

Risk Averse International Investors, Wealth Effects and Sovereign Risk

Sandra Valentina Lizarazo[†]

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Abstract

This paper develops a quantitative model of debt and default for small open economies that interact with risk averse international investors. The model developed here extends the work of Eaton and Gersovitz (1981) on the analysis of endogenous default risk to the case in which international investors are risk averse agents with decreasing absolute risk aversion (DARA). By incorporating risk averse lenders who trade with a single emerging economy, the present model offers two main improvements over the standard case of risk neutral investors: i.) the model exhibits a better fit of debt-to-GDP ratio and ii.) the model explains a larger proportion of the spread between sovereign bonds and riskless assets. The paper shows that if investors have DARA preferences, then the emerging economy's default risk, capital flows, bond prices and consumption are a function not only of the fundamentals of the economy-as in the case of risk neutral lenders-but also of the level of financial wealth and risk aversion of the international investors. In particular, as lenders become wealthier or less risk averse, the emerging economy becomes less credit constrained. As a result the emerging economy's default risk is lower, and its bond prices and capital inflows are higher. Additionally, with risk averse lenders, the risk premium in the asset prices of the sovereign countries can be decomposed in two components: a base premium that compensates the lenders for the probability of default (as in the risk neutral case) and an "excess" premium that compensates them for taking the risk of default.

[†]Department of Economics, Duke University, Durham, NC 27708-0120 Email: svl@duke.edu
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1 Introduction

This paper extends the work of Eaton and Gersovitz (1981) on the analysis of endogenous default risk to the case in which international investors are risk averse agents whose preferences exhibit decreasing absolute risk aversion. The current paper develops a model of debt and default for a small open economy that interacts with risk averse international investors. This model is used to account for seven stylized facts regarding emerging financial markets:

- (i) Emerging economies experience a sudden loss of access to international capital markets and large reversals of their current account deficits in times of crises¹.
- (ii) Emerging economies' domestic interest rates are counter-cyclical².
- (iii) Default on sovereign debts occurs in equilibrium³.
- (iv) Emerging economies' credit ratings are negatively correlated with their income level and their growth rate, and positively correlated with the size of their external debt⁴.
- (v) Emerging economies' estimated default probabilities do not account for all of the yield spreads in their sovereign bonds⁵.
- (vi) The proportion of sovereign yield spreads explained by emerging economies' own fundamentals is smaller for riskier sovereign bonds than for investment grade bonds⁶.
- (vii) Investors' financial performance and their net foreign asset position in emerging economies are positively correlated⁷.

¹The literature on "sudden-stops" has focused on explaining the dynamics of the sudden loss of access to international capital markets that emerging economies experience during periods of crises. Examples of this literature includes Mendoza(2002), Mendoza and Smith (2002), Christiano, Gust and Roldos (2003), and Edwards (2004).

²Uribe and Yue(2003), and Neumeyer and Perri (2004) focus on the counter-cyclical behavior of domestic interest rate for emerging markets.

³Kaminsky and Reinhart (1998) and Reinhart, Rogoff and Savastano (2003) document and empirically analyze default events.

⁴Cantor and Pecker (1996) analyzes the determinants of credit ratings.

⁵Westphalen (2001) and Huang and Huang (2003) have considered bond spreads and the role of the probability of default (estimated based on the economy's fundamentals) in the determination of such spreads.

⁶See, for example, Cantor and Pecker (1996), Cunningham, Dixon and Hayes (2001), Westphalen (2001), and Kamin and von Kleist (1999).

⁷See for example Goldberg (2001), and Hernandez, Mellado and Valdes (2001), Calvo, Fernandez-Arias, Reinhart, Talvi (2001), and Kang, Kim, Kim and Wang (2003), FitzGerald, and Krolzig (April 2003), and Mody and Taylor (2004).

- (viii) Emerging economies' credit spreads are positively correlated with spreads of corporate junk bonds from developed countries⁸.
- (ix) Sovereign bond spreads across emerging economies are highly correlated⁹.

In the model presented here, three types of agents interact through international financial emerging markets: developed economies' agents, emerging economies' agents, and international financial intermediaries. Financial intermediaries or investors take the form of mutual funds, hedge funds, pension funds, etc. These agents invest in emerging financial markets in the name of developed economies' agents—i.e., developed economies' agents are able to invest in emerging market assets by holding shares of mutual funds, pensions or hedge funds. Since intermediaries act in tandem with developed economies' agents, these two actors will not be modeled separately. Therefore only two types agents will be explicitly modeled, the agents of the emerging economies and international investors.

It is assumed that all of the agents of the emerging economy are identical, all the international investors are identical, and that none of these agents follow mixed strategies. Under these assumptions, it is possible to focus on the representative agent of each type. For her part, the representative investor is a risk averse agent. This agent solves a dynamic portfolio problem in which she decides the optimal allocation of her portfolio between bonds of the emerging economy and riskless assets denominated as T-Bills. On the other side of the market, the representative agent of the emerging economy is also a risk averse agent who solves a dynamic optimization problem. Each period, this agent receives an stochastic endowment and chooses her consumption and savings subject to her budget constraint. The emerging economy borrows or saves by trading one-period non-contingent bonds with the representative investor. The interaction between the two parties determines the equilibrium price of the bonds in the emerging economy.

On the side of the emerging economy, there is limited liability. While the representative investor is able to commit to repay any debt that she might have, the representative agent of the emerging economy is not. In this case, the emerging economy might default on her debts. However if she defaults, under the framework presented here, she is forever excluded from international credit markets.¹⁰

⁸See, for example, FitzGerald and Krolzig (2003), Ferruci, Herzberg, Soussa, and Taylor (2004), and Mody and Taylor (2004).

⁹See, for example, Valdes (1996), Baig and Goldfajn (1998), Baig and Goldfajn (2000), and Forbes and Rigobon (1999).

¹⁰The assumption of permanent exclusion is merely a simplifying assumption. This assumption is relaxed in the numerical section of the paper by assuming that each period there is an exogenous constant probability

The representative agent of the emerging economy is assumed to have market power: because the sovereign bonds that she issues are a differentiated product, the emerging economy is aware that her borrowing decisions affect the equilibrium price of her bonds. Furthermore, she knows the optimal response of the representative investor to her actions. The market structure can be seen as derived from a Stackelberg duopoly type game where the emerging economy acts as the leader, who knows the optimal response function of the representative investor; the representative investor acts as the follower who takes the equilibrium price of the sovereign bonds as given.

While the investor acts as a price taker, investors are not competitive agents. As a group, investors collude to punish any deviant borrower that defaults on a debt contract with any individual investor. Investors must also collude to punish any deviant investor that helps out a borrower who has previously defaulted.

The assumption of non-competitive investors builds on the work of Kletzer and Wright (2000), Wright (2002) and Paasche and Zin (2001). If investors were competitive agents, then once a borrower defaulted with one investor there would be no incentives for the remaining investors to exclude the defaulting borrower from financial markets. In such a case, it is not possible to support sovereign borrowing in equilibrium—it is not sufficient to merely assume reputation losses of any borrower that does not pay back.

Under a non-competitive market structure, both risk averse and risk neutral investors may obtain economic profits from their lending activity. However, under this market structure, which is implicit in much of the previous sovereign debt literature, only risk averse investors are strictly better off by trading with the emerging economy. This result follows from the concavity of the periodic utility function—any risk averse agent will accept at least a small amount of any risk that is actuarially favorable. In contrast, for a risk neutral investor, once the price of the sovereign bond is adjusted by the probability of default, the investor is indifferent between trading or not trading with the emerging economy. This clarification is important because it helps to emphasize that the main departure from previous literature is the assumption of risk averse investors, and not the assumption of non-competitive investors.

By relaxing the assumption of risk neutrality, and allowing for wealth effects on the side of the international investors, the model presented here attempts to better match the stylized facts of international financial markets during the last two decades of the 20th century. These stylized facts are only partially explained in the existing sovereign debt literature.

that a defaulting country can regain access to credit markets.

Under the assumption that investors are risk neutral, previous models of endogenous sovereign risk have explained stylized facts (i) through (iv)¹¹. As a result of incorporating risk averse lenders with decreasing absolute risk aversion, the model presented here endogenously explains all of the stylized facts listed above.

The present model explains stylized facts (v) through (ix) as follows. First, international investors demand an excess risk premium in order to willingly take the risk of default embodied in the emerging economies' sovereign bonds (i.e. a risk averse agent would only take a risk that is actuarially favorable.). Therefore the present model is able to account for stylized fact (v): the price of the emerging economy's bonds is lower than the world price of riskless bonds adjusted by the emerging economy's default probability. This result is consistent with the findings of the empirical finance literature on sovereign bond spreads. Those findings suggest that under the assumption of risk neutral investors and competitive financial markets, the price of sovereign bonds cannot be completely explained by the estimated probabilities of default.¹²

Second, as risk averse agents, international investors demand a higher risk premium for higher levels of risk—above the premium predicted solely by the probability of default. With risk averse lenders, the risk premium can be decomposed in two components: a base premium that compensates the lenders for the probability of default and an excess premium that compensates them for taking the risk of default.¹³ Therefore the present model is able to account for stylized fact (vi): The proportion of sovereign yield spreads explained by default probabilities is smaller for riskier sovereign bonds than for less risky bonds. This result is consistent with the empirical regularity reported in several empirical papers: that spreads in investment grade bonds can be explained to a larger extent by emerging economies' fundamentals than spreads in speculative grade bonds.

Third, since investors preferences exhibit decreasing absolute risk aversion, these agents are able to tolerate more default risk the wealthier they are. Therefore the present model can account for stylized fact (vii): there is a positive correlation between the representative lender's wealth and the lender's investment in the emerging economy. This result is

¹¹This literature begins with Eaton and Gersovitz (1981). More recent examples include Arellano(2003) and Aguiar and Gopinath (2004).

¹²An alternative explanation exists which does not depend on risk aversion. Sovereign bonds could be mispriced under the assumption that international investors do not take prices as given. However this assumption only explains stylized fact (v). Stylized facts (vi) through (ix) cannot be accounted for by a model in which portfolio allocations to each emerging country are independent of the wealth of the investors and the overall risk of the portfolio.

¹³Models with risk neutral lenders only capture the base premium.

consistent with empirical findings which demonstrate a positive relation between proxies of investors wealth (like developed economies' GDP or stock indexes) and capital flows to emerging economies.

Fourth, the endogenous credit limits faced by the emerging economy become increasingly tight when the lender's risk aversion increases. This tightening occurs because a more risk averse investor demands a higher risk premium in order to accept default risk. Therefore, for any given level of risk aversion of the representative investor, the set of financial contracts available to the emerging economy is always a subset of the set of contracts available to an identical economy trading with a less risk averse lender.¹⁴ This result is consistent with stylized fact (viii): whenever investors' willingness to take risk changes, there must be a change in the spreads of all risky assets. As a consequence, the spreads of emerging economies' sovereign bonds and the spreads of industrialized economies' junk bonds should exhibit some co-movement.

Fifth, under decreasing absolute risk aversion, investors have a higher tolerance for risk when they are wealthier. Therefore at higher levels of wealth, these agents demand a smaller risk premium than at lower levels of wealth in order to take the same amount of default risk. Furthermore, a smaller risk premium in the emerging economy's bonds increases the benefits for the economy of fulfilling its contract. Since these effects reinforce each other, the equilibrium price of sovereign bonds is an increasing function of investors' wealth levels. This result is consistent with the empirical literature on the determination of sovereign credit spreads for emerging economies,¹⁵ and implies that the current model can explain stylized fact (ix): sovereign bond spreads across emerging economies are highly correlated because the equilibrium price of the emerging economy's bonds varies with the representative investor's wealth.¹⁶

In addition to the results consistent with the stylized facts above, two other results follow from the model.

