Understanding Inflation Persistence: a comparison of different Models.*

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Abstract

This paper adopts the Impulse-Response methodology to understand inflation persistence. It has often been argued that existing models of pricing fail to explain the persistence that we observe. We adopt a common general framework which allows for an explicit modelling of the distribution of contract lengths and for different types of price setting. In particular, we find that allowing for a distribution of contract lengths can yield a more plausible explanation of inflation persistence than indexation.

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1 Introduction

The persistence of inflation is a central issue both for economic policy makers and for theorists. For policy makers the issue is how far should they look forward and how rapidly their policy actions take effect. For theorists, the issue is to what extent the theories are consistent with the empirical evidence on persistence. There are two main sources of evidence raised to support the idea that inflation is persistent. One is the autocorrelation of inflation. If you regress inflation on itself the coefficients on lagged inflation will be quite high. Clark (2005) found for the US that the sum of the AR coefficients for the aggregate inflation series are about 0.9. Pivetta and Reis (2004) found evidence that inflation persistence has been high and approximately unchanged in the United States between 1947 and 2001. Batini (2002) found that for the Euro zone 1970-2002, AR(5) coefficients sum to around 0.7: this varied at individual country level. Whilst some studies argue that the coefficient is reduced if you allow for structural breaks and regime switches[1], few would argue for coefficients near to zero. Second, there is the evidence of VARS. These introduce another dimension: the shape and timing of the response of inflation to monetary policy. It is widely agreed that there is a delayed response of inflation to monetary policy: the maximum effect of policy occurs sometime after the policy: there is a hump-shaped response.

Views about the timing of the peak differ. The traditional view was put forward by Friedman: monetary policy has "long and unpredictable lags": the impact on inflation would be as long as 8 quarters or even more. Certainly, this is the view taken by the Bank of England: when setting monetary policy, the MPC looks 8 quarters ahead[2]. The ECB takes the view that the maximum impact is 6 quarters. Different researchers have estimated the response from 4 quarters (Smets and Wouters 2003) to 12 quarters (Nelson (1998), Batini and Nelson (2001), Batini (2002)).

We can summarise these observations by three stylised facts or features:

[1] Levin and Piger (2004) evaluate persistence in inflation series for twelve industrial countries within the context of a model that allows for structrual breaks. They find that the degree of persistence of the process in terms of the sum of the AR coefficients is less that 0.7 for the seven countries. Similarly, Taylor (2000) found evidence that US inflation persistence has been lower during the period 1982-1999, compared to the period 1960–1979. Similiar conclusion has been reached by Cogley and Sargent (2001), who provide evidence that inflation persistence had varied widely over time and recently fallen considerably.

[2] Although oddly enough the Bank of England's own model (BEQM) has the peak impact at 6 quarters.

Feature 1 The biggest effect is not on impact (Hump)

Feature 2: The biggest effect is (a) after 4Q, (b) after 8Q, (c) after 12 Q (timing of Hump).

Feature 3: After 20 Q, the effect on inflation is (a) 1%, (b) 5% of the maximum.(persistence).

In the case of Feature 2, we take three different values for the timing, corresponding to the moderate view (8Q), the Hawkish view (12Q) and the rapid view (4Q). Likewise for Feature 3, we have two thresholds.

Another issue we tackle is the relationship of the inflation response and the output response. Again, there is a common view that inflation peaks *after* output, and the gap between the two peaks is about 4 Quarters.

In this paper, we take a standard model of wage-setting behaviour with nominal rigidity, as in Dixon and Kara (2005b). This is a Dynamic Stochastic General Equilibrium model which is microfounded, and can be calibrated using standard reference values. We then embed in the model a range of different dynamic pricing models, which include all of the main models that have constant steady-state distributions of durations (see Dixon (2005))[3]. Our approach is to model the steady-state distribution of contract lengths in a way that enables us to isolate the effect of the different models of dynamic wage-setting. There are basically 4 different types of "contract":

- The wage is set in nominal terms for a fixed and known period (e.g Taylor (1980),Fuhrer and Moore (1995))
- The wage is set in nominal terms with the duration being random (e.g.Calvo (1983))
- There is a fixed or uncertain contract length, and the firm/union sets the wage for each period at the beginning of the contract (e.g. Fischer (1977), Mankiw and Reis (2002)).
- The initial wage is set, but through the contract length the nominal wage is updated by recent inflation (Indexation) [4]: (e.g.Woodford

[3]The main model that does not have a steady state distribution of durations is the state-dependent pricing model such as Wolman (1999)

[4]A variation on the indexing model can also be seen as a rule-of-thumb approach to wage and price setting as in Gali and Gertler (1999): part of the economy optimises, the other part just updates using past inflation.

(2003)[5]).

The main innovation of this paper is to put these models of wage-setting into a general framework that allows for an explicit and consistent examination of the distribution of contract lengths and its relation to the persistence of inflation. We consider 3 groups of models. First, we have the Generalised Taylor Economy (GTE) set out in Dixon and Kara (2005b), in which there are many sectors, each with a Taylor contract of a particular length. Second, the Generalised Fischer Economy (GFE) in which there are many sectors each one with a Fischer contract of a particular length. The Mankiw-Reis Sticky-information (SI) model is a special case of the GFE with the Calvo distribution of contract lengths. Thirdly, we have Hybrid Phillips Curve models (HPC) which can arise either through Taylor-type contracts (as in the Fuhrer and Moore (1995)) or through indexation/bounded rationality (Woodford (2003), Kiley (2005)).

The approach of the paper is to compare a range of models in a consistent way and to see what is the role of key parameters in changing the properties of the models in terms of inflation persistence. We ask the question in two stages. First we take standard calibrations of the SDGE model and see to what extent the different models meet the three Stylised features (in either weak or strong forms). The second approach is to ask what would be required in terms of contract length, distribution of contract lengths or parameter values for the model to satisfy the Features.

The conclusions of the paper can be briefly summarised as follows. On Feature 1, all of the models have hump-shaped responses except for Calvo, which never has a hump. On Feature 2, at standard parameter values, the only way to get hump at 4, 8 or 12 quarters is to have average contract lengths of that magnitude: the easiest way to have this is the simple Taylor model. Of course, whilst 4Q is plausible as an empirical magnitude, most would believe 8Q or more to be far too long. This is a necessary but not sufficient condition: Calvo can have long average lifetimes of contracts, but sill never develop a hump in the first place, and others such as the Sticky-information model will have humps that peak before the average lifetime.

[5]Other examples includes Christiano, Eichenbaum and Evans(CEE, 2005) and Smets and Wouters(2003). In CEE(2005), indexing occurs in periods in which wage/price setters are not allowed to re-optimize their prices. Smets and Wouters(2003) instead use a specification in which a fraction of the economy optimises and the other part uses the CEE indexation mechanism.

Second, we take the key parameter γ, which measure the sensitivity of the optimal flexible wage to output. We find that as this becomes lower, models with a distribution of contract lengths have the hump later if they have Taylor, Fischer or Indexed contracts but *not* in the case of Calvo contracts. For example, at the very low value put forward by Fuhrer and Moore (1995), several models have humps at 8Q and beyond. It is also possible to have the hump occurring after the average contract length.

Lastly, we consider the timing of the inflation hump in response to the output hump. Again, the Calvo model does worst: inflation always peaks before output. We find that for γ is important again: it determines the degree of inflationary pressure in response to an increase in output: for lower values of γ we find that several models have peaks in output around $4Q$ before the peak in inflation.

We believe that the distribution of contract lengths plays a vital role in understanding the response of inflation to monetary policy. On its own it does not solve the "puzzle", but it suggests a better way forward than the ad hoc and unrealistic device of grafting on Indexation to the Calvo model[6]. The best approach is to choose the model of wage or price setting that best characterises behaviour and model the distribution of contract lengths using empirical data.

The rest of the paper is organised as follows. Section 2 describes the basic structure of the Economy and outlines the GTE framework. Section 3, 4, 5 consider the three groups of models: the Generalised Taylor Economy (GTE), the Generalised Fischer Economy (GFE), Hybrid Phillips Curve models (HPC), respectively and each section examines to what extent the model meet the three stylised features and also discusses what would be required in terms of contract length for the model to satisfy the features. Section 6 considers what happens as the key parameter γ varies. Section 7 examines the timing of the inflation hump in response to the output hump. Section 8 concludes. The details of the derivation of the structural equations for each model are in the appendix.

[6]As Woodford (2006) points out when discussing Calvo with indexation "there are number of reasons to doubt the correctness of this model as an explanation of US (or euro-area) inflation dynamics. One is the lack of direct microeconomic evidence for the indexation of prices..."

2 The Model

The model that we use is the GTE framework of Dixon and Kara (2005b), which can be interpreted as the log-linearized equilibrium conditions of a DGE model in which there are potentially many sectors, each with a Taylor contract of a particular length. The unique feature of the GTE framework is that it allows us to model any distribution of co ntract lengths, including the one generated by the Calvo model. The details of the derivation of the structural equations can be found in Dixon and Kara (2005b) and we provide a brief summary is in the appendix. The main difference is that we generalise the framework to allow for different types of contract from the Taylor case of a fixed nominal wage for the duration of the contract.

2.1 Structure of the Economy

The Detailed structure of the model is outlined in Dixon and Kara (2005b) and a brief summary is in the appendix. In the model economy, there is a continuum of firms $f \in [0, 1]$, each producing a single differentiated good, which are combined to produce a final consumption good. The production of intermediate goods requires labour as the only input. Corresponding to the continuum of firms f there is a unit interval of household-unions. The economy is divided into many sectors where the $i-$th sector has a contract length of i periods. The share of each sector is given by α_i with $\sum_{i=1}^{N} \alpha_i = 1$. Within each sector i, each firm is matched with a firm-specific union and there are i cohorts of equal size. The representative household-union derives utility form consumption, real money balances and leisure. The representative household-union in each sector chooses the reset wage to maximize lifetime utility given labour demand and the additional constraint that the contract will be in force for i periods. In any given period, in each sector a cohort will come to the end of its contract period and set the new contract. There are three types of contract considered:

- The wage is set in nominal terms for a fixed and known period (Taylor, Fuhrer and Moore))
- The wage is set in nominal terms with the duration being random (Calvo);

- There is a fixed and known contract length, and the firm/union sets the wage for each period at the beginning of the contract;

- The initial wage is set, but through the contract length the nominal is updated by recent inflation.

In Dixon and Kara (2005a), we considered the first type of contract only and focussed on the GTE. We start first with the case of GTE and then consider different type of contracts.

2.2 The log-linearised economy.

In the appendix, we provide a full description of the model and the different wage-setting equations under the 4 different types of contract. In this section we will simply present the log-linearised macroeconomic framework common to all approaches (which differ only in the type of contract considered).

Sectoral price level is given by the average wage set in the sector, and the wage is averaged of the i cohorts in sector i:

$$p_{it} = w_{it} = \frac{1}{i} \sum_{j=1}^{i} w_{ijt}$$

The sectoral output level y_{it} can be expressed as a function of the sectoral price relative to the aggregate price level p_t and aggregate output y_t where the coefficient θ is the elasticity of demand:

$$y_{it} = \theta(p_t - p_{it}) + y_t \tag{1}$$

The linearized aggregate price index in the economy is the average of all sectoral prices:

$$p_t = \sum_{i=1}^{N} \alpha_i p_{it} \tag{2}$$

The inflation rate is given by $\pi_t = p_t - p_{t-1}$.

Aggregate demand is given by a simple Quantity Theory relation:

$$y_t = m_t - p_t \tag{3}$$

The money supply follows a $AR(1)$ process,

$$m_t = m_{t-1} + \ln(\mu_t), \quad \ln(\mu_t) = v. \ln \mu_{t-1} + \xi_t \tag{4}$$

where $0 < v < 1$ and ξ_t is a white noise process with zero mean and a finite variance.

2.3 Wage setting rules.

All the models we use share the common macroeconomic framework embodied in the log linearised equations of the previous section. They differ in the wage-setting rules implied by the different nature of contracts. In this section we will briefly outline the main rules (details are in the appendix).

Before defining the optimal wage setting rules, let us define the optimal wage which would occur if wages were perfectly flexible: "the optimal flex wage". The optimal flex wage in each sector[7] is given by

$$w_t^* = p_t + \gamma y_t \tag{5}$$

where the coefficient on output γ is:

$$\gamma = \frac{\eta_{LL} + \eta_{cc}}{1 + \theta \eta_{LL}} \tag{6}$$

Where $\eta_{cc} = \frac{-U_{cc}C}{U_c}$ is the parameter governing risk aversion, $\eta_{LL} = \frac{-V_{LL}H}{V_L}$ is the inverse of the labour elasticity, θ is the elasticity of substitution of consumption goods.

In the GTE, the reset wage in sector i is simply the average (expected) optimal flex wage over the contract length (the nominal wage is constant over the contract length).

$$x_{it} = \frac{1}{i} \sum_{s=0}^{i-1} E_t w_{t+s}^* \tag{7}$$

Note that the reset wages will in general differ across sectors, since they take the average over a different time horizon.

