Entrance and Fiscal Policies in a Monetary Union. A Theoretical Analysis

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Abstract

This paper examines the macroeconomic implications of a small open economy which joins a monetary union. Using an overlapping generations, general equilibrium model we show that when a highly indebted country enters a monetary union and the real interest rate falls, the following results hold: Output and capital increase while public debt falls. Consumption and private debt depends on the discount factor and the parameter which captures the production technology. Low capitalised countries with high discount factors are found to be net debtors in the steady state while countries with high discount factor and capital share are net creditors. Since the government's balance is improving after the accession, the paper investigates the efficiency of different fiscal schemes which use the additional revenue and proposes the lump sum tax which maximizes individuals' utility.

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1. Introduction

The current members of the European Monetary Union (EMU) are becoming increasingly integrated from an economic and financial standpoint. In the very near future some of the new members of the European Union (EU) will be ready to adopt the single currency. How does the European Monetary Union (EMU) membership affect the private and public finances of these countries through a different, lower interest rate? In addition, what kind of fiscal policies the new members are able to pursue?

These are the questions examined by this paper. More specifically, we examine the macroeconomic implications for a small open economy which joins a monetary union like EMU. We focus on the effects of a lower and stable interest rate following the EMU accession on output, consumption, capital accumulation, public debt, private debt and current account. We consider best ways of transition to the new steady state and examine the optimality of fiscal schemes in terms of output.

Our economy is small enough not to influence the union's interest rate. The union has adopted a common currency and a common interest rate which is determined by the Central Bank. Up to the time of the entrance the government is free to exercise its fiscal tools in a discretionary way to smooth the effects of an adverse shock and to stabilize output. After entrance, the country must submit its monetary sovereignty to the union's Central Bank and should exercise its fiscal policy under certain constraints or criteria. The dynamic and steady state effects are shown through a general equilibrium, small-open economy model. Some of the main features of this continuous time model are overlapping generation agents with finite horizons, exogenous labour supply, distortionary taxes on firms gross output, adjustment costs on investment and government's fiscal policies which affect the households decisions.

We find that lower interest rates boost the level of output and capital stock to higher levels as expected. *The effects on consumption* *and private debt are very interesting* as they can be different from the Keynesian ones. This is because they depend on two factors: Firstly, they depend on the discount factor and how impatient people are to consume. Secondly, the private debt depends on the production technology and the share of capital in the economy. Regarding public debt, a fall on its level is observed which allows higher flexibility for public policy and variety of fiscal schemes.

Mongelli (1997) has studied the effects of EMU on the public finances of the participating countries through two channels: the interest rate spending and the taxation channel. He amends a model of Blanchard and Fisher (1989) to analyse the two channels and also provides empirical evidence on tax harmonization. His paper states that the highly indebted and high tax countries pursuing prudent fiscal policies could benefit the most from EMU. He shows that highly indebted countries which will have less space to raise taxation after their entrance as they will follow a tax harmonization process. However as the interest rates are expected to fall rapidly while the tax harmonization will proceed slowly, those countries do not need to increase their tax levels.

An empirical study about the effects of a country which enters a union can be found in Giavazzi and Blanchard (2002) etc. They suggest that a small open economy with low level of output per capita which enters a low interest rate environment like EMU will experience large current account deficits due to convergence and integration. This is the explanation that Blanchard and Giavazzi give about the increasing current account deficits of Greece and Portugal by running conventional Feldstein-Horiokka regressions of investment on saving over different time periods. They suggest that since the huge current account deficits originate from private and not public investment there is no reason for macro policy intervention. They claim that the person who takes a mortgage in Portugal is the same who is going to repay it and this does not affect future generations. Apparently higher future incomes are expected.

The present analysis departs from earlier studies about the effects from joining EMU in several respects: Firstly, the model is a combination of Blanchard's (1985) and Tobin's (1969) models which provides complete and detailed analysis of the demand side and a realistic supply side with adjustment costs in investment. Even though this makes it more challenging to be solved analytically it offers some new aspects compared to Blanchard's original model. For example the steady state values of consumption and private debt do not only depend on how heavily people discount the future but also on how much capital the economy uses in its production. In this model it is possible for labour intensive economies to have positive debt in equilibrium even though they may have high rate of time preference. This happens because agents may choose to save by investing in domestic capital and maintain a certain level of consumption by foreign borrowing.

2. The Small Open Economy Model

We consider a small open economy, in a one good world with perfect competition and perfect international capital mobility. There is a single good produced domestically which is used for both consumption and investment purposes and it is a perfect substitute for foreign goods. An important assumption is that the interest rate enters exogenously in the model as we assume a small open economy where the accumulation of assets and capital stock does not affect the union's interest rate. Therefore $r_{dom} = r_{union} = r$. For the sake of simplicity we assume that the union's interest rate is constant: $r^u(t) = \bar{r}$.

The economy consists of three sectors: households, firms and the government. Households optimize their utility intertemporally and firms maximize their profits according to their respective constraints. Their behavior is determined on the microeconomic level. The government is not an optimizing entity in this study and faces only an intertemporal budget constraint. Government expenditure and tax policy are exogenous to the production and the consumption decisions of the private agents. All sectors operate under perfect competition.

The household sector draws upon Blanchard's (1985) demographic structure of agents with uncertain lifetime in a continuous time set up. This set up assumes that there are overlapping generations of individuals that have finite lives but the economy as a whole has infinite horizon due to the continuous entry of newly born cohorts. These cohorts face a constant probability of death at any point of time and the size of the population remains constant overtime. Another feature of this structure is that there are no inter-generational bequests so newly born individuals enter without owning any financial wealth. The absence of bequests coupled with finite horizons forces individuals to insure themselves with independent insurance companies. In return they receive a premium on their financial wealth to leave their assets to these companies after death. We depart from other frameworks by allowing households to hold private and public bonds. In addition, wage income is endogenous and is determined by the profits of the firms.

The production sector consists of a large number of identical firms owned entirely by households and produce a single good under conditions of perfect competition. The firms maximize their profits and face distortionary taxes on their output. Distortionary taxation is another departure from Blanchard's framework where less realistic lump sum taxes are assumed. In our analysis the economy does not experience perpetual growth but production is subject to the law of diminishing returns. The firms invest and buy capital from perfect capital markets. Capital markets are integrated and capital is perfectly mobile across countries. However firms cannot adjust their capital stocks instantaneously and without cost from one period to the other. This is because of the presence of adjustment costs which make the price and the cost of installing capital a rising function of the level of investment. Including installation costs to our model allows us to provide well defined investment and saving decisions where investment is not a passive function of savings. Labour supply is fixed inelastically at unity and is assumed to be perfectly immobile across countries.

The government has, unlike households, an infinite life-span. It finances its expenditure by levying proportional taxes on output. Distortionary taxation can affect the allocation of resources in economy. The government can also borrow from the private sector and from abroad to finance its expenditure. In the present model we assume that the domestic private sector buys all the public debt in order to capture the full effects of fiscal policy on aggregate demand. In this structure the role of fiscal policy and the timing of taxes are very important. The infinitely lived individual is indifferent to the way the government finances its deficit because higher future taxes are expected. However the finite lived agent strongly prefers government debt finance because one may not live to experience a future tax increase. In this case the timing of taxes influences consumption and investment decisions. The finite horizon of individuals together with the distortionary taxation collapse the Ricardian equivalence.

The rest of the paper is structured as follows: Section three analyses the demand side of the economy and solves the maximization problem of households. More specifically, the section starts by stating the maximization problem of the individual with finite horizon and derives the optimal consumption. The second part aggregates the variables across all cohorts and derives aggregate per capita values. The fourth section sets up the maximization problem of firms taking into account adjustment costs and distortionary taxes on output. Section five describes the government and its fiscal policy. The long run equilibrium of the economy is derived and its dynamics are demonstrated through phase diagrams in sections six and seven. In section eight we consider the comparative statics by examining changes in the tax rate, lump sum tax and interest rate. Section nine describes the macroeconomic implications through a lower real lower interest rate. Different fiscal responses are investigated and the more efficient fiscal plan regarding lump sum taxes is proposed. Section ten concludes.

3. The Demand Side

The household sector is a modified version of Yaari (1965) and Blanchard (1985) overlapping generations economy. An important feature of this framework is the probability of death that each individual faces and perfectly competitive life insurance companies. This section first describes individual behavior and then proceeds to the aggregate behavior.

3.1. Individual's Behaviour

There are overlapping generations of rational agents who have finite horizons. Individuals face a probability of death γ from one period to the other which is independent of their age and it is constant through their lifetime. In continuous time, the probability of death can take any values between zero and infinity. This is because the probability is a rate per unit time and does not need to be bounded between zero and one as it would be in the case of a discrete version of this framework[1]. A person, who is born at time s, is alive at time t $(s < t)$ with a probability $e^{-\gamma(t-s)}$. Since the probability of death remains constant throughout his life, our individual faces the probability $1 - e^{-\gamma(t-s)}$ of being dead at time t. The probability density function for death at time t is $f_x(t) = \gamma e^{-\gamma t}$. The expected life time can be calculated as: $E(X) = \int_0^\infty t\gamma e^{-\gamma t}dt = [-te^{-\gamma t}]_0^\infty + \int_0^\infty e^{-\gamma t}dt = \frac{1}{\gamma}$.

We assume that at every instant of time a new cohort of people is born with the same probability of death γ. Even though each person faces uncertainty about the length of his life, the whole cohort is large enough to decline through time with the probability γ. A cohort born at time zero has a size of e$^{-\gamma t}$. The size of population at any time is

[1] Blanchard and Fisher,"Lectures on Macroeconomics" 1990, p.149. The constant probability means that the random variable $X =$ *time until death* has an exponential distribution. Let X be this variable with its density funtion given:
$f_x(t) = \gamma e^{-\gamma t}$
Its expected value is given by
$E(X) = \int_0^\infty t\gamma e^{-pt}dt = [-te^{-\gamma t}]_0^\infty + \int_0^\infty e^{-\gamma t}dt = \gamma^{-1}$

$\int_{-\infty}^{t} \gamma e^{-\gamma(t-s)} ds = 1$. Hence population growth is zero and the size of population at any time is constant.

We assume that our agents are not altruistic and do not leave bequests to their decedents. They do not maximize their utility taking into account heirs and do not receive anything from their ancestors. This implies that individuals are born without financial wealth[2]. Furthermore, individuals do not die indebted or leave wealth. Due to the probability of death and the lack of any bequests, it is optimal for individuals to insure themselves in life insurance companies. These companies pay a premium to individuals while alive in exchange for having their financial wealth after death. In case of death the insurance companies receive any financial wealth left and compensate the creditors for any outstanding debt. In exchange, the companies must pay the (discounted) lump sum amount of $(r+\gamma)\, v(z)$ to the individuals while alive for receiving their equities $v(z)$ after their death. Apart from the world's interest rate r, the probability of death is added to the premium paid by the companies.

The companies also provide loans b^p and charge individuals the world's interest rate plus a risk premium in case the borrower dies and leaves unpaid debt. The total return received is $(r+\gamma)\, b^P(z)$. The insurance companies operate in a perfect competitive market with free entry and zero profits. The companies receive interest income $\gamma b^p(z)$ as a fraction γ of the individuals who survive each period to repay the loan. Since the proposition of individuals who enter is the same with those who die there is no revenues uncertainty for the insurance companies.

An unrealistic assumption of this framework is that all individuals have the same propensity to consume independently of their age but this feature has been introduced in order to facilitate aggregation.

[2]One can wonder how new individuals enter this economy. This particular assumption of new born peolpe owning nothing fits better to cases involving poor immigrants or families that practice selective altruism, in other words leave bequests only to some children.

