Policy-Induced Mean Reversion in the Real Interest Rate?

Zisimos Koustas* Jean-François Lamarche†

Department of Economics
Brock University
May 2005

Abstract

This paper utilizes tests for a unit root that have power against nonlinear alternatives to provide empirical evidence on the time series properties of the *ex-post* real interest rate in the G7 countries. We find that the unit-root hypothesis can be rejected in the presence of a nonlinear alternative motivated by theoretical literature on optimal monetary policy rules. This represents a reversal of the results obtained using standard linear unit-root and cointegration tests. Tests for linearity reject this hypothesis for Canada, France, Germany, Italy, and the US. For these countries we estimate nonlinear models representing the dynamics of the *ex-post* real interest rate.

Keywords: Fisher Effect; Unit Roots; Self-Exciting Threshold Autoregression;

JEL classification: E40; E50; C32

*Corresponding author. St. Catharines, Ontario L2S 3A1; Phone: (905) 688-5550; Fax: (905) 688-6388; E-mail: zisimos.koustas@brocku.ca

†Lamarche gratefully acknowledges support from SSHRC.

1 Introduction

The Fisher effect occurs when fully anticipated inflation has a unit effect on nominal interest rates; thus, leaving real interest rates unchanged. Theoretical support for the Fisherian conjecture is not universal. The proposition holds in models (e.g. Sidrauski, 1967) in which the real interest rate is determined by a relation like the modified golden rule and therefore does not depend on monetary variables. However, it is violated in models in which the Tobin effect applies - see Tobin (1965). In such models, an increase in the inflation rate leads to an increase in the opportunity cost of holding real money balances and a reallocation of wealth towards capital. With constant returns in the production technology, the marginal product of capital (the real interest rate) falls. Recent theoretical literature continues to be divided on this issue. For example, Cooley and Hansen (1989) support the Fisher effect while Drazen (1981), Weiss (1980), and Romer (1986) oppose it.[1]

It is clear that empirical evidence must be brought to bear on this issue in order to provide guidance for future theoretical modelling. Unfortunately, the voluminous empirical literature on this issue has not shown signs of convergence. While long-run money neutrality has received some empirical support, the evidence on long-run money superneutrality and the Fisher effect is mixed - see.Bullard (1999).

Recent work on the Fisher effect has focused on the time series properties of the macrovariables. Nominal interest rates and inflation rates appear to be nonstationary variables - i.e. contain a unit root. However, a linear combination of these variables - such as their difference - may be stationary. In such case, the real interest rate will be mean reverting and the two variables will be cointegrated with a cointegrating vector $(1, -1)$. Mishkin (1992) used post-war US data and found that the nominal interest rate and inflation are cointegrated. Nonetheless, he points out that the results were sensitive to the choice of time period. Crowder and Hoffman (1996), using tax-adjusted US data, found strong support for a cointegrating relationship between nominal interest rates and inflation. Further, they were unable to reject the hypothesis that the cointegrating vector was $(1, -1)$; thus, supporting a one-to-one relationship between inflation and interest rates.

Alternative testing strategies for long-run neutrality propositions based

[1] Orphanides and Solow (1990) provide an in-depth review of the earlier inflation-and-growth literature.

on bivariate structural vector autoregressions were developed by Fisher and Seater (1993) and King and Watson (1997). The King and Watson tests have been applied to international data sets and provided mostly negative results for the Fisher effect and long-run money superneutrality - see Weber (1994), Koustas (1998), Koustas and Serletis (1999).

One possible explanation for the seemingly conflicting empirical results on the Fisher effect is that there may have been one or more structural breaks during the sample period. Ignoring structural breaks, biases the unit-root tests in favour of accepting the null hypothesis of a unit root and against finding cointegration. Perron (1989) produced Monte Carlo evidence confirming this bias in the unit-root tests. Garcia and Perron (1996), among others, have tested for regime shifts in the real interest rate using US data for the period 1961-1985. They found evidence in favour of the Fisher effect once they allowed for occasional mean shifts in the real interest rate. However, Phillips (2005) points out that numerous mean shifts are required in order to characterize the data when longer samples on the real interest rate are used. One drawback of this approach is that it can lead to curve fitting with *ex post* rationalization of the regime shifts.

In this paper, we re-examine the empirical evidence on the Fisher effect by employing unit-root tests that are designed to have more statistical power against nonlinear alternatives. The tests used in most of the previous empirical literature are based on linear autoregressions in which the null hypothesis of a unit root is tested against a linear stationary alternative. Such tests have been shown to have low power in distinguishing between the unit-root model and a nonlinear but stationary alternative - see Pippenger and Goering (1993), Balke and Fomby (1997), Caner and Hansen (2001). The nonlinear alternative used in this paper is motivated by theoretical work on optimal monetary policy rules by Orphanides and Wilcox (2002).