¹⁴A financial contract in this context is the combination of the bond prices and quantities that the emerging economy can borrow or save.

¹⁵For example, Warther (1995), Ferruci, Herzberg, Soussa, and Taylor (2004), FitzGerald, and Krolzig (April 2003), and Westphalen (2001).

¹⁶This result of the model is consistent with the literature on financial contagion. A large body of empirical literature presents evidence that financial links play a significant role in explaining simultaneous financial crises and correlated spreads across emerging economies. See, for example, Kaminsky and Reinhart (1998), Van Rijckeghem and Weder (1999), Kaminsky, Lyons and Schmukler (1999), Kaminsky Lyons and Schmukler (2000), Kaminsky and Reinhart (2000), and Hernandez and Valdes (2001).

First, the likelihood of observing default in equilibrium is a function not only of the emerging economy fundamentals but also of the investors' characteristics such as wealth and risk aversion. While the model does not give a definitive answer regarding likelihood of default a priori, in the numerical simulations documented in this paper, default is more likely to be an equilibrium outcome when the investor's initial wealth is low or when the investor is more risk averse.

Second, this model presents a theoretical framework that can account for the non-robustness of empirical findings regarding the role of international interest rate in the determination of sovereign bond spreads and capital flows to emerging economies. In the current model changes in the world interest rate have two opposing effects on the set of financial contracts available to emerging economies:¹⁷ On the one hand, an increase in the world interest rate increases the cost of borrowing for emerging economies, increasing their incentives to default and their default risk. On the other hand, an increase in this rate increases the level of wealth of non-leveraged investors, this wealth effect would tend to increase the set of financial contracts available to emerging economies.

It is important to remark that the assumption of risk aversion on the side of the investors seems to be justified by the characteristics of the foreign players in emerging financial markets. These players are both individuals and institutional investors such as banks, mutual funds, hedge funds, pension funds and insurance companies. For the case of individual investors, it is straightforward to assume that these agents are risk averse. These agents can be treated as the representative agent of developed economies; it is standard practice in the literature to treat these agents as risk averse. In the case of institutional investors the assumption of risk aversion is somewhat more difficult, but nevertheless quite plausible. For institutional investors, risk aversion may follow from two sources: regulations over the composition of their portfolio and the characteristics of the institutions' management. Regarding the first source, banks face capital adequacy ratios; mutual funds face restrictions in their access to leverage against their asset holdings; and pension funds and insurance companies face strict limits in their exposure to risk. Regarding the second source, for each class of institutional investor, managers ultimately make the portfolio allocation decisions. These agents, as individuals can also be treated as representative agents of developed

¹⁷See, for example, Calvo, Leiderman, and Reinhart (1993), Fernandez-Arias (1996), Cline and Barnes (1997), Chohan, Claessens, Mamingi (1998), Eichengreen and Mody (1998), Kamin and von Kleist (1999), Herrera and Perry (2000), Arora and Cerisola (2001), Calvo, Fernandez-Arias, Reinhart and Talvi (2001), Goldberg (2001), Hernandez, Mellado and Valdes (2001), Kaminsky and Schmulker (2001), Bekeart, Harvey and Lusmadaine (2002), FitzGerald and Krolzig (2003), Kang, Kim, Kim and Wang (2003), Ferruci, Herzberg, Soussa, and Taylor (2004), and Mody and Taylor (2004).

economies. Additionally, in general the remuneration—and therefore the wealth—of these agents is closely related to the performance of the portfolio that they manage. These factors suggest that portfolio choices of institutional investors will be consistent with the choices of risk averse agents, whose preferences exhibit decreasing absolute risk aversion.

This paper is organized as follows: section 1 is the introduction; section 2 presents the theoretical model; section 3 characterizes the equilibrium of the model; section 4 discusses the quantitative implications of the model; and section 5 concludes. The appendix presents the proofs of the propositions in the main text.

2 THE MODEL

The model is a discrete time, infinite horizon model. There are two types of agents in the model, a representative agent small open economy, and a representative risk averse international investor. In each period, the emerging economy receives a stochastic endowment of tradable goods. The representative agent of this economy may smooth her consumption across periods by trading non-contingent discount bonds with the representative investor. For her part, the representative investor may trade assets with the emerging country or with industrialized countries. Thus the investor must choose an optimal allocation of her portfolio between the bonds of the emerging economy and bonds of the industrialized countries, denominated hereafter as T-Bills.

The market for T-bills, θ^{TB} , will not be modeled explicitly. Since debt contracts between the representative investor and industrialized countries are assumed to be enforceable, the representative investor is a price taker in the market for T-Bills. The price of T-Bills, q^f , which is not determined endogenously in this context, is assumed to be deterministic. Therefore T-Bills are riskless assets.

Bonds of emerging economies, b , on the other hand, are risky assets because debt contracts between the representative investor and the emerging economy are not enforceable. As a consequence, there is a one sided commitment problem. While the representative investor is able to commit to honor her debt obligations with the emerging economy, the representative agent of the emerging country is not able to commit to honor her obligations with international investors. Therefore in each period, the representative agent of the emerging economy compares the costs and benefits derived from the repayment of her obligations. The decision between repayment or default is made individually by each agent of the emerging economy. Each agent of this economy makes her decision, taking as given

the decision of the other agents. However given that all agents are identical who do not follow mixed strategies, it is possible to focus attention on the problem of the representative agent.

If the economy defaults, international investors are able to collude to punish her. As a consequence of default, it is assumed that investors will collude to exclude the defaulting country forever from the financial markets. Since all investors behave in the same exact way, it is possible to focus on the representative international investor.

The representative lender takes as given the price of the emerging economy's non-contingent discount bonds, q . On the other hand, the representative agent of the emerging economy internalizes the fact that because of the limited enforceability of debt contracts, her demand for borrowing affects the equilibrium price of the bonds that she issues.¹⁸

As laid out here, the asset market is imperfect in three different ways. First, there is a one-sided commitment problem which implies that debt contracts with emerging economies are not enforceable. Second, markets are incomplete because the only traded assets are one period no-contingent bonds, and risk free T-Bills. Therefore the representative investor is not able to insure away the income uncertainty specific to the emerging country. Third, the market structure of the financial market is non-competitive in two ways: the emerging economy behaves non competitively as the first mover in a Stackelberg type duopoly model; and for their part, investors form a cartel that colludes to punish any deviant investor or borrower.

2.1 International investors

There are a large but finite number of price-taking identical investors. Investors collude in order to punish any borrower that defaults in her debts or any investor that lends to a borrower who has previously defaulted, so that a defaulting country is permanently excluded from the financial markets.¹⁹

¹⁸Because sovereign bonds have a country risk that is specific to each economy, this type of assets can be seen as differentiated product. This differentiation can be used to justify the assumption of market power by the emerging economy.

The assumption of market power also serves a technical purpose. If the agents of the emerging economy were assumed to be price takers in the market of sovereign debt, it would be necessary to differentiate between individual asset holdings of the agents of the emerging economy and aggregate asset holdings of the economy. This inclusion would expand the set of state variables from three (endowments, asset position of emerging economy and wealth of the investors) to four state variables.

¹⁹ As in the papers on sovereign debt literature of Kletzer and Wright (2000), and Wright(2002), no investor will deviate from this punishment as long as deviations are punished by the remaining investors.

The representative investor is a risk averse agent whose preferences over consumption are defined by a constant relative risk aversion (CRRA) periodic utility function with parameter $\gamma^L > 0$. The investor has perfect information regarding the income process of the emerging economy, and in each period the investor is able to observe the realizations of this endowment.

The representative investor maximizes her discounted expected lifetime utility from consumption

$$\underset{c_t^L}{Max} E_t \sum_{t=0}^{\infty} \beta_L^t v(c_t^L) \quad (1)$$

where c_t^L is the investor's consumption. The period utility of this agent is given by $v(c_t^L) = \frac{(c_t^L)^{1-\gamma^L}}{1-\gamma^L}$. The representative investor is endowed with some initial wealth W_0 , at time 0, and in each period, the investor receives an exogenous income X .

Because the representative investor is able to commit to honor her debt, she can borrow or lend from industrialized countries (which are not explicitly modeled here) by buying T-Bills at the deterministic risk free world price of q^f . The representative investor can also invest in non-contingent bonds of the emerging economy. These bonds have an endogenously determined stochastic price of q . In each period the representative investor faces the budget constraint

$$W + X = c_t^L + Dq\theta' + q^f\theta^{TB'} \quad (2)$$

where W is investors wealth at time t , θ' is the portfolio allocation to the emerging country and $\theta^{TB'}$ is the investor's allocation to the riskless asset. D is a variable that determines the default/repayment state of the emerging economy in the current period: $D = \prod_{m=\tau}^t d_m$, and d_m is an indicator function that represents the emerging economy's repayment/default decision at period m . d_m takes the value of 1 in period m when the small open economy chooses to repay its debts, and takes a value of 0 otherwise. Therefore $D = 1$ if the emerging economy has never defaulted up to the current period, and $D = 0$ if the emerging economy has defaulted in some previous period. For $D = 0$ the country is in permanent financial autarky.

Punishment is achieved by inducing the representative agent of the emerging economy to default in her debts with the deviant investor. The colluding investors induce the emerging economy by offering her a new financial contract with slightly better terms. In this case investors will never deviate.

As discussed in the introduction, the assumption of permanent exclusion is merely a simplifying assumption. Empirical evidence suggest that once a country defaults, that country is excluded from the credit market for an average of 5.4 years (Gelos, Sahay, and Sandleris 2003).

It is assumed that investors cannot go short in their investments with emerging economies. Therefore whenever the emerging economy is saving, the representative international investor receives these savings and invests them completely in T-Bills. The representative investor does not use these resources to go long in T-Bills. This assumption implies that $\theta' \geq 0$ for all t .²⁰

The law of motion of the representative investor's wealth is given by

$$W' = D'\theta' + \theta^{TB'} \quad (3)$$

The optimization problem that the representative investor faces can be described as one in which in each period t the representative international investor optimally chooses her portfolio according to her preferences in order to maximize her discounted expected lifetime utility from consumption, subject to her budget constraint, the law of motion of her wealth, and given W_0 . This dynamic problem can be represented recursively by the Bellman Equation

$$V^L(s) = \max_{\theta', \theta^{TB'}} v(c^L) + E\beta_L V(s') \quad (4)$$

where s is defined as follows:

Definition 1 *The state of the world, s , is given by the realization of the emerging economy's endowment, y , the emerging economy's asset position, b , the representative investor's asset position or wealth, W , and the variable D which states whether or not the emerging economy is in default.*

The stochastic dynamic problem for the representative investor is characterized by the first order conditions for this optimization problem:

For $\theta^{TB'}$

$$q^f v_{c^L}(c^L) = \beta_L E \left[v_{c^L}(c^{L'}) \right] \quad (5)$$

For θ'_j

$$D \left[-q v_{c^L}(c^L) + \beta_L E \left[v_{c^L}(c^{L'}) d' \right] \right] = 0. \quad (6)$$

²⁰This assumption does not seem to be inconsistent with reality. For example, while mutual funds are strictly restricted by The Investment Company Act in their ability to leverage or borrow against the value of securities in their portfolio, hedge funds and other types of investments face no such restrictions. Since international investments like hedge funds are not subject to these type of regulations, it seems reasonable to have the simplifying assumption that international investors are able to leverage the riskless asset, θ^{TB} , but must have a non-negative position on the emerging economy's asset.