In a GFE, the trajectory of wages is set at the outset of the contract. Suppose an i period contract starts at t, the sequence of wages chosen from

[7] Note that the optimal flex wage in each sector is the same: this is because it is based on the demand relation (1) which has the same two aggrgegate variables $\{p_t, y_t\}$ for each sector.

t to $t+i-1$ is $\left\{E_t w^*_{t+s}\right\}_{s=0}^{s=i-1}$. Hence, the average wage in sector i at time t is

$$w_{it} = \frac{1}{i}\sum_{s=0}^{i-1} E_{t-s} w^*_t$$

which is the best guess of each cohort for the optimal flex wage to be holding at t. This embodies the "sticky information" idea in Fischer contracts: part of current wages are based on old information.

In both the Calvo model and its extension with (full) indexation, when the wage is set it is not known how long the contract will last[8]: thus in all sectors the reset wage is the same. With indexation the reset wage equation is:

$$x_t = \omega\left(p_t + \gamma y_t\right) + (1-\omega)\, x_{t+1} - (1-\omega)\, \pi_t$$

which is the standard Calvo reset equation with the additional inflation term. The evolution of the aggregate wage index is given by

$$w_t = \omega x_t + (1-\omega)\left(w_{t-1} + \pi_{t-1}\right)$$

Again, indexation introduces the additional inflation term into the standard Calvo equation. In the appendix we also derive the Fuhrer-Moore wage setting rule and the GTE with indexation.

2.4 The Choice of parameters.

We have chosen a range of parameters and performed a grid search: there are two main parameters which determine the shape of the inflation response: the value of γ (the effect of the output on wage-setting) and the value of ν (the serial correlation of monetary growth). As discussed in Dixon and Kara (2005b), there are a range of values that are consistent with a microfounded model. We take as our reference point $\gamma = 0.2$. However, if we think in terms of a lower level of what is plausible, we take $\gamma = 0.1$ We can keep $\sigma = \eta_{CC} = 1$ and $\eta_{LL} = 4.5$: if $\theta = 10$ (as in Chari, Kehoe and McGrattan (2000)) we get a value of $\gamma = 0.12$. Throughout the paper, there are standard or reference values of $\gamma = 0.2$ and $\nu = 0.5$ are used unless specified otherwise. It is important to note that non-microfounded econometric estimates of γ

[8] As argued in Dixon and Kara (2005b), the Calvo setup can be seen as a game of incomplete information, where firms or unions do not know which sector they belong to when they set the price/wage.

tend to be much smaller: Taylor (1980) estimates $\hat{\gamma} = 0.05$, Coenen and Levin (2004) $\hat{\gamma} = 0.003 - 0.01$,Fuhrer and Moore (1995) $\hat{\gamma} = 0.005$[9]. We will also report on these values when appropriate as a point of reference. So, our reference set for γ are $\{0.2, 0.1, 0.05, 0.01, 0.005\}$. It should be noted that more recent papers have argued that the presence of firm-specific capital can lead to lower values of γ: Altig, Christiano, Eichenbaum and Linde (2004), Coenen and Levin (2004), Eichenbaum and Fisher (2004), Smets, Wouters and de Walque (2005), Woodford (2003).

When it comes to the serial correlation of money growth ν, this has been estimated to be ν =0.57 by Chari et al. (2000): Mankiw and Reis (2002) use the value of ν =0.5, Huang Lui and Phaneuf (2004) use a value of $\nu = 0.75$. We have simulated both extreme values $\nu \in \{0.5, 0.75\}$ and have found it makes little difference in terms of our three features. In the paper, all of the reported simulations are undertaken with a low value of $\nu = 0.5$.

3 Inflation Persistence in a Generalised Taylor Economy (GTE).

Before we explore the general case of the GTE, we can recap on the standard cases of the Calvo model and the simple Taylor (ST) model. The Calvo (1983) pricing model has a single parameter: the reset probability or hazard rate ω, which gives the non-duration dependent probability that a firm/union will have the option to reset its wage in any period. As it is well known (see Woodford (2003) for a discussion), even though the Calvo model can generate inflation persistence, it fails to generate the delayed response to monetary shocks. The increase in inflation is largest when the shocks first hits. As illustrated in figure 1, which is the impulse response function of inflation to a one percent innovation in the money supply.

Figure 1

As is well known, the Calvo model cannot deliver a hump shaped inflation response: the maximum is always in the first period (unless one imposes some ex-ante pricing). *Feature 3* can be met if we choose the reset probability low enough. Certainly, for any ω, the exponential-like nature of the decay means

[9] See Roberts (2005) for a survey and an attempt to reconcile the differences in publised estimates.

that the effect does not disappear even after 20 quarters: as shown with the popular value of $\omega = 0.25$ and $\omega = 0.4$ (which correspond to average contract lengths of 7 and 4 quarters respectively as shown in Dixon and Kara (2005a)). The value $\omega = 0.25$ there is a enough persistence to meet the stronger criteria (a); with $\omega = 0.4$, enough to meet the weaker criterion (b).

The failure of the Calvo model to generate the observed responses of inflation suggests that the missing element might be backward-lookingness. The intertemporal backward-looking and forward-looking effects in Taylor model are emphasized by Taylor (1980). We now investigate this possibility. Figure 2 displays the impulse response function of inflation in the Taylor's staggered contract model: for contract lengths $T = 2, 4, 6, 8$:

$$Figure 2$$

As figure 2 shows the maximum inflation response in Taylor's model is indeed delayed for several quarters and reaches its peak $T - 1$ quarters after the first period in which the shock occurs[10]. There is a hump shape of sorts, but a rather jagged one. Hence Features 1 and 2 can be met. However, the simple Taylor contract will only generate a hump at around 2 years if the contract lasts for that length of time ($T = 8$). Most authors think that $T = 4$ is more realistic and few would argue that the economy consists of $2-$year contracts with constant nominal wages. Thus Feature 2 can only be met if there is an implausibly long contract length. Furthermore, if we turn to Feature 3, inflation dies away rapidly T periods after the shock. In particular, for $T = 4$, the effects of the shock are almost gone by 15 periods and certainly fails to meet even the weak criterion.

3.1 Generalised Taylor Economy (GTE).

What the previous analysis of simple Taylor contracts suggests is if we want to generate a smoother hump-shaped impulse response which dies away more slowly, we need to have a distribution of contract lengths, using the GTE as in Dixon and Kara (2005b). The economy is divided into many sectors where the $i-$th sector has a simple Taylor contract length of $T = i$ periods

[10] Every cohort setting their wage from the period of the shock sets the wage knowing the innovation in the money supply. The last cohort to set its wage without this information is the one which set its wage the period before the innovation. Its contract ends $T - 1$ periods after the shock: inflation peaks when this cohort resets its wage.

and a share of α_i in the economy. Let the $n-$vector of contract lengths be denoted $\alpha \in \Delta^{n-1}$. This vector characterizes the GTE, which can be written as $GTE\left(\boldsymbol{\alpha}\right)$. We will now consider some special $GTEs$

3.1.1 Calvo-GTE.

Let us first consider the Calvo-GTE, in which the share of each duration is the same as generated by the Calvo model:

$$\alpha_i = \omega^2 i\left(1-\omega\right)^{i-1} : i = 1...\infty$$

For computational purposes, we truncate the distribution at $i = 20$ and put all of the mass of the contracts $j \geq 20$ onto $i = 20$. We show the Calvo-GTE for $\omega = 0.25$, which has a mean contract length of 7 quarters and a modal length of 3 and 4 quarters. The inflation impluse-response is depicted for this in Fig.3, with the corresponding Calvo impulse-response from Figure 1 superimposed.:

$$Figure 3.$$

The two economies in Figure 3 have the same distribution of contract lengths (except for the fact that the Calvo-GTE is truncated at 20 quarters). They differ in the wage-setting decisions: in the calvo model the wage-setters do not know he length of heir contract, but have a probability distribution over contract lengths and hence all wage-setters set the same price. In the Calvo-GTE each wage-setter knows its contract length when it sets the wage. As we can see, in terms of Feature 1, the Calvo GTE does have a hump shape. However, with $\omega = 0.25$, the hump appears in the third quarter. However, reflecting the long tail of the contract lengths in the Calvo distribution, there is lots of persistence in the tail with inflationary effects lasting 20 quarters and beyond. In order to get the hump at the 8 quarter a value of $\omega = 0.1$ is required, which implies an average contract length of 19 quarters which is implausibly long. Hence moving from the Calvo pricing rule to the Taylor approach with the same distribution of contract lengths lets us satisfy Feature 1 in addition to Feature 3. The timing of the hump is still well below the average lifetime of contracts in the economy.

3.1.2 Taylor's (1993) US Economy.

Whilst the Calvo-GTE is a useful reference point, it probably includes too many long contracts. Can we get a reasonable hump with a simpler specification? One possibility is Taylor's US economy as employed in Dixon and Kara (2005b) based on Taylor (1993). This has an empirical distribution of wage contract durations from 1 to 8 quarters based on the US economy[11] which has an average duration of 3.6 quarters.

$$Figure 4$$

Note that the Taylor's US economy GTE does have a hump shape, but it peaks in the third quarter and the effect is almost entirely gone by the 16th quarter. So Feature 3 is not satisfied: unlike the Calvo-GTE there are no contracts lasting longer than 8 quarters. The timing of the hump is similar to the Calvo-GTE despite it having an average lifetime just half as long. However, just like the Calvo distribution, the two most common contract lengths are 3rd and 4th quarters.

3.1.3 Bils-Klenow Distribution: $BK - GTE$.

We can construct a distribution of duration data using the Bils and Klenow (2004) data set. This is for *price* data, but we use it as an illustrative data set. The data is derived from the US Consumer Price Index data collected by the *Bureau of Labor statistics*. The period covered is 1995-7, and the 350 categories account for 69% of the CPI. The data set gives the average proportion of prices changing per month for each category. We assume that this is generated by a simple Calvo process within each sector. We can then generate the distribution of durations for that category using Dixon and Kara (2005a). We can then sum over all sectors using the category weights. We depict the distribution in terms of quarters in Figure 5.

$$Fig5 : BK - GTE : \text{Distribution.}$$

Note that the mean contract length is 4.4 quarters. There is a very long tail, with some very long contracts: over 3% of weighted categories have less than 5% of prices changing per month, implying average contract lengths of over 40 months (13.5Q). However, the most common contracts durations is 1 Quarter; the distribution looks a bit like a geometric distribution, at least in the feature that the longer the duration, the lower its share of the population.

$$Fig6 : BK - GTE : I - R$$

[11] In Taylors US economy, the sizes of the sectors are $\alpha_1 = 0.07$, $\alpha_2 = 0.19$, $\alpha_3 = 0.23$, $\alpha_4 = 0.21$, $\alpha_5 = 0.15$, $\alpha_6 = 0.08$, $\alpha_7 = 0.04$, $\alpha_8 = 0.03$.

If we consider the IR for the $BK - GTE$, this gives us a hump shape that peaks at the second quarter but dies away gradually so that Feature 3 is met. If we compare the Calvo-GTE and the Taylor's US$-GTE$, we can see that the longer average lifetime of the former does not imply that the hump is later. Indeed, the greater presence of shorter contracts (1 and 2 quarters) seems to be the key factor here. Thus, we can see that the timing of the hump seems to be close with the most common contract durations: this is perhaps not surprising, since it is at this moment when the most wages are being reset after the innovation.

4 Generalised Fischer Economy (GFE).

In this section we consider an economy with many sectors, in which contract lengths can differ. As in the GTE, this can be represented by a vector of sectoral shares $\alpha \in \Delta^{n-1}$ where sector $i = 1...n$ have contract lengths i. The difference between a GFE and a GTE is in the nature of the contract: with a Fischer contract, the wage-setter chooses a trajectory of wages, one for each period for the whole length of the contract as in Fischer (1977). The wages set are thus conditional on the information the agent has when it sets the wages, so that as the contract gets older the information will get more out of date. An alternative interpretation is that the firm sets its wage or price optimally each period, but that it only updates its information infrequently. We believe the latter interpretation is less plausible changes in demand should be obvious to the firm

There are two general points that need to be understood when interpreting the Fischer contracts. First, the IR functions are generated by a single innovation in the initial period. The initial shock is perpetuated because we assume that money follows an $AR(1)$ process. However, in terms of information, any new contract that starts after the initial shock will be fully informed. Once all contracts have been renewed after the shock, the economy will behave as if there is full information and flexible wages/prices: i.e. the only inflation will be generated by what remains of the monetary $AR(1)$ process. The second point is that *the length of the contract has no influence on the wages chosen for any specific period*. This is because a separate wage can be chosen for each period within the contract. So, it makes no difference to the wage chosen for period 2 of the contract whether the contract last for 2 periods or 1000. Its period 2 wage will be its best guess at what the optimal wage is going to be in period 2 of the contract.