Individuals maximize their utility under uncertainty due to the probability of death they face across their life. They derive utility directly by the consumption of both private and public goods. An individual who was born at time s i.e. belongs to the generation s, maximizes his expected utility at time t:

$$E\left[\int_t^{\infty} u(c(s,z),G_z)e^{-\gamma(z-t)}dz/t\right]$$

If the person is dead at time z, the utility becomes zero. However, if the person is alive he derives utility $u(c(z),G)$, and does not face any other source of uncertainty about his financial and human wealth. For this reason the utility that a person of generation s derives at time t is equivalent to:

$$U_{s,t}=\int_t^{\infty} e^{-(\rho+\gamma)(z-t)}\left(\log c(s,z)+\psi\log G\right)dz \tag{1}$$

$U_c>0$, $U_{cc}<0$, $U_G>0$ and $U_{GG}<0$. The concavity assumption indicates individual's effort to smooth consumption during his lifetime. For simplicity a logarithmic utility has been adopted which implies unit elasticity of substitution between consumption across periods. Consumption and government expenditure are added separately which means that government's decisions do not interact with the private ones. The parameter $0<\psi<1$ is the impact of the government consumption on the utility and ρ is the constant rate of time preference and expresses the degree of individuals' impatience to consume at the current period. The effective rate of time preference for a person with finite horizons is $\rho+\gamma$,where γ is the constant probability of death. When $\gamma=0$, the model collapses to a representative agent infinite horizon one with a $e^{-\rho t}$ discount rate. Here the effective discount factor $e^{-(\rho+\gamma)(z-t)}$ is bigger than the one in infinite horizon models which means that individuals are more impatient to consume today than tomorrow because of the uncertainty of being alive in the next period. Another characteristic of the constant probability of death is that the individual's utility does not change over time. Since the individual does not get older there is no reason for changing the initial optimal programs.

The individual's income[3] at time t consists of wages $y(s,z)$, dividends on equity wealth holdings $v(s,z)$ and public debt $b^G(z)$ after a lump sum tax τ_{1s}. Each individual supplies labour inelastically and earns wage income y_t each period. The wage income is assumed to remain the same across lifetime, in contrast to the two-period Diamond models where wage income falls in the second period of individual's lifetime. Therefore there is no life-cycle-pattern of wealth and savings. If an individual has nonhuman wealth $v(z)+b^G(z)$ at time t, he receives $r\left[v(z)+b^G(z)\right]$ in interest payments and a premium from the insurance companies of $\gamma\left[v(z)+b^G(z)\right]$. In total he receives the amount of $(r+\gamma)\left[v(z)+b^G(z)\right]$. The individual can also borrow at period t and has to pay previous period debt plus the interest rate discounted by the probability of death γ: His total debt is $(r+\gamma)b^P(z)$ which is the private debt.

The budget constraint of an individual who was born at time s, and is of age $s+t$ is:

$$c_{s,t}(z)=y_{s,t}(z)-\tau_{1s}+(r+\gamma)\left[v_{s,t}(z)+b^G_{s,t}(z)-b^P_{s,t}(z)\right]-\left[\dot{v}_{s,t}(z)+\dot{b}^G_{s,t}(z)-\dot{b}^P_{s,t}(z)\right] \quad (2)$$

$c_{s,t}$ stands for the consumption of the individual of the generation s at time t who will be alive at a time z in the future and $v_{s,t}$ is the shares of the firms' profits that he holds at time t while the term $(r+\gamma)\left[v(z)+b^G(z)\right]$ captures the premium paid by the insurance companies to the individual while alive in exchange for his financial wealth after his death. $b^P_{s,t}$ denotes private borrowing that takes place at period t while $(r+\gamma)b^P_{s,t}$ is the risk premium that the individual has to pay to the insurance companies which consists of the market interest rate and the insurance premium.

[3] Time span:
s——————t——————————z
(born) present time future time

Agents can buy a share of the firm's future profits in the stock market. A firm's market value at time t, v_t, is the present discounted value of the dividends paid by the firm to shareholders over the future:

$$V_t = \int_t^{\infty} e^{-rt} d(z) dz \tag{3}$$

The dividends that a firm has to pay to its shareholders equal the difference between the firm's after tax output and the sum of its investment and wage expenditure:

$$D_t = (1-\tau)Q_t - y_t L_t - I_t(1 + \frac{gI_t}{2K_t}) \tag{4}$$

Self fulfilling speculative asset price bubbles are ruled out:

$$\lim_{z\to\infty} v(z) e^{-\int_t^z (r+\gamma) d\mu} = 0 \tag{5}$$

The Non-Ponzi condition for preventing an individual who is still alive at time z from borrowing indefinitely is:

$$\lim_{z\to\infty} b^P(z) e^{-\int_t^z (r+\gamma) d\mu} = 0 \tag{6}$$

The above expression implies that an individual cannot accumulate debt forever at an interest rate higher than the risk premium he has to pay when borrowing. If we integrate the individual's dynamic budget constraint and use the Non Ponzi conditions we can derive the intertemporal budget constraint:

$$\int_t^{\infty} e^{-\int_t^z (r+\gamma) d\mu} c(s,z) dz =$$
$$= \int_t^{\infty} e^{-\int_t^z (r+\gamma) d\mu} [y(s,z) - \tau_1(s,z)] dz + v(s,t) + b^G(s,t) - b^P(s,t) = w(s,t) \tag{7}$$

The above equation states that the lifetime total wealth of an individual of age $s+t$ at time t, w_t consists of three components: First, the human wealth which is the present discounted value of the individual's future wage income after taxes, $\int_t^{\infty} e^{-\int_t^z (r+\gamma) d\mu} [y(s,z) - \tau_1(s,z)] dz$ and disappears from the system after the individual dies. The second element of the total wealth is the financial wealth which is the equity wealth and government's bonds $v(t) + b^G(t)$ and the third is the private debt $b^P(t)$. We can define the financial wealth as the difference between the assets that a person holds and his debt: $f(s,t) = v(s,t) + b^G(s,t) - b^P(s,t)$. For simplicity we can also define human wealth as $h(s,t) = \int_t^{\infty} e^{-\int_t^z (r+\gamma)d\mu}[y(s,z) - \tau_1(s,z)]dz$. Therefore the intertemporal budget constraint can be expressed as:

$$\int_t^{\infty} e^{-\int_t^z (r+\gamma)d\mu} c(s,z)dz = h(s,t) + f(s,t) = w(s,t) \tag{8}$$

The individual's problem is to maximize his expected utility subject to his budget constraint (2) and the solvency requirements. The Hamiltonian function becomes:

$$H = e^{-(\rho+\gamma)(z-t)}[\log c(s,z) + \psi \log G] + \lambda_t \left[y(s,z) - \tau_1 + (r+\gamma)f(s,z) - c(s,z)\right] \tag{9}$$

The solution to the maximization problem is:

$$\frac{dc(s,z)}{dz} = c(s,z)(r-\rho) \tag{10}$$

The first order condition states that the change in consumption depends not only on the market interest rate but also on the subjective discount factor. Hence, when the interest rate exceeds the subjective discount factor, consumption rises over time in the optimal path. An important aspect is that individual's consumption pattern does not depend on the probability γ. This is because the probability of death does not change through time since the individual is not getting old. The next step is to solve the above differential equation so we can obtain an expression for $c(t)$:

$$c(z) = c(t)e^{(r-\rho)(z-t)} \tag{11}$$

Substituting for $c(z)$ into consolidated budget (8) we find an expression for consumption as a function of the total (human and financial) wealth:

$$c(s,t) = (\rho+\gamma)[h(s,t)+f(s,t)] = (\rho+\gamma)[h(s,t)+v(s,t)+b^G(s,t)-b^P(s,t)] \tag{12}$$

where $\rho+\gamma$ is the propensity to consume out of wealth. We can see that the marginal propensity to consume out of wealth depends on the subjective discount factor ρ and the probability of death γ and not on the market interest rate. The absence of the market interest rate from the intertemporal consumption function is due to logarithmic utility function assumption. The property that the individual's consumption depends on the probability of survival independently of his age permits a simple aggregation of the individual's consumption function.

3.2. Aggregate Behaviour of Households

After having defined the individual's consumption pattern we aggregate across generations in order to evaluate the aggregate per capita values of consumption and wealth. Population is normalized so that at birth every cohort consists of one individual who is assumed to be born without debt. Due to the probability of death, the size of each cohort which was born at time s has at time t the size of $\gamma e^{-\gamma(t-s)}$. Frenkel and Razin (1996) justify the equality between the probability of death of a given cohort and its frequency relative to its initial size from the law of large numbers. Since at each period there are $\gamma e^{-\gamma(t-s)}$ members of a cohort of age t-s, the constant aggregate size of population at any time t is:

$$\int_{-\infty}^{t} \gamma e^{-\gamma(t-s)} ds = 1$$

In order to estimate aggregate per capita values (X_s) we first integrate the individual values $x(s,t)$ over the cohorts and then we divide them by the size of the population which is 1. The relation between aggregate and individual variables is given by:

$$X(t) = \int_{-\infty}^{t} x(s,t)\gamma e^{-\gamma(t-s)} ds$$

We denote the aggregate consumption, private and public debt, human, financial, equity and total wealth at time t as $C(t), B^P(t),\ B^G(t), H(t), F(t), V(t),$

and $W(t)$ respectively. The aggregate consumption in period t is the sum of consumption of individuals from all cohorts:

$$\begin{aligned} C(t) &= \int_{-\infty}^{t} c(s,t)\gamma e^{-\gamma(t-s)}ds = \int_{-\infty}^{t} (\rho+\gamma)[h(s,t)+f(s,t)]\gamma e^{-\gamma(t-s)}ds \Rightarrow \\ C(t) &= (\rho+\gamma)[H(t)+F(t)] = (\rho+\gamma)W(t) \end{aligned} \quad (13)$$

The above expression states that the aggregate per capita consumption at any period t is a positive function of total wealth where the parameters of the subjective discount factor and the probability of death are the propensity to consume out of wealth. Invariance of the individual spending propensity with respect to age is reflected in the equality between the individual and the aggregate spending propensities.

When deriving the aggregate per capita human wealth we take into account the assumption that wage income remains the same across all the individuals regardless of their age. The aggregate per capita human wealth $H(t)$ is the present discounted value of future labour income after taxes:

$$\begin{aligned} H(t) &= \int_{-\infty}^{t} \gamma e^{-\gamma(t-s)}h(s,t)ds = \int_{-\infty}^{t} \gamma e^{-\gamma(t-s)}\{\int_{t}^{\infty} e^{-\int_t^z (r+\gamma)d\mu}\left[y(s,z)-\tau_1(s,z)\right]dz\} \\ &= \int_{t}^{\infty}\{\int_{-\infty}^{t} \gamma e^{-\gamma(t-s)}\left[y(s,z)-\tau_1(s,z)\right]ds\}e^{-\int_t^z (r+\gamma)d\mu}dv = \\ &= \int_{t}^{\infty}\left[Y(v)-T_1(v)\right]e^{-\int_t^z (r+\gamma)d\mu}dv \end{aligned}$$

In the second line, the term in the parenthesis aggregates the labour income of all the individuals who are still alive until period t. Then we integrate the future labour income of those individuals and we discount it by the rate of market's interest rate and the probability of death. In order to derive the law of motion for human wealth we differentiate (15) with respect to time and also use the assumption that:

$$\lim_{z\to\infty} Y(z)e^{-\int_t^z (r+\gamma)d\mu} = 0 \quad (15)$$

The dynamics for human wealth, which also depend on the market's interest rate and the probability of death, become:

$$\frac{dH(t)}{dt} = (r+\gamma)H(t) - [Y(t) - T_1] \tag{16}$$

The above expression states that the level of human wealth is the discounted value of disposable future wage income of the individuals currently alive plus its change through the time.