According to Equation (5), the investor chooses an allocation to the riskless asset such that the discounted expected marginal benefit of future consumption equals the marginal cost of current consumption. Equation (6) determines the allocation of the investor's resources to the emerging country. Unless the emerging country has never defaulted, i.e. $D = 1$, the emerging country does not belong in the investment set of the international investors. If the country has never defaulted, then Equation (6) also equates the marginal cost of allocating wealth to bonds issued by the emerging country to the discounted expected marginal benefit of this investment. The benefit of this investment is realized only in those periods in which the emerging economy optimally chooses to repay its debts ($d' = 1$).

For the case in which $D = 1$, equation (6) highlights the fact that the endogenous risk of default by the emerging economy—i.e. the case for which $d' = 0$ for some state of the world in the next period—will reduce the representative investor's expected marginal benefit of investing in the emerging economy. Everything else equal, this result will tend to reduce the allocation of resources to the emerging economy relative to the case where the emerging economy could commit to repayment.

To understand the role that risk aversion plays in this model, it is instructive to analyze in detail the determination of the equilibrium price of the emerging economy's bonds. Define

$$Ed' = 1 - \delta$$

where δ is the probability that the emerging economy will default in the next period.

Define q^{RN} as the equilibrium price of the emerging economy's bonds that would prevail in a world with risk neutral lenders. For a risk neutral investor, the present value of one unit of a bond issued by a emerging economy that cannot commit to repay is given by

$$-q^{RN} + q^f Ed' \tag{7}$$

where q^f is used as the discount factor. This factor represents the opportunity cost for the representative investor of her investment in emerging economy bonds. Given the assumption that the investor is a price taker and that the borrower knows the optimal response function of the representative investor, a risk neutral representative investor would make zero profits.²¹ Therefore (7) implies

$$q^{RN} = q^f (1 - \delta)$$

²¹Recall that the lender is a Stackelberg follower. In this case, as Stackelberg leader, the emerging economy chooses a price which makes the lender indifferent between participation and non-participation. For the risk neutral lender, this price implies zero profits.

which establishes that for the case of a representative risk neutral investor, the price of a discounted one period non-contingent bond is equal to its opportunity cost.

It is possible to manipulate equation (6) to get

$$\begin{aligned}
q &= \frac{\beta_L E \left[v_{c^L} \left(c^{L'} \right) d' \right]}{v_{c^L} (c^L)} \\
&= \beta_L \frac{Cov \left[v_{c^L} \left(c^{L'} \right) d' \right] + E v_{c^L} \left(c^{L'} \right) E d'}{v_{c^L} (c^L)} \\
&= \frac{\beta_L Cov \left[v_{c^L} \left(c^{L'} \right) d' \right]}{v_{c^L} (c^L)} + q^f (1 - \delta) \\
&= \frac{\beta_L Cov \left[v_{c^L} \left(c^{L'} \right) d' \right]}{v_{c^L} (c^L)} + q^{RN}.
\end{aligned} \tag{8}$$

Equation (8) highlights two important features of the model: i) unless the probability of default is positive, the price of the emerging economy's bonds is equal to the price of the bonds of industrialized countries; and ii) taking as given the degree of default risk (i.e. the probability of default), the price of the bonds issued by a emerging economy trading with a risk averse investor will be lower or at best equal to price of those same bonds traded with a representative risk neutral investor. This latter implication holds true because

$$Cov \left[v_{c^L} \left(c^{L'} \right) d' \right] \leq 0.$$

When the emerging economy does not find it optimal to default at $t + 1$ in any state of the world, then $d' = 1$ for all states. Therefore $Cov \left[v_{c^L} \left(c^{L'} \right) d' \right] = 0$. On the other hand, when at $t + 1$ there exist states of the world in which the emerging economy would optimally choose to default, then for the states in which it is not optimal to default, $d' = 1$. In this case, the wealth of the representative investor at $t + 1$ is given by

$$[W' | (d' = 1)] = \theta' + \theta^{TB'}$$

and the wealth of the representative investor at $t + 1$ for the states in which the emerging economy finds it optimal to default ($d' = 0$) is given by

$$[W' | (d' = 0)] = \theta^{TB'}.$$

It is obvious that

$$[W' | (d' = 1)] > [W' | (d' = 0)].$$

Therefore it must hold that

$$\left[c^{L'} \mid (d' = 1) \right] \geq \left[c^{L'} \mid (d' = 0) \right]$$

and by concavity of the investor's utility function

$$\left[v_{c^L} \left(c^{L'} \right) \mid (d' = 1) \right] \leq \left[v_{c^L} \left(c^{L'} \right) \mid (d' = 0) \right].$$

As a consequence, for higher d' , we have lower $v_{c^L} \left(c^{L'} \right)$. Clearly for this case

$$Cov \left[v_{c^L} \left(c^{L'} \right) d' \right] < 0.$$

It is important to note that the equilibrium probability of default is different in the case of a risk neutral investor, $\delta(s, b')$, compared to the case of a risk averse investor, $\delta^{RN}(s, b')$. For any given s and b' , the probability of default is an increasing function in investor's degree of risk aversion. (This result will be studied in detail in the next section.) Therefore it is possible to say that for s and b' given, the price of the bonds issued by the emerging economy trading with a risk averse investor $q(\delta(s, b'))$ is always lower or at best equal to price of the same bonds traded with a representative risk neutral investor $q^{RN}(\delta^{RN}(s, b'))$.

Compared to the case of risk neutral investors, the introduction of risk averse investors is a step forward in explaining the risk premium in the returns of bonds from emerging economies. This risk premium seems to be supported empirically since the price of emerging economies' bonds seems to be determined by much more than just the opportunity cost of the funds adjusted by the probability of default of such economies.²² Risk aversion can help explain this phenomena since such a investor would have to be compensated beyond the probability of default-adjusted rate of return in order to face the risk of a default by an emerging economy. The higher the degree of risk aversion, the higher the bond spread.

2.2 The Emerging Economy

The representative agent of the emerging economy maximizes her discounted expected life-time utility from consumption

$$\max_{\{c_t\}_{t=\tau}^{\infty}} E_{\tau} \sum_{\tau}^{\infty} \beta^{t-\tau} u(c_t) \quad (9)$$

²²This phenomena is discussed in Cantor and Pecker (1996) and Cunningham, Dixon and Hayes (2001) among others.

where $0 < \beta < 1$ is the discount factor and c is the emerging economy's consumption at time t . The emerging economy's periodic utility takes the functional form

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

where $\gamma > 0$ is the coefficient of relative risk aversion.

In each period, the economy receives a stochastic stream of consumption goods y . This endowment is non-storable; realizations of the endowment are assumed to have a compact support; and the endowment follows a Markov process drawn from probability space $(y, Y(y))$ with a transition function $f(y' | y)$.

In each period, based on the stochastic endowment y , the economy decides how much to consume c . The economy can consume $c > y$ by trading one period non-contingent discount bonds b' at a price q with international investors. The economy may only trade bonds if up to period t the economy has never defaulted.

In equilibrium, the price of bonds is determined by both investors and the emerging economy. However, while the price of bonds is taken as given by international investors, the price of bonds is not taken as given by the emerging economy—the emerging economy internalizes the fact that due to her inability to commit to repay her debts, the price of bonds depends on her own demand for borrowing. And since she knows the optimal response of the representative investor to her actions, by choosing her demand for borrowing the economy can choose bond prices.

As a consequence of the commitment problems, the price of the emerging economy's bond might be different depending on whether the economy is saving or borrowing. If $b' > 0$, the country is saving, and because the international investor is able to commit, there is no risk of default on such a bond. In this case, the emerging economy's bond is identical to the bonds issued by industrialized markets; therefore, because the representative investor is a price taker, in equilibrium the bond price of a emerging economy with no default risk is the same as the bond price of industrialized countries. Consequently, the price of a bond with a positive face value is equal to the price of a T-Bill, so $q = q^f$.

If $b' = 0$, the emerging economy is not borrowing and there is no risk of default because it is not optimal for the emerging economy to declare default on a debt of size 0. If the economy were to declare default in this circumstance, there would be no change in the present pattern of consumption, but a reduction in the opportunities of consumption smoothing in the future.

If $b' < 0$ the emerging country is borrowing. In this case, because emerging economies

cannot bind themselves to honor their debts, the emerging country might default next period. There might be values of $b' < 0$, for some given state of the world, s , such that the representative agent of the economy never finds it optimal to default. In this case the bonds issued by the emerging economy do not involve any default risk, and therefore $q = q^f$. However for the same state of the world, s , some other values of $b' < 0$ might imply that the emerging economy will find it optimal to default on her debts in some states of the world next period s' . In this case, in order to induce international investors to buy the emerging economy's bonds, the price of such bonds needs to be lower than the price of a T-Bill, $q < q^f$. Finally, for the same state of the world, s , there might be values of $b' < 0$ such that once the debt is due the economy would not choose to repay in any state of the world next period, s' . In this case $q = 0$.

Based on this logic, the price of the emerging economy's bonds is a function not only of the state of the world, s , but also of b' . By internalizing this fact, the emerging economy knows that at every state of the world s she faces a different price depending on her demand for borrowing. Furthermore, it is assumed that she knows the optimal response of the representative lender to her actions. Therefore she is able to observe the menu of equilibrium bond prices for each level of borrowing b' for every state of the world, s .

The resource constraint of the emerging economy is given by

$$c = y + D(b - qb') \quad (10)$$

where D , which has been defined in the investor's section, describes the state of economy with respect to participation in international financial markets. If $D = 1$, the economy has never defaulted. If $D = 0$ the emerging economy is in default and this country is in permanent financial autarky. Once a country defaults (even if the default is partial), that country is permanently excluded from access to the credit market, so that the country remains in a state of default forever. In that case the country is not able to smooth its consumption, and it is limited to consume its stochastic endowment forever.

Definition 2 *The value for the emerging economy of default is given by*

$$V^A(y) = u(y) + \beta EV^A(y' | y).$$

This equation represents the value for the economy of remaining in autarky forever.²³

²³The term autarky is used loosely here. Autarky refers only to the fact that a country does not participate in the asset market. It might however continue to trade goods, but is obligated to keep a zero trade balance.

Under this framework, the optimization problem of the emerging country can be represented recursively by the following Bellman equation

$$V(s) = \max \left\{ V^C(s), V^A(y) \right\} \quad (11)$$

and

$$\begin{aligned} V^C(s) &= \max_{c, b'} u(c) + \beta EV(s' | y) \\ \text{s.t.} \quad c &= y + b - qb' \\ \& \quad b' &\geq \underline{b} \end{aligned} \quad (12)$$

where $V^C(s)$ is the value of not defaulting and $V^A(y)$ is the value of defaulting in the current period.

For the emerging country the decision of default/repayment depends on the comparison between the continuation value of the credit contract, $V^C(s)$, versus the value of opting for financial autarky $V^A(y)$. The decision of current default/repayment takes the functional form:

$$d = \begin{cases} 1 & \text{if } V^C(s) > V^A(y) \\ 0 & \text{otherwise} \end{cases} \quad (13)$$

Equation (12) corresponds to the “natural” debt limit discussed in Ayagari (1993), which prevents the representative agent of the emerging economy from running ponzi games. In the current model, this constraint would not be binding. Instead a tighter credit limit is determined endogenously in the model.

The stochastic dynamic problem for the emerging economy is characterized by the Euler equation (conditional on not defaulting in the current period):

$$u_c(c(s)) q \left(1 + \frac{\partial q(s; b')}{\partial b'(s)} \frac{b'(s)}{q(s; b')} \right) = \beta E [u_c(c'(s')) d'(s')] \quad (14)$$

and equations (10) and (13).

The Euler equation (14) equates the marginal benefit of one unit of current consumption to the discounted expected marginal cost of giving up one unit of future consumption. Because of the commitment problem, this cost is experienced only in those states in which the emerging economy optimally chooses to repay its debt, i.e. only on those states in which $d' = 1$.