4.1 Simple Fischer Economy

We will start out by considering an economy in which there is only one contract length, analogous to the simple Taylor economy. We depict the IR functions for he cases where contracts last for $T = 4, 6, 8, 10$ quarters:

$$Fig7 : SF - IR$$

If we compare Figure 7 with Figure 2, there are similarities and differences. The similarity is the shape: we have a jagged hump, with a peak $T - 1$ periods after the shock. The difference is that the ascent to the peak accelerates more with the SF economy, and the drop from the peak is even more precipitous. The second feature is easy to understand. $T - 1$ periods after the shock, all contracts have been renewed since the shock, so are now fully informed. Wage inflation simply follows the money growth. The first difference is simple as well: recall that in the steady state prior to the monetary shock at time t there is zero inflation, so all wage-plans involve constant wages. In period t, when the shock hits, the current cohort revise their wage plans: although the money supply has increased, $\frac{T-1}{T}$ of firms are still setting the old wage, so this holds down the wage reset for period t : in period $t + 1$ $\frac{T-2}{T}$ of firms are still setting the new prices. There are now two cohorts who are able to set prices reflecting the monetary innovation at time t, so they plan a higher price. In each subsequent period fewer wages at the old steady-state wage until $T - 1$ periods after the shock, no wages reflect the pre-shock steady state and wages are at the fully flexible values and output is at its natural rate.

The SF economy will fail Feature 3: unless it has contracts in excess of 20 quarters, inflation just follows what is left of monetary growth: with an autoregressive coefficient of 0.5 this will be negligible after 20quarters. The SF will satisfy Feature 1, and also Feature 2 but only if the contracts are long enough.

4.2 Mankiw and Reis's sticky information (SI) model.

Mankiw and Reis's Sticky Information model (SI) is a GFE where the distribution of contract lengths is Calvo with their choice of $\omega = 0.25$, resulting in an average length of 7 quarters. The parameter ω is presented as a "re-plan" probability: just as in the Calvo model, when the trajectory of wages is chosen at the outset of the contract, the wage-setter does not know how long it will last but has a subjective distribution over the lifetime. However, as we have noted, the length of the contract has no influence on the wage-setting behaviour. Hence the SI model as presented by Mankiw and Reis is exactly the same as a Calvo-GFE: an economy where there is a Calvo distribution of contract lengths but in which each wage setter knows exactly how long the contract will run for. With Fischer contracts, the Calvo reset probability is only important in generating the distribution: nothing else.

$$Fig8 : SF - T = 7; SI - \omega = 0.25$$

In Figure 8 we depict the IR functions for two GFE.s: the SI model with $\omega = 0.25$ and the SF with the same mean contract length. The SF has the jagged peak property as in Figure 7, with negligible inflation after 7 quarters. In contrast, the SI model has a smooth hump, peaking at the 4th quarter, and inflation dies away slowly so that Feature 3 is satisfied by the weak criterion. The reason for this shape is the distribution of contract lengths and in particular the longer contracts that let inflation persist. As we have noted before, with a 1% monetary innovation at t, and an AR with coefficient 0.5, the total cumulative effect of inflation in all models is 2%. In the long-run money is neutral. The long tail of inflation persistence must imply a lower peak, since adding up inflation over all periods gives the same answer for all models: 2%. Note that in Mankiw and Reis (2002), they parameterise $\gamma = 0.1$, whilst Figure 8 is based on our baseline value $\gamma = 0.2$, and leads to an earlier peak. We will discuss the role of γ later in the paper.

5 Hybrid Phillips Curve Models.

There has been much empirical work done on the New Keynesian Phillips curve. As is well known, it does not do well in explaining the data (see for example Gali and Gertler (1999). Empirically, a model which does much better is the hybrid Phillips curve, which takes the form

$$\pi_t = a\beta E_t \pi_{t+1} + (1 - a)\,\pi_{t-1} + by_t \tag{8}$$

where $a \in [0, 1]$ and $a = 1$ gives the New Keyesian Phillips curve. This has given rise to several attempts to have a theoretical model that can give something like this. The theoretical justification can be found by adding indexation to the Calvo model (see for example Christiano, Eichenbaum and Evans (2005), Smets and Wouters (2003)), or by the Fuhrer and Moore model. Although indexation is often used, many people are not comfortable with it: it is more of a regrettable necessity than a "microfoundation", since there is little evidence of the sort of indexation required occurring.

5.1 Fuhrer and Moore's(1995) inflation persistence model.

In Fuhrer and Moore's model wages are set in nominal terms as in Taylor contracts, but the objective function is different: the, *relative real wage* is targeted. As Driscoll and Holden (2000) have argued, the model is not plausible in this respect and if done in a more consistent manner does not yield the same result. However, here we take the model as given and merely seek to explore the quantitative dynamic properties displayed by it. In the special case of two period Taylor contracts, the FM model gives rise to a hybrid Phillips curve of the form

$$\pi_t = \frac{1}{2}E\pi_{t+1} + \frac{1}{2}\pi_{t-1} + \gamma y_t$$

In fact, in the empirical model they specify an *age distribution* of contracts: at any moment of time there is a proportion α^s of contracts which are s periods old. As shown in Dixon (2005), this corresponds to a distribution of completed contract lengths of α_i.with a mean length $\bar{T} = 3.2$

Age α^s	**Lifetime** α_i
$\alpha^1 = 0.37$	$\alpha_1 = 0.08$
$\alpha^2 = 0.29$	$\alpha_2 = 0.16$
$\alpha^3 = 0.21$	$\alpha_3 = 0.24$
$\alpha^4 = 0.13$	$\alpha_4 = 0.52$

Table 1. Contract Durations in FM

Fig 9: FM and FM-GTE

In Figure 9 we depict the IR for the FM model and the FM-GTE, where the latter has the standard wage-setting rule, but with the same distribution of contract lengths as the FM model. As we can see, the FM model has a hump shape which reaches a peak at quarters 3 and 4 (there is a flat top to the hump). So, in this case the FM model satisfies Feature 1 and the weakest form of Feature 2. However, the effect falls of rapidly and becomes negative, so that there is not enough persistence to satisfy Feature 3. In contrast the FM-GTE clearly peaks at 3 quarters but also dies away so that Feature 3 is not satisfied.

5.2 Calvo with Full Indexation: Woodford (2003).

The second hybrid Phillips curve approach is to allow for the full indexation of contracts in between reset decisions. That is, at the beginning of the contract the nominal wage is set. For the rest of the contract duration this is updated by the previous periods inflation. This gives rise to a HPC of the following form

$$\pi_t = \frac{1}{2}E\pi_{t+1} + \frac{1}{2}\pi_{t-1} + \gamma\frac{\omega^2}{2(1-\omega)}y_t$$

In figure 10, we display the response of inflation to a monetary shock for $\omega = 0.25$ and $\gamma = 0.2$. As the figure illustrates introducing backward looking indexation can affect the impact of the shock on inflation and leads to a hump shape response. The model can satisfy Feature 1 and the weakest form of Feature 2. However, even though the average contract length in this economy is quite long i.e. 7 quarters, the model fails to generate enough persistence to satisfy Feature 3.

Fig 10: Calvo with Full Indexation.

5.3 *GTE* with Full Indexation

We now consider a modification to the above model and replace Calvo style contracts with Taylor style contracts. In Figure 11, we report the response of inflation in the simple Taylor economy.

Fig 11: Simple Taylor 4 with Full Indexation

As the figure shows, allowing for the indexation of contracts does not affect the timing of the hump in the simple Taylor with our reference value of $\gamma = 0.2$, which stands in sharp contrast to that obtained in the Calvo Model. As discussed earlier, the main difference between the two models is their contract structures. We can use the Calvo-GTE approach, which has exactly the same contract structure as in the Calvo Model to see whether the presence of a range of contract lengths in the Calvo Model is the main reason behind this result .

Fig 12: GTE with Full Indexation

Figure 12 reports the response of inflation in the $Calvo - GTE$ with indexation. We assume that $\omega = 0.25$, as in the Calvo with indexation. We also include the response of inflation in the IC. As the figure shows in both models inflation peaks at the same time, which indicates that a presence of a distribution in the Calvo model is the main reason behind the result.

As in the IC, the model can satisfy Feature 1 and the weakest form of Feature 2 but fails to generate enough persistence to satisfy Feature 3. However, allowing for a lower value of γ can potentially affect the results. We will discuss the role of γ later in the text.

5.4 Kiley (2005): Calvo with Indexation based on a moving average.

In Kiley, a slightly different approach to indexation is adopted. Rather than have Calvo resetting with indexation in between resets, Kiley proposes a model with two different types of wage-setters: a proportion $(1 - a)$ are Calvo wage setters of the orthodox kind, and a proportion a are "rule of thumb" agents who update using lagged inflation, where lagged inflation is a moving average over the last b periods. Kiley's model is a model formulated for econometric estimation and so is not directly comparable to the other models presented. The main difference is that he uses marginal cost rather than the output gap. However, we can reformulate his model into a form directly comparable to the others in the paper by assuming that the Calvo resetters base the wage-setting decision on the MRS between consumption and leisure as in Woodford's 2003 model. This results in a HPC of the form:

$$\pi_t = (1 - \alpha) E_t \pi_{t+1} + \frac{a}{b} \sum_{j=1}^{b} \pi_{t-j} + \gamma \frac{\omega^2}{(1 - \omega)} y_t$$

in which Woodford (2003) is a special case when $b = 1$ and $a = 0.5$. Our adaptation of Kiley's approach has 4 parameters: $\{a, b, \gamma \, \omega\}$ This is a relatively larger parameter space than the other models considered. We will take the value of $\gamma = 0.2$ and consider the two cases considered by Kiley for $\{a, b\} = \{0.24, 1\}$ and $\{0.17, 4\}$ with two values of $\omega = 0.25$ and 0.4.

Fig 13: Kiley's Moving average indexation.

As we can see from Figure 11, there is a hump, but it peaks well before $8Q$ and does not even make $4Q$ even when $\omega = 0.25$ with an average contract length of $7Q$.

6 What is the effect of γ?

In this section, we consider what happens as γ varies. Now, as we have discussed previously, the values of γ put forward in different studies vary by huge magnitudes: with price-setting values well over 1: with wage-setting microfounded values should be calibrated at around 0.2 or 0.1 perhaps; FM estimates a value of 0.005. In this section we want to take all of the models together and sytematically show how the change in γ influences the models in terms of Features 1-3. In Table 2 we show how Features 1-3 fare for each of the models at different reference levels of γ : the benchmark level 0.2, $0.1, 0.05, 0.01$, and 0.005. Where there are weak and strong criteria (Features 2 and 3) the more ticks indicates the stronger criteria being met.

$Table2$ Features 1-3 as γ varies.

Let us first take the benchmark case, which we have already explored. The only models to satisfy all three Features are the $T = 8$ simple Taylor case and the SI with $\omega = 0.25$, although the SI only meets the weak Feature 2 (peaks at $4Q$). Thus having long contracts is necessary but certainly not sufficient to satisfy the stronger version of F2: the weaker $4Q$ version is satisfied by several, but they all fail F3 except for SI with $\omega = 0.25$. Turning to $\gamma = 0.1$, the value adopted by Mankiw and Reis (2002), we see that SI meets the strong criterion for Features 2 and 3. The Calvo-GTE and Calvo with Indexation (both with $\omega = 0.25$) both satisfy the $4Q$ peak and the strong version of Feature 3.

Jumping to $\gamma = 0.005$, the FM value, something interesting happens. At this low value of γ many models pass all three features: Strong Feature 2 and 3 are satisfied by: Simple Taylor 8, Calvo-GTE ($\omega = 0.25$), $BK - GTE$,

$SI(\omega = 0.25)$, FM, Calvo with Indexation ($\omega = 0.4, \omega = 0.25$), Simple Taylor 4 with indexation. Weak Feature 2 and strong 3 are also met by Simple Taylor 4, Taylor's US economy, Calvo-GTE ($\omega = 0.4$), $FM - GTE$. The important thing to note is that at low values of γ, models without long average contract lengths and backward looking indexation are showing peaks at $8Q$ or beyond. The main conclusion to note is that this only happens in models where there is a distribution of contract lengths: for example the $BK - GTE$ has an average of 4.4 quarters, but peaks at 10 quarters. Indeed, this is best illustrated if we just look at the peak for different values of γ in Table 3:

$Table3$: The peak response.

Note is that as γ varies some models do not change peak at all. These are the Calvo model: the peak is always at 1, and the simple models with just one contract length: simple Taylor and simple Fischer. In all other cases, as γ decreases, the peak gets pushed further back: and in all other cases there is a *distribution* of contract lengths. In just three models is the distribution truncated: in FM and FM-GTE there are no contracts longer than 4Q, and in Taylor-US GTE there are none greater than 8. In all others, the SI, Calvo-GTE, Indexed Calvo, BK-GTE, Calvo-GTE there are some very long contracts which for practical computational purposes are truncated.

This brings us to our conclusion about γ. When there is a distribution of contracts, an decrease in γ will tend to delay the maximum impact if there is already a hump shape. Note also that the assumption of indexation can affect the timing of the inflation peak in models with one type of contract length if γ is sufficiently low but it has a larger effect on the inflation peak in models with a distribution.