It is assumed that the monetary authority implements policy through its influence on a short-term interest rate such as the federal funds rate. Modest deviations of inflation from its long-run target do not trigger policy action due to the uncertainty surrounding policy activism. Within a policy inaction band the real interest rate may exhibit weak or no mean reversion. On the contrary, incipient inflation prompts monetary policy action that re-enforces mean reversion in the real interest rate. Balke and Fomby (1997) studied mean reversion in the real exchange rate using a similar nonlinear framework. By allowing the real interest rate to exhibit different degrees of mean reversion over time, the nonlinear model provides a theoretically motivated and structured approach to understanding real interest rate persistence.

We obtain rejections of the unit-root hypothesis in the *ex-post* real interest rate for all of the G7 countries when we use a Self-Exciting Threhold Autoregression (SETAR) model as the alternative. This reverses the results from linear unit-root and cointegration tests. Our tests for linearity reject this hypothesis for five of the G7 economies for which we estimate SETAR models and test hypotheses of interest.

The next Section summarizes standard linear tests for the Fisher effect and outlines the proposed nonlinear alternative. Section 3 discusses the econometric methodology used to obtain the empirical results that are presented in Section 4. Finally, Section 5 closes with a brief summary and conclusion.

2 The Fisher Effect

2.1 The Linear Framework

Irving Fisher (1930) introduced the idea of the *ex-ante* real interest rate - nominal interest rate minus expected inflation - as the compensation of lenders based on their beliefs about inflation. If i_t^m denotes the m-period nominal interest rate at time t, $\pi_t^{e,m}$ the expected rate of inflation from time t to time $t+m$, and $r_t^{e,m}$ the corresponding *ex-ante* real interest rate, then the Fisher equation is

$$1 + i_t^m = (1 + r_t^{e,m})\,(1 + \pi_t^{e,m}) \tag{1}$$

For low rates of inflation, the Fisher equation can be approximated by

$$i_t^m = r_t^{e,m} + \pi_t^{e,m} \tag{2}$$

where both $r_t^{e,m}$ and $\pi_t^{e,m}$ are unobservable. Defining π_t^m as the m-period inflation rate, we obtain the following relation between the *ex-post* (r_t^m) and the *ex-ante* real interest rate:

$$r_t^m = i_t^m - \pi_t^m = r_t^{e,m} + (\pi_t^{e,m} - \pi_t^m) = r_t^{e,m} + \epsilon_t \tag{3}$$

where the inflation forecast error (ϵ_t) has a zero mean, is uncorrelated over time and orthogonal to any other economic variables under rational expectations.

Equation (3) suggests that the gap between the *ex-post* and the *ex-ante* real interest rate is stationary. Consequently, if it is found that the *ex-post* real interest rate is stationary, one may conclude that the *ex-ante* real interest rate is also stationary and the Fisher effect holds.

In the context of cointegration, equation (2) suggests that if the *ex-ante* real interest rate is stationary with mean α, the coefficient β should be unity in a regression of the form

$$i_t^m = \alpha + \beta \pi_t^{e,m} + \eta_t \tag{4}$$

and the residuals η_t should be stationary. The *ex- post* real interest rate could then be written as

$$r_t^m = i_t^m - \pi_t^m = \alpha + (\pi_t^{e,m} - \pi_t^m) + \eta_t = \alpha + \epsilon_t + \eta_t \tag{5}$$

which implies stationary fluctuations around a constant level α. Thus, in the linear context real interest rates may deviate from their mean only temporarily as a result of uncorrelated inflation forecast errors or stationary shocks, when the Fisher effect holds.

2.2 A Nonlinear Framework

This subsection draws heavily from Orphanides and Wilcox (2002) who postulate a conventional macromodel consisting of an aggregate demand relationship and an expectations-augmented Phillips curve. Their model is summarized as follows:

$$y_t = \rho y_{t-1} - \sigma(r_t - r^*) + u_t \tag{6}$$

$$\pi_t = \pi_t^e + \alpha y_t + e_t \tag{7}$$

where y_t is the deviation of output from potential (measured in logarithms). The deviation of the real interest rate from its steady-state level is denoted by $r_t - r^*$, σ is a positive parameter and u_t is an aggregate demand shock. Further, π_t and π_t^e are inflation and expected inflation respectively, and e_t is an aggregate supply shock. The parameter ρ is a positive fraction capturing the persistence of the output gap and α is assumed to be positive.

The policy maker is assumed to suffer loss from deviations of inflation from its intermediate target $\pi_t - \widetilde{\pi}$ (the long-run target is zero), and deviations of output from potential according to the following loss function:

$$L = (\pi_t - \widetilde{\pi})^2 + \gamma y_t^2 + \psi \left| y_t \right| \tag{8}$$

where $\gamma \geq 0$, and $\psi \geq 0$. It is further assumed that the intermediate target for inflation is a positive fraction λ of the lagged inflation rate (i.e. $\widetilde{\pi} = \lambda \pi_{t-1}$). The lower the value of λ, the more aggressively the intermediate inflation target is adjusted to its long-run value of zero. It is shown that if neither the demand nor the supply shock can be anticipated, and inflation expectations are static ($\pi_t^e = \pi_{t-1}$) the optimal rule for the policy maker is opportunistic regarding its response to inflation:

$$i_t = \begin{cases} \pi_{t-1} + r^* + \frac{\rho}{\sigma} y_{t-1} + \frac{\alpha(1-\lambda)}{\sigma(\alpha^2+\gamma)} (\pi_{t-1} - \overline{\pi}_{t-1}) & \text{if } \pi_{t-1} > \overline{\pi}_{t-1} \\ \pi_{t-1} + r^* + \frac{\rho}{\sigma} y_{t-1} & \text{if } \underline{\pi}_{t-1} \leq \pi_{t-1} \leq \overline{\pi}_{t-1} \\ \pi_{t-1} + r^* + \frac{\rho}{\sigma} y_{t-1} + \frac{\alpha(1-\lambda)}{\sigma(\alpha^2+\gamma)} (\pi_{t-1} - \underline{\pi}_{t-1}) & \text{if } \pi_{t-1} < \underline{\pi}_{t-1} \end{cases} \tag{9}$$

where $\overline{\pi}_{t-1} = \frac{\psi}{2\alpha(1-\lambda)}$ and $\underline{\pi}_{t-1} = -\frac{\psi}{2\alpha(1-\lambda)}$ are an upper and a lower threshold respectively.

The opportunistic monetary policy rule suggests that as long as inflation is within the band defined by the lower and upper threshold $\left[\underline{\pi}_{t-1}, \overline{\pi}_{t-1}\right]$, it does not warrant a policy response. Once inflation exceeds the upper threshold, it triggers an increase in the nominal interest rate intended to bring the real interest rate up toward its long-run level. An analogous argument can be made for rates of inflation that are below the lower threshold. From an empirical standpoint, this model suggests that optimal monetary policy reinforces the mean-reverting properties of the *ex post* real interest rate when lagged inflation lies outside a certain band.

The values of the upper and lower threshold determine the width of the policy-inaction band. A monetary authority with a gradualist approach to disinflation (high λ) will have a relatively wide band. The same is true for a monetary authority that faces a strong output-inflation trade-off (low α) and/or experiences a high loss from deviations of output from potential (high ψ).

3 The Econometric Methodology

Linear unit root tests on the demeaned *ex post* real interest rate (z_t) are based on the following auxiliary regression:

$$\Delta z_t = \mu + \rho z_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta z_{t-i} + \epsilon_t \tag{10}$$

where μ is a drift term and ϵ_t is a white noise disturbance. The null hypothesis of a unit root is tested by imposing the restriction $\rho = 0$ against the alternative $\rho < 0$ and testing it using the Dickey and Fuller (1981) τ_μ or τ_τ statistics. If the null cannot be rejected using appropriate critical values, it is concluded that deviations of z_t from its mean are infinitely persistent.

In the context of the optimal monetary policy rule described in the previous section, real interest rate deviations from its mean will be more persistent - ρ will be closer to zero - inside the policy inaction band. When real interest rates are outside the policy inaction band, they trigger a monetary policy response that strengthens mean reversion - ρ becomes more negative in the outer regimes. A linear specification like (10) amounts to the pooling of observations from three different regimes and leads to biased estimates of ρ.

To deal with this issue, we use the following 3-regime SETAR model:

$$\Delta z_t = \eta_t + \begin{cases} \mu_1 + \rho_1 z_{t-1} + \sum_{i=1}^{p} \alpha_{1i} \Delta z_{t-i} & \text{if } z_{t-d} > \theta \\ \mu_2 + \rho_2 z_{t-1} + \sum_{i=1}^{p} \alpha_{2i} \Delta z_{t-i} & \text{if } \theta \geq z_{t-d} \geq -\theta \\ \mu_3 + \rho_3 z_{t-1} + \sum_{i=1}^{p} \alpha_{3i} \Delta z_{t-i} & \text{if } -\theta > z_{t-d} \end{cases} \tag{11}$$

where η_t is a white noise disturbance common across regimes and the dynamics of the process are captured by Δz_{t-i}. The transition variable is z_{t-d} with a delay, d, that is unknown. The threshold, θ, is assumed to be symmetric around zero but unknown. Within the band $[-\theta,\ \theta]$ real interest rate deviations from its long-run value may be infinitely persistent ($\rho_2 = 0$) because the monetary authority tolerates them. However, ρ will be negative if the Fisher effect holds. Bec, Ben Salem, and Carrasco (2004) have shown that a process like (11) will be globally stationary - under certain conditions - even if it has a unit root in the middle regime. The same authors propose a test of the unit-root hypothesis that is specifically designed to have increased statistical power when the alternative hypothesis is a SETAR model like (11).