The Euler equation (14) highlights some important features of the model. First, the decision not to default in the current period does not imply that the economy will not

default in the future (i.e. d' might be 0 for some states of the world). Second, for an economy which has the possibility of default, the optimal path of consumption is different from the optimal path of consumption of an otherwise identical economy which cannot default. To see this difference, consider the following. A small economy that can commit to repay its debts takes the price of the bonds that issues as given, and it is able to borrow or lend always at that price. The Euler equation for such an economy is simply given by

$$q^f u_c(c) = \beta E u_c(c'). \quad (14)$$

Comparison of equations (14) and (14) shows that there are two additional effects which modify the intertemporal savings decision for an economy which cannot commit to repayment. First, the cost of borrowing in the current period is only experienced in states of the world for which future repayment is optimal. All else equal, this effect suggests that an economy that cannot commit to repayment will tend to borrow more than an economy that can commit. Second, for an economy that cannot commit, the price of the bonds depends on the borrowing decisions of that economy. This result can be seen in equation (6). From equation (6), it is clear that the borrower's limited liability reduces the investor's incentive to invest in the emerging economy.

Equations (14) and (6) make clear that for the case of an economy that cannot commit to repayment, when there exist levels of b' in which the emerging economy finds it optimal to default in some states of the world, then the price of bonds depends not only on the emerging economy's fundamentals, but on the representative investor's level of wealth and risk aversion. This case is very different from the case of a small open economy that can commit. In the latter environment, the assets of the emerging economy are riskless from the point of view of the representative investor. Therefore, as long as the representative investor is a price taker, the price of emerging economy's bonds is equal to the price of the industrialized countries' bonds. And bond prices are independent of the investors' wealth.

Another feature of this model, which is shared with models of the same kind in which investors are risk neutral, is that the emerging economy only defaults when it is facing capital outflows. In this case, $d(s) = 0$ implies that for all the financial contracts available to the economy $b - q(s; b'(s)) b'(s) < 0$. Intuitively, whenever the emerging economy decides to default, the value of default must be at least as good as the value of the optimal financial contract available to this country ($V^C(s) \leq V^A(y)$). However if any available financial contracts allows for capital inflows to the emerging economy, then by choosing that contract the economy not only can consume more in the current period than under autarky ($c > y$), but in the next period the economy is guaranteed at least the same level of satisfaction as under autarky (because the economy has the option of defaulting in the

next period). Therefore for any state of the world s , whenever there are financial contracts $\{q(s; b'(s)), b'(s)\}$ such that $b - q(s; b'(s)) b'(s) > 0$, default is not an optimal decision.

3 Characterization of the Equilibrium

The recursive equilibrium in this model is given by prices q^f , and q and quantities $c, b', d, \theta', \theta^{TB'}$ which solve the emerging economy's problem using her knowledge of the representative investor's optimal response function, (6). At the same time, the prices q^f , and q and quantities $c, b', d, \theta', \theta^{TB'}$ solve the representative investor's problem, taking the price of all assets in her portfolio as given.

The equilibrium prices and quantities must also clear asset markets:

$$\begin{aligned} b' &= -\theta' (W + X) & \text{if } b' < 0. \\ 0 &= -\theta' (W + X) & \text{if } b' \geq 0. \end{aligned} \tag{15}$$

Equations (15) and (15) imply that in equilibrium the emerging economy and the representative investor agree on a financial contract, b' and q , that is optimal for both agents.

Definition 3 *For a given level of wealth, W , the default set $D(b | W)$ consists of the equilibrium set of y for which default is optimal when emerging economy's asset holdings are b :*

$$D(b | W) = \left\{ y \in Y : V^C(s) \leq V^A(y) \right\}. \tag{16}$$

Equilibrium default sets, $D(b' | W'(s))$, are related to equilibrium default probabilities, $\delta(b', y' | s)$, by the equation

$$\delta(b', y' | s) = 1 - E d'(b', y' | s) = \int_{D(b' | W'(s))} f(y' | y) dy' \tag{17}$$

If the default set is empty for b' , then for all realizations of the economy's endowment $d' = 1$ and the equilibrium default probability $\delta(b', y' | s)$ is equal to 0. In this case, it is not optimal for the economy to default in the next period for any realization of its endowment, $Cov[v_{cL}(c^{L'}) d'] = 0$ and $q = q^f$. On the other hand, if the default set includes the entire support for the endowment realizations, i.e. $D(b' | W'(s)) = Y$, then $d' = 0$ for all realizations of the economy's endowment. As a consequence, the equilibrium default probability $\delta(b', y' | s)$ is equal to 1, and $Cov[v_{cL}(c^{L'}) d'] = 0$, so $q = 0$.

Otherwise when the default set is not empty but does not include the whole support for the endowment realizations $0 < \delta(b', y' | s) < 1$. In this case, which was analyzed in the previous section describing the investors optimization problem, $Cov \left[v_{c^L} \left(c^{L'} \right) d' \right] > 0$, so $q < q^f$.

3.1 Characterization of Default Sets

The characterization of default sets is the characterization of incentives to default and therefore the characterization of endogenous default risk. In this model default risk is a function of both the emerging economy's fundamentals—the economy's endowment process and its asset position—and the characteristics of the international investor—the investor's risk aversion and wealth.

Default Sets and Risk Aversion of International Investors The degree of investors' risk aversion is an important determinant of access of emerging economies to credit markets, and of the risk of default of the economy. In this model, the more risk averse are international investors, the higher is the default risk and the tighter is the endogenous credit constraint faced by all emerging economies.

Proposition 1 *For any state of the world, s , as the risk aversion of the international investor increases, the emerging economy's incentives to default increase.*

Proof. See Appendix. ■

The economic intuition behind the result is straightforward. For the emerging economy, while the value of autarky is not a function of the investor's risk aversion; the value of maintaining access to credit markets is decreasing in the lender's degree of risk aversion. In order to induce a very risk averse investor to hold sovereign bonds, the representative agent of the emerging economy has to forgo much more current consumption—i.e., has to accept a very low price for her bonds. Other things equal, with lower bond prices, incentives to default are stronger. Therefore for any given state of the world, s , the degree of risk in the economy is increasing in the degree of risk aversion of international investors.

As the degree of risk in the economy changes, so too will the capital flows to the economy. In order to see how the capital flows change, it is necessary to define two concepts, the *endogenous credit constraint* given by the model and the *maximum safe level of debt*. To define these concepts, note that the stochastic process for the endowments has a compact

support. Also note that, conditional on W , the value of the credit contract is monotonically decreasing in b . These facts imply that conditional on W , there exists a unique level of assets, $\underline{b}(W)$, that is low enough such that no matter what the realization of the endowment, default is the optimal choice and $D(\underline{b}(W) | W) = Y$. Also, since default can be optimal only if $b < 0$, the compactness of the endowments support and the characteristic that the value of the contract is monotonically decreasing in b also imply that conditional on W , there exists a unique level of assets $\bar{b}(W)$ for which staying in the contract is the optimal choice for all realizations of the endowment. In this case, $D(\bar{b}(W) | W) = \emptyset$. Based on this discussion, it is obvious that

$$\underline{b}(W) \leq \bar{b}(W) \leq 0 \quad \forall W.$$

Given some current level of investors' wealth, any investments in the emerging economy's bonds in excess of $\underline{b}(W)$ imply a probability of default equal to 1. These investments will have a price of 0. On the other hand, all investments in the emerging economy's bond of an amount lower than $\bar{b}(W)$ imply a zero probability of default. These investments will have a price of q^f .

Definition 4 *For a given level of investor's wealth, W , $\underline{b}(W)$ is the endogenous credit constraint given by the model. This credit constraint ensures that in equilibrium only investments with some probability of repayment are made.*

Definition 5 *For a given level of investor's wealth, W , $\bar{b}(W)$ is the maximum safe level of debt. This value is the highest level of debt for which the probability of repayment is 1.*

Corollary 6 *For $\gamma_L^1 < \gamma_L^2$ Proposition 1 implies that*

$$D(b | W; \gamma_L^1) \subseteq D(b | W; \gamma_L^2).$$

Therefore, it must hold that

$$\begin{aligned} \underline{b}(W; \gamma_L^2) &\geq \underline{b}(W; \gamma_L^1). \\ \bar{b}(W; \gamma_L^2) &\geq \bar{b}(W; \gamma_L^1). \end{aligned}$$

This equation shows that endogenous credit constraints $\underline{b}(W)$ for the emerging economy are tighter the more risk averse are international investors—some contracts that are feasible under less risk adverse investors are not feasible under more risk averse investors.

The result in Proposition 1 is consistent with empirical findings which characterize the role of investor's risk aversion in the determination of country risk and sovereign yield.²⁴

Default Sets and Investor's Wealth In the present model, the economic performance of the emerging economy cannot be explained by the fundamentals of the emerging economy alone, i.e. by the economy's asset position and stochastic process of the endowment. The investor's wealth also affects the emerging economy's performance. This result is formalized in Proposition 2.

Proposition 2 *Default sets are shrinking in assets of the representative investor. For all $W_1 < W_2$, if default is optimal for b in some states y , given W_2 then default will be optimal for b for the same states y , given W_1 therefore $D(b | W_2) \subseteq D(b | W_1)$*

Proof. See Appendix. ■

The intuition for Proposition 2 is simple: given some default risk, it is less costly (in terms of current utility) for the investor to invest in the emerging economy when she is wealthy than when she is poor. So keeping constant the degree of risk that the investor faces, any investment that she is willing to undertake when she is poor she also will be willing to undertake when she is rich. Intuitively, financial contracts available to the representative agent of the emerging economy when investors are relatively rich have to be at least as good as feasible contracts when investors are relatively poor. Additionally, the previous effect implies that the emerging economy faces stronger incentives to default when the wealth of the investors is relatively low. Therefore default risk is decreasing in the wealth of the investors. These two effects amplify and reinforce each other.

²⁴Much empirical evidence supports Proposition 1: Using the spread between the yield of three month T-bills and the U.S. federal funds rate as a proxy for market turbulence, Arora and Cerisola (2001) find that heightened macroeconomic uncertainty in the U.S., has a positive significant effect on sovereign credit spreads for emerging markets. Using high-low yield spreads on U.S. corporate bonds as a proxy for risk aversion of U.S. investors, Ferruci et. al. (2004) and FitzGerald and Krolzig (2003) find that sovereign bond spreads increase when the risk aversion of U.S. investors increases. Similarly, Cunningham, Dixon and Hayes (2001), Westphalen (2001), and Kamin and von Kleist (1999) find evidence that risk premium in sovereign bonds increases more than proportionally when default risk increases.

Related empirical evidence supports Corollary 1: Examples include Mody and Taylor (2004), Ferruci et. al. (2004), and FitzGerald and Krolzig (2003). According to the findings in these papers, risk aversion of U.S. investors is an important determinant of capital flows to emerging economies: a higher U.S. high-low yield spread—interpreted as a reduction in investor risk appetite—results in a reduced supply of capital to emerging economies.

Corollary 7 *Proposition 2 implies that for $W_1 < W_2$ it must hold*

$$\begin{aligned}\underline{b}(W_1) &\geq \underline{b}(W_2) \\ \bar{b}(W_1) &\geq \bar{b}(W_2)\end{aligned}$$

and therefore the endogenous credit limit that the emerging economy faces is tighter for lower levels of wealth of the investor ($\underline{b}(W_1) \geq \underline{b}(W_2)$).

The previous result is a consequence of the fact that for investors the marginal cost of investing in sovereign bonds in terms of current consumption is decreasing in investors' wealth. Given that these agents are risk averse, investing in the sovereign bonds when their wealth is low is too costly; so when the wealth of the investor falls, the resources available to the emerging economy become scarce, reducing the value for the emerging economy of participating in credit markets. In turn, because the sovereign country has increasing incentives to default, some loans or portfolio investments that are feasible when the investor is wealthy cannot be an equilibrium outcome when the investor is poor.