The aggregate financial wealth can be defined as the amount of equities and public bonds held by the households minus their debt:

$$\begin{aligned} F(t) &= \int_{-\infty}^{t} f(s,t)\gamma e^{-\gamma(t-s)}ds = \int_{-\infty}^{t} [v(s,t) + b^G(s,t) - b^P(s,t)]\gamma e^{-\gamma(t-s)}ds = \\ &= V(t) + B^G(t) - B^P(t) \end{aligned} \tag{17}$$

In the previous section we defined the amount of equities that individuals hold as the present discounted value of their future dividend income. In aggregate, the amount of equities in the economy is:

$$\begin{aligned} V(t) &= \int_{-\infty}^{t} v(s,t)\gamma e^{-\gamma(t-s)}ds = \int_{-\infty}^{t} \gamma e^{-\gamma(t-s)}[\int_{t}^{\infty} e^{-\int_t^z (r+\gamma)d\mu} d(s,z)dz]ds \\ &= \int_{t}^{\infty}[\int_{-\infty}^{t} \gamma e^{-\gamma(v-s)} d(s,z)ds]e^{-\int_t^z (r+\gamma)d\mu}dv = \int_{t}^{\infty} D(v)e^{-\int_t^z (r+\gamma)d\mu}dv \end{aligned} \tag{18}$$

By following the some methodology we differentiate $V(t)$ with respect to time to obtain the dynamics for the aggregate equity wealth:

$$\frac{dV(t)}{dt} = rV(t) - D(t) \tag{19}$$

The aggregate equities change at rate r instead of $r+\gamma$ which is the case with the individuals. This is because the amount γV is a transfer from those individuals who die to those who are alive.

At this stage we can define the aggregate per capita net holdings (B^N) in bonds by aggregating and dividing the individual's bonds by the population:

$$B^N(t) = B^P(t) - B^G(t) = \int_{-\infty}^{t} \left[b^P(s,t) - b^G(s,t)\right]\gamma e^{-\gamma(t-s)}ds \tag{20}$$

By differentiating $B^P(t) - B^G(t)$ with respect to time we obtain the dynamics for net private debt:

$$\begin{aligned} \frac{d\left[B^N(t)\right]}{dt} &= \int_{-\infty}^{t} \frac{d\left[b^P(s,t) - b^G(s,t)\right]}{dt} \gamma e^{-\gamma(t-s)} ds - \\ & -\gamma\left[B^P(t) - B^G(t)\right] + \gamma\left[b^P(t,t) - b^G(t,t)\right] \quad (21) \end{aligned}$$

The first term in the left hand side represents the evolution of the net private debt of people currently alive. The second term is the debt of people who die at time t while the last one is the debt of people just born which is by assumption equal to zero. If we substitute for $b^P(s,t)$ from the individual's dynamic budget constraint and use equation (20) we obtain the dynamics for the private debt B_t^P:

$$\begin{aligned} \frac{d\left[B^N(t)\right]}{dt} &= C(t) - Y(t) + T_1 - D(t) + r\left[B^P(t) - B^G(t)\right] \\ &= C(t) - Y(t) + T_1 - D(t) + r\left[B^N(t)\right] \quad (22) \end{aligned}$$

From the above three dynamics we see that only the human wealth depends on the probability of death and this is because it disappears from the system once the individual dies. The other two forms of wealth, however, remain in the system and are transferred to the insurance companies. For this reason their dynamics depend only on the market interest rate.

Lastly we deal with the dynamics of aggregate per capita consumption. In order to derive the law of motion for consumption we differentiate the aggregate consumption equation (14) and eliminate $H(t)$ by using (17) in order to express the dynamics as a function of consumption, equities and net private indebtedness.

$$\frac{dC(t)}{dt} = (r - \rho)C(t) - \gamma(\rho + \gamma)[V(t) + B^G(t) - B^P(t)] \quad (23)$$

Again the above expression reflects the importance of the uncertainty about the life span as the probability of death enters in the evolution of aggregate consumption. In an infinite horizon framework where $\gamma = 0$, the same relation becomes:

$$\frac{dC(t)}{dt} = (r - \rho)C(t)$$

In a small open economy with infinitely lived agents, the stability of the system requires that the real interest rate must be equal to the rate of time preference ($r = \rho$). When $r > \rho$ the agents save indefinitely and the small open economy ends up owing too much wealth that it violates the small country assumption. On the other hand if $r < \rho$, the small open economy becomes so impatient that it ends up owning no wealth at all and consumption shrinks for ever. This is the reason why the steady state of an infinite horizon model requires $r = \rho$ which implies that the aggregate per capita consumption (and wealth) is flat. In this model instead, the condition $r = \rho$ is not needed for a stable consumption path. Individuals' consumption may change overtime according to whether $r < \rho$, $r > \rho$, but aggregate per capita consumption is constant. Even when $r > \rho$ individual consumption is rising over time but this does not imply that aggregate per capita consumption is not constant. This is because new born individuals have no financial wealth so their consumption is low at the beginning of their life.

4. The Production Sector (Firms)

The production sector consists of identical firms owned by domestic and foreign households which produce a single traded good in perfect markets. Firms employ labour and capital to produce output by using a linearly homogeneous Cobb-Douglas production function of the form:

$$Q(t) = AK^{a}L^{1-a}$$

where $0 < a < 1$. The intensive form of the production function becomes:

$$Q(t) = AK(t)^a \tag{24}$$

The economy does not allow for perpetual growth in the long run due to diminishing returns of scale which apply to physical capital and labour. This means that government expenditure and taxes have only level effects in the long run: they can influence growth only along the transition but not on the long run steady state. Furthermore there is no technological progress through time. The production function is subject to constant returns of scale. The firms own their own capital stock and households have a claim on the firm's profits in the form of dividends. Capital's accumulation is defined by:

$$\dot{K}(t) = I(t) \tag{25}$$

where $I(t)$ is gross investment. For the sake of simplicity we omit depreciation of capital. The above relation states that the change of capital must be equal to the gross investment for every period. Following the Tobin's q theory of investment, private investment is subject to increasing and convex costs of adjustment. The cost of investment includes an adjustment cost which is an increasing function of investment in relation to capital. Firm's investment pattern is given by the adjustment cost function: $I(.) = I + \frac{gI^2}{2K}$ where g is the rate at which adjustments cost rise with investment ($I'(.) > 0$ and $I''(.) > 0$).

The firm maximizes the present discounted value of its profits which is the difference between its after tax output and its wage liabilities and expenditure on investment by choosing the optimal level of labour and investment:

$$V_o = \int_{t=0}^{\infty} e^{-rt} \left[(1-\tau)Q(t) - y(t)L(t) - I(t)(1 + \frac{gI(t)}{2K(t)}) \right] \tag{26}$$

subject to the following constraints:

$$\dot{K}(t) = I(t)$$

$$K_0 = K(0) \tag{27}$$

$$\lim_{t->\infty} e^{-rt}\mu_t q_t K_t = 0 \tag{28}$$

where $\mu_t q_t$ is the costate variable or the shadow price of capital in period t. The transversality condition implies that the capital stock should reach zero when time tends to infinity. The Hamiltonian function becomes:

$$H_t = e^{-rt}\left[(1-\tau)AK_t^a L_t^{1-a} - Y_t L_t - I_t - \frac{gI_t^2}{2K_t} + \mu_t q_t I_t\right] \tag{29}$$

From the three first order conditions we derive the expressions for per capita wage, investment and Tobin's q:

$$Y_t = (1-\tau)(1-a)AK_t^a \tag{30}$$

The above relation is the usual profit maximization condition according to which the firm employs labour up to the point where the after tax marginal product of labour equals to the wage rate. The second condition indicates the shadow price of capital must be equal to purchase value which is unity plus the marginal installation cost $\frac{gI_t}{2K_t}$. q is the marginal investment.

$$q_t = 1 + \frac{gI_t}{K_t} \tag{31}$$

By using the above expression we can also define the optimal investment decision of the firm. The investment decision depends positively on the shadow price of capital:

$$I_t = \left(\frac{q_t - 1}{g}\right) K_t \tag{32}$$

The motion of K and the above condition indicates that the firm at the margin equates an additional unit of capital with its marginal cost, or the growth rate of capital is determined by the marginal cost of capital, q. Moreover, the higher the shadow price of capital, the higher the rate of investment relative to the capital stock. In order to have positive investment the shadow price of capital must exceed unity. In the steady state, when investment is zero, the shadow price of capital is one.

$$\dot{K}_t = I_t = \left(\frac{q_t - 1}{g}\right) K_t \Rightarrow \frac{\dot{K}_t}{K_t} = \left(\frac{q_t - 1}{g}\right) \tag{33}$$

Lastly, the first order condition associated with capital stock states the dynamics for the shadow price of capital:

$$-\frac{\partial H_t}{\partial K_t} = \frac{d\left(\mu_t q_t e^{-rt}\right)}{dt} \Rightarrow \frac{dq_t}{dt} = rq_t - a(1-\tau)AK_t^{a-1} + \frac{g}{2}\left(\frac{I_t}{K_t}\right)^2 \tag{34}$$

According to Hayashi (1982) the shadow price of the installed capital q is also defined as the ratio of the stock market value of capital to capital's replacement cost as long as the production function is subject to constant returns of scale which is the case in this model.

$$q_t = \frac{V_t}{K_t} \tag{35}$$

In order to find the intertemporal shadow price of capital we can use the dynamics of q. By integrating (35) and using the transversality condition (29) q is equal to:

$$q_t = \int_{t=0}^{\infty} \left[a(1-\tau)AK_t^{a-1} + \frac{g}{2}\frac{I_t^2}{K_t^2}\right] e^{-rt} dt \tag{36}$$

Equation (37) states that the intertemporal shadow price of capital is equal to the present discounted value of the marginal product of capital and the marginal reduction in adjustment costs from an increase in the capital stock. Given the interest rate, the investment decision does not depend on the utility function, the consumption decision of the households, or the level of private and public debt. However, since $V = qK$, the shadow price of capital affects not only the investment decision of firms but also the consumption pattern of households through their equities holdings.

5. The Government

The government gains revenues by taxing the gross output of the firms and the households and by borrowing from the foreign and domestic private sector. In return it pays interest rate equal to the exogenous real interest rate so agents are indifferent in holding public or private bonds. The government faces the following dynamic budget constraint:

$$\frac{dB^G(t)}{dt} = \bar{G} - T(t) + rB^G(t) \tag{37}$$

where B^G is the per capita government debt, T are the revenues from gross output taxes firms and lump sum taxes on households ($T(t) = T_a + \tau Q(t)$). Proportional taxes on output indicates that labour and capital are taxed in a uniformly way. $\bar{G}$ is the constant government expenditure in public goods which offers utility to households. By integrating the budget constraint and ruling out bubbles in the public bonds we derive the consolidated budget constraint:

$$\int_0^{\infty} e^{-\int_0^t r d\mu} T(t) dt = \int_0^{\infty} e^{-\int_0^t r dv} \bar{G} dt + B_0^G \tag{38}$$

For time t the above expression becomes:

$$B^G(t) = \int_t^{\infty} e^{-\int_t^z r d\mu} (T_z - \bar{G}_z) dz \tag{39}$$

which states that the current level of government's debt must be equal to the present discounted value of surpluses. In other words if the government runs a budget deficit now it must run primary surpluses sometime in the future.

The non-Ponzi condition is:

$$\lim_{t \to \infty} e^{-\int_0^t r d\mu} B^G(t) = 0 \tag{40}$$

5.1. The Capital Account

From the dynamics of the net private debt (23) we can define the foreign assets position of the private sector. We assume that all government debt is bought by the domestic households hence the country's foreign asset position is identical with the private sector's debt to foreigners $B^P(t)$. By using the dynamics for government's bonds, the definitions of dividends and wage income into equation (23) we derive the capital account's dynamics:

$$\frac{dB^P(t)}{dt} = C(t) + I(.) + \bar{G} + rB^P(t) - Q(t) \tag{41}$$

The above expression also provides the equilibrium in the goods market:

$$Q(t) = C(t) + I(.) + \bar{G} - \frac{dB^P(t)}{dt} + rB^P(t) \tag{42}$$

The gross domestic product is equal to the domestic absorption and the capital account surplus.