The testing procedure that we follow in the paper is outlined below. Define the indicator functions

$$I_t = \begin{cases} 1 & \text{if } z_{t-d} > \theta \\ 0 & \text{if } z_{t-d} \leq \theta \end{cases} \tag{12}$$

$$J_t = \begin{cases} 1 & \text{if } z_{t-d} < -\theta \\ 0 & \text{if } z_{t-d} \geq -\theta \end{cases} \tag{13}$$

Write:

$$\Delta z_t = I_t \left(\mu_1 + \rho_1 z_{t-1} + \sum_{i=1}^{p} \alpha_{1i} \Delta z_{t-i} \right) + (1 - I_t - J_t) \left(\mu_2 + \rho_2 z_{t-1} + \sum_{i=1}^{p} \alpha_{2i} \Delta z_{t-i} \right) \tag{14}$$
$$+ J_t \left(\mu_3 + \rho_3 z_{t-1} + \sum_{i=1}^{p} \alpha_{3i} \Delta z_{t-i} \right) + \eta_t$$

The following steps describe the estimation and testing process:

1. The data on z_t (in absolute value) is sorted in ascending order.

2. The minimum value for the threshold, $\underline{\theta}$, is selected in such a way that 15 percent of the observations lie below it. The maximum value for the threshold, $\overline{\theta}$, is selected in such a way that 15 percent of the observations lie above it. This procedure guarantees that at least 15 percent of the observations on z_t will lie inside or outside the middle regime. This way, the estimated SETAR model is not unduly influenced by a few important outliers. Moreover, the Likelihood Ratio and Wald tests described below will have distributions that are free of nuisance parameters.

3. The optimal threshold and delay parameters are selected by conducting a grid search for θ in the interval $\left[\underline{\theta}, \overline{\theta}\right]$ and for d in the interval $[1, 8]$. Equation (14) is repeatedly estimated by nonlinear least squares for different values of these parameters and the pair that minimizes the sum of squared residuals is selected.

4. Conditional on the estimated optimal threshold and delay, we compute the following likelihood-ratio test statistic:

$$Sup\ LR_T(\widehat{\theta}, \widehat{d}) = T \ln \left(\frac{\widetilde{\sigma}^2}{\widehat{\sigma}^2} \right) \tag{15}$$

   where $\widetilde{\sigma}^2$ is the restricted estimate of the variance (imposing $\rho_1 = \rho_2 = \rho_3 = 0$) and $\widehat{\sigma}^2$ is the unrestricted estimate.

5. The estimate of the $Sup\ LR_T(\widehat{\theta}, \widehat{d})$ is compared to its critical value. If the test statistic exceeds the critical value, the unit root hypothesis is rejected.

6. For countries that reject the unit-root null, the analysis proceeds with Sup Wald tests for linearity suggested by Hansen (1996, 1997). We test the null hypothesis of linearity by imposing the following restrictions $\alpha_i = \alpha$, $\mu_i = \mu$, $\rho_i = \rho$ for $i = 1, 2, 3$. Since the distribution of this test under the null depends on nuisance parameters, we simulate the p values for the $Sup\ Wald_T(\widehat{\theta}, \widehat{d})$ using 1000 replications (see Hansen 1997).

4 Estimation Results

The data set used in the estimation consists of the 3-month Treasury Bill rate (or other short-term interest rate in a few cases) and the Consumer Price Index for the G7 countries. The data was converted to quarterly from a monthly frequency through averaging. The annual rate of inflation was computed as the log-difference of the quarterly CPI times 400. Finally, the rate of inflation was aligned with the appropriate quarterly observation of the interest rate. The sample spans the period 1960 to 2004 in most cases. However, the sample for Germany starts in 1962 while data for France and the UK are available after 1970. The US data were obtained from the Federal Reserve on-line data base FRED. The rest of the data were obtained from Datastream.

Table 1 presents some standard unit root and cointegration tests for comparability with previous work. The results from the Augmented Dickey Fuller (ADF) test for a unit root are reported in columns 2 and 5. The auxiliary autoregressions include a constant and have a lag length that was selected using the Akaike information criterion (AIC). The p values for the test - reported in parentheses - indicate the probability of rejecting the unit-root null when it is in fact true. In all cases, the p values are greater than the 5 percent significance level rejecting stationarity for both the nominal interest rate and inflation. These results were confirmed by the Generalized Least Squares version of the ADF test (GLS-ADF) suggested by Elliot et al. (1996). This test can increase power by obtaining estimates of the parameters for the deterministic regressors and using them to detrend the data prior to the estimation of the ADF auxiliary autoregression. In all cases, the reported t statistics were less (in absolute terms) than the 5 percent critical value rejecting stationarity.

Kwiatkowski et al. (1992) argue that the way in which classical hypothesis testing is carried out ensures that the null hypothesis is accepted unless there is strong evidence against it. They have proposed tests (known as the KPSS tests) of the hypothesis of stationarity against the alternative of the unit root. The p values for the KPSS tests - reported in columns 4 and 7 - are less than 5 percent in all cases except the German interest rate. Thus, with the possible exception of the German interest rate, the null hypothesis of stationarity can be rejected.