Findings of several empirical papers on the literature regarding the determinants of capital flows and sovereign bonds spreads of emerging economies are consistent with the results in Proposition 2 and Corollary 7. See, for example, Warther (1995), Westphalen (2001), Kang et al (2003), FitzGerald and Krolzig (2003), Mody and Taylor (2004), and Ferruci et al (2004).

The results in Proposition 2 and Corollary 7 are also consistent with the evidence regarding financial contagion across countries who share investors. See for example Kaminsky and Reinhart (1998), Van Rijckeghem and Weder (1999), Kaminsky and Reinhart (2000) and Hernandez, and Valdes (2001).²⁵

Default Sets and the Asset Position of the Emerging Economy In the model, a highly indebted economy is more likely to default than an economy with lower debt. And

²⁵For the period 1984 to 1993, Warther (1995) finds that an inflow to corporate bonds funds of around 1% of the mutual fund's assets results in a permanent increase of 2.1% in those bond prices (i.e., reduces the cost of borrowing for those issuing those bonds). Using world and U.S. equity indexes respectively as proxies for the business climate (an increase in these indexes is associated with a better business climate), Westphalen (2001) and Ferruci et. al. (2004) find a negative relation between economic expansion in the investors' countries and sovereign yield spreads of emerging economies. Kang et. al. (2003) finds that a 1% increase in the world GDP growth rate improves Korea's financial account by 4.73% of the trend GDP. FitzGerald and Krolzig (2003) find a positive and significant relationship between US GDP and capital inflows to emerging economy. Finally, Mody and Taylor (2004) find that a higher growth in industrial production in US has a positive effect on the supply of capital to emerging economies.

as in models of the same type where lenders are risk neutral, default sets are shrinking in assets.

Proposition 3 *Default sets are shrinking in assets of the emerging economy. For all $b_1 < b_2$, if default is optimal for b_2 in some states y , given W , then default will be optimal for b_1 for the same states y , given W . Therefore $D(b_2 | W) \subseteq D(b_1 | W)$.*

Proof. See Appendix. ■

This result is analogous to the result in Arellano (2003), and closely related to the results in Eaton and Gersovitz (1981) and Chatterjee, et. al. (2002). The main difference in the present paper is that the result is conditioned on the level of wealth of the representative investor. The economic intuition is as follows. While the value for the economy of fulfilling the contract is increasing in b , the outside value of the economy is not—the value of autarky does not depend on b . Therefore as the indebtedness of the economy increases, the value of the contract decreases, while the value of default remains unchanged. As a consequence, starting from an asset position b in which default is the optimal choice, it is clear that if the assets shrink, the value of the contract also falls. As the value of the contract falls, default will continue to be the optimal choice.

This result is consistent with the empirical literature on the determination of credit ratings and yield-bond spreads. See for example, Cantor and Pecker (1996), Cunningham, Dixon and Hayes (2001), Durbin and Ng (1999), and Merrick (2000)).

Default Sets and Endowment Realization Default sets also depend on the realization of income. As in Arellano (2003), it is possible to show analytically that if the endowment process is *i.i.d.*, for given W , then default incentives are stronger for lower levels of income. The numerical solution of the present model extends this result to the case in which the stochastic process of the endowments follows a Markov chain with persistence.

Proposition 4 *If the endowment process is i.i.d., default incentives are stronger the lower the endowment. For all $y_1 < y_2$ if $y_2 \in D(b | W)$ then $y_1 \in D(b | W)$.*

Proof. See Appendix. ■

The intuition for this result follows Arellano (2003). Again the main difference is that in the present context, the result is conditioned on the level of wealth of the investors. The logic behind this results follows from the fact that default is only optimal if under all

feasible financial contracts the emerging economy experiences capital outflows. In the case of a recession, capital outflows are extremely costly in terms of the welfare of a risk averse agent (because the concavity of the periodic utility); therefore at sufficiently low levels of the endowment realization, the credit market becomes a less effective tool for consumption smoothing than default.

This result is also consistent with the empirical literature on the determination of credit ratings and sovereign yields. In this literature, sovereign yield spreads increase when the economy's fundamentals deteriorate, mainly when GDP falls.

Additionally, this result implies that because default risk is counter-cyclical, domestic interest rates are also counter-cyclical. Counter cyclicity is consistent with the stylized facts of financial emerging markets (see Neumeyer and Perri (2004), and Uribe and Yue (2003)).

3.2 Default as an equilibrium outcome of the model

In this model, default can be an equilibrium outcome if the emerging economy ever finds it optimal to choose b' such that $D(b' | W'(s)) \neq \emptyset$. In other words, to observe default at equilibrium it must hold that beginning from an asset position b such that $D(b | W) = \emptyset$, then there exists a sequence of endowment shocks such that this economy ends up borrowing b' such that $D(b' | W'(s)) \neq \emptyset$. As in the case in which international investors are risk neutral, studied in Arellano (2003), this outcome is possible only if the equilibrium price function does not decrease “too fast” when assets decrease. Default is a possible outcome at equilibrium only if by increasing its borrowing to levels for which there is default risk, the emerging economy is able to increase current period capital inflows $b - q(s, b') b'$. In this case, by borrowing more and more, the economy achieves a higher level of consumption even though the economy has to accept a lower price for its bonds in order to compensate the investors for taking the risk of default.

Proposition 5 *Given $\bar{b}(W'(s); f(y' | y), \gamma^L)$, default at equilibrium is a possible outcome of the time series of this model if for $b' = \bar{b}(W'(s); f(y' | y), \gamma^L)$*

$$\frac{\partial c}{\partial \bar{b}(\cdot)} = -\frac{\partial q(\cdot) \bar{b}(\cdot)}{\partial \bar{b}(\cdot)} < 0.$$

In other words, default can be an equilibrium outcome if for $b' = \bar{b}(W'(s); f(y' | y), \gamma^L)$, it holds $\frac{\partial q(\cdot) \bar{b}(\cdot)}{\partial \bar{b}(\cdot)} > 0$, so that by increasing its borrowing, the emerging economy is able to increase its consumption.

The sign of this derivative is ambiguous depending on both the emerging economy's fundamentals and investors' characteristics:

$$\begin{aligned} \frac{\partial q(\cdot) \bar{b}(\cdot)}{\partial \bar{b}(\cdot)} &= \overbrace{q(s, \bar{b}(W'(s)); f(y'|y), \gamma^L)}^{>0} \\ &+ \underbrace{\bar{b}(W'(s); f(y'|y), \gamma^L)}_{\leq 0} \underbrace{\frac{\partial q(s, \bar{b}(W'(s)); f(y'|y), \gamma^L)}{\partial \bar{b}(W'(s); f(y'|y), \gamma^L)}}_{\geq 0} \leq 0 \end{aligned} \quad (18)$$

given

$$\frac{\partial q(s, \bar{b}(W'(s)); f(y'|y), \gamma^L)}{\partial \bar{b}(W'(s); f(y'|y), \gamma^L)} = \beta_L E \frac{v_{c_L}(c'_L)}{v_{c_L}(c_L)} \left[\left(\frac{\gamma_L}{c'_L} + \frac{\gamma_L}{c_L} q \right) d' + \frac{\partial d'}{\partial \bar{b}} \right] \quad (18a)$$

$$q(s, \bar{b}(W'(s)); f(y'|y), \gamma^L) = q^f. \quad (18b)$$

Roughly speaking, the smaller is the equilibrium maximum safe level of borrowing, $\bar{b}(W'(s); f(y'|y), \gamma^L)$, the higher is the chance that this derivative turns out to be positive. Intuitively, because investors must be compensated in order to induce them to take some default risk, this risk imposes an additional cost of borrowing for the emerging economy. For the borrower, the cost of borrowing beyond the maximum safe level must be paid over the total amount of resources borrowed, and not only over the marginal amount of borrowing. Therefore, the larger is the base over which this additional cost of borrowing has to be paid—i.e. the larger is the maximum safe level of borrowing—the higher is the cost of default risk and the lower is the likelihood that the economy would ever choose to borrow beyond safe level of debt.

Role of W in the determination of the sign of $\frac{\partial q(\cdot) \bar{b}(\cdot)}{\partial \bar{b}(\cdot)}$ A priori, it is not possible to determine the manner in which the sign of $\frac{\partial q(\cdot) \bar{b}(\cdot)}{\partial \bar{b}(\cdot)}$ changes with the level of investors' wealth.

First, because of Proposition 2, $\frac{\partial \bar{b}(\cdot)}{\partial W'(\cdot)} < 0$, i.e., a higher level of investor's wealth allows the emerging economy to borrow more. This effect implies that when investors are wealthier, other things equal, default risk imposes a larger additional cost of borrowing beyond the safe level of debt, (i.e. $\bar{b}(\cdot) \frac{\partial q(\cdot)}{\partial \bar{b}(\cdot)}$ is larger). In this case, any change in the price of the sovereign bonds will be felt over a larger base of borrowing. As a result, for the emerging economy there is potentially less to gain from accepting a lower price for these bonds in order to further increase borrowing. This effect makes it more difficult for the economy to increase

consumption by risking default. Consequently, this effect implies that it should be easier to observe default as an equilibrium outcome when international investors are relatively constrained financially compared to when investors are relatively solvent.

Second, a higher level of investors' wealth reduces the absolute risk aversion of these agents, $\left(\frac{\gamma_L}{c_L} \& \frac{\gamma_L}{c_L}\right)$. As a consequence, because the investors demand a relatively small *excess* risk premium, sovereign bond prices change "more slowly"—that is, $\frac{\partial q(\cdot)}{\partial b(\cdot)}$ is smaller in absolute terms.

The two effects go in opposite directions. Analytically, it is not possible to determine which effect predominates. However, numerical results in the model presented here establish that the first effect is the dominant one. Other things equal, the observation of default in the time series of the model is more likely when the investors wealth is relatively low. This result constitutes a testable implication of the model.

Role of γ_L in the determination of the sign of $\frac{\partial q(\cdot)\bar{b}(\cdot)}{\partial b(\cdot)}$ It is also not possible to determine analytically the manner in which $\frac{\partial q(\cdot)\bar{b}(\cdot)}{\partial b(\cdot)}$ changes with the investor' level of risk aversion.

First, Proposition 1 establishes that $\frac{\partial \bar{b}(\cdot)}{\partial \gamma_L} > 0$, i.e., the more risk averse investors are, the less the economy is able to borrow and the lower is the maximum safe level of borrowing for any given state of the world. Therefore, other things equal, if the investor is very risk averse, the cost of a change in the price of the bonds is felt over a smaller borrowing base. In this case, there is potentially more to gain from accepting a lower price for these bonds in order to further increase borrowing. Therefore this effect makes default a more likely outcome of the model.

Second, larger risk aversion of the investor also implies a larger response of $q(\cdot)$ to changes in the borrowing level. Other things equal, the more risk averse is the investor, the larger is the excess risk premium that she demands in order to take default risk.

As in the relationship between W and $\frac{\partial q(\cdot)}{\partial b(\cdot)}$, the two effects here also go in opposite directions. Again, without a numerical examination of the issue, it is not possible to determine which effect would dominate. But numerical results of the model suggest that for higher levels of the investor's risk aversion, default is more likely.