6. Steady State

The steady state of the economy is derived from the dynamics by setting $\frac{dX(t)}{dt} = 0$

6.1. The Supply Side: Investment and Capital

In steady state the dynamics of capital are equal to zero. Therefore investment is also zero and the shadow price of capital (q) becomes unity from (34):

Investment

$$\dot{K}_t = 0 \Rightarrow \bar{I} = 0 \tag{43}$$

Shadow price of capital

$$\bar{q} = 1 \tag{44}$$

Using the third first order condition and setting $\bar{I} = 0$ and $\bar{q} = 1$ we derive the usual relation which states that in steady state the marginal product of capital must be equal to the world's real interest rate:

$$r^w = a(1-\tau)A\bar{K}^{a-1} \tag{45}$$

where r is the exogenous interest rate. Solving for K we derive the equilibrium value of the capital stock:

Capital

$$\bar{K} = \left(\frac{r}{aA(1-\tau)}\right)^{\frac{1}{a-1}} \tag{46}$$

This is a closed form expression for K as it depends only on exogenous parameters which are the world's interest rate r, the elasticity of capital α, the productivity parameter A and the tax rate τ. We notice that the investment decision of the firms and the level of capital do not depend on households' consumption decision. Wages and output are also derived from the production sector as the following:

Net Output (intensive form)

$$\bar{Q}^N = (1-\tau)A\bar{K}^a \tag{47}$$

Wage

$$\bar{Y} = (1-\tau)(1-a)A\bar{K}^a = (1-\tau)MPL \tag{48}$$

6.2. The Demand Side: Households

By setting the dynamics of human and equity wealth and private debt equal to zero ($\frac{dH(t)}{dt} = \frac{dV(t)}{dt} = \frac{d\left[B^G(t)-B^P(t)\right]}{dt} = 0$) we derive their equilibrium values:

Human Wealth

$$\bar{H} = \left(\frac{1}{r+\gamma}\right)(\bar{Y} - T_a) \tag{49}$$

Equity Wealth

$$\bar{V} = \frac{\bar{D}}{r} \tag{50}$$

where D̄ is the steady state value of dividends $\bar{D} = (1-\tau)\bar{Q} - \bar{Y}$.

The steady state consumption is also derived by setting $\frac{dC(t)}{dt} = 0$

Consumption

$$\bar{C} = \frac{\gamma(\rho+\gamma)(\bar{V} + \bar{B}^G - \bar{B}^P)}{r-\rho} \tag{51}$$

The steady state level of consumption can also be expressed as a function of income $\bar{Y}$ if we eliminate $\bar{V}$ and $\bar{B}^P$ by their steady state values:

$$\bar{C} = \frac{\gamma(\rho+\gamma)\left(\bar{Y} - T_a\right)}{\gamma(\rho+\gamma) - r(r-\rho)} = \frac{\gamma(\rho+\gamma)\left(\bar{Y} - T_a\right)}{(r+\gamma)(\gamma+\rho-r)} \tag{52}$$

For positive consumption in the equilibrium we need the condition $\gamma + \rho > r$ to hold and intuitively the lump sum tax T_a to be smaller than the wage income Y. If the market's interest rate is higher than $\gamma + \rho$ then the future consumption would increase at a rate higher than γ which means that it will increase forever.

Net Private Debt

$$\begin{aligned} \bar{B}^N &= \bar{B}^P - \bar{B}^G = \frac{\bar{D} + \bar{Y} - T_a - \bar{C}}{r} = \\ &= \left(\frac{a\gamma(\rho+\gamma) - r(r-\rho)}{a(r+\gamma)(\gamma+\rho-r)}\right) K + \frac{(r-\rho)}{(r+\gamma)(\gamma+\rho-r)} T_a \end{aligned} \tag{53}$$

Net private indebtedness can be positive or negative in the steady state indicating that the economy can be a debtor or a lender. The private debt is positive when $a\gamma(\rho+\gamma) - r(r-\rho) > 0$ or when $-\sqrt{\alpha\gamma(\rho+\gamma)} < r < \rho + \sqrt{\alpha\gamma(\rho+\gamma)}$. Note that the net asset position of the private sector depends on two characteristics of the economy: First is the discount factor which signifies how impatient people are to consumet. The second important factor is the share of capital in the production function which plays a crucial role in determining the equilibrium net debt. Even if the interest rate is equal to the discount rate the individuals still prefer to borrow and to consume than to save. This implies that economies with similar discount factors and the rest of the macroeconomic variables will have different levels of private debt if the share of capital in the production is different. In other words, the interest rate must be high enough to provide a strong incentive to save at the present and to consume in the future.

The way the share of capital affects the net private debt is not the same for every economy. If there is technological progress (α increases), private debt can follow any direction, according to agents' preferences: As domestic output rises, agents may choose to increase their debt level, or to increase the rate of borrowing and maintain the same level of debt stock, or leave the debt stock to fall in lower levels. If we look at the effect of a higher a $\left[\frac{\vartheta \bar{B}^N}{\vartheta a} = \left(\frac{r(r-\rho)}{a^2(r+\gamma)(\gamma+\rho-r)}\right) K + \left(\frac{a\gamma(\rho+\gamma)-r(r-\rho)}{a(r+\gamma)(\gamma+\rho-r)}\right)\frac{\partial K}{\partial a}\right]$ we see that whilst the first term is always positive, the second one depends on how big a is, relatively to r and ρ $\left(\frac{\partial K}{\partial a} < 0\right)$. If $a < \frac{r(r-\rho)}{\gamma(\rho+\gamma)}$ then $\frac{\vartheta \bar{B}^N}{\vartheta a} > 0$.

This is a result in addition to the original Blanchard's model where the demand side of an open economy is examined with exogenous labour income and output.

We can define the private sector's foreign debt which is also the capital account by substituting for the dynamics of government bonds into equation (54):

$$\bar{B}^P = \frac{T - G - \bar{C} + \bar{D} + \bar{Y}}{r} \tag{54}$$

6.2.1. *The Government*

In equilibrium the evolution of public debt is zero $\left(\frac{dB^G(t)}{dt} = 0\right)$ hence taxes must be equal to expenditure and the interest payments on debt:

Government Bonds

$$\bar{B}^G = \frac{T-G}{r} = \frac{T_a + \tau Q - G}{r} \quad (55)$$

7. The convergence path to the steady state

The dynamics of the households, the firms and the government characterize the economy of our country. In order to analyze the dynamic system described by the non linear equations (23), (24), (34), (35),(38) we can use their linear approximations near the steady state which sufficiently describe the qualitative behavior of the trajectories around this point. Therefore we need their linear approximations near the steady state, $K = \bar{K}$, $\bar{q} = 1$, $C = \bar{C}$, $B^P = \bar{B}^P$, $B^G = \bar{B}^G$. Commencing with the supply side, the linear functional forms of capital and Tobin's q near their steady state values are:

$$\dot{K} = 0(K - \bar{K}) + \frac{\bar{K}}{g}(q-1) \quad (56)$$

$$\dot{q} = [(1-a)\frac{r}{\bar{K}}](K - \bar{K}) + r(q-1) \quad (57)$$

Consumption can be expressed as a function of private debt, capital and its shadow price by eliminating the equities V since $V = qk$. Therefore the dynamics for consumption become:

$$\dot{C} = (r-\rho)C - \gamma(\rho+\gamma)qK + \gamma(\rho+\gamma)B^N \quad (58)$$

Note that we have defined $B^P - B^G = B^N$ for notational simplicity. The linear approximation around $\bar{C}$ is:

$$\dot{C} = (r-\rho)(C-\bar{C}) + \gamma(\rho+\gamma)(B^N - \bar{B}^N) - \gamma(\rho+\gamma)(K-\bar{K}) - \gamma(\rho+\gamma)\bar{K}(q-1) \quad (59)$$

The dynamics of the net private debt as a function of B^N, C, K and q are:

$$\dot{B}^N = C - (1-\tau)AK^a + \frac{K}{2g}(q^2-1) + T_a + rB^N \quad (60)$$

The linearized form around the steady state becomes:

$$\begin{aligned}\dot{B}^N &= (C-\bar{C}) + r(B^N - \bar{B}^N) - a(1-\tau)A\bar{K}^{a-1}(K-\bar{K}) + \frac{\bar{K}}{g}(q-1) \quad (61)\\ &= (C-\bar{C}) + r(B^N - \bar{B}^N) - r(K-\bar{K}) + \frac{\bar{K}}{g}(q-1)\end{aligned}$$

We note that fiscal policy enters in the linearized near the steady state dynamics for consumption and net private debt through the tax rate which is included in the capital and not through lump sum taxes.

Finally the linearized dynamics for government debt are:

$$\dot{B}^G = -a\tau A\bar{K}^{a-1}(K-\bar{K}) + r(B^G - \bar{B}^G) \qquad (62)$$

The dynamic system described by the four differential equations is the following:

$$\begin{pmatrix}\dot{K}\\ \dot{q}\\ \dot{C}\\ \dot{B}^N\end{pmatrix} = \begin{pmatrix} 0 & \frac{\bar{K}}{g} & 0 & 0 & 0\\ (1-a)\frac{r}{\bar{K}} & r & 0 & 0 & 0\\ -\gamma(\rho+\gamma) & -\gamma(\rho+\gamma)\bar{K} & (r-\rho) & \gamma(\rho+\gamma) & 0\\ -r & \frac{\bar{K}}{g} & 1 & r & 0\end{pmatrix}\begin{pmatrix}K(t)-\bar{K}\\ q(t)-1\\ C(t)-\bar{C}\\ B^N(t)-\bar{B}^N\end{pmatrix}$$

We observe that the dynamics of K and q depend only on the capital stock and the shadow price of capital endogenously, and for this reason we can study the dynamics of investment and capital separately and solve for their time paths.

7.1. *Investment and Capital*

The solution to the linear system of K and q is given by the following system of differential equations:

$$\begin{pmatrix}\dot{K}\\ \dot{q}\end{pmatrix} = \begin{pmatrix}0 & \frac{\bar{K}}{g}\\ (1-a)\frac{r}{\bar{K}} & r\end{pmatrix}\begin{pmatrix}K(t)-\bar{K}\\ q(t)-1\end{pmatrix}$$

the characteristic equations becomes $\kappa^2 - r\kappa + \frac{r(a-1)}{g} = 0$ and the roots are:

$$\kappa_{1,2} = \frac{r +/- \sqrt{r^2 + \frac{4r(1-a)}{g}}}{2} \qquad (63)$$

The determinant of the coefficient matrix A is $detA = -(1-a)\frac{r}{g} < 0$ and $\kappa_1\kappa_2 = detA < 0$. The roots are both real and distinct with one root stable corresponding to the predetermined capital stock and one root unstable corresponding to the forward looking price variable q. The behavior of the linearized system is that of a saddle point equilibrium in the neighborhood of the steady state. Since $1 + \sqrt{\frac{4(1-a)r}{g}} > 0$ the first root κ_1 is the unstable one and κ_2 is the one that leads the system to convergence. By using the initial condition for capital ($K(0) = K_0$), the particular solution[4] of the system which rules out the explosive root is:

$$K(t) = (K_0 - \bar{K})e^{\kappa_2 t} + \bar{K} \tag{64}$$

$$q(t) = \frac{\kappa_2 g}{\bar{K}}(K_0 - \bar{K})e^{\kappa_2 t} + 1 \tag{65}$$

Phase Diagram for K and q

The q isocline is obtained by setting $\dot{K} = 0 \Rightarrow \bar{q} = 1$. By the same way the K isocline comes from $\bar{q} = 0 \Rightarrow q = \frac{1}{r} - (1-a)\frac{K}{\bar{K}} + (1-a)$. The K isocline has a negative slope: $\frac{dq}{dK} = -\frac{(1-a)}{\bar{K}}$

[4] Set the coefficient of the explosive root $C_1 = 0$. Furthermore the initial conditions for K and q should satisfy the following equation:

$$q(t) = \frac{\kappa_2 g}{\bar{K}}(K_0 - \bar{K}) + 1$$

The locus of points (K and q) defined by this equation is the saddlepath.