Having found that nominal interest rates and inflation rates are probably integrated - $I(1)$ - variables, we test for cointegration by checking the residuals from a regression of the nominal interest rate on inflation for a unit root. The ADF tests on the residuals and their associated p-values are reported in column 8. In all cases the p values exceed the 5 percent significance level thus rejecting stationarity in the residuals (rejecting cointegration). Further, we test for the full Fisher effect by restricting the cointegrating vector to be $(1, -1)$. This is equivalent to testing for a unit root in the *ex post* real interest rate. The ADF tests, reported in column 9, reject cointegration in all cases. It should be noted that inability to reject the unit-root null in the real interest rate is very pronounced - the minimum p value is 15 percent and the maximum is 47 percent.

These results are not surprising as the ADF tests for a unit root have been shown to have low power against stationary but nonlinear alternatives. In Table 2, we report the results from unit root tests on the ex post real interest rate that are designed to have increased power against nonlinear alternatives. The nonlinear alternative hypothesis is the SETAR model described by equation (13). This model is more general than restricted versions considered by Kapetanios and Shin (2002) who impose a unit root in the middle regime, $\rho_2 = 0$, and zero drift in all regimes, $\mu_i = 0$ for $i = 1, 2, 3$. Further, the model encompasses the Band-TAR model that has been studied by a number of authors (e.g. Balke and Fomby 1997, Taylor 2001). The Band-TAR model is a special case of the present setting obtained by imposing, $\mu_1 = -\mu_3$, $\rho_1 = \rho_3$ and $\alpha_{1i} = \alpha_{2i} = \alpha_{3i}$ for $i = 1, 2, \cdots, p$ on equation (13). We follow the testing strategy that was outlined in the previous section which follows Bec, Ben Salem and Carrasco (2004).

The *Sup* LR test statistics, reported in column 3, exceed their simulated critical values (reported in Table 4) at the 5 percent level of significance in all cases except Canada and the US. For these two countries the test statistics are slightly lower than the 5-percent but greater than the 10- percent critical values. We conclude that in all cases the unit-root null can be rejected at the 10 percent level of significance in favour of a very general nonlinear alternative. For five of the G7 economies the unit root hypothesis can be rejected at the 5 percent level of significance. This represents almost a complete reversal of the results obtained under a linear alternative hypothesis.

The number of lagged differences (Δz_{t-i}) that were used to describe the dynamics of the nonlinear model was determined by the AIC applied on the linear model - equation (12). The simulated critical values for the LR tests show little sensitivity with respect to the value of the delay parameter. The critical values for these tests were obtained from the simulation of the linear model

$$\Delta z_t = \mu + \alpha_1 \Delta z_{t-1} + \alpha_2 \Delta z_{t-2} + \epsilon_t$$

which is model (12) under the unit-root null ($\rho = 0$) and $\epsilon_t \sim N(0, \sigma^2)$. The model was estimated for the G7 economies and the parameters μ, α_1, α_2, and σ were set at the average over all countries. This yielded the following parameter values for the simulations: $\mu = 0$, $\alpha_1 = -0.5$, $\alpha_2 = -0.2$, and $\sigma = 0.03$. The sample size for the simulations was set to 200 to reflect the available sample size and the number of replications was set to $10,000$. Empirical sizes for the LR test were simulated using the same model. The results, reported in Table 5, suggest that the Sup LR test performs quite well and support its use in the present context.

Having rejected the unit-root null in the *ex post* real interest rate, we proceed with tests for linearity. The *Sup* Wald tests, reported in column 4 of Table 2, test the null hypothesis of linearity against the SETAR model (13) by imposing the following restrictions: $\mu_1 = \mu_2 = \mu_3$, $\rho_1 = \rho_2 = \rho_3$, and $\alpha_{1i} = \alpha_{2i} = \alpha_{3i}$ for $i = 1, 2, \cdots, p$. The p values for the test suggest that linearity can be rejected at the 10 percent significance level in all cases except Japan and the UK which are dropped from further analysis

For the countries that rejected the linearity restrictions, the real interest rate is characterized by a general SETAR model, equation (13). The last four columns in Table 2 report tests for a number of restrictions on the SETAR model. We find that the nonstationary middle regime hypothesis, $\rho_2 = 0$, cannot be rejected for Canada and France, but it is rejected for Germany and the the US at the 5 percent significance level and for Italy at the 10 percent level. This finding suggests that the full Fisher effect is probably valid inside the central bank's policy inaction band for these three countries while, for Canada and France, the real interest rate is globally stationary but locally nonstationary.

Kapetanios and Shin (2002) proposed a version of the present SETAR model that imposes the restriction of zero drift in all regimes. Wald tests for the restriction $\mu_1 = \mu_2 = \mu_3 = 0$, reported in column (10), reject it except in the case of Italy. Wald tests for the Band-TAR hypothesis, $\mu_1 = -\mu_3$, $\rho_1 = \rho_3$, are unable to reject it for Canada, France, Germany, and Italy. The Band-TAR hypothesis is rejected for the US suggesting asymmetric mean reversion in the US real interest rate. Finally, the hypothesis of common dynamics across regimes, $\alpha_{1i} = \alpha_{2i} = \alpha_{3i}$ for $i = 1, 2, \cdots, p$, cannot be rejected in all cases

Estimates of the restricted SETAR models that were suggested by our previous tests are reported in Table 3. For Canada, France, Germany, and Italy we report estimates of a Band-TAR model, while for the US we report estimates of an unrestricted three-regime TAR model. In addition, we impose common dynamics across regimes.