Role of the Stochastic Process of the endowment in the determination of the sign of $\frac{\partial q(\cdot)\bar{b}(\cdot)}{\partial b(\cdot)}$ Finally, it is necessary to examine properties of the hazard function of the

probability distribution of the endowments $\frac{f(y'|y)}{1-F(y'|y)}$, in order to determine the response of default risk to changes in borrowing by the emerging economy $\frac{\partial d'(\cdot)}{\partial \bar{b}(\cdot)}$. For the case of the risk neutral representative investor, the main determinant of the sign of $\frac{\partial q(\cdot)\bar{b}(\cdot)}{\partial \bar{b}(\cdot)}$ the main determinant of the sign of $\frac{\partial q(\cdot)\bar{b}(\cdot)}{\partial \bar{b}(\cdot)}$ is the stochastic process of the endowments. In this case $\frac{\partial q(\cdot)\bar{b}(\cdot)}{\partial \bar{b}(\cdot)}$ takes the form of

$$\frac{\partial q(\cdot)\bar{b}(\cdot)}{\partial \bar{b}(\cdot)} = \overbrace{q(y, \bar{b}; f(y'|y))}^{>0} + \underbrace{\bar{b}(y; f(y'|y))}_{\leq 0} \underbrace{\frac{\partial q(y, \bar{b}; f(y'|y))}{\partial \bar{b}(y; f(y'|y))}}_{\geq 0} \stackrel{\leq 0}{\geq} 0 \quad (19)$$

given

$$\frac{\partial q(y, \bar{b}; f(y'|y))}{\partial \bar{b}(y; f(y'|y))} = q^f E \left[\frac{\partial d'(y, \bar{b}; f(y'|y))}{\partial \bar{b}(y; f(y'|y))} \right].$$

Definition 8 For some level of borrowing b' , the upper-bound of the default set $y^*(b' | s; f(y'|y), \gamma^L)$ is the unique value of the next period endowment realization for which, given the current state of the world s , the emerging economy is indifferent between staying in the credit contract and defaulting. $y^*(b' | s; f(y'|y), \gamma^L)$ is defined such that $V^A(y^* | y) = V^C(b', y^* | s; f(y'|y), \gamma^L)$. The definition of default sets implies that $D(b | W) = [y, y^*(s; f(y'|y), \gamma^L)]$. In the case of risk neutral investors this upper-bound is a function only of the current realization of the emerging economy's endowment—i.e. $y^{*RN}(\cdot) = y^*(b' | y; f(y'|y))$.

Following Arellano (2003), using the continuity of the support space for the endowment realizations, the definitions of the default probability, δ , and the upper-bound of the default sets y^* , it is possible to manipulate (19) so that for the case of risk neutral investors, the following expression holds:

$$\frac{\partial q(\cdot)\bar{b}(\cdot)}{\partial \bar{b}(\cdot)} = q^f [1 - F(y^{*RN}(\cdot))] \left[1 - \bar{b}(\cdot) \frac{f(y^{*RN}(\cdot))}{1 - F(y^{*RN}(\cdot))} \frac{\partial y^{*RN}(\cdot)}{\partial \bar{b}(\cdot)} \right].$$

For this case, bond prices change in response to changes in the level of borrowing beyond the maximum safe level of debt; this price change depends only on the interaction of the hazard function $\frac{f(y^{*RN}(\cdot))}{1 - F(y^{*RN}(\cdot))}$ in the neighborhood of $y^*(\bar{b} | y; f(y'|y))$, relative to how fast the upper bound of the default sets increases with debt.

On the other hand, consider the case where investors are risk averse. Like the case of risk neutral investors, for the case of risk averse lenders equilibrium is determined by the

interaction of the hazard function $\frac{f(y^*(\cdot))}{1-F(y^*(\cdot))}$ in the neighborhood of $y^*(\bar{b} | s; f(y' | y), \gamma^L)$ relative to how fast the upper bound of the default set increases. Additionally however, for the case of risk averse investors, investors' characteristics matter in determining the magnitude with which $y^*(\bar{b} | s; f(y' | y), \gamma^L)$ increases with debt.

3.3 Default Risk and the Price of Riskless Assets

In the current model there are two channels through which changes in the world interest rate affect the emerging economy's incentives to default.

The first channel is the substitution effect of a change in the world interest rate. This effect is the only effect present when investors are risk neutral. The substitution effect implies that when the international interest rate falls (i.e. the price of T-Bills increases²⁶), borrowing for the emerging economy becomes less costly. From the investor's point of view, sovereign bonds and T-Bills are substitute assets. Therefore, when the return on T-Bills falls these assets become less desirable relatively to sovereign bonds. This effect implies that the set of feasible financial contracts for high values of the world interest rate is a subset of the set of those contracts for low values of this interest rate. As a consequence, for the emerging economy the value of participating in credit markets increases when the world interest rate decreases and *therefore* default incentives are weaker.

The second channel is the wealth effect of a change in the world interest rate. This effect is not present in a model where investors are risk neutral. The effect works as follows. A change in the world interest rate modifies the level of financial wealth of the investors. If the representative investor is holding positive positions of the riskless asset, a reduction in the return of this asset reduces the agent's lifetime wealth. From Proposition 2, this effect would increase the risk of default of the emerging economy. The increased default risk reduces the set of feasible financial contracts to the emerging economy. This effect suggests a positive correlation between capital inflows to emerging economies and the international interest rate.

If the substitution effect of a change in the world interest rate dominates, then an increase in the price of T-Bills, q^f , reduces the total premium that the economy pays on its

²⁶The gross world interest rate is the inverse of the price of the bonds. That is

$$(1 + r^f) = \frac{1}{q^f}.$$

bonds $(q - q^f)$ and the set of feasible financial contracts available to the emerging economy expands. In this case the emerging economy is at least as well off as it was under a lower price T-Bills. If the wealth effect is stronger, then when international interest rates fall, the premium on the emerging economy's sovereign bonds increases, and the set of available financial contracts to the emerging economy shrinks. In such a case the emerging economy is more credit constrained than it was under a higher international interest rate.

When the international interest rate changes, the following expression shows how the opposing effects determine the change in the premium on sovereign bonds:²⁷

$$\frac{\partial (q - q^f)}{\partial q^f} = \beta_L E \left[\frac{v_{c^L}(c^{L'})}{v_{c^L}(c^L)} \gamma^L \left(\frac{1}{c^{L'}} \frac{\partial c^{L'}}{\partial q^f} - \frac{1}{c^L} \frac{\partial c^L}{\partial q^f} \right) (d' - 1) + \frac{v_{c^L}(c^{L'})}{v_{c^L}(c^L)} \frac{\partial d'}{\partial q^f} \right]$$

Using equation (5), it is possible to say that when q^f increases

$$\frac{\partial}{\partial q^f} \left(\frac{1}{c^{L'}} \frac{\partial c^{L'}}{\partial q^f} - \frac{1}{c^L} \frac{\partial c^L}{\partial q^f} \right) < 0.$$

In words, when the price of T-Bills increases investors reduce their savings, mainly by modifying their position in T-Bills. This decision implies an increase in their current consumption and a reduction in next period consumption. From equation (6), we see that due to the concavity of the investor's utility function, an increase in the investor's current consumption implies a decrease in the investor's marginal cost (in terms of utility) of investing in the emerging economy's bonds. This relationship corresponds to the previously discussed substitution effect of a change in the world interest rate. This effect suggests a positive correlation between the international interest rate and domestic interest rates of emerging economies. The effect also suggests a negative correlation between the international interest rate and capital inflows to emerging economies.

With respect to changes in the world interest rate, the relative strength of the substitution and income effects determines the sign of $\frac{\partial d'}{\partial q^f}$. The derivative $\frac{\partial d'}{\partial q^f}$ indicates how

²⁷From Equations 5 and 6, the premium on sovereign bonds is given by

$$q - q^f = \beta_L E \left[\frac{v_{c^L}(c^{L'})}{v_{c^L}(c^L)} (d' - 1) \right].$$

Because for all b s.t. $D(b | W) \neq \emptyset$, $q - q^f < 0$, then if an increase in q^f reduces the premium on sovereign bonds, it must be the case that $\frac{\partial (q - q^f)}{\partial q^f} > 0$.

default risk changes when the price of T-Bills changes. If the wealth effect is stronger than the substitution effect then $\frac{\partial d'}{\partial q^T} < 0$. If the change in the default risk is sufficiently large, then it is possible that the price of emerging economy bonds falls when the price of the T-Bills increases. Under this circumstance, the correlation between emerging economies' domestic interest rates and the international interest rate would be negative. Also in this case, capital inflows to emerging economies would be positively correlated with international interest rates.

The empirical evidence regarding the effect of changes in developed countries' interest rates in emerging economies credit spreads and capital flows is mixed. See, for example, Calvo, Leiderman and Reinhart (1993), Fernandez-Arias (1996), Cline and Barnes (1997), Chohan, Claessens, Mamingi (1998), Kamin and von Kleist (1999), Eichengreen and Mody (1998), Herrera and Perri (2000), Calvo, Fernandez-Arias, Reinhart and Talvi (2001), Goldberg (2001), Arora and Cerisola (2001), Hernandez, Mellado and Valdes (2001), Bekeart et al (2002), FitzGerald and Krolzig (2003), Kang et al (2003), Ferruci et al (2004), and Mody and Taylor (2004)²⁸

Preliminary numerical results in this paper suggest that for the parameters of a typical emerging economy and a typical international investor in emerging markets, the substitution effect dominates the wealth effect of a change in world interest rates.

4 Numerical Solution

The model in this paper is not calibrated to match the business cycle statistics of any particular developing country. Instead the model is solved for a hypothetical *typical* emerging economy.²⁹ For the typical economy, the model is solved numerically in order to establish its

²⁸Arora and Cerisola (2001) and FitzGerald and Krolzig (2003) find evidence of a positive relationship between world interest rates and emerging economies' sovereign spreads. However, Cline and Barnes (1997) and Kamin and von Kleist (1999) find no significant relationship between U.S. interest rates and emerging economy spreads. Using primary market data on sovereign bonds spreads, Eichengreen and Mody (1998) find that an increase in the U.S. interest rate increases spreads for Latin America. For Asia, these authors find mixed evidence: U.S. rate increases correspond to increase for floating-rate Asian bonds; at the same time U.S. rate increases correspond to decreases in the spread of fixed-rate Asian bonds. Kaminsky and Schmulker (2001) find a positive and significant relationship between U.S. interest rates and emerging economy sovereign spreads for the case of fragile economies. Finally, Ferruci et. al. (2004) find that short term U.S. interest rates have a positive relationship with emerging economies sovereign spreads, but long term U.S. interest rates have a negative relationship.

²⁹Countries which might be considered similar to the "typical" emerging economy include Argentina, Brazil, Chile, China, Colombia, Czech Republic, Hong-Kong, Hungary, India, Indonesia, Korea, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Russia, Singapore, Slovak, Sri Lanka, Taiwan, Thailand and

qualitative and quantitative implications for the case in which the endowment process follows a persistent Markov process. The numerical exercise is performed quarterly. The model parameters are chosen to replicate some features of both typical emerging economies, and typical international investors in emerging economies. Table 1 gives the parameters which are considered in the numerical analysis of the model. For the benchmark calibration, the emerging economy’s coefficient of risk aversion is 5, a standard value considered in business cycle literature. The representative investor’s coefficient of risk aversion is set at 0.5, a low value in comparison to standard values considered in business cycle literature for risk averse agents from developed countries. This parameter is purposely chosen to be low in order to determine the impact of departing (even by a little) from the assumption of investors’ risk neutrality. Somewhat more standard values of 1 and 2 will also be used to determine the impact of changing investors’ risk aversion.

The mean income of the emerging economy is normalized to 1. The representative investor, on the other hand, receives a deterministic income of 0.05, (or 5% of the emerging economy’s mean income) in each period.³⁰ This parameter is chosen so that the equilibrium wealth level of the representative investor is consistent with observed values of the average asset positions of international mutual funds specialized in emerging economies.³¹ Total Net Assets of US Mutual funds for the period 1994-1999 are taken from the “2004 Mutual Fund Book”, published by Investment Company Institute. For mutual funds investing in emerging markets in Latin America, Europe, and Asia, during 1994-1999, the average annual net asset position is US\$196.2 Bn (94.3% of the average emerging economy GDP)³².