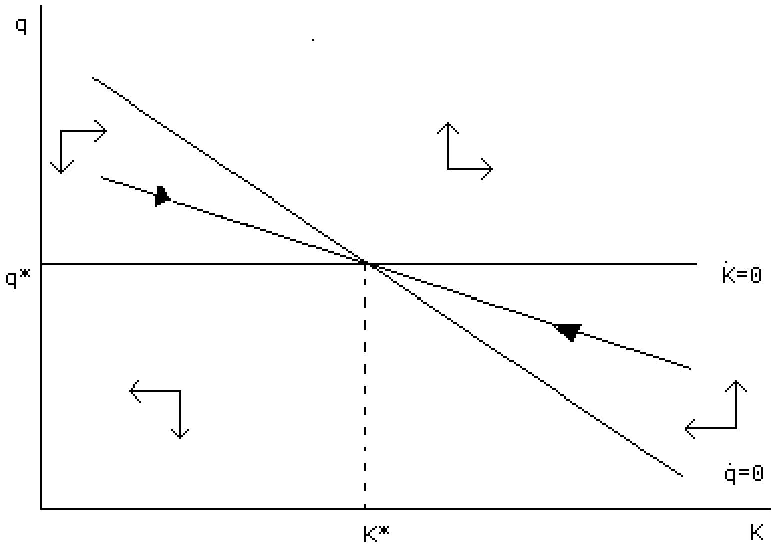

Figure 1: Dynamics for Investment and Capital

Capital is rising when $q > 1$ as we can see from the dynamics of K and is decreasing for $q < 1$. The shadow price of capital is increasing in the isosector above the $q = 0$ and is decreasing in the area below. Since the system is a saddle point stable there is only a unique path that converges to steady state meaning that there is only one value for q for any initial capital stock that places the firm on the stable adjustment path. Only on the stable path the q bubbles are excluded.

7.2. Consumption and Net Private Debt

Having solved for the time paths of K and q we next deal with the dynamics of C and B^N which are both dependent on K and q time paths:

$$\begin{pmatrix} \dot{C} \\ \dot{B}^N \end{pmatrix} = \begin{pmatrix} (r-\rho) & \gamma(\rho+\gamma) \\ 1 & r \end{pmatrix} \begin{pmatrix} C_t - \bar{C} \\ B_t - \bar{B} \end{pmatrix} + \begin{pmatrix} -\gamma(\rho+\gamma)(1+g\kappa_2)(K_0-\bar{K})e^{\kappa_2 t} \\ -(r-\kappa_2)(K_0-\bar{K})e^{\kappa_2 t} \end{pmatrix}$$

The determinant of the coefficient matrix is negative by using the condition for a positive $\bar{C}$, indicating a saddle point equilibrium. From the system's characteristic equation $\lambda^2-\lambda\left(2r-\rho\right)+r^2-\rho r-\gamma(\rho+\gamma)=0$ we derive two real and distinct roots λ_1 and λ_2.

$$\begin{aligned}\lambda_{1,2} &= \frac{(2r-\rho)+\sqrt{(2r-\rho)^2-4(r^2-pr-\gamma(\rho+\gamma))}}{2} \Rightarrow \\ &\Rightarrow \lambda_1 = r+\gamma \text{ and } \lambda_2 = r-\gamma-\rho\end{aligned}$$

The first eigenvalue is $\lambda_1 = r+\gamma$ and it is always positive. The second eigenvalue is $\lambda_2 = r-(\gamma+\rho)$ and it is negative only if we impose the assumption that $r-(\gamma+\rho)<0$ or $r<(\gamma+\rho)$ which is the necessary and sufficient condition for positive consumption in the steady state. This assumption is also necessary because if the interest rate was bigger than the probability of death and the discount factor, the aggregate consumption would increase for ever. The time paths for C and B^N are:

$$\begin{aligned}C(t) &= C_1 e^{(r+\gamma)t}+C_2 e^{(r-\gamma-\rho)t}+\beta_1(K_0-\bar{K})e^{\kappa_2 t}\\ B^N(t) &= \frac{C_1 e^{(r+\gamma)t}}{\gamma}-\frac{C_2 e^{(r-\gamma-\rho)t}}{\gamma+\rho}+\beta_2(K_0-\bar{K})e^{\kappa_2 t}\end{aligned}$$

The particular solution corresponding to the stable arm is:

$$\begin{aligned}C(t) &= \left(\tilde{C}_0-\bar{C}\right)e^{(r-\gamma-\rho)t}+\beta_1(K_0-\bar{K})e^{\kappa_2 t}\\ B^N(t) &= -\frac{\left(\tilde{C}_0-\bar{C}\right)e^{(r-\gamma-\rho)t}}{\gamma+\rho}+\beta_2(K_0-\bar{K})e^{\kappa_2 t} \qquad (66)\end{aligned}$$

where $\beta_1=\frac{g\gamma(r+\rho)(r-\kappa)\kappa}{(r+\gamma-\kappa)(r-\gamma-\rho+\kappa)}=\frac{g\gamma(r+\rho)(r-\kappa)\kappa}{(r+\gamma-\kappa)(\lambda+\kappa)}>0$,and $\beta_2=\frac{(r-\kappa)(\rho-r+k)+\gamma(r+\rho)(1+g\kappa)}{-(r+\gamma-\kappa)(\lambda+\kappa)}>0$. Note that κ and λ are the stable eigenvalues for the systems of K and q and C and B^N respectively.

At this point we draw the phase diagrams taking into account two possible scenarios:

In the first scenario subjective discount factor is bigger than the market's interest rate $(r<\rho+\sqrt{(\alpha\gamma(\rho+\gamma))})$. Individuals prefer to consume more and de cumulate assets. The economy is a net debtor in the steady state (see figure 3a).

The system of $\bar{B}^N$ and $\bar{C}$ is the following:

$$\bar{B}^N = \frac{(1-\tau)\,A\bar{K}^a - T_a - \bar{C}}{r}$$
$$\bar{C} = \frac{\gamma(\rho+\gamma)(\bar{K}-\bar{B}^N)}{r-\rho}$$

When $r < \rho + \sqrt{(\alpha\gamma(\rho+\gamma))}$, the level of net private debt is positive in the steady state : $\bar{B}^N = \left(\frac{a\gamma(\rho+\gamma)-r(r-\rho)}{a(r+\gamma)(\gamma+\rho-r)}\right) K + \frac{(r-\rho)}{(r+\gamma)(\gamma+\rho-r)} T_a > 0$ Net private debt is increasing in the region above the isocline $\tilde{B}^N$=0 while it is decreasing in the area below. The $\dot{B}^N = 0$ isocline has a negative slope since $\frac{dC}{dB} = -r < 0$ in both scenarios while the $\dot{C} = 0$ isocline has a positive slope $\frac{dC}{dB} = \frac{-\gamma(\rho+\gamma)}{r-\rho}$ in this case.

In the second case the market's interest rate is higher than the subjective discount factor $\left(r > \rho + \sqrt{\alpha\gamma(\rho+\gamma)}\right)$ and agents are accumulating assets over their life and the level of private debt is negative in equilibrium. Surprisingly enough the share of capital in the production function plays a crucial role in a country's indebtedness when the agents are not impatient to consume. If it is very small then agents choose to save and invest. However, if the economy is highly capitalized then agents would rather consume and dis-save.

$$\bar{B}^N = \left(\frac{a\gamma(\rho+\gamma) - r(r-\rho)}{a(r+\gamma)(\gamma+\rho-r)}\right) K + \frac{r\,(r-\rho)}{r\,(r+\gamma)\,(\gamma+\rho-r)} T_a < 0$$

In the steady state the private sector is a net creditor to the rest of the world given a high interest rate (see figure 3b).

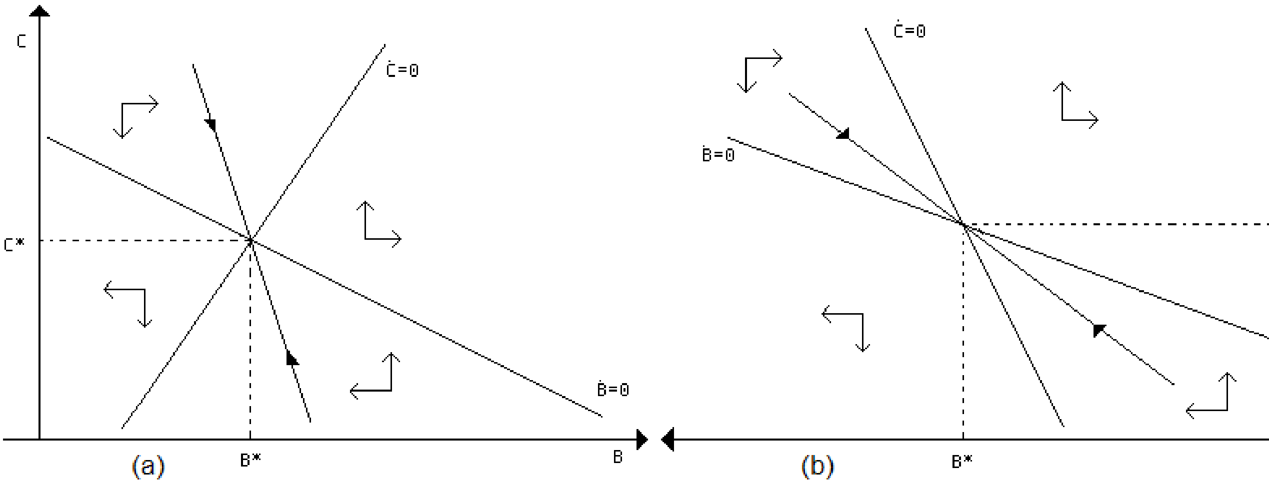

Figure 3: Dynamics for Consumption and Private Debt

8. Comparative statics

8.1. Steady state effects of a lower tax rate

A lower tax rate tends to boost market activity, that is to encourage aggregate consumption and investment and therefore aggregate demand. A permanent decline in the tax rate raises the marginal product of capital which in turn leads to a higher demand for equities and rises the market price of capital. Due to higher investment in the short run, the capital stock and the level of equities rise to a new equilibrium:

Capital

$$\frac{\partial \bar{K}}{\partial \tau} = \frac{\partial \bar{V}}{\partial \tau} = -\frac{\bar{K}}{(1-a)(1-\tau)} < 0$$

In equilibrium, the after tax marginal product of capital is equal to the world's interest rate : $r^w = a(1-\tau)A\bar{K}^{a-1} = MPK$. For a given r^w, a lower τ raises the after tax return from investment to a higher level than the world's interest rate. The economy experiences capital inflow. A lower tax rate requires a higher capital stock which will generate a sufficient decrease in the marginal product of capital. A new lower tax rate in the steady state illustrates a higher capital stock. Furthermore the elasticity of $\bar{\mathrm{K}}$ with respect to $(1-\tau)$ is $\frac{1}{1-a} > 1$ which illustrates how volatile the capital can be with respect to the tax rate.

