. The well-known formula for the evolution of the Home manufacturing price is given by

$$p_t = \frac{\pi_p}{1+\beta\pi_p^2} p_{t-1} + \frac{\beta\pi_p}{1+\beta\pi_p^2} E_t p_{t+1} + \frac{(1-\pi_p)(1-\beta\pi_p)}{1+\beta\pi_p^2} p_t^{opt},$$

where p_t^{opt} is the optimal flexible price (as shown in the previous sections). For illustration, I choose the time-invariant probability of being allowed to set the price or wage to be $\pi_p = \pi_w = 0.5$.

Figure 6 shows the results for the case of nominal rigidities when the exchange rate shock is temporary and Home is rich. The basic message of the figures is that nominal rigidities do not play an important role in exchange rate pass-through in this case. As I showed in the previous sections, small nominal and real frictions (such as money in the utility and oligopolistic behavior) are enough to induce quite limited exchange rate pass-thrugh.

The interesting difference between the fully flexible case and this is that Home's import competing production now also expands initially, together with the export sector. Also, the real effects of the exchange rate shocks are bigger, i.e. the production changes are larger and also more volatile. This difference can potentially be used to evaluate the importance of nominal rigidities on actual data.

6 Conclusion

In this paper I developed a tractable, two-country dynamic general equilibrium model with two sectors and oligopolistic conduct. The model matches some important stylized facts reported in the literature on exchange rate pass-through: it is incomplete in the short-run and gradual, and pass-through is significantly smaller in advanced economies. I also presented evidence on the key assumption of the model, which is that advanced countries trade more differentiated products, while developing countries specialize in homogeneous goods.

The challenge for future research is both to provide more detailed evidence on trade patterns, and to explore the relative role of price discrmination, nominal rigidities and non-tradables in exchange rate pass-through in more detail. The importance of this lies not so much to increase the quantitative predicting power of the model, but in disentangling the effects of the three main channels for incomplete exchange rate pass-through. Given the simple structure of the current model, this should be a relatively simple task, and should form a useful addition to the current model.

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A The log-linearized model

In order to solve the system, I log-linearize the dynamic and static equations. The log-linearized equations are as follows:

- Current account

$$\begin{aligned} M(m_t - m_{t-1}) &= Ts_t - \alpha P p_t + P^* X_f^*(s_t + p_t^* + x_{f,t}^*) - P X_f(p_t + x_{f,t}) \\ M^*(m_t^* - m_{t-1}^*) &= -\alpha P^* p_t^* + P X_f(p_t + x_{f,t} - s_t) - P^* X_f^*(p_t^* + x_{f,t}^*) \end{aligned}$$

- Money demand

$$\begin{aligned} p_t &= \beta E_t p_{t+1} + (1-\beta) m_t \\ p_t^* &= \beta E_t p_{t+1}^* + (1-\beta) m_t^* \end{aligned}$$

- Labor supply

$$\begin{aligned} w_{it} - p_t &= \varphi l_{it} \\ w_{it}^* - p_t^* &= \varphi l_{it}^* \end{aligned}$$

- Labor demand

$$
\begin{aligned}
l_{i,t} &= x_{i,t} \\
l_t^* &= x_{i,t}^*
\end{aligned}
$$

- Prices and production

$$
\begin{aligned}
p_t &= \frac{(W/A)\, w_t + \tau W^*(w_t^* + s_t)}{P(2-1/\sigma)} \\
p_t^* &= \frac{\tau W/A(w_t - s_t) + W^* w_t^*}{P^*(2-1/\sigma)} \\
x_{h,t} &= x_t - p_t + \frac{P p_t - (W/A)\, w_t}{P - W/A} \\
x_{f,t} &= x_t - p_t + \frac{P p_t - \tau W^*(w_t^* + s_t)}{P - \tau W^*} \\
x_{h,t}^* &= x_t^* - p_t^* + \frac{P^* p_t^* - W^* w_t^*}{P^* - W^*} \\
x_{f,t}^* &= x_t^* - p_t^* + \frac{P^* p_t^* - \tau W/A(w_t - s_t)}{P^* - \tau W/A}
\end{aligned}
$$