Bec, Guay, and Guerre (2004) have proposed adaptive unit-root tests in the context of the Band-TAR model. The key difference between their approach and the one that we have followed so far, is in the choice and construction of the threshold set. The adaptive tests allow the threshold sets to adapt to the hypothesis that is being tested. For countries in our sample that do not reject the Band-TAR hypothesis, we compute the adaptive unit-root tests and report them in column 2 of Table 3. The reported test statistics are greater than their 5 percent critical value rejecting the unit-root null and confirming our previous results from the unrestricted model. Interestingly, the estimates of the thresholds based on the adaptive tests are similar to our previous results with the exception of Italy.

5 Concluding Remarks

Standard tests for a unit root in the real interest rate of the G7 economies - against linear alternatives - reveal that the former is not mean reverting. This implies that the nominal interest rate and inflation can drift apart from one another indefinitely, thus invalidating the Fisher effect. However, it is well documented that linear unit root and cointegration tests lack power against nonlinear alternatives.

In this paper, we have used recent methodological advances to test the unit- root hypothesis against a nonlinear alternative hypotheses. Our use of a three-regime SETAR model was motivated by recent theoretical literature suggesting different degrees of mean reversion in the real interest rate depending on whether monetary policy is actively pursued or in a stand-by mode. Unit-root tests of the type used in the paper have been shown to have a increased power against nonlinear alternatives. This was confirmed by our empirical results. We found that, with only one exception, the unit-root in the real interest rate can be rejected in favour of a SETAR nonlinear model.

Further, our linearity tests suggest that the real interest rate follows a stationary nonlinear process in the case of Canada, Germany, Italy and the US. The symmetric-three-regime TAR model seems to describe the dynamics of the real rate well for these economies. The nonlinear model that we proposed is motivated by recent theoretical literature on optimal monetary policy rules.

Overall, our empirical findings are encouraging for the Fisher effect in a statistical sense. Nevertheless, they indicate that in some cases, mean reversion in the real interest rate may be the result of monetary policy reaction and not a bond market outcome as envisioned by Irving Fisher. In view of this likelihood, macro modelling may require an explicit description of monetary policy. The empirical evidence presented seems to be unfavourable to theoretical models - particularly short run - that have an embedded Fisher relationship assumed to hold at all times.

References

[1] Balke, N. S. and T. B. Fomby. “Threshold Cointegration,” *International Economic Review* 38 (1997), 627-645.

[2] Bec F., M. B. Salem, and M. Carrasco. “Tests for Unit-Root Versus Threshold Specification With an Application to the Purchasing Power Relationship.” *Journal of Business and Economic Statistics* 22 (2004), 382-395.

[3] Bec F., A. Guay, and E. Guerre. “An Adaptive Consistent Unit-Root Test Based on Autoregressive Threshold Model.” Working Paper, CREST (2004), France.

[4] Bullard, J.. “Testing Long-run Monetary Neutrality Propositions: Lessons from the Recent Research”. *Federal Reserve Bank of St. Louis Review* 81 (1999), 57-77.

[5] Caner, M. and B. E. Hansen. “Threshold Autoregression With a Unit Root.” *Econometrica* 69 (2001), 1555-1596.

[6] Cooley, T. F. and G. D. Hansen. “The Inflation Tax in a Real Business Cycle Model.” *American Economic Review* 79 (1989), 733-748.

[7] Crowder, W. and D. Hoffman. “The Long-run Relationship Between Nominal Interest Rates and Inflation: The Fisher Equation Revisited.” *Journal of Money, Credit and Banking* 28 (1996), 102-118.

[8] Dickey, D. A. and W. A. Fuller. “Likelihood Ratio Statistics for Autoregressive Time Series With a Unit Root.” *Econometrica* 49 (1981), 1057-1072.

[9] Drazen, A. “Inflation and Capital Accumulation Under a Finite Horizon.” *Journal of Monetary Economics* 8 (1981), 247-260.

[10] Elliot, G., T. J. Rothenberg, and J. H. Stock. “Efficient Tests for an Autoregressive Unit Root.” *Econometrica* 64 (1996), 813-836.

[11] Fisher, I. *The Theory of Interest.* New York: Macmillan, 1930.

[12] Fisher, M. E. and J. J. Seater. "Long-run Neutrality and Superneutrality in an ARIMA Framework." *American Economic Review* 83 (1993), 402-415.

[13] Garcia, R. and P. Perron. "An Analysis of the Real Rate of Interest Under Regime Shifts." *Review of Economics and Statistics* 78 (1996), 111-125.