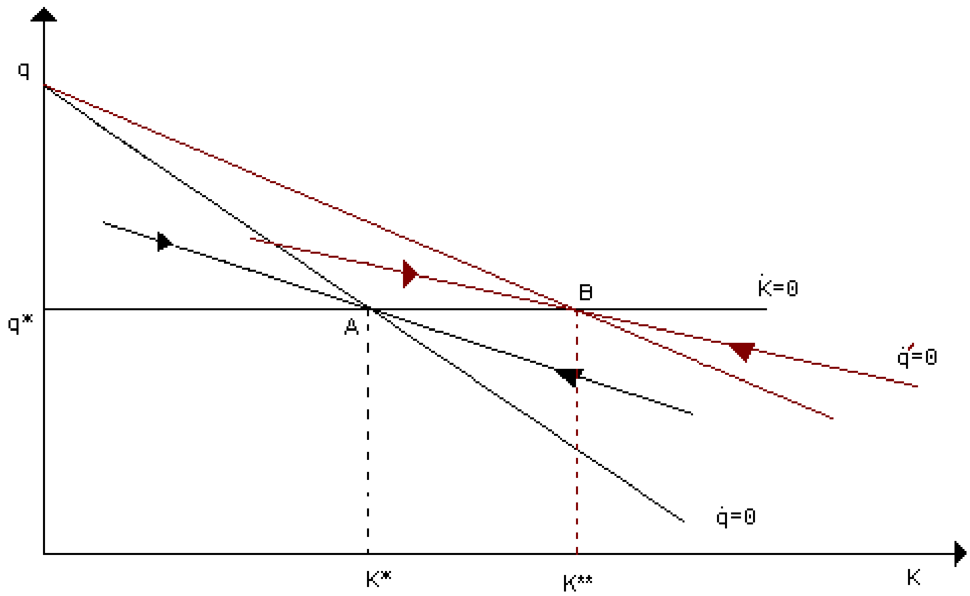

Figure 4: The effects of a lower tax rate on K and q

Figure 4 illustrates the case of a permanent decrease of τ. The tax rate enters only in the $\dot{q} = 0$ locus and affects its slope but not its intercept. The new isocline $\dot{q} = 0$ is flatter and provides a higher level of capital stock. The fall of the tax rate increases capital's shadow price q which jumps initially to the new saddle path increasing the marginal product of capital. Since q is higher than unity and the marginal product of capital is higher, firms increase investment until the capital stock reaches its new long run equilibrium.

Moving on to the other economic variables, we see that a lower tax rate has positive effects on wage income, dividends, output and human wealth:

Wage income

$$\frac{\partial \bar{Y}}{\partial \tau} = -A\left(\frac{r}{aA(1-\tau)}\right)^{\frac{a}{a-1}} = -\bar{Q} < 0$$

The equilibrium wage is equal to the marginal product of labour. A lower corporate tax rate raises the marginal product of labour and therefore the real wage.

Dividends

$$\frac{\partial \bar{D}}{\partial \tau} = -\frac{rK}{(1-\tau)} < 0$$

Output

$$\frac{\partial \bar{Q}}{\partial \tau} = -\frac{A}{1-a}[\frac{r}{aA(1-\tau)}]^{\frac{a}{a-1}} = -\frac{A}{1-a}\bar{K}^a = -\frac{\bar{Q}}{1-a} < 0$$

Human Wealth

$$\frac{\partial \bar{H}}{\partial \tau} = -\frac{A\Omega}{(1-a)(r+\gamma)}[\frac{r}{aA(1-\tau)}]^{\frac{a}{a-1}} = -\frac{A\Omega}{(1-a)(r+\gamma)}\bar{K}^a < 0$$

Consumption

$$\frac{\partial \bar{C}}{\partial \tau} = -A\left[\frac{\gamma(r+\gamma)}{(r+\gamma)(\gamma+\rho-r)}\right]\left[\frac{r}{aA(1-\tau)}\right]^{\frac{a}{a-1}} = -A\bar{K}^a[\frac{\gamma(r+\gamma)}{(r+\gamma)(\gamma+\rho-r)}] < 0$$

The impact of a change in the tax rate on consumption is negative. The necessary and sufficient condition for positive consumption $(\gamma+\rho-r>0)$ has been used to sign the above derivative.

Net Private Debt

$$\frac{\partial \bar{B}^N}{\partial \tau} = \frac{K}{a(1-\tau)}\left[\frac{1}{1-a} - \frac{\gamma(\rho+\gamma)}{(r+\gamma)(\gamma+\rho-r)}\right]$$

The above derivative illustrates that the effect of a lower tax rate on net private debt can be positive or negative. The total effect is positive $\left(\frac{\partial \bar{B}^N}{\partial \tau} > 0\right)$ when the parenthesis in the right hand side is positive or $r > \rho + \sqrt{\alpha\gamma(\rho+\gamma)}$. This is exactly the case when the country is a net creditor: As the tax rate falls the return of capital increases and agents prefer to hold more equities or to increase consumption instead of acquiring foreign bonds. The incentive for lending to the rest of the world is weaker in the new tax rate regime. The opposite holds for the debtor economy with a high discount rate: the higher value of equities increases disposable income and agents can afford not only a higher level of consumption but a higher level of private debt as well.

Figure 5a depicts the impact of lower taxation on consumption and private debt when considering a debtor private sector. A change in the tax rate affects both isoclines: Their slopes do not change but both curves shift upwards in a parallel way. Correspondingly, the saddle path shifts rightward. The $\dot{C} = 0$ locus shifts up as the value of equities rises and agents have higher disposable income and higher consumption. Individuals have two choices: they can increase investment and capital stock in the long run or they can just borrow more for even higher consumption, thus increasing their private debt. The second scenario where agents have a high discount rate $\left(r < \rho + \sqrt{\alpha\gamma(\rho+\gamma)}\right)$ has been depicted in figure 5b. In a creditor economy where the interest rate is high enough $\left(r > \rho + \sqrt{\alpha\gamma(\rho+\gamma)}\right)$ agents have a strong incentive to invest rather than to lend to the rest of the world as the returns of capital are higher than the returns on bonds. The $\dot{B}^N = 0$ locus shifts upwards substantially showing a lower level of private lending.

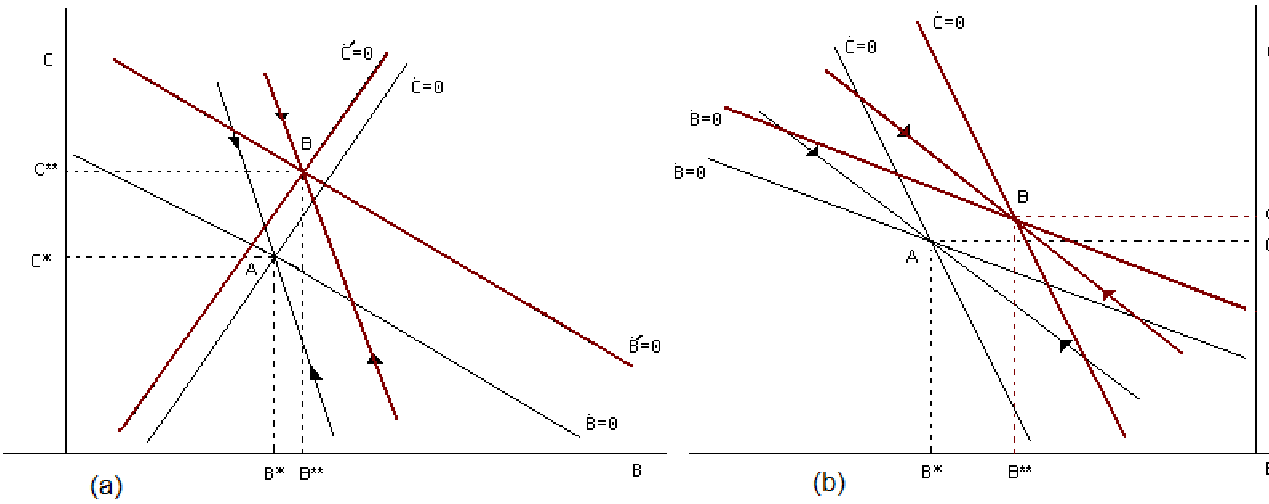

Figure 5: The impact of a lower tax rate: the case of a debtor (a) and a creditor (b) countr

Government Debt

The government debt can increase or decrease depending on the tax revenues. Only if the tax revenues increase ceteris paribus, the public debt falls.

$$\frac{\partial \bar{B}^G}{\partial \tau} = \frac{-\left(1-a-\tau\right)AK^a}{r(1-a)(1-\tau)}$$

Tax Revenues

We have defined tax revenues as: $T_t = T_{1t} + \tau_2 Q_t$. An increase of the tax rate would boost tax revenues unless domestic output falls substantially.

$$\frac{\partial T}{\partial \tau} = \frac{\left(1-a-\tau\right)AK^a}{(1-a)(1-\tau)}$$

The effect of a higher tax rate on government's revenues depends on the negative response of real taxable income to the tax rate. Tax revenues increase only if $1-a-\tau > 0$ or $a+\tau < 1$ which means that the after tax elasticity of capital must be less than one (or capital must exhibit increasing returns of scale).

8.2. Steady state effects of a lower lump sum tax

The lump sum tax does not enter the dynamics of K and q and a change in this parameter does not affect the equilibrium.

Consumption and Net Private Debt

Steady state effect:

A fall in the lump sum tax will increase steady state consumption by $\frac{-\gamma(\rho+\gamma)T_1}{\gamma(\rho+\gamma)-r(r-\rho)}$ while net private borrowing depends on the external asset position of the country.

$$\frac{\partial \bar{C}}{\partial T_1} = \frac{-\gamma(\rho+\gamma)}{(r+\gamma)(\gamma+\rho-r)} < 0 \quad \text{and} \quad \frac{\partial \bar{B}^N}{\partial T_1} = \frac{(r-\rho)}{(r+\gamma)(\gamma+\rho-r)}$$

The locus $\dot{C} = 0$ is:

$C_t = -\frac{\gamma(\rho+\gamma)}{r-\rho}\left(B_t^N - \bar{B}^N\right) + \bar{C} + \gamma(\rho+\gamma)(1+g\kappa_2)(K_0 - \bar{K})e^{\kappa_2 t}$

and the locus $\dot{B}^N = 0$ becomes:

$C_t = -r\left(B_t^N - \bar{B}^N\right) + \bar{C} + (r-\kappa_2)(K_0 - \bar{K})e^{\kappa_2 t}$

A change in the lump sum tax does not affect the slope or the intercept of the $\dot{C} = 0$ locus whilst $\dot{B}^N = 0$ locus changes its intercept but not its slope. A fall of the lump sum tax shifts the $\dot{B}^N = 0$ schedule upwards in a parallel way. The effect on the intercept is $\frac{\partial(r\bar{B}^N)}{\partial T_1} = \frac{-\gamma(\rho+\gamma)+(r-\rho)r}{(r+\gamma)(\gamma+\rho-r)} = -1$. Both in the case of a creditor and a debtor economy, consumption and the absolute value of net debt increases (see figure 6).

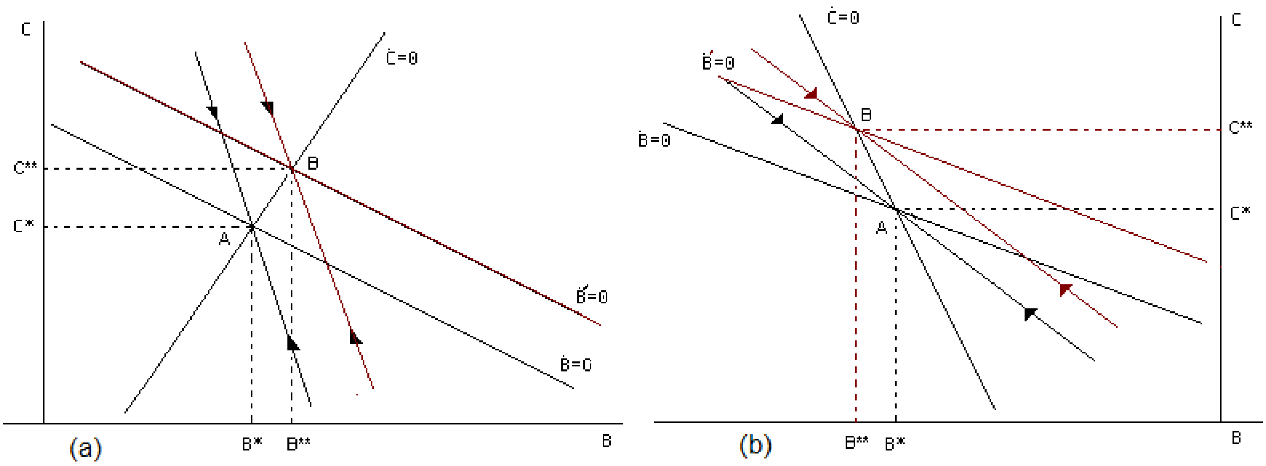

Figure 6: The effects of a lower lump sum tax on a debtor (a) and on a creditor (b) country

Public Debt

A higher lump sum tax allows for a higher level of public debt ($\frac{\vartheta \bar{B}^P}{\vartheta T_1} = \frac{1}{r}$).