6.1 Output and inflation.

> "We cannot be precise about the size or timing of all these channels. But the maximum effect on output is estimated to take up to about one year. And the maximum impact of a change in interest rates on consumer price inflation takes up to about two years" the Bank of England's webpage[12].

Up until now we have been looking at inflation persistence on its own. However, it has long been argue that inflation peaks after output: the initial

[12] http://www.bankofengland.co.uk/monetarypolicy/how.htm

increase in output generates the upward pressure on prices. For example, Christiano et al. (2005) find that inflation peaks at 8Q and output at 4Q. The Bank of England Quarterly Model has inflation peaking at 6Q and output at 3Q. The above quote from the Bank of England on monetary policy reflects this conventional view.

Clearly, the dynamic of output and inflation are governed at the aggregate level by the quantity identity: (3): the sum of inflation and output growth must add up to the rate of monetary growth in each period. The role of γ is very important: it determines the inflationary pressure on wages and prices resulting from an increase in output. A low value of γ means that this inflationary pressure works through more slowly so that the reaction of inflation to output growth becomes slower. This also means that growth in output can be sustained for longer because the inflationary response is being delayed. The cumulative effect on inflation is fixed at 2%, since money is neutral in the long run. However, the cumulative affect on output will vary: the slower the response of inflation, the longer output can be above the long-run equilibrium.

For both Calvo and simple Taylor, whilst the timing of the peak impact on inflation does not depend on γ, the peak impact on output does. In the case of Calvo, inflation always peaks on impact, and the output hump peaks later with lower γ. For all of the models with a distribution of contract lengths (except Calvo), γ affects both the timing of the output and inflation humps, having a larger effect on the inflation peak.

$Table 4$: output and inflation

In Table 4 we state the timing of the peak output growth and the difference between the peak inflation and output. In the first table we can see that in all models, the lower is γ, the more delayed is the response of output. In the case of Calvo, inflation always peaks on impact, and the output hump peaks later with lower γ. This indicates that not only does the standard Calvo model generate the wrong inflation dynamics, but also reverses the relative timing of output and inflation peaks providing yet another reason against the use of the Calvo model. In the simple Taylor and Fischer models, inflation peaks at the length of the contract, output peaks earlier with a smaller gap when γ is lower.

The most interesting thing to note about Table 4 is that for all of the models *except* simple Taylor, Simple Fischer and Calvo, not only is the peak inflation after the output hump, but also the gap between the humps increases as γ gets smaller. We can illustrate this with the BK-GTE.

$Fig14$: output and inflation in $BK - GTE$

In Fig 14a we depict the IR for output and inflation with the reference value of $\gamma = 0.2$ there is a hump in inflation which peaks at $2Q$ and output at $3Q$. However, with the lower value of $\gamma = 0.01$, inflation peaks at $7Q$ and output at $4Q$.

7 Conclusion

We have used the Generalized Taylor Economy, GTE, framework to evaluate the performance of competing approaches for modelling nominal rigidities in the DGE models based on their potential in accounting for the observed inflation dynamics. Recent empricial studies find that a monetary shock leads to a hump shaped persistent inflation response. Due to the lack of consensus in the literature about the timing of the hump peak, we allow for three different values which corresponds to the moderate view (8Q), the Hawkish view (12Q) and the rapid view (4Q). We show how the assumptions regarding the distribution of contract lengths and the key parameters affect the models' implications on inflation dynamics. Our findings can be summarised as follows:

- In models with one type of contract length inflation always reaches its peak $T - 1$ quarters after the first period in which shock occurs. However, this type of model fails to generate enough persistence to satisfy feature 3 with our reference value of $\gamma = 0.2$. When we allow for a low value of γ, which can be obtained, for example, by introducing firm specific capital, the problem goes away in the case of Taylor contracts. If you take the view that the hump peaks at 4 quarters, then the Simple Taylor economy with average contract length of 4 can meet the three stylised facts. However, going beyond the 4 quarters would only be possible if the model has long contracts. On the other hand, the simple Fischer model never generates enough persistence to satisfy Feature 3 unless it has contract lengths in excess of 20 quarters.

- The presence of a distribution of contract lengths has significant implications on the models' performance in accounting for the inflation dynamics. In particular, these models do not have problem in generating persistent inflation response with plausible parameter values. The main problem in this class of models is the timing of the hump. When $\gamma = 0.2$, most models cannot even make $4Q$. The timing of the hump seems to be close with the most common contract durations. However, with a low value of γ,these models can generate a hump at $8Q$ and even beyond. In this case, the predictions of such models are also consistent with the common view that inflation peaks after output, and the gap between the two peaks is about 4 Quarters.

- The Calvo model does not capture inflation dynamics: inflation always peaks on impact, and this precedes the peak in output. As γ gets smaller, it does not affect the timing of the peak in inflation, but it does affect the shape of the IR, and low values make the affect die off more slowly after the peak. This model should not be used to model inflation dynamics.

- The assumption of indexation can affect the timing of the inflation hump, having a larger effect on the inflation peak in models with a distribution. In particular, IC and $IC - GTE$ have very similar implications on inflation dynamics. However, the lack of direct evidence for the indexation makes this class of models less plausible.

These findings lead us to conclude that models with a distribution, along with other frictions which helps to lower γ, can be fairly successful in explaining the inflation dynamics following a monetary shock. In fact, this finding, to a large extent, explains the result obtained by Coenen and Levin (2004). Coenen and Levin (2004) show that a model with Taylor style contracts that allows for firm specific capital and has a distribution of contract lengths fits the German data very well without needing the assumption of backward-looking indexation. The same results also indicate that stronger microfoundations are required in future work to include the distribution of contract lengths.

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Appendix.

A Model

In this section, we describe the GTE framework in detail and then discuss the modifications required to the model when we consider different assumption regarding the wage-setting.

A.1 Generalised Taylor Economy (GTE).

A.1.1 Firms

There is a continuum of firms $f \in [0,1]$, each producing a single differentiated good $Y(f)$, which are combined to produce a final consumption good Y. The production function here is CES with constant returns and corresponding unit cost function P

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{\theta-1}{\theta}} df\right]^{\frac{\theta}{\theta-1}} \tag{9}$$

$$P_t = \left[\int_0^1 P_{ft}^{1-\theta} df\right]^{\frac{1}{1-\theta}} \tag{10}$$

The demand for the output of firm f is

$$Y_{ft} = \left(\frac{P_{ft}}{P_t}\right)^{-\theta} Y_t \tag{11}$$

Each firm f sets the price P_{ft} and takes the firm-specific wage rate W_{ft} as given. Labor L_{ft} is the only input so that the inverse production function is

$$L_{ft} = \left(\frac{Y_{ft}}{\alpha}\right)^{\frac{1}{\sigma}} \tag{12}$$

Where $\sigma \leq 1$ represents the degree of diminishing returns, with $\sigma = 1$ being constant returns. The firm chooses $\{P_{ft,} Y_{ft}, L_{ft}\}$ to maximize profits subject to (11,12), yields the following solutions for price, output and employment at the firm level given $\{Y_t, W_{ft}, P_t\}$

$$P_{ft} = \left(\frac{\theta-1}{\theta}\right)\frac{\alpha^{-1/\sigma}}{\sigma}W_{ft}Y_{ft}^{\frac{1-\sigma}{\sigma}} \tag{13}$$

$$Y_{ft} = \kappa_1\left(\frac{W_{ft}}{P_t}\right)^{-\sigma\varepsilon}Y_t^{\frac{\varepsilon\sigma}{\theta}} \tag{14}$$

$$L_{ft} = \kappa_2\left(\frac{W_{ft}}{P_t}\right)^{-\varepsilon}Y_t^{\frac{\varepsilon}{\theta}} \tag{15}$$

where $\varepsilon = \frac{\theta}{\theta(1-\sigma)+\sigma} > 1 \quad \kappa_1 = \left(\frac{\theta-1}{\theta}\right)^{-\sigma\varepsilon}\sigma^{-\sigma\varepsilon}\alpha^{-\varepsilon} \quad \kappa_2 = \left(\frac{\theta-1}{\theta}\right)^{-\varepsilon}\sigma^{\varepsilon}\alpha^{\varepsilon\left(\frac{\theta-1}{\theta}\right)}$.

Price is a markup over marginal cost, which depends on the wage rate and the output level (when $\sigma < 1$): output and employment depend on the real wage and total output in the economy.

A.1.2 Uniform GTE: the structure of contracts.

A.1.3 Household-Unions and Wage Setting

Households $h \in [0,1]$ have preferences defined over consumption, labour, and real money balances. The expected life-time utility function takes the form

$$U_h = E_t\left[\sum_{t=0}^{\infty}\beta^t u(C_{ht}, \frac{M_{ht}}{P_t}, \underbrace{1-H_{ht}}_{L_{ht}})\right] \tag{16}$$

where C_{ht}, $\left(\frac{M_{ht}}{P_t}\right)$, H_{ht}, L_{ht} are household $h's$ consumption, end-of period real money balances, hours worked, and leisure respectively, t is an index for time, $0 < \beta < 1$ is the discount factor, and each household has the same flow utility function u, which is assumed to take the form

$$U(C_{ht}) + \delta\ln(\frac{M_{ht}}{P_t}) + V(1-H_{ht}) \tag{17}$$

Each household-union belongs to a particular sector and wage-setting cohort within that sector (recall, that each household is twinned with firm $f = h$). Since the household acts as a monopoly union, hours worked are demand determined, being given by the (15).

The household's budget constraint is given by

$$P_tC_{ht}+M_{ht}+\sum_{s_{t+1}} Q(s^{t+1} \mid s^t)B_h(s^{t+1}) \leq M_{ht-1}+B_{ht}+W_{ht}H_{ht}+\pi_{ht}+T_{ht} \quad (18)$$

where $B_h(s^{t+1})$ is a one-period nominal bond that costs $Q(s^{t+1} \mid s^t)$ at state s^t and pays off one dollar in the next period if s^{t+1} is realized. B_{ht} represents the value of the household's existing claims given the realized state of nature. M_{ht} denotes money holdings at the end of period t. W_{ht} is the nominal wage, π_{ht} is the profits distributed by firms and $W_{ht}H_{ht}$ is the labour income. Finally, T_t is a nominal lump-sum transfer from the government.

The households optimization breaks down into two parts. First, there is the choice of consumption, money balances and one-period nominal bonds to be transferred to the next period to maximize expected lifetime utility (16) given the budget constraint (18). The first order conditions derived from the consumer's problem are as follows:

$$u_{ct} = \beta R_t E_t \left(\frac{P_t}{P_{t+1}} u_{ct+1} \right) \quad (19)$$

$$\sum_{s_{t+1}} Q(s^{t+1} \mid s^t) = \beta E_t \frac{u_{ct+1} P_t}{u_{ct} P_{t+1}} = \frac{1}{R_t} \quad (20)$$

$$\delta \frac{P_t}{M_t} = u_{ct} - \beta E_t \frac{P_t}{P_{t+1}} u_{ct+1} \quad (21)$$

Equation (19) is the Euler equation, (20) gives the gross nominal interest rate and (21) gives the optimal allocation between consumption and real balances. Note that the index h is dropped in equations (19) and (21), which reflects our assumption of complete contingent claims markets for consumption and implies that consumption is identical across all households in each period $(C_{ht} = C_t)$[13].

Household h in sector i chooses wage to maximize lifetime utility given labour demand (15) and the additional constraint that nominal wage will be fixed for T_i periods in which the aggregate output and price level are given$\{Y_t, P_t\}$. Since the reset wage at time t will only hold for T_i periods, we have the following first-order condition:

[13] See Ascari (2000).

$$X_{it} = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \left[\frac{E_t \sum_{s=0}^{T_i-1} \beta^s \left[V_L \left(1 - H_{t+s}\right) \left(K_{t+s}\right) \right]}{E_t \sum_{s=0}^{T_i-1} \beta^s \left[\frac{u_c(C_{t+s})}{P_{t+s}} K_{t+s} \right]} \right] \tag{22}$$

where $K_t = \kappa_2 P_t^{\varepsilon} Y_t^{\frac{\varepsilon}{\theta}}$ collects all of the terms in (15) which the union treats as exogenous.

Equation (22) shows that the optimal wage is a constant "mark-up" (given by $\frac{\varepsilon}{\varepsilon-1}$) over the ratio of marginal utilities of leisure and marginal utility from consumption within the contract duration, from t to $t+T_i-1$. When $T_i = 2$, this equation reduces to the first order condition in Ascari (2000).

A.2 Generalised Fischer Economy (GFE)

In a model with Fischer (1977) contracts, household h in sector i chooses chooses a trajectory of wages, one for each period for the whole length of the contract. The first order condition now becomes:

$$X_{it+s} = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \left[\frac{E_t \left[V_L \left(1 - H_{t+s}\right) \right]}{E_t \left[\frac{u_c(C_{t+s})}{P_{t+s}} \right]} \right] \tag{23}$$

where $s = 0...T_i - 1$. Note that the equation is identical to the the wage level in which wages are fully flexible.