The standard deviation of the income process for the emerging economy’s endowment is set to 6.5%. This value is close to the standard deviation of the tradable sector in Argentina³³ but somewhat higher than the variability of the GDP for most emerging markets³⁴. The auto-correlation of the endowment process is assumed to be 0.65, roughly the

Venezuela.

³⁰The parameter X is important in the current model. This parameter is the main determinant of the natural credit limit faced by international investors, i.e., the no-ponzi condition. The larger this parameter is, the looser the credit limit, and the wealthier are the investors; consequently when this parameter is larger, the smaller is the impact of changes in wealth over the optimal investors’ portfolio.

³¹More specifically, ‘average asset positions’ here refer to net asset position as a proportion of the “average” emerging economy GDP.

³²Annual average emerging market GDP is computed using the “Global Development Finance and World Development Indicators” of the World Bank. This number takes the average GDP of 25 emerging countries in which mutual funds invested for the period 1994-1999.

³³Arellano (2003) reports the standard deviation of the tradable sector in Argentina at around 5.6%.

³⁴Valderrama (2002) reports standard deviations in GDP ranging from 1.6% for Brazil to 5.7% for Peru. Emerging countries within this range include Argentina, Brazil, Korea, Mexico, Peru, Thailand, and Turkey.

Table 1: Parameter Values

Parameter	Baseline Value	Sensitivity Analyses	
Emerging Economy's Risk Aversion γ	5		
Representative investor's Risk Aversion γ^L	0.5	1	2
Emerging Economy's Mean Income $E[y]$	1		
Std. Dev. Emerging Economy's Income $std[y]$	0.065		
Autocorr. Emerging Economy's Income Process	0.65		
Emerging Economy's Discount Factor β	0.95	0.93	0.89
Representative Investor's Discount Factor β^L	0.9875	0.98	
Risk Free Interest Rate $r^f = \frac{1}{q^f}$	0.01	0.0125	
Representative investor's Income X	0.05	1	10

value seen in Brazil, Peru, Slovak Republic and Turkey.³⁵

The free interest rate is set to 1%, to match the quarterly US interest rate. However, in order to determine the effect of changes in the world interest rate, the model is also simulated with a quarterly interest rate of 1.225%. Also, in order to determine how the equilibrium of the model is modified by other types of uncertainty (other than default risk), we separately analyze the case where the world interest rate follows a stochastic process. In this exercise, the mean of the world interest rate is assumed to be 1%, its variability 1%, and its auto-correlation 0.95.³⁶

The representative investor's discount factor is set to 0.9875 which is in the range commonly used in business cycle studies of industrialized countries. A sensitivity analysis is performed considering a discount factor of 0.98. The emerging economy's discount rate is chose to allow the model to exhibit default as an equilibrium outcome of the time series of the model.

4.1 Solution Method

This model is solved numerically using an interpolation method. This method interpolates between discrete values in order to provide a solution to the model which is continuous in b and W .

³⁵See Aguiar and Gopinath (May 2004). The average autocorrelation for emerging economies reported in that paper is 0.73. By choosing a value below average (but within in the range of observed values), the relatively high variability of the income process is offset.

³⁶The value considered for the variability of the world interest rate is very close to the estimated 1.08% in Neumeyer and Perry (2004). The value assumed for the autocorrelation is a standard value within the business cycles literature.

The methodology proceeds as follows. Initially, the state space of the model is discretized for each of the state variables of the model, b , y , and W . The continuous stochastic endowment process is approximated with a discrete Markov chain that allows for 5 possible realizations of the original process distribution. This approximation is done using the methodology of Hussey and Tauchen (1991). For the emerging economy's debt position, b , the asset space takes 200 possible discrete values. Finally, investors' wealth, W , takes 10 possible discrete values. By interpolating over the grid points, the solution algorithm allows a de facto continuous range for both b , and W .

The solution algorithm has the following steps:

- (i) Make an initial guess for the emerging economy's value function, $V^0(s)$, next period asset position, $b'^0(s)$, default/repayment decision $d^0(s)$ and *equilibrium* price function $q^{APC,(0)}(s)$. The initial guesses are the value function, the policy function functions and the equilibrium price function that result from an analogous model with risk neutral investors $V^{RN,0}(s)$, $b'^{RN,0}(s)$, $d^{RN,0}(s)$ and $q^{RN,(0)}(y; b'(y))$ respectively.
- (ii) Taking $b'^{*,(-i)}(s)$, $d^{*,(-i)}(s)$ and $q^{APC,(-i)}(s)$ as given, and assuming equilibrium in emerging credit markets given by

$$\theta'^{*,(i)}(s) = \begin{cases} b'^{*,(-i)}(s) & \text{if } b'^{*,(-i)}(s) < 0 \\ 0 & \text{if } b'^{*,(-i)}(s) \geq 0 \end{cases},$$

iterate on the representative investor's Bellman equation (4) to solve for the optimal value function $V^{L(i)}(s)$ and the optimal policy functions $W'^{*,(i)}(s)$

- (iii) Iterate on the emerging economy's Bellman equation (11) to solve for the optimal value function $V^{(i)}(s)$, the optimal policy functions $b'^{*,(i)}(s)$, and $d^{*,(i)}(s)$ and the corresponding equilibrium price function $q^{EE(i)}(s; b'^{(i)}(s))$. This iteration involves the next sub-steps:

- (a) Take $q^{APC,(-i)}(s)$ and $W'^{*,(i)}(s)$ as given to compute $c_L^{(i)}(s; b')$.

- (b) Given $c_L^{(i)}(s; b')$ and $W'^{*,(i)}(s)$, compute

$$A^{(i)}(s, b') = \beta_L \int \left(c_L^{L'}\right)^{-\gamma^L} f(y' | y, W'^{*,(i)}(s)) dy'$$

- (c) For any s, b' solve for $q^{(i)}(s, b')$ by solving the non-linear equation on $q^{(i)}(s, b')$ that is derived from (6):

$$q(s, b')^{-\gamma^L} - b' A^{(i)}(s, b') q(s, b') - \tilde{c}_L^{(i)}(s; b') A^{(i)}(s, b') = 0$$

where $\tilde{c}_L^{(i)}(s; b') = X + W - W'^{*,(i)} q^f - b' q^f$.

(d) For any s, b' given $W'^{*,(i)}(s)$ compute

$$\beta \int V^{C(i)}(s; b') f(y' | y, W'^{(i)}(s)) dy'.$$

(e) Maximize

$$u(y + b - b'q(s, b')) + \beta \int V^{C(i)}(s; b') f(y' | y, W'^{(i)}(s)) dy'$$

with respect to b' to find $V^{C(i)}(s)$ and the associated $b'^{*,(i)}(s)$ and $q^{(i)}(s, b'^{(i)}(s))$.

(f) Determine $d^{*,(i)}(s)$ by comparing $V^{C(i)}(s)$ to V^A .

(g) Determine the equilibrium price of bonds by setting

$$q^{EE(i)}(s; b'^{(i)}(s)) = \begin{cases} q^{(i)}(s, b'^{(i)}(s)) & \text{if } d^{(i)}(s) = 1 \\ 0 & \text{otherwise} \end{cases}.$$

(iv) If $|q^{EE(i)}(s; b'^{(i)}(s)) - q^{APC,(-i)}(s; b'^{(-i)}(s))| < \varepsilon$ stop. Otherwise, set $q^{APC,(i)}(s; b'^{(i)}(s)) = q^{EE(i)}(s; b'^{(i)}(s))$, and repeat steps 2 to 4.

4.2 Policy Functions

Numerical results of this exercise confirm the analytical results previously discussed.

Investors' Risk Aversion and Policy Functions One of the implications from considering risk averse investors is that sovereign bond prices carry two type of premiums: a default probability premium and a pure risk premium. Given the level of investors' wealth, W , Figure 1 shows a comparison between the price of sovereign bonds if only default probability was taken in account vis a vis the price inclusive of the risk premium.

The model also predicts that default risk, proxied by default probabilities, is always higher for more risk averse investors. Figure 2 shows default probability functions for two identical economies that trade with two different type of investors, one risk neutral and the other risk adverse³⁷. For a given level of wealth, Figure 2 shows that when investors are risk averse, the probability of default is greater than or equal to the probability of default associated with the same levels of debt when investors are risk neutral. This result holds for all realizations of the economy's endowment and all levels of debt.

³⁷For the economy trading with risk averse investors, the probability of default shown corresponds to the equilibrium probability of default for the highest level of wealth considered in the exercise.

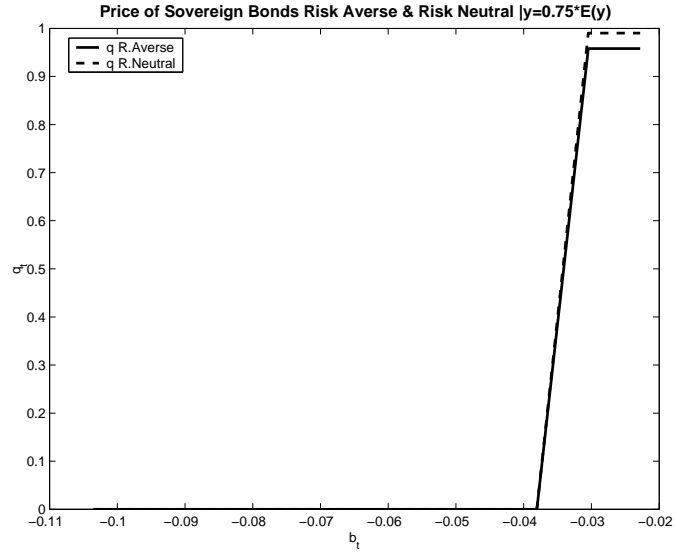


Figure 1: The Risk Premium in Sovereign Bond Prices.