[14] Hansen, B. E.. "Inference When a Nuisance Parameter is not Identified Under the Null Hypothesis." *Econometrica* 64 (1996), 413-430.

[15] Hansen, B. E. "Inference in TAR Models." *Nonlinear Dynamics and Econometrics* 2 (1997), 1-14.

[16] Kapetanios, G. and Y. Shin."Unit Root Tests in Three-Regime SETAR Models." Unpublished Manuscript, Bank of England, 2002.

[17] King, R. G. and M. W. Watson. "Testing Long-run neutrality." Federal Reserve Bank of Richmond *Economic Quarterly* 83 (1997), 69-101.

[18] Koustas, Z. "Canadian Evidence on Long-Run Neutrality Propositions." *Journal of Macroeconomics* 20 (1998), 397-411.

[19] Koustas, Z. and A. Serletis. "On the Fisher Effect." *Journal of Monetary Economics* 44 (1999), 105-130.

[20] Kwiatkowski D., P. C. B. Phillips, P. Schmidt, and Y. Shin. "Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root." *Journal of Econometrics* 54 (1992), 159-178.

[21] Mishkin, F. "Is the Fisher Effect for Real?:A Re-examination of the Relationship Between Inflation and Real Interest Rates." *Journal of Monetary Economics* 30 (1992), 195-215.

[22] Orphanides, A. and D. Wilcox. "An Opportunistic Approach to Disinflation." *International Finance* 5 (2002), 47-71.

[23] Orphanides, A. and R. M. Solow. "Money, Inflation and Growth." in Benjamin Friedman and Frank Hahn, eds., *Handbook in Monetary Economics*. Amsterdam: North-Holland, 1990.

[24] Park, J. Y. "Bootstrap Unit Root Tests." *Econometrica* 71 (2003), 1845-1985.

[25] Perron, P."The Great Crash, the Oil Price Shock, and the Unit Root Hypothesis." *Econometrica* 57 (1989), 1361-1401.

[26] Phillips, P. C. B. "Econometric Analysis of Fisher's Equation." *The American Journal of Economics and Sociology* 64 (2005), 125-168.

[27] Pippenger, M. K. and G. E. Goering. "A Note on the Empirical Power of Unit Root Tests Under Threshold Processes."*Oxford Bulletin of Economics and Statistics* 55 (1993), 473-481.

[28] Romer, D. "A Simple General Equilibrium Version of the Baumol-Tobin model." *Quarterly Journal of Economics* 101 (1986), 663-685.

[29] Sidrauski, M. "Rational Choice and Patterns of Growth." *American Economic Review Papers and Proceedings* 57 (1967), 534-544.

[30] Taylor, A. "Potential Pitfalls for the PPP Puzzle? Sampling and Specification Biases in Mean-Reversion Tests of the LOOP." *Econometrica* 69 (2001), 473-498.

[31] Tobin, J. "Money and Economic Growth." *Econometrica* 33 (1965), 671-684.

[32] Weber, A. "Testing Long-run Neutrality: Empirical Evidence for G7 Countries with Special Emphasis on Germany." *Carnegie-Rochester Conference Series on Public Policy* (1994), 67-117.

[33] Weiss, L. "The Effects of Money Supply on Economic Welfare in the Steady State." *Econometrica* 48 (1980), 565-576.

Table 1. Unit root and cointegration tests against linear alternatives

	i_t			π_t			Cointegration Tests	
Country	ADF	$ADF-GLS$	$KPSS$	ADF	$ADF-GLS$	$KPSS$	$i_t = \alpha + \beta\pi_t + e_t$	$i_t - \pi_t = \alpha + e_t$
Canada	−1.78	−1.51	0.59	−2.13	−1.44	0.68	−2.11	−2.30
	(0.46)		(0.02)	(0.27)		(0.01)	(0.27)	(0.20)
France	−1.05	−1.48	1.40	−1.44	−1.50	1.99	−1.45	−2.24
	(0.68)		(0.00)	(0.51)		(0.00)	(0.61)	(0.25)
Germany	−3.04	−1.69	0.38	−2.80	−2.24	0.71	−2.39	−2.24
	(0.06)		(0.08)	(0.07)		(0.01)	(0.19)	(0.24)
Italy	−1.33	−1.05	0.73	−1.48	−1.18	0.68	−1.52	−2.24
	(0.63)		(0.00)	(0.60)		(0.01)	(0.56)	(0.24)
Japan	−1.14	−0.36	2.17	−1.72	−1.67	1.49	−1.47	−2.62
	(0.61)		(0.00)	(0.43)		(0.00)	(0.50)	(0.15)
UK	−1.26	−2.06	1.12	−1.69	−1.54	1.70	−1.67	−1.80
	(0.62)		(0.00)	(0.49)		(0.00)	(0.51)	(0.47)
US	−2.02	−1.69	0.52	2.70	−2.24	0.51	−2.14	−2.37
	(0.31)		(0.03)	(0.10)		(0.04)	(0.27)	(0.18)

Notes: For the ADF and KPSS tests, values in parentheses are bootstrap p values (see Park 2003). The null hypothesis for the ADF tests is the presence of a unit root. The null hypothesis for the KPSS tests is of level stationarity. The 5% critical value for the ADF-GLS test is -2.93. For the cointegration tests the null is of a unit root in the residuals implying no cointegration. The number of lagged differences used for the tests was found using the AIC.