A.3 Calvo with Indexation: Woodford (2003)

As in the simple Calvo (1983) pricing model, there is the reset probability or hazard rate ω, which gives the non-duration dependent probability that a firm/union will have the option to reset its wage in any period and household h chooses wage to maximize lifetime utility given labour demand (15) during the lifetime of the contract. But now maximization problem also includes lagged inflation due to the assumption of indexation. The first order condition is given by:

$$X_t = \left(\frac{\varepsilon}{\varepsilon - 1}\right) \left[\frac{E_t \sum_{s=0}^{\infty} ((1-\omega)\beta)^s \left[V_L \left(1 - H_{t+s}\right) \left(K_{t+s}\right) \right]}{E_t \sum_{s=0}^{\infty} ((1-\omega)\beta)^s \left[\frac{u_c(C_{t+s})}{P_{t+s}} \left(\frac{P_{t-1+i}}{P_{t-1}}\right)^a K_{t+s} \right]} \right] \tag{24}$$

where $0 \leq a \leq 1$ measures the degree of indexation to the past inflation rate. $a = 0$ gives the wage setting rule for the simple Calvo Economy. Note that the index i is dropped in this equation due reason that in the Calvo pricing model there is only one reset wage (see Dixon and Kara (2005b) for a discussion).

A.4 The *GTE* with Indexation

When we allow for indexation in the GTE, the optimal wage rule changes from (22) to:

$$X_{it} = \left(\frac{\varepsilon}{\varepsilon - 1}\right)\left[\frac{E_t \sum_{s=0}^{T_i-1} \beta^s \left[V_L \left(1 - H_{t+s}\right)\left(K_{t+s}\right)\right]}{E_t \sum_{s=0}^{T_i-1} \beta^s \left[\frac{u_c(C_{t+s})}{P_{t+s}} \left(\frac{P_{t-1+i}}{P_{t-1}}\right)^a K_{t+s}\right]}\right] \quad (25)$$

B Linearized Model

This section presents equation which are the linearized counterparts to the equations outlined in the pervious section of the appendix. We start with the wage-setting rules for different contracts. Lower-case letters denotes log-deviations of variables from the steady state

B.1 Generalised Taylor Economy (GTE).

The linearized version of the equations described in the previous section are follows. As in Dixon and Kara (2005b), the wage setting rule is summarised for the reset wage which is set by the cohort changing contracts at t and remains in force for i periods.

$$x_{it} = \frac{1}{\sum_{s=0}^{T_i-1} \beta^s}\left[\sum_{s=0}^{T_i-1} \beta^s \left[p_{t+s} + \gamma y_{t+s}\right]\right] \quad (26)$$

where

$$\gamma = \frac{\eta_{LL} + \eta_{cc}(\sigma + \theta(1-\sigma))}{\sigma + \theta(1-\sigma) + \theta\eta_{LL}} \quad (27)$$

The rest of the equations are given by

$$w_{it} = \sum_{j=0}^{T_i-1} \frac{1}{T_i} x_{it-j} \tag{28}$$

$$p_{it} = w_{it} + \left(\frac{1-\sigma}{\sigma}\right) y_{it} \tag{29}$$

$$y_{it} = \theta(p_t - p_{it}) + y_t \tag{30}$$

$$p_t = \sum_{i=1}^{N} \alpha_i p_{it} \tag{31}$$

$$y_t = m_t - p_t \tag{32}$$

In addition, the money supply follows a AR(1) process,

$$m_t = m_{t-1} + \ln(\mu_t), \quad ln(\mu_t) = v\mu_{t-1} + \xi_t \tag{33}$$

where $0 < v < 1$ and ξ_t is a white noise process with zero mean and a finite variance.

B.2 Generalised Fischer Economy (GFE).

Log-Linearising version of (23) is given by

$$x_{it+s} = p_{t+s} + \gamma y_{t+s} \tag{34}$$

Sectoral wage index can be expressed as

$$w_{it} = \sum_{s=0}^{T_i} \frac{1}{T_i} E_{t-s} x_{it} \tag{35}$$

Replacing (26) and (28) with (34) and (35) from the above equations, respectively, gives the equilibrium conditions in the GFE.

B.3 Calvo with Indexation: Woodford (2003).

Log-Linearising (24) and putting for simplicity $\beta = a = 1$ yields

$$x_t = \omega\left(p_t + \gamma y_t\right) - (1-\omega)\,\pi_t + (1-\omega)\,x_{t+1} \tag{36}$$

The evolution of the aggregate wage index is given by

$$w_t = \omega x_t + (1-\omega)\left(w_{t-1} + \pi_{t-1}\right) \tag{37}$$

Finally, the aggregate price index is

$$p_t = w_t + \left(\frac{1-\sigma}{\sigma}\right) y_t \tag{38}$$

By inserting (38) and (37) into (36), after some algebra, we obtain the hybrid phillips curve reported in the main text.

B.4 GTE with Indexation.

When we allow partial indexation in the GTE, the wage setting rule changes to

$$x_t = \frac{1}{\sum_{s=0}^{T_i-1}\beta^s}\left[\sum_{s=0}^{T_i-1}\beta^{si} p_{t+s} + \gamma\sum_{s=0}^{T_i-1}\beta^s y_{t+s} - \sum_{s=0}^{T_i-2}\sum_{k=s+1}^{T_i-1}\beta^k \pi_{t+s}\right] \tag{39}$$

Sectoral wage index changes from (28) to

$$w_{it} = \sum_{j=0}^{T_i-1}\frac{1}{T_i}\left(x_{it-j} - \pi_{t-1-j}\right)$$

B.5 Fuhrer and Moore (1995) and $FM - GTE$.

Fuhrer and Moore's model does not have microfoundations. We modified the equations to allow for an economy with many sectors, each with a Fuhrer and Moore contract of a particular length. In this case, unions care about relative real wages within the contract duration.

$$x_{it} - p_t = \sum_{s=0}^{T_i - 1} f_s v_{t+s} + \gamma \sum_{i=0}^{T_i - 1} f_s y_{t+s} \tag{40}$$

where $f_s = \frac{1}{T_i}$.
The aggregate index of real wages is given by

$$v_t = \sum_{i=1}^{N} \alpha_i v_{it} \tag{41}$$

where

$$v_{it} = \sum_{j=0}^{T_i - 1} \frac{1}{T_i} (x_{it-s} - p_{t-s})$$

Equations (40) and (41) along with the equations (28)-(32) characterize the equilibrium in the FM-GTE. When $T_i = 4$ and $f_s = 0.25 + (1.5 - s)q$, $q \in (0, \frac{1}{6})$, (41) reduces to the wage setting rule as in Fuhrer and Moore (1995).

	γ=0.2			γ=0.1			γ=0.05			γ=0.01			γ=0.005		
	Feature1	Feature2	Feature3	Feature1	Feature2	Feature3	Feature1	Feature2	Feature3	Feature1	Feature2	Feature3	Feature1	Feature2	Feature3
Calvo; ω=0.40			✓			✓✓			✓✓			✓✓			✓✓
Calvo; ω=0.25			✓✓			✓✓			✓✓			✓✓			✓✓
STE; T=4	✓	✓		✓	✓		✓	✓	✓	✓	✓	✓✓	✓	✓	✓✓
STE; T=8	✓	✓✓	✓✓	✓	✓✓	✓✓	✓	✓✓	✓✓	✓	✓✓	✓✓	✓	✓✓	✓✓
Taylor-US GTE	✓			✓		✓	✓	✓	✓✓	✓	✓	✓✓	✓	✓	✓✓
Calvo-GTE; ω=0.40	✓			✓		✓	✓	✓	✓✓	✓	✓	✓✓	✓	✓	✓✓
Calvo-GTE; ω=0.25	✓		✓✓	✓	✓	✓✓	✓	✓	✓✓	✓	✓✓	✓✓	✓	✓✓	✓✓
BK-GTE	✓		✓	✓		✓✓	✓		✓✓	✓	✓	✓✓	✓	✓✓	✓✓
GTE(with FM distr)	✓			✓			✓	✓	✓	✓	✓	✓✓	✓	✓	✓✓
FM	✓	✓		✓	✓		✓	✓	✓	✓	✓		✓	✓✓	✓✓
SI; ω=0.40	✓			✓	✓		✓	✓		✓	✓✓	✓	✓	✓✓	✓
SI; ω=0.25	✓	✓	✓	✓	✓✓	✓✓	✓	✓✓	✓✓	✓	✓✓✓	✓✓	✓	✓✓✓	✓✓
SFE; T=4	✓	✓		✓	✓		✓	✓			✓		✓	✓	
SFE; T=8	✓	✓✓		✓	✓✓		✓	✓✓		✓	✓✓		✓	✓✓	
IC; ω=0.40	✓	✓		✓	✓		✓	✓		✓	✓✓	✓✓	✓	✓✓	✓✓
IC; ω=0.25	✓	✓		✓	✓	✓✓	✓	✓✓	✓✓	✓	✓✓✓	✓✓	✓	✓✓✓	✓✓
IC-GTE; ω=0.40	✓	✓		✓	✓		✓	✓		✓	✓✓	✓✓	✓	✓✓	✓✓
IC-GTE; ω=0.25	✓	✓		✓	✓		✓	✓✓	✓✓	✓	✓✓✓	✓✓	✓	✓✓✓	✓✓
Indexed-STE; T=4	✓	✓		✓	✓		✓	✓		✓	✓		✓	✓✓	✓✓

Table 2. Features 1-3 as γ varies

γ	0.2	0.1	0.05	0.01	0.005
Calvo; ω=0.40	1	1	1	1	1
Calvo; ω=0.25	1	1	1	1	1
STE; T=4	4	4	4	4	4
STE; T=8	8	8	8	8	8
Taylor-US GTE	3	3	4	5	5
Calvo-GTE; ω=0.40	2	3	4	5	6
Calvo-GTE; ω=0.25	3	4	5	9	11
BK-GTE	2	2	2	7	10
GTE(with FM distr)	3	3	4	4	4
FM	4	4	4	7	8
SI; ω= 0.40	3	5	6	9	11
SI; ω= 0.25	5	8	11	16	19
SFE; T=4	4	4	4	4	4
SFE; T=8	8	8	8	8	8
IC; ω= 0.40	4	5	6	9	11
IC; ω= 0.25	5	6	8	13	16
IC-GTE; ω= 0.40	4	4	6	9	11
IC-GTE; ω= 0.25	5	6	8	13	16
Indexed-STE; T = 4	4	4	5	7	9

Table 3. The peak response of inflation (in quarters)

γ	0.2	0.1	0.05	0.01	0.005
Calvo; ω=0.40	-1	-2	-2	-3	-4
Calvo; ω=0.25	-2	-2	-3	-4	-4
STE; T=4	2	2	1	1	0
STE; T=8	5	5	4	3	3
Taylor-US GTE	1	1	1	2	1
Calvo-GTE; ω=0.40	0	0	1	1	2
Calvo-GTE; ω=0.25	0	1	1	4	6
BK-GTE	-1	-1	-1	3	5
GTE(with FM distr)	1	1	2	1	1
FM	2	2	2	4	5
SI; ω= 0.40	1	2	3	5	6
SI; ω= 0.25	2	4	7	10	13
SFE; T=4	2	2	1	1	1
SFE; T=8	4	4	3	2	2
IC; ω= 0.40	2	3	3	5	7
IC; ω= 0.25	2	3	5	9	11
IC-GTE; ω= 0.40	2	2	3	5	6
IC-GTE; ω= 0.25	2	3	5	8	11
Indexed-STE; $T = 4$	2	2	2	4	5