8.3. *Steady state effects of a lower interest rate*

A decrease in the interest rate will have the following effects on the stock of capital and its shadow price:

Capital

$$\frac{\vartheta \bar{K}}{\vartheta r} = \frac{1}{a-1}\left(\frac{1}{r}\right)^{\frac{a}{a-1}}\left(\frac{1}{aA(1-\tau)}\right)^{\frac{1}{a-1}} = \frac{1}{a-1}\left(\frac{1}{r}\right)^{\frac{a}{a-1}} K < 0 \quad (67)$$

The interest rate only enters in the $\dot{q} = 0$ locus and it shifts to the right. The new steady state capital stock is higher than the previous one because the required rate of return on capital has fallen. As a result the saddle path shifts rightward. The capital stock is a predetermined variable and cannot change in the short run. However, the shadow price of capital can adjust immediately. Thus the fall of the interest rate causes the new equilibrium to shift from the point A to point B on the new saddle path (see figure 7). The initial response to the information of a future permanent fall in the interest rate will result in an increase of capital's shadow price, followed by a gradual process of higher investment and a higher level of capital stock over time.

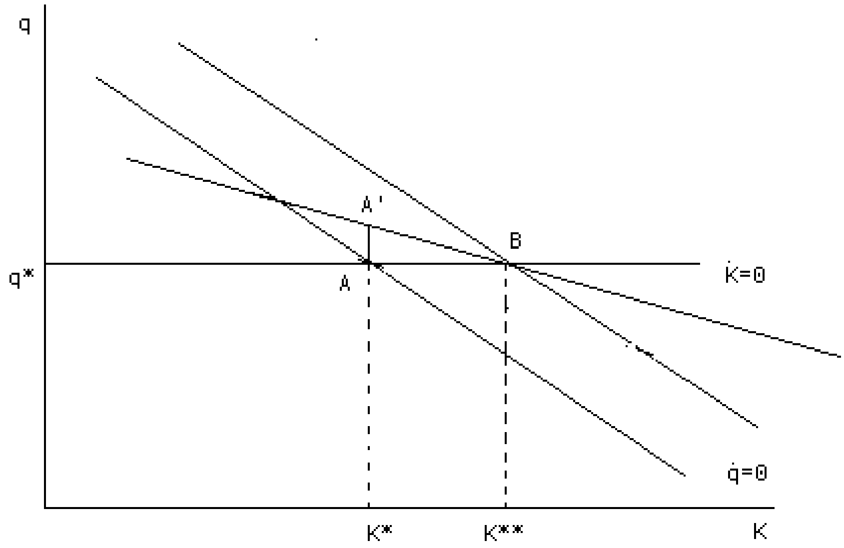

Figure 7: A foreseen permanent fall of the interest rate

The lower interest rate has a positive impact on equilibrium output, wages, dividends, equities and human wealth. This is shown by the derivatives listed below:

Output

$$\frac{\vartheta Q}{\vartheta r} = \frac{-K}{1-a} < 0$$

Wage

$$\frac{\vartheta Y}{\vartheta r} = -K < 0$$

Dividends

$$\frac{\vartheta D}{\vartheta r} = -\frac{a}{1-a}K < 0$$

Human Wealth

$$\frac{\vartheta \bar{H}}{\vartheta r} = \frac{-K(r+a\gamma)}{a(r+\gamma)^2} < 0$$

Equity Wealth

$$\frac{\vartheta V}{\vartheta r} = \frac{\vartheta \bar{K}}{\vartheta r} = \frac{1}{a-1}\left(\frac{1}{r}\right)^{\frac{a}{a-1}} K < 0$$

Consumption

The response of equilibrium consumption to a change in the interest rate is given by:

$$\frac{\vartheta \bar{C}}{\vartheta r} = \frac{\gamma(r+\gamma)\left\{\frac{\partial Y}{\partial r}\left[\gamma(\rho+\gamma) - r(r-\rho)\right] + (2r-\rho)\,Y\right\} + (2r-\rho)\,T_1}{\left[\gamma(\rho+\gamma) - r(r-\rho)\right]^2}$$

It is not clear what happens to equilibrium consumption when the interest rate falls. First, it depends on whether the economy in question is a net saver or a net debtor. If the economy is a net creditor, then its revenues from interest payments fall and downward pressure on consumption is expected. The opposite occurs when the economy is a debtor: the agents have to pay less interest on their debt and more income is left to consume. Another important factor which affects consumption and private debt is public debt: Provided that the public sector is always a debtor to the private sector, public debt falls in both types of economies.

In the case of a creditor economy, lower returns from holdings in public bonds will further constrain private consumption and a lower consumption level is expected. In the case of a debtor economy it is not very clear which effect prevails. On the one hand agents pay lower interest on their private debt but on the other hand they hold less public bonds. If the negative effect of public debt overcomes the positive effect of the lower interest rate then consumption may not increase. On the contrary, it may even fall.

The economy experiences higher consumption and net private debt only when individuals discount the future heavily, i.e. only when $r < \frac{\rho}{2}$ then $\frac{\vartheta \bar{C}}{\vartheta r} < 0$. In this case, even though financial wealth from public bonds falls, agents increase borrowing substantially so that their new steady state consumption is higher. In this case not only the substitution effect is positive but also the income effect.

Net private debt

We define the net private debt as: $\bar{B}^N = \frac{D}{r} - \frac{(r-\rho)Y}{\gamma(\rho+\gamma)-r(r-\rho)} + \frac{T_1}{\gamma(\rho+\gamma)-r(r-\rho)}$

$$\begin{aligned} \frac{\vartheta \bar{B}^N}{\vartheta r} &= \frac{r\frac{\partial D}{\partial r} - D}{r^2} - \frac{\left[Y + (r-\rho)\frac{\partial Y}{\partial r}\right]\left[\gamma(\rho+\gamma) - r(r-\rho) - (r-\rho)Y(\rho-2r)\right]}{\left[\gamma(\rho+\gamma) - r(r-\rho)\right]^2} \\ &\quad + \frac{(\rho-2r)\,T_1}{\left[\gamma(\rho+\gamma) - r(r-\rho)\right]^2} \end{aligned}$$

The first term reflects the effect of a higher interest rate on private debt through the effect on dividends and it is always negative. The second term is the effect on income when agents have finite horizons and clearly depends on the relationship between the discount factor and the interest rate. This term is positive when the discount factor is double the interest rate $\left(\frac{\rho}{2} > r\right)$. The third term represents the effect through lump sum taxes and its effect on the net indebtedness depends on the relative size of the discount factor to the interest rate. If the interest rate increases and the lump sum tax remains unchanged, a very impatient private sector $(\rho > 2r)$ will increase its net debt and the third term turns out to be positive. We can tell that in the case of $\rho > 2r$ the effect of higher interest rate on net debt is negative $(\frac{\vartheta \bar{B}}{\vartheta r} < 0)$. This of course, assumes that the positive effect from the lump sum tax is smaller than the impact from dividends and income.

To sum up, when $\frac{\rho}{2} > r$ then both consumption and net private debt increase as a result of a fall in the interest rate. The new steady state is higher than the old one. In this case the country is a net debtor. Figure 8 illustrates the new phase diagram for a permanent fall of the interest rate.

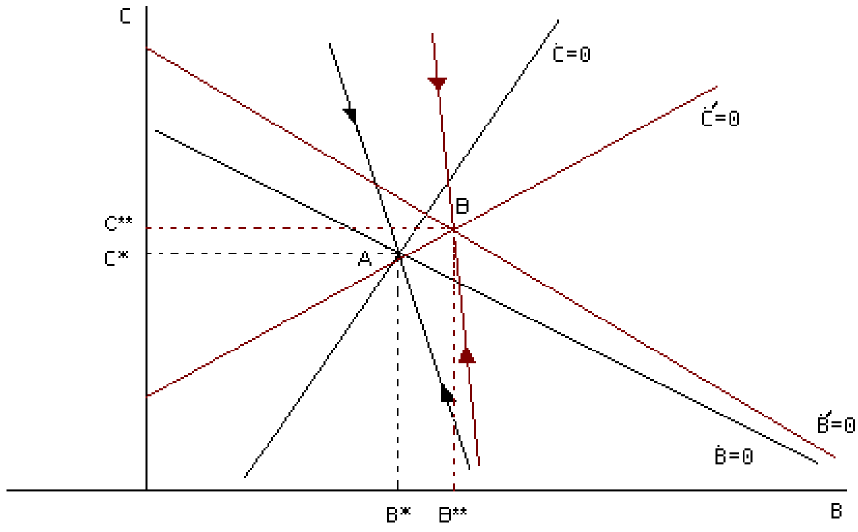

Figure 8: The effects of a lower interest rate on consumption and private debt

We can demonstrate the shift of the isocline and the new steady state for C and B^N only in the case of $\frac{\rho}{2} > r$. When the interest rate falls both isoclines shift up: The consumption isocline shifts up and becomes flatter and less sensitive to debt while the net debt isocline shifts up and becomes steeper. The new steady state is higher as both consumption and net private debt are higher. Consumption initially overshoots as interest payments fall and agents can afford a higher level of debt. When the interest payments increase again due to the higher level of debt, consumption falls again reaching the new steady state which is nevertheless higher than the previous one.

Public debt

The impact of a lower interest rate on public debt ($\bar{B}^G = \frac{T_a+\tau Q-G}{r}$) is twofold: First, there is a direct impact on public debt through lower interest payments. A lower primary budget surplus is required for a given government debt stock. The second impact is through higher tax revenues. The lower EMU interest rate boosts the steady state capital and in turn the steady state output and tax revenues increase. Algebraically, the effect of the interest rate on government's debt is negative:

$$\frac{\vartheta \bar{B}^G}{\vartheta r} = \frac{\tau \frac{\partial Q}{\partial r} r - (\tau Q + T_a - G)}{r^2} < 0$$

The first term in the expression above is negative as a higher real interest rate has a negative effect on output. The term in the parenthesis is the primary surplus of the public sector. An indebted government, like the one in this model, must hold a primary surplus in equilibrium in order to meet its interest payments. For this reason the primary surplus is positive and the whole expression is negative.

9. The entrance in a monetary union

9.1. The impact of a lower interest rate

The economy is initially in steady state. When the country enters the monetary union it does not face the world's interest rate (which includes a risk premium) any longer but the union's interest rate set by the union's Central Bank. It is intuitive to assume that the members of the monetary union borrow from the rest of the world at a lower interest rate than they used to as the risk premium falls. This is also because the union members have to fulfil the fiscal convergence criteria and demonstrate fiscal discipline which implies that they have to run an almost balanced budget or surplus.

Our analysis discusses the long run equilibrium effects of a permanent interest rate decline and the optimal fiscal response. As the interest payments become smaller and the tax revenues increase, the government faces the following options:

Case 1:

The government chooses to take no action. In this case public debt continues to fall and eventually becomes negative following an explosive route to minus infinity. We have seen that around the steady state a lower interest rate leads to a lower level of public debt: $\dot{B}^G = -a\tau A\bar{K}^{a-1}(K_2 - \bar{K}_1) + r_2(B_2^G - \bar{B}_1^G)$ given that the government does not change its spending behaviour. Clearly government's intervention is required at a certain point.

Case 2:

The government targets a higher level of public debt. It increases its borrowing until it reaches a higher level of debt stock. Nevertheless, given that such a scheme would violate the fiscal criteria for EMU membership and any fiscal constraints set by the members (like the previous Stability and Growth Pact), this case is rather unrealistic.