Table 2. Unit root and linearity tests on the real interest rate against a threshold alternative

Country	Lagged Differences	*Sup* LR	*Sup* Wald	Threshold	Delay	% of obs. in Lower	% of obs. in Middle	% of obs. in Upper	Wald tests for Zero Drift	Wald tests for Band-TAR	Wald tests for Common Dynamics	t test for Unit Root in Middle Regime
Canada	3	16.49	82.70	± 4.34	z_{t-6}	11	81	8	11.69	15.15	5.36	-1.42
			(0.01)						(0.01)	(0.06)	(0.50)	(0.16)
France	1	23.63	18.24	± 0.83	z_{t-3}	40	15	45	18.82	0.31	0.19	-0.44
			(0.06)						(0.00)	(0.99)	(0.91)	(0.66)
Germany	0	43.29	13.61	± 0.78	z_{t-2}	34	30	36	10.48	3.77		-2.50
			(0.09)						(0.01)	(0.15)		(0.01)
Italy	1	41.08	37.54	± 5.27	z_{t-2}	9	81	10	0.98	6.55	0.41	1.86
			(0.00)						(0.81)	(0.16)	(0.81)	(0.06)
Japan	4	25.75	32.43		z_{t-1}							
			(0.42)									
UK	4	19.23	32.02		z_{t-4}							
			(0.86)									
US	2	15.95	39.62	± 3.53	z_{t-7}	9	82	9	37.58	34.58	7.49	-1.92
			(0.01)						(0.00)	(0.00)	(0.11)	(0.05)

Notes: The Sup LR test considers the unit root null. Its simulated critical values are reported in Table 4. The Sup Wald test considers the linear null. The numbers in parentheses are asymptotic p values for the tests. The threshold variable, z_{t-d}, is the real interest rate and d is the delay parameter.

Table 3. Estimates of threshold models

Band-TAR Country	Adaptive *Sup* LR	μ_1	μ_2	$\mu_3 = -\mu_1$	α_1	α_2	α_3	ρ_1	ρ_2	$\rho_3 = \rho_1$	Threshold	% of obs. in Lower	Middle	Upper
Canada	16.75	1.60	−0.18	−1.60	−0.29	−0.35	−0.25	−0.51	−0.12	−0.51	±4.59	8	84	8
		(0.50)	(0.19)	(0.50)	(0.09)	(0.08)	(0.07)	(0.11)	(0.07)	(0.11)				
France	25.55	1.25	−0.05	−1.25	−0.09			−0.42	−0.31	−0.42	±1.14	38	23	39
		(0.32)	(0.26)	(0.32)	(0.09)			(0.10)	(0.16)	(0.10)				
Germany	48.39	0.38	−0.31	−0.38				−0.55	−0.50	−0.55	±0.60	39	22	39
		(0.17)	(0.26)	(0.17)				(0.19)	(0.08)	(0.19)				
Italy	39.64	2.38	−0.04	−2.38	0.04			−0.17	−0.61	−0.17	±1.56	34	26	40
		(0.60)	(0.41)	(0.60)	(0.15)			(0.25)	(0.16)	(0.25)				

TAR Country		μ_1	μ_2	μ_3	α_{*1}	α_{*2}	α_{*3}	ρ_1	ρ_2	ρ_3	Threshold	% of obs. in Lower	Middle	Upper
US		1.91	−0.23	1.55	−0.33	−0.34		−0.41	−0.11	0.08	±3.74	9	84	7
		(0.71)	(0.11)	(0.73)	(0.10)	(0.10)		(0.21)	(0.06)	(0.20)				

Notes: Robust standard errors are reported in parentheses.

Table 4. Empirical critical values for Sup LR

Delay	Quantiles	85%	90%	95%	99%
$d = 1$		14.962	16.239	18.561	22.599
$d = 2$		14.617	15.940	18.021	22.392
$d = 3$		14.405	15.725	17.618	22.532
$d = 4$		13.584	14.935	17.218	22.027
$d = 5$		13.684	14.961	17.193	21.534
$d = 6$		13.507	14.828	16.999	21.250
$d = 7$		13.563	14.882	17.018	21.827
$d = 8$		13.546	14.854	16.874	21.774

Table 5. Empirical sizes of Sup LR

Nominal Size	Delay	$d = 1$	$d = 2$	$d = 3$	$d = 4$	$d = 5$	$d = 6$	$d = 7$	$d = 8$
0.05		0.043	0.050	0.053	0.047	0.048	0.045	0.045	0.054

Figure 1: Ex-Post Real Interest Rates and Monetary Policy Regimes