Table 4. the difference between the peaks

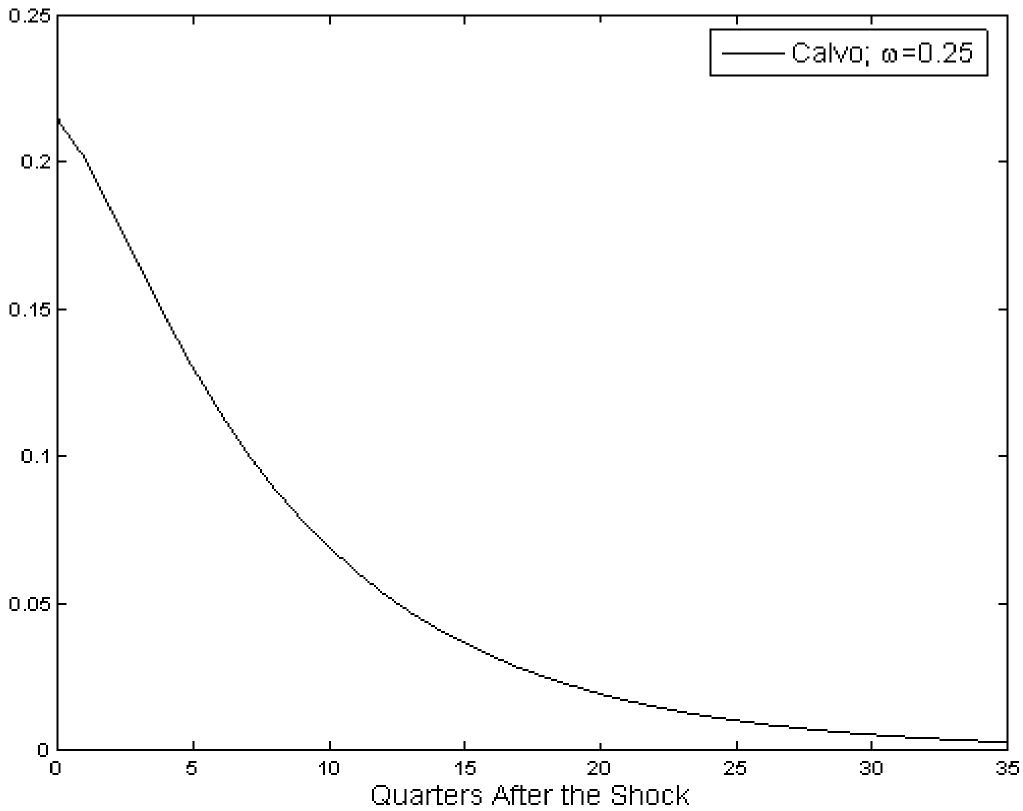

Figure 1: Response of Inflation in the Calvo Economy

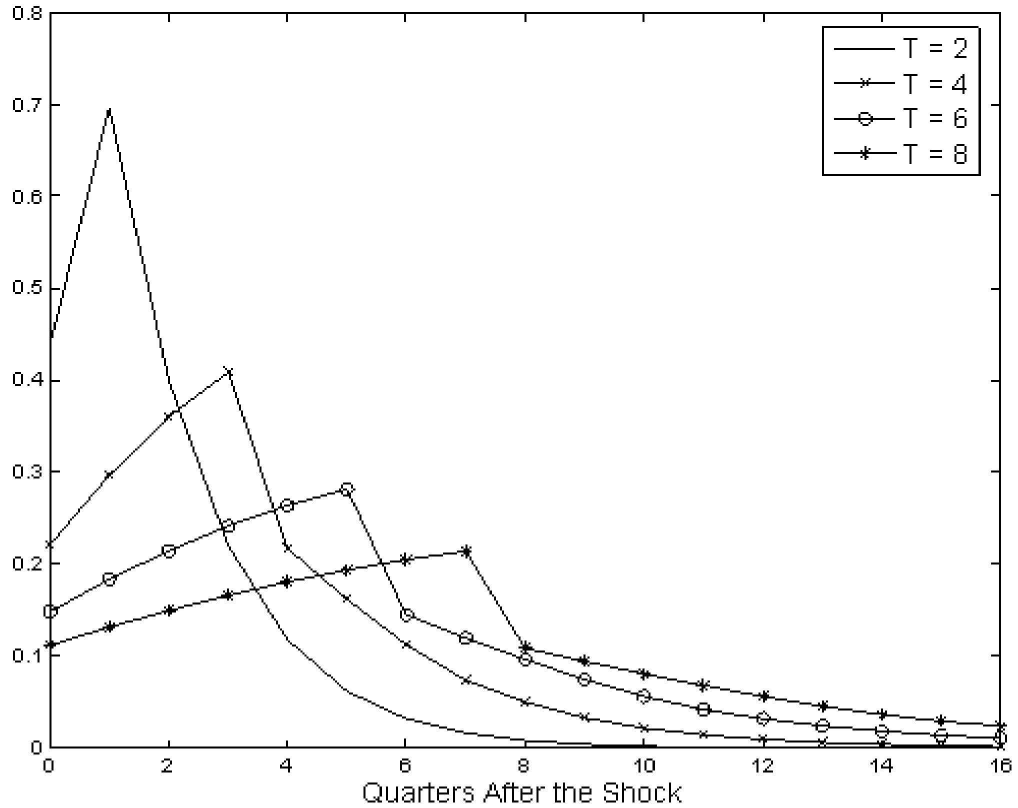

Figure 2: Response of Inflation in the Simple Taylor Economy

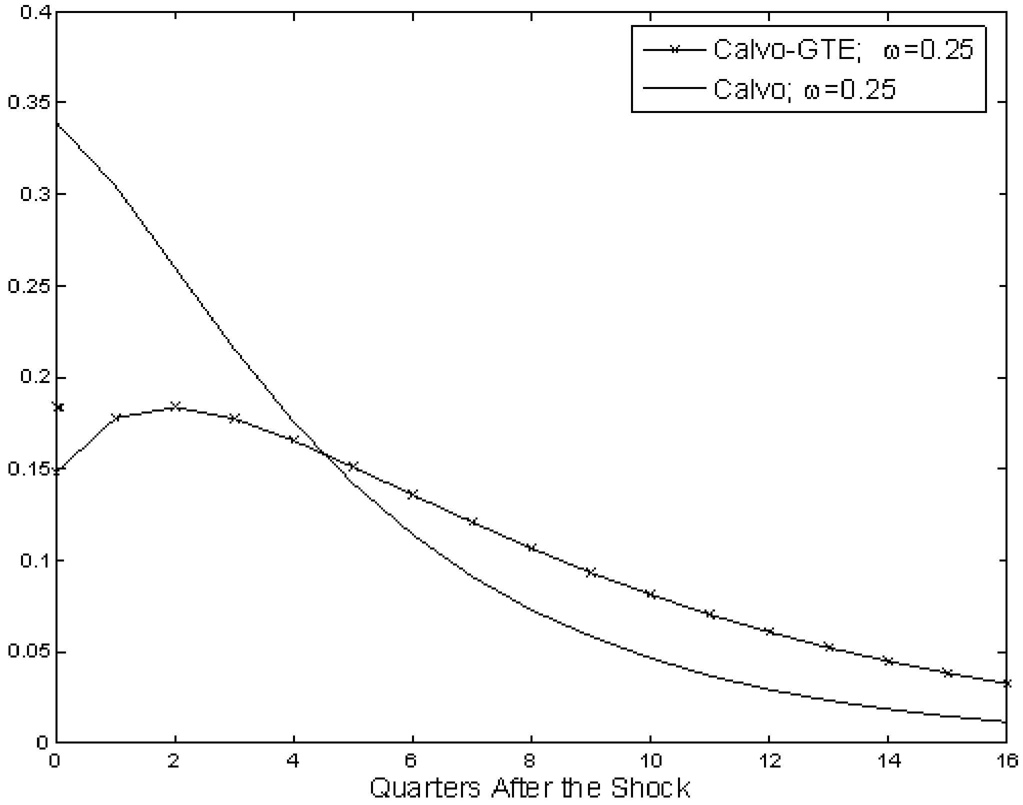

Figure 3: Response of Inflation in the Calvo-GTE and the Calvo Economy

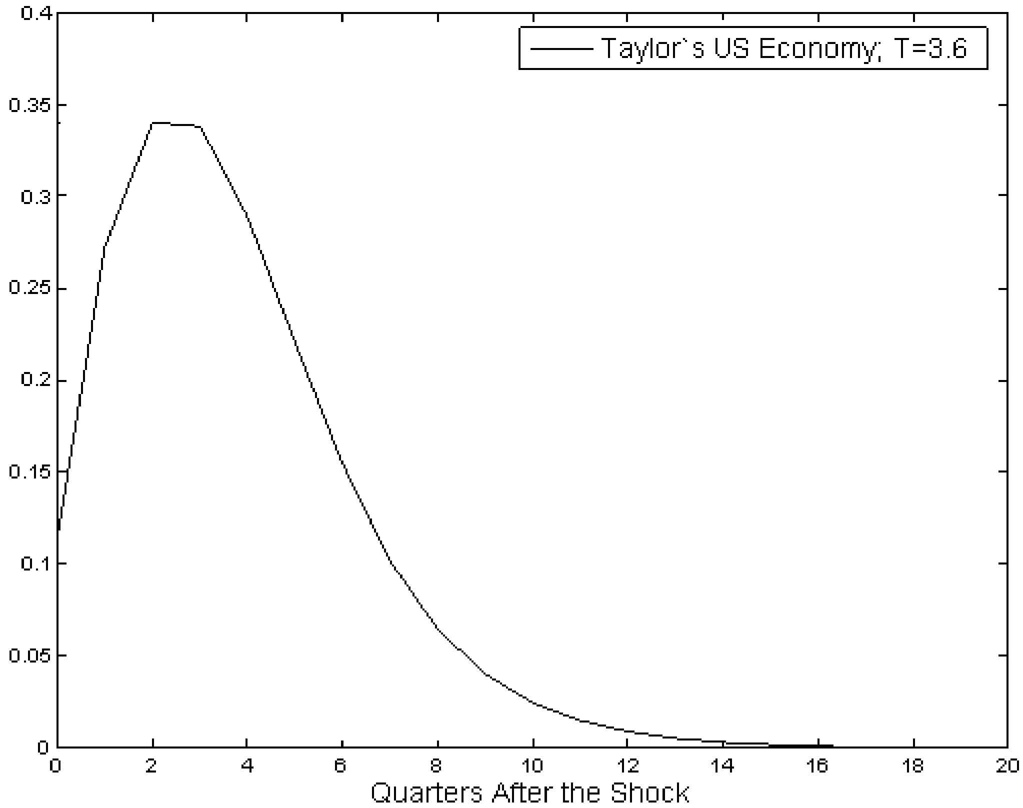

Figure 4: Response of Inflation in the Taylor's US Economy