Case 3:

A more realistic plan of action, which is in accordance with the EMU fiscal constraints, is to target a lower level of public debt $\bar{B}_2^G$ ($\bar{B}_2^G < \bar{B}_1^G$) and to adjust the taxation schedule accordingly. As soon as the target level of debt is reached, the government switches to a new primary surplus which supports the new debt level. This can be achieved by changing the lump sum tax T_a[5]. In this case, lump sum taxes are negative and can take the form of transfers. Lets assume that the lump sum tax was T_a in the initial steady state: $\bar{B}_0^G = \frac{T_a + \tau Q_0 - \bar{G}}{r_1}$ and the government sets a lower target debt level after entering EMU: $\bar{B}_2^G = \frac{T_b + \tau Q_2 - \bar{G}}{r_2}$. Then, the new lump sum tax which supports the new equilibrium debt must be: $T_b = r_1 \bar{B}_1^G - \tau Q_1 + \bar{G}$.

The net gain in taxes that individuals save from entering the EMU when the economy reaches the new debt level is:

$$T_b - T_a = (r_1 \bar{B}_1^G - r_0 \bar{B}_0^G) - \tau(Q_1 - Q_0)$$

[5] The lump sum tax T_1 becomes T_a, T_b or T_c according to the fiscal plan followed.

Note that the target level of the debt stock and the new real interest rate are both lower which makes the first term negative. In addition, output is higher in the new equilibrium and the second term is positive. The government has two options:

Firstly, it may choose not to change the taxation scheme initially: After entrance in the monetary union, the government may allow public debt to fall because of lower interest payments and higher tax revenues (arising from higher output, ceteris paribus), until it reaches the lower target level. When this level has been reached, some gains can be returned to the agents in the form of lump sum transfers.

Alternatively, the government may prefer to return gains before decreasing public debt reaches its target. This is more suitable in cases when the fall of public debt to its new level requires a long period of adjustment which makes it politically costly for the government. For these reasons the public sector may allow some kind of transfers before the economy reaches the new level. However the initial transfers must be lower than T_b in order to keep the public debt in a downward trajectory. The same results hold in case the government decides to increase its public spending.

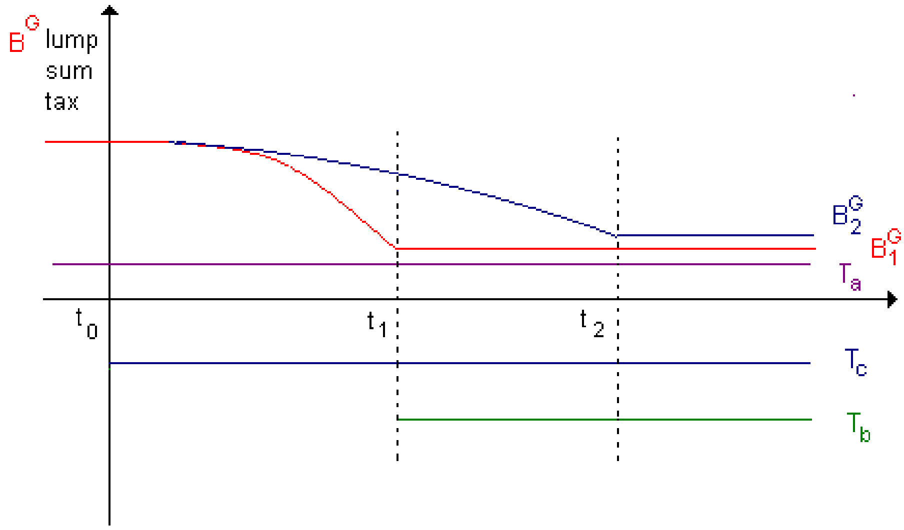

Figure 9: Three different fiscal responses

The two governments' options with respect to the lump sum tax have been depicted in figure 9: The first policy option is to reach a target level of debt $(\bar{B}_1^G)$ and then start providing transfers to citizens. The country enters the monetary union at time zero (t_0) and reaches the new target level of debt $(\bar{B}_1^G)$ at time one (t_1). The new level of public debt will be determined by a higher level of output and the lower interest rate. Therefore the amount of transfers will be: $T_b = r_1\bar{B}_1^G - \tau\bar{Q}_1 + \bar{G}$. The net present value of tax savings (TS_1) that individuals enjoy after time t_1 is the following:

$$\begin{aligned} NPV(TS_1) &= \left[\int_{t_1}^{\infty} (T_b - T_a)\, e^{-r_1(t-t_1)}dt\right] e^{-r_2(t_1-t_0)} = \\ &= e^{-r_1(t_1-t_0)}\left[r_0\bar{B}_0^G - r_1\bar{B}_1^G - \frac{\tau}{a}\left(r_1\bar{K}_1 - r_0\bar{K}_0\right)\right] \end{aligned}$$

In this model where agents have finite horizons and face the probability of death, the timing of taxes is very important. The above taxation scheme will benefit the individuals who are alive at time t_1

and enjoy not only public transfers but higher income as well. However it is not optimal for the individuals who are alive at the time of the entrance (t_0) and may not be alive at time t_1 to receive the governmental transfers. Given that the government tries to maximize the intertemporal utility of its citizens, it will choose a tax scheme which starts providing transfers at the time of entrance even though the new target level of debt has not been reached yet $\left(\bar{B}_2^G\right)$. In this case a lower debt level will be reached later than the first plan but individuals can receive transfers earlier than in the previous case and those alive at time t_o are better off.

The new transfers starting at time t_o are: $T_c = r_1 \bar{B}_2^G - \tau \bar{Q}_2 + \bar{G}$. The net present value of the tax savings for individuals, discounted at time t_o is:

$$NPV(TS_2) = \int_{t_0}^{\infty} (T_c - T_a)\, e^{-r_1(t-t_0)} dt = \left[r_0 \bar{B}_0^G - r_1 \bar{B}_2^G - \tfrac{\tau}{a}\left(r_1 \bar{K}_2 - r_0 \bar{K}_0\right)\right]$$

The net present value of both schemes is the same so we can solve for the T_c:

$$NPV(TS_1) = NPV(TS_2) \Rightarrow \left[\int_{t_1}^{\infty} (T_b - T_a)\, e^{-r_1(t-t_1)} dt\right] e^{-r_2(t_1-t_0)} =$$

$$\int_{t_0}^{\infty} (T_c - T_a)\, e^{-r_1(t-t_0)} dt \Rightarrow$$

$$\Rightarrow T_c = T_a - \left[r_0 \bar{B}_0^G - r_1 \bar{B}_1^G - \tfrac{\tau}{a}\left(r_1 \bar{K}_1 - r_0 \bar{K}_0\right)\right] e^{-r_1(t_1-t_0)}$$

10. Conclusions

In this paper we have analysed the macroeconomic effects that a country encounters while entering a monetary union. An overlapping generations general equilibrium model has been used and solved analytically. Steady state closed form solutions and time paths of the main macroeconomic variables have been found. Whilst the standard results in the existing literature about current accounts in a finite horizon framework show that the net asset position of an economy depends on how heavily the agents discount the future, we find that in a more realistic environment with capital adjustment costs, net private debt does not depend only on the discount factor but also on the share of capital in the production function.

When we examine the effects on the economy entering a union, we find that the immediate effect is lower interest rates imposed by the creditors since the country, as a member of a larger union, has a lower probability of default. Lower interest rate will boost the equilibrium consumption in higher levels and also encourage the private sector to borrow more. However the impact on private debt is more complex: A country increases its private debt depending on the net wealth effect which consists of the interest payments effect, the lower public bond holdings and the higher holdings of equities. We find that a debtor country chooses a higher private debt level when individuals discount the future heavily.

Although based on a number of simplifying assumptions, our analysis can provide useful insights into the appropriate design of taxation policy. The most interesting conclusion that emerges from our analysis is that after the entrance in a monetary union a benevolent central planner should provide some sort of transfers to individuals. This is because after joining a low interest rate club of countries there is a constant improvement in the government's budget balance. If the country does not experience an adverse shock no budgetary reform is necessary. The government should return some of the revenues otherwise its level of debt will decrease continuously. At this point we evaluate different taxation options and propose a lump sum transfer from the time of the accession which is intertemporally effective for all the generations.

11. APPENDIX

11.1. The Economy's variables as a function of K and closed form solutions

This appendix presents the steady state as a function of capital and then derives a steady state closed form solution for the open economy general equilibrium model developed in this paper.

Supply Side

Having derived a closed form solution for K, closed form for the remaining endogenous variables of the system can be found by appropriate substitution.

- Capital

$$\bar{K} = \left(\frac{r}{aA(1-\tau)} \right)^{\frac{1}{a-1}}$$

- Production Function (intensive form)

$$\bar{Q} = \left(\frac{r}{a} \right) \bar{K} \qquad (68)$$

substituting for K we obtain the closed form solution:

$$\bar{Q} = \left(\frac{r}{a} \right)^{\frac{a}{a-1}} [A(1-\tau)]^{\frac{-1}{a-1}}$$

- Wage

$$\bar{Y} = r\bar{K} \left(\frac{1-a}{a} \right) \qquad (69)$$

Closed form:

$$\bar{Y} = r \left(\frac{1-a}{a} \right) \left(\frac{r}{a(1-\tau)A} \right)^{\frac{1}{a-1}}$$

- Investment

$$\bar{I} = 0 \tag{70}$$

- Tobin's q

$$\bar{q}^T = 1 \tag{71}$$

- Dividends

$$\bar{D} = r\bar{K} \tag{72}$$

the closed form is:

$$\bar{D} = r\left(\frac{r}{aA(1-\tau)}\right)^{\frac{1}{a-1}}$$

Demand Side

- Consumption

$$\bar{C} = \left(\frac{\gamma(\gamma+\rho)}{(r+\gamma)(\gamma+\rho-r)}\right)[\left(\frac{1-a}{a}\right)r\bar{K} - T_a] \tag{73}$$

The closed form is:

$$\bar{C} = \left(\frac{1-a}{a}\right)\left(\frac{\gamma(\gamma+\rho)}{(r+\gamma)(\gamma+\rho-r)}\right)\{[a(1-\tau)A]^{\frac{-1}{a-1}}r^{\frac{a}{a-1}} - T_a\}$$

- Human Wealth

$$\bar{H} = \left(\frac{1}{r+\gamma}\right)[\frac{(1-a)r}{a}\bar{K} - T_a] \tag{74}$$

and the closed form solution is:

$$\bar{H} = \left(\frac{1}{r+\gamma}\right)[\frac{(1-a)r}{a}\left(\frac{r}{aA(1-\tau)}\right)^{\frac{1}{a-1}} - T_a]$$

- Equity Wealth

$$\bar{V} = \frac{D}{r} = \bar{K} \tag{75}$$

The closed form is:

$$\bar{V} = \left(\frac{r}{aA(1-\tau)}\right)^{\frac{1}{a-1}} = \bar{K}$$

- Net Private Indebtedness

$$\begin{aligned} \bar{B}^N &= B^P - B^G = \frac{(1-\tau)\, AK^a - T_a - C}{r} = \\ &= \left(\frac{a\gamma(\rho+\gamma) - r(r-\rho)}{a(r+\gamma)(\gamma+\rho-r)}\right) K + \frac{(r-\rho)}{(r+\gamma)\,(\gamma+\rho-r)} T_a \end{aligned} \tag{76}$$

The closed form is:

$$\bar{B} = \left(\frac{a\gamma^2 + r\rho + a\gamma\rho - r^2}{a(r+\gamma)(\gamma+\rho-r)}\right)\left(\frac{r}{aA(1-\tau)}\right)^{\frac{1}{a-1}} + \frac{(r-\rho)}{(r+\gamma)\,(\gamma+\rho-r)} T_a$$

- Government Debt

$$\bar{B}^G = \frac{T_a - G}{r} + \bar{K}\left(\frac{\tau}{a(1-\tau)}\right) \tag{77}$$

The closed form is:

$$\bar{B}^G = \frac{T_a - G}{r} + \frac{\tau}{a(1-\tau)}\left(\frac{r}{aA(1-\tau)}\right)^{\frac{1}{a-1}}$$

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