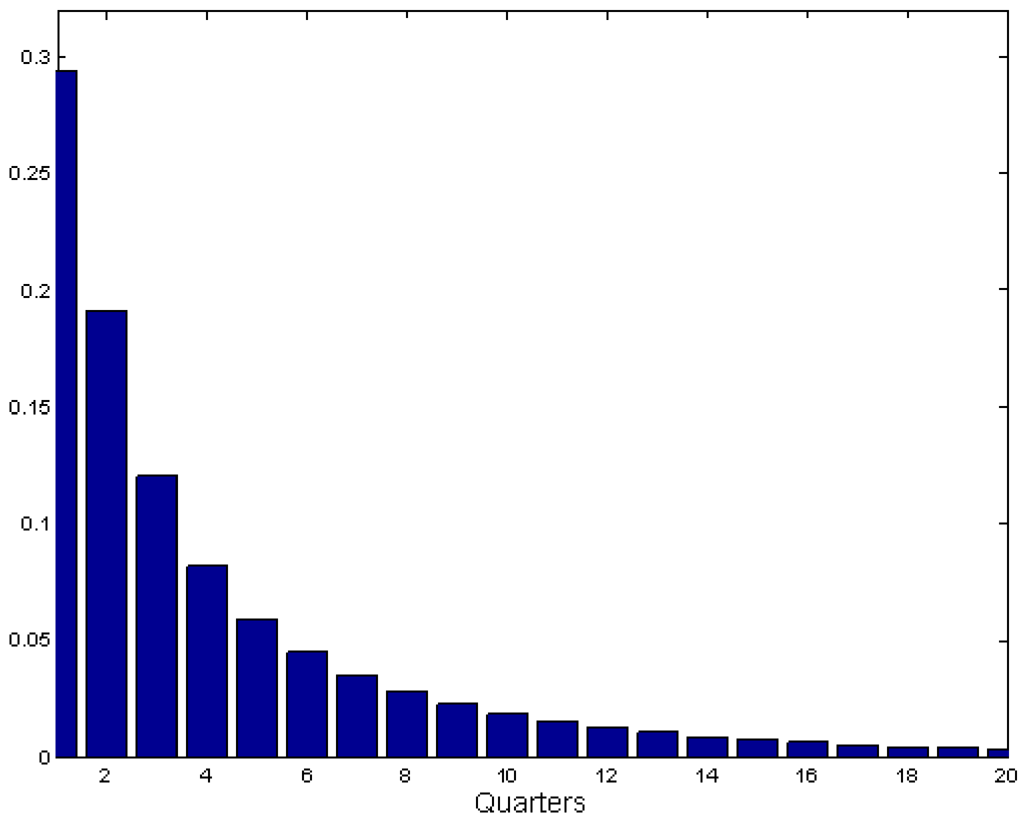

Figure 5: $BK - GTE$:Distribution

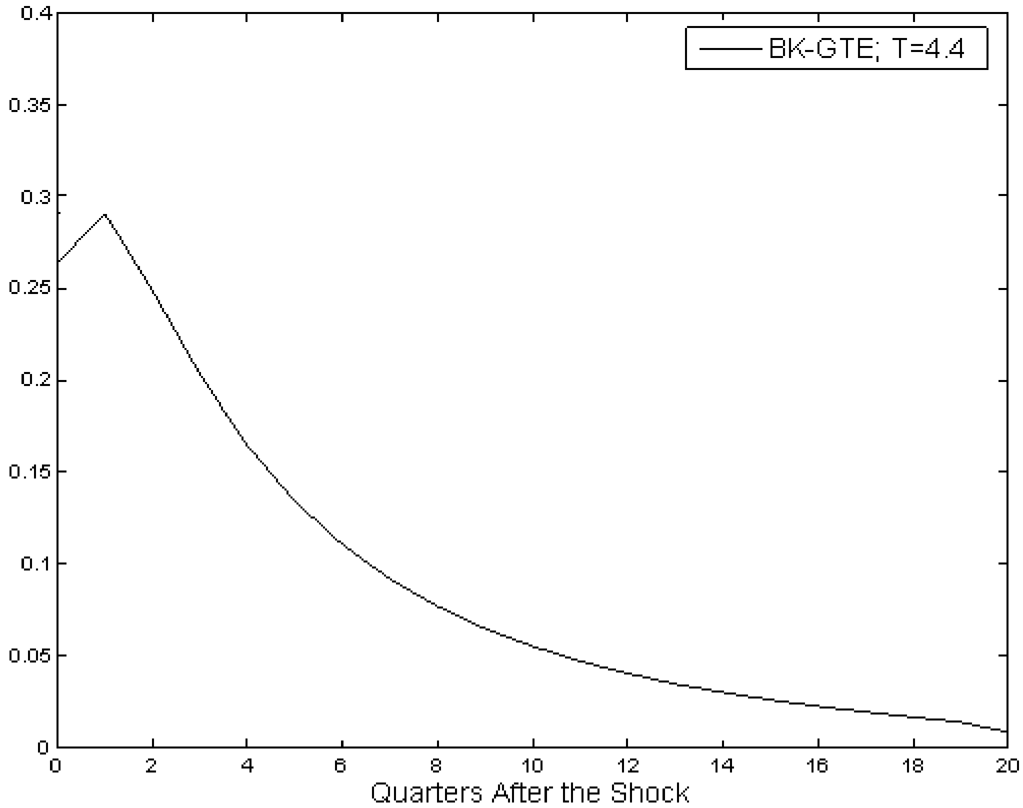

Figure 6: Response of Inflation in the BK-GTE

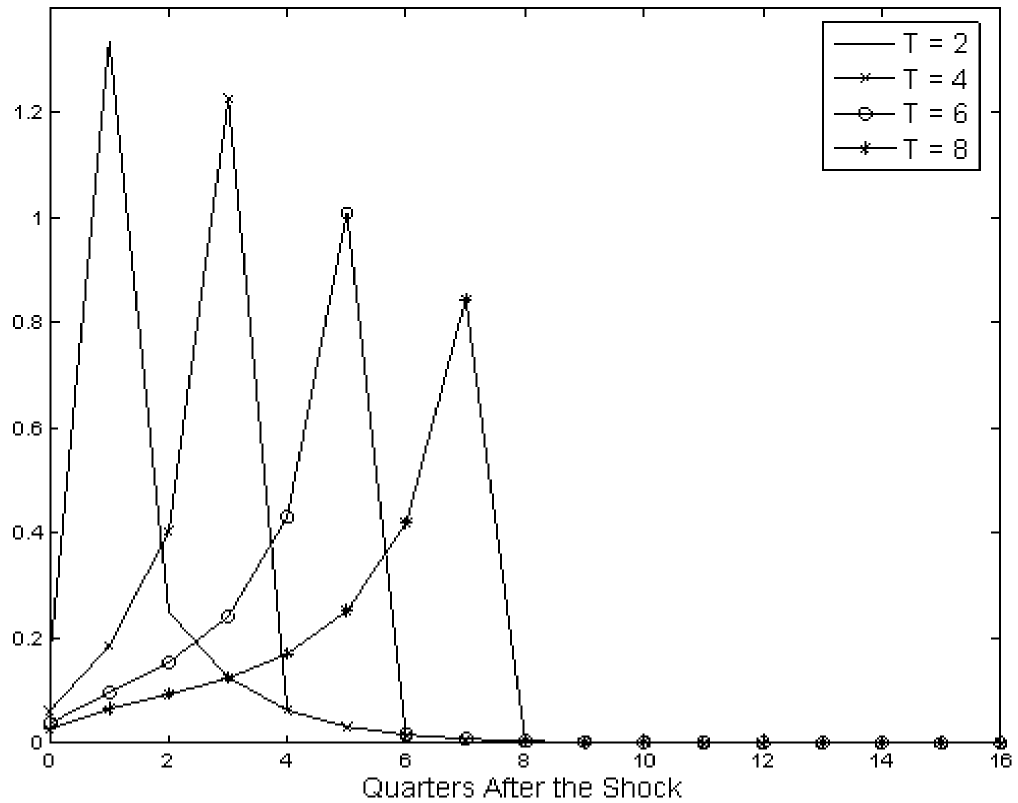

Figure 7: Response of Inflation in the Simple Fischer Economy

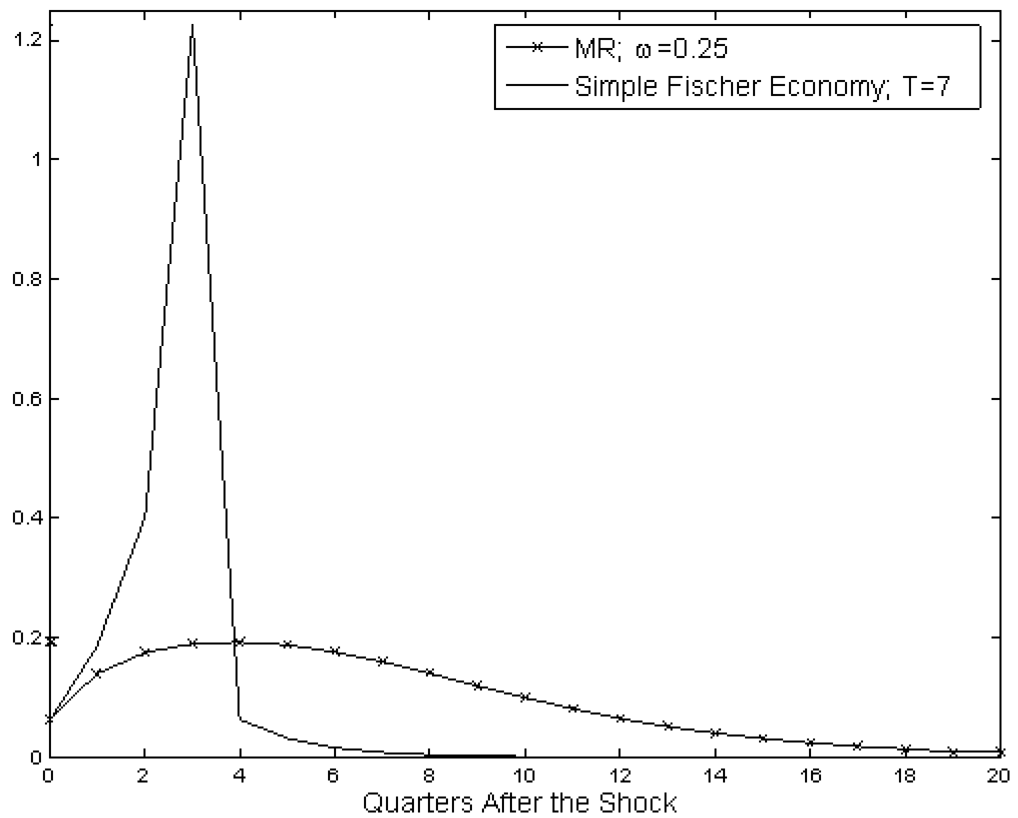

Figure 8: Response of Inflation in the Simple Fischer Economy and *SI*

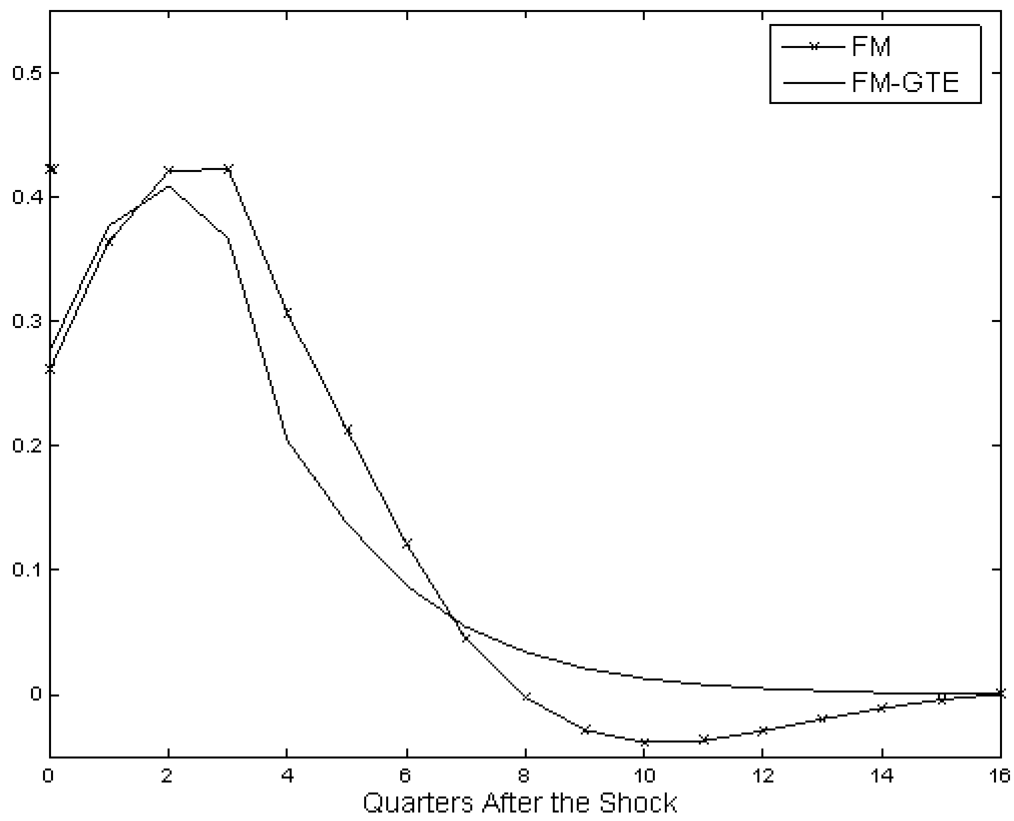

Figure 9: Response of Inflation in the FM and FM-GTE

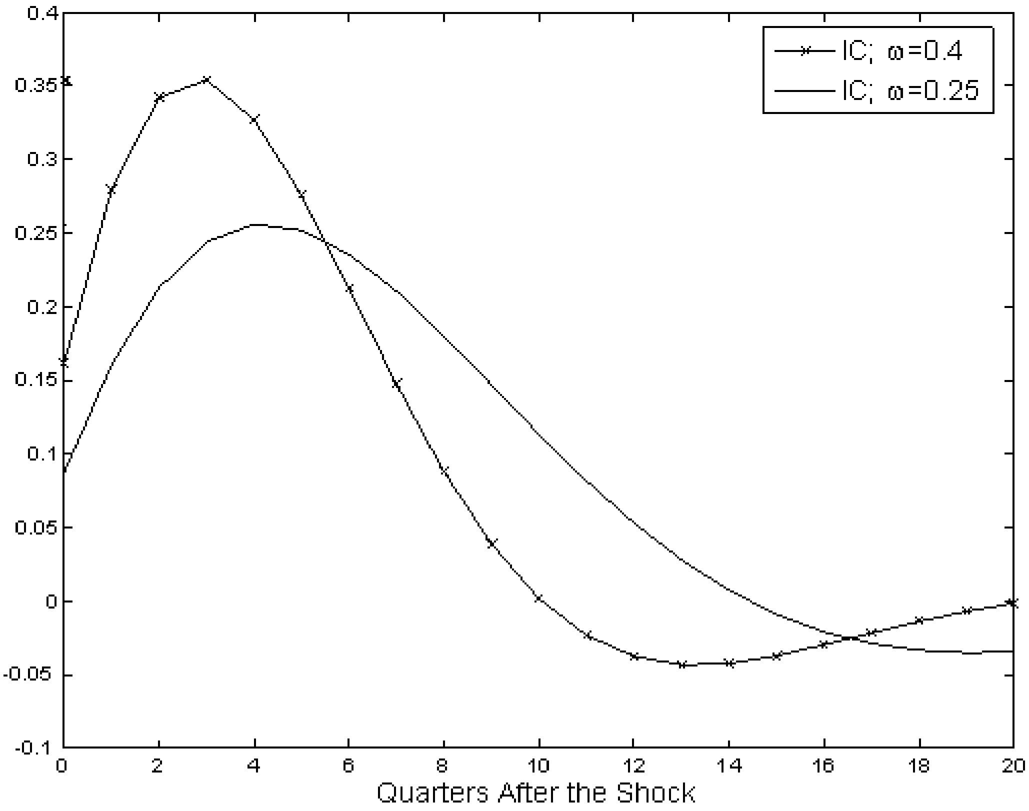

Figure 10: Response of Inflation in the Indexed-Calvo

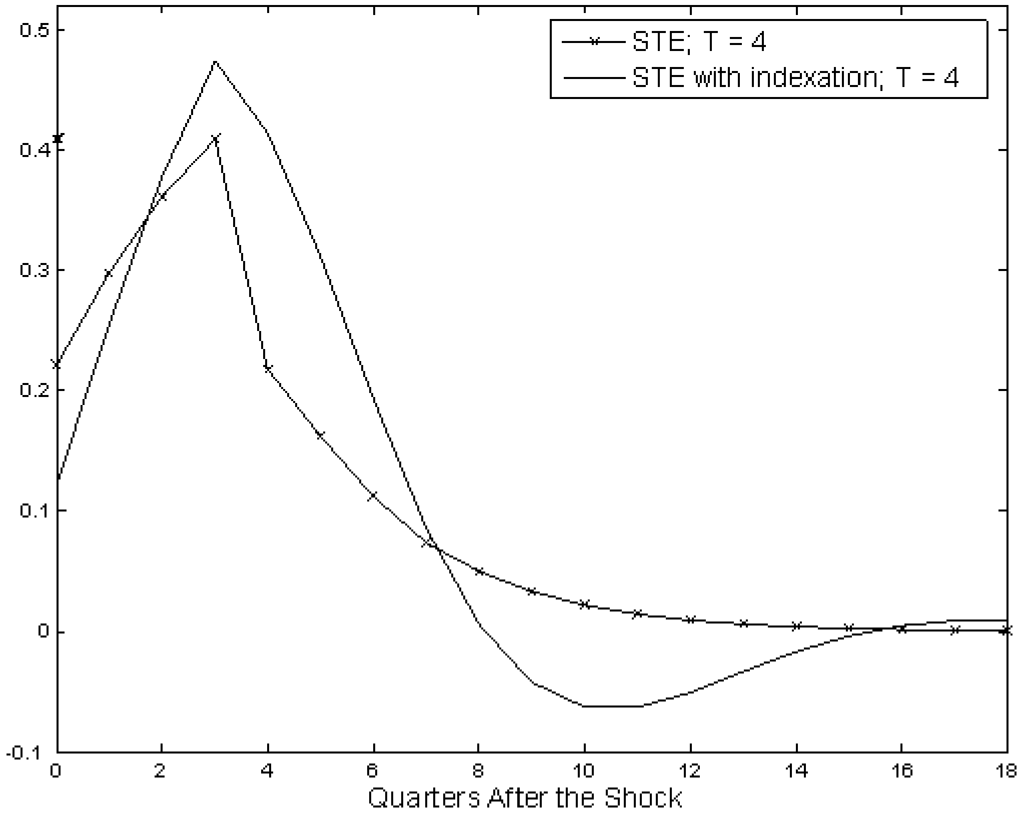

Figure 11: Response of Inflation in the Indexed-Taylor

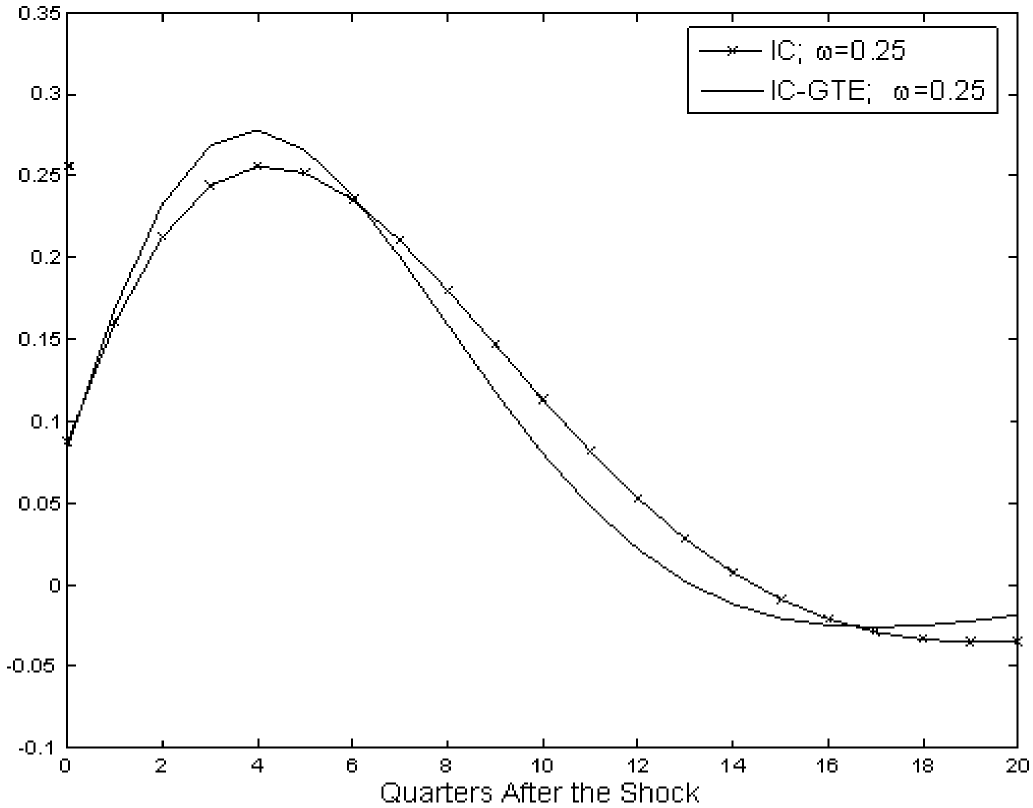

Figure 12: Response of Inflation in the IC-GTE and IC

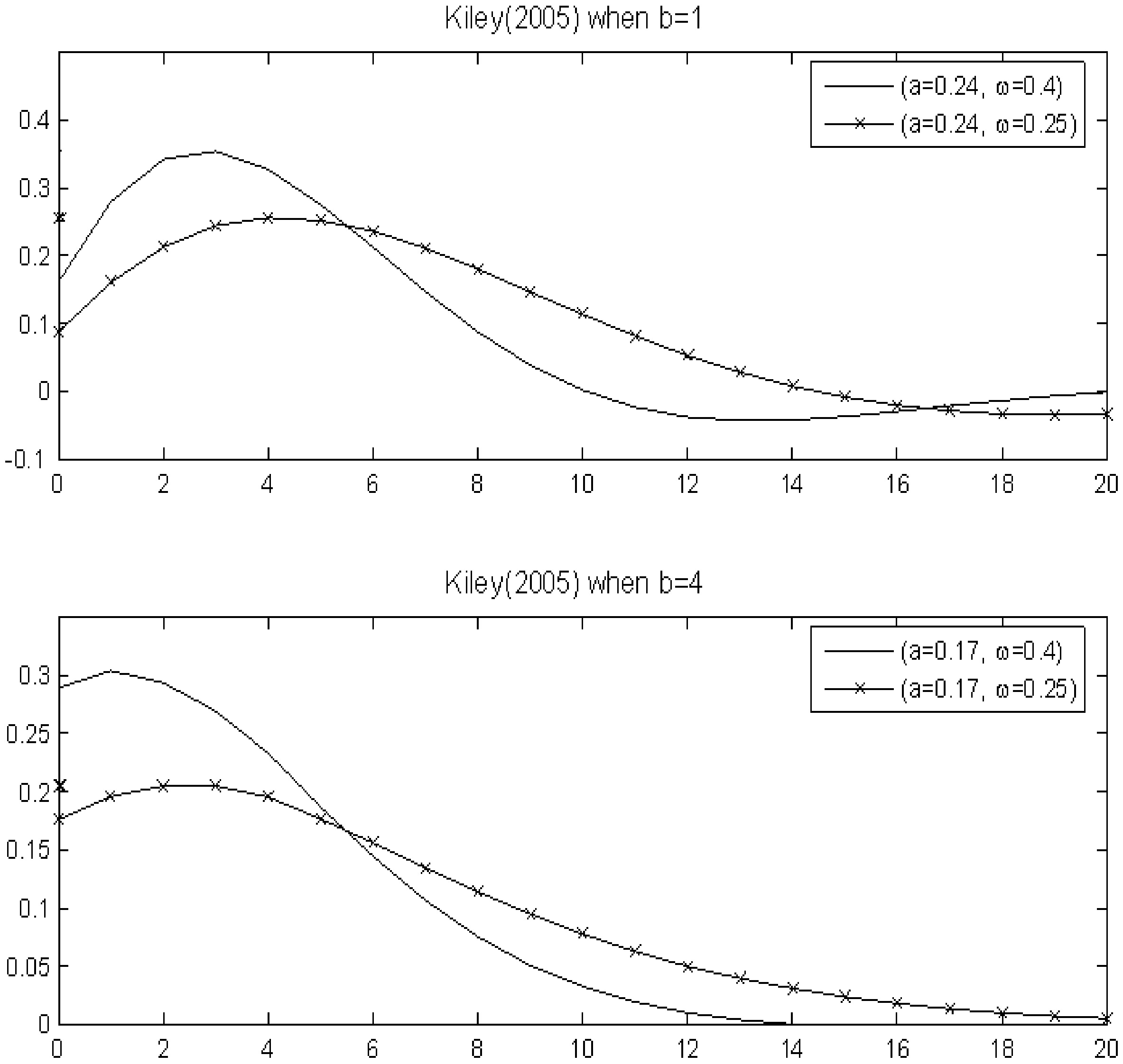

Figure 13: Kiley's Moving average indexation

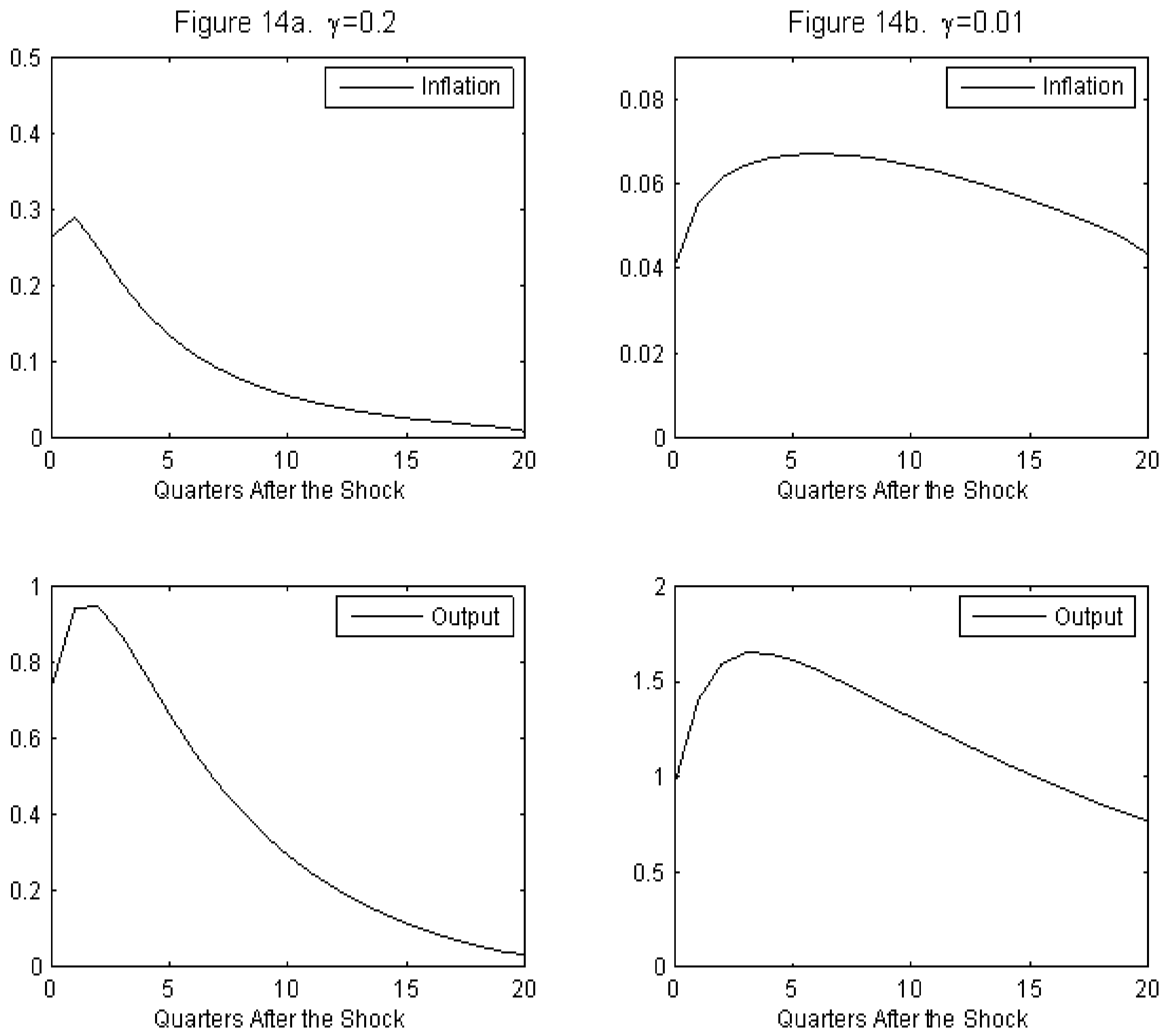

Figure 14: Output and Inflation Responses in $BK - GTE$