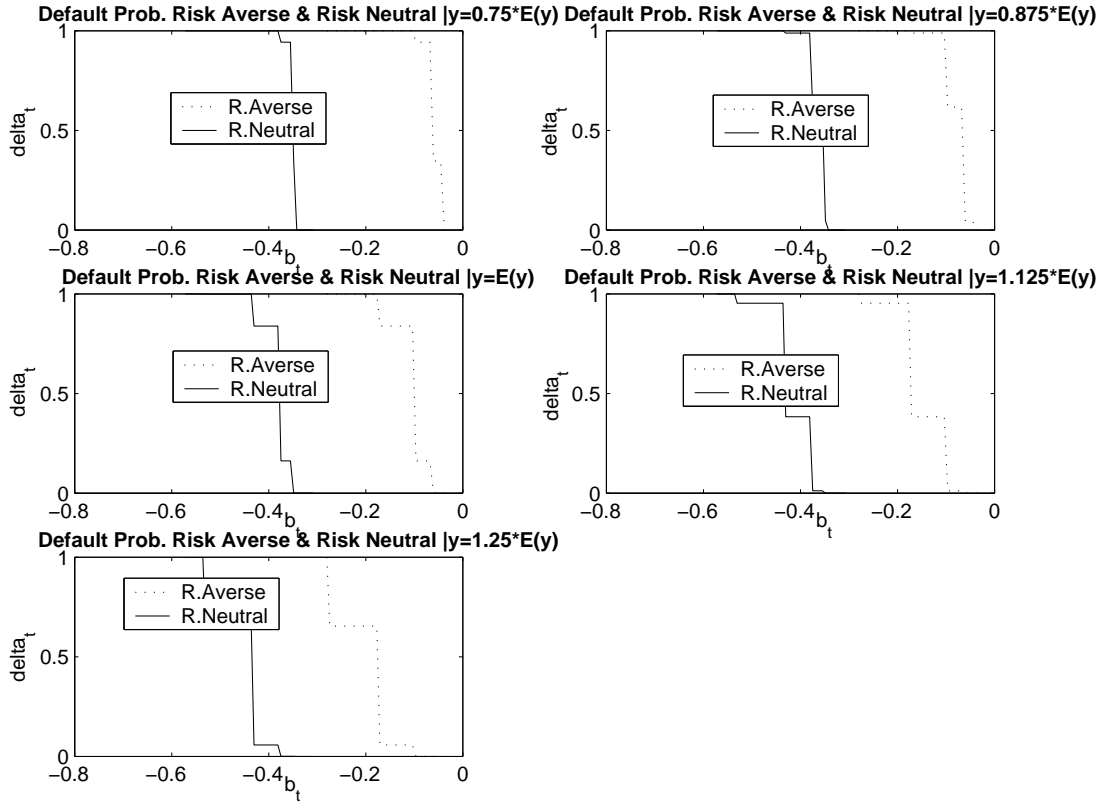


Figure 2: Default Probabilities: Risk Neutral vs. Risk Adverse Lenders

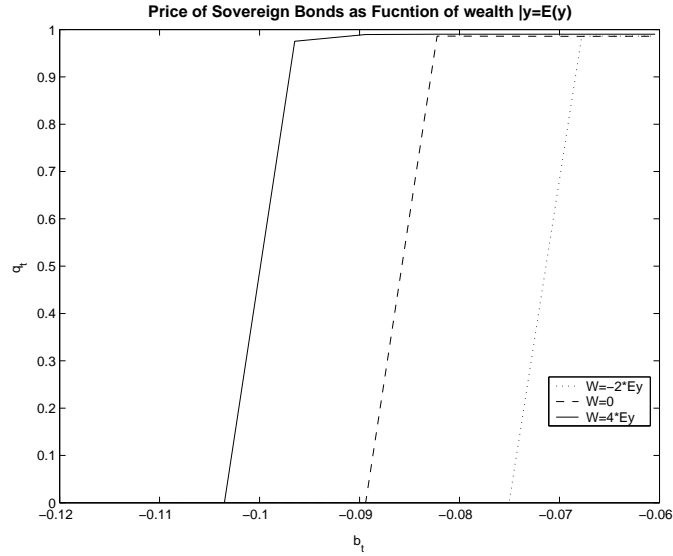


Figure 3: Bond Prices as a Function of Wealth

Table 2: Credit Limits and Investor's Wealth

Credit Limits as a function of Investors' Wealth									
Wealth Level	-3.76	-3.01	-2.55	-2.04	-1.42	-0.67	0.29	1.66	4
Credit Limit	-0.28	-0.29	-0.30	-0.31	-0.33	-0.33	-0.35	-0.35	-0.37

Investors' Wealth and Policy Functions Higher wealth implies higher bond prices and higher capital inflows to emerging economies. For a given realization of the emerging economy's endowment, Figures 3 and 4 shows bond prices and borrowing as a function of wealth. It can be seen in Figure 3 that for any level of the economy's debt, the equilibrium bond price is increasing in investors' wealth. Likewise in Figure 4, for any level of the economy's debt, the equilibrium level of borrowing is increasing in investor's wealth.

Figure 4 also shows that the credit limit of the economy tightens when investors' wealth is lower. When investors' wealth is around 4 times the economy's average income, the economy can borrow up to the point where its external debt is around 10% of its average GDP. On the other hand, if investors' wealth is around 2 times the economy's average GDP, then the economy is only able to borrow up to the point where its external debt is around 6% of its average income.

Additionally, the numerical results of the model confirm the previous analytical results: debt limits tighten when wealth falls. Table 2 shows the equilibrium default probabilities and debt limits of the model as a function of investors' initial wealth.

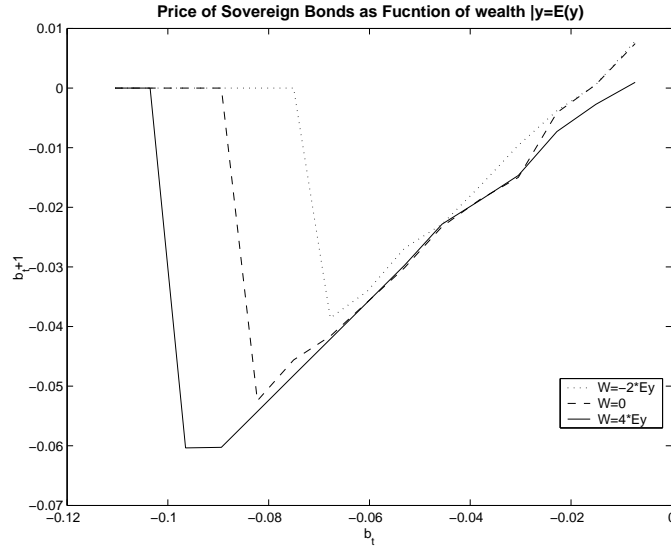


Figure 4: Borrowing as a Function of Wealth

Emerging Economy's Fundamentals and Policy Functions The analytical results of this paper imply that bond prices and the emerging economy's borrowing are increasing in income and decreasing in the debt level of the economy. For a given level of wealth, Figure 5 illustrates these results for the case of an economy with persistent income process. Figure 5 shows that the equilibrium bond price function is increasing in the economy's endowment and decreasing in the economy's debt level. As a consequence, domestic interest rates (which are roughly the inverse of bond prices) are lower when the economy's income is larger.

Figure 5 also shows that the credit limit of the economy tightens when the economy's endowment realization is lower. When the endowment realization is the highest, the economy can borrow up to the point where its external debt is around 25% of its average GDP. When the endowment realization is the lowest, the economy can only borrow up to the point where its external debt is less than 5% of its average income.

World Interest Rate and Policy Functions To measure the impact of the world interest rate on policy functions, we compare two otherwise identical economies with discount rate 0.89, the first facing world interest rate of 1%, the second facing world interest rate of 1.225%. From this comparison, the economy facing the higher rate experiences reduced capital flows and increased sovereign yield spreads. Furthermore, the reduction in capital inflows is relatively larger when the economy faces recession than when the economy faces expansion.

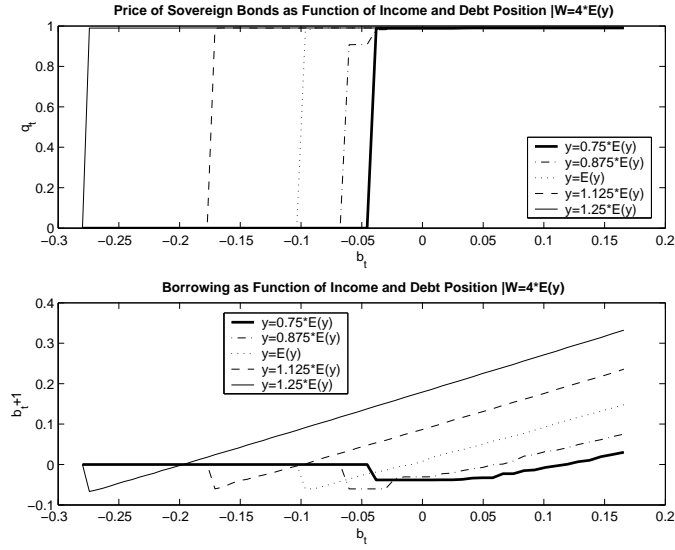


Figure 5: Bond Prices for an Economy with a Persistent Income Process.

Table 3: Credit Limits and World Interest Rates

Credit Limits as a function of World Interest Rate									
Wealth	-2.00	-1.42	-1.08	-0.67	-0.19	0.39	1.14	2.19	4.00
$r^f = 1\%$	-0.079	-0.079	-0.080	-0.080	-0.080	-0.082	-0.082	-0.082	-0.082
$r^f = 1.23\%$	-0.076	-0.076	-0.076	-0.077	-0.079	-0.079	-0.079	-0.080	-0.082

Table 3 compares credit limits and shows that an economy trading with risk averse agent will be more credit constrained for higher levels of the world interest rate.

4.3 Simulations

By allowing for risk aversion on the part of investors, the simulations presented here better replicate the following observed dynamics of sovereign yield spreads, and capital flows to emerging economies: i) sovereign risk premium is high during recessions, or when the economy is highly indebted; ii) default is observed when the fundamentals of the economy deteriorate, and iii) in periods previous to default the economy experiences capital outflows and collapses in consumption.

The main results that follow are derived under the assumption that the punishment for default is a permanent exclusion of the credit market. Therefore the asset distributions of the emerging economy and the investors are degenerate—i.e. as long as default can be a result of the time series, default will occur in finite time in which case both the economy

and the investor will remain in permanent autarky. This feature of the model makes the simulations (and therefore the business cycle statistics) sensitive to initial conditions. The alternate assumption of temporary exclusion was also considered using two punishments for default: i.) exclusion from the market with some probability of re-entry each period and ii.) the seizure of a proportion of the economy's endowment. Some of the results of the model were found to be sensitive to this exclusion assumption, such as the equilibrium debt-to-GDP ratio and the countercyclicality of the domestic interest rates. However, the main results of the model regarding the role of investors characteristics are robust to this modification—i.e. default probabilities are decreasing in investors' wealth and increasing in investors' risk aversion.³⁸ The results shown below, under permanent exclusion, assume for initial conditions that the economy begins with its mean income, and zero debt. The statistics shown below are the average for 100 simulations of 100 periods each (i.e. 25 years).

The simulations presented here show that considering risk averse lenders provides a better match to the risk premium of sovereign bond prices as well as to the level of borrowing by emerging economies. An empirical weakness of risk neutral models is that, in order to match the observed time series behavior of default events, those models need to use values for the emerging economy's discount rate which are too low—typical values required to match the time series behavior of default events in those models are in the range of 0.79 to 0.89. The model in the current paper is able to use a more standard value of 0.95. This larger discount rate allows this model to support higher levels of debt at equilibrium—which are closer to the observed levels. Furthermore, because the risk premium in the asset prices has to be large enough to compensate the investor not only for the probability of default, but also for taking the risk of default, the model simulated here is able to account for a larger proportion of credit spreads than models with a representative risk neutral investor.

The business cycle statistics for the benchmark model are given in Table 4. The first feature of the observed dynamics of sovereign yield spreads that the model reproduces is the counter-cyclical behavior of domestic interest rates. The numerical solution of the

³⁸Temporary exclusion modifies the equilibrium results as follows: First, other things equal, as the probability of re-entry after default increases, the equilibrium borrowing levels decrease. Second, allowing re-entry after default has a larger effect on equilibrium levels of borrowing when the emerging economy has a larger discount rate. When the discount rate of the economy is 0.89, borrowing hardly changes due to exclusion; when the discount rate is 0.95 borrowing changes dramatically. Third, for larger re-entry probabilities, in order to obtain counter-cyclical domestic interest rates, it is necessary to have a smaller first-period punishment. For example, suppose the economy loses 2% of its endowment after a default episode and that the probability of re-entry is greater than 5%. In this case, if the 2% loss is spread across periods, domestic interest rates are counter-cyclical. However, if the 2% loss is restricted only to the period of default, domestic interest rates are not counter-cyclical.

Table 4: Benchmark Model: Business Cycles Statistics

Business Cycles Statistics for the Benchmark Model ($\beta = 0.95$)				
	Mean	Std. Dev	Corr. Y	Corr.W
Interest Rates	0.0352	0.0111	-0.2692	-0.1334
Emerging Economy's Consumption	1.0398	0.0050	0.7146	0.4828
Investor's Consumption	0.0632	0.0081	0.0628	0.3274
Capital Account	0.0320	0.0006	-0.3893	0.4410
Mean. Default Probability (for $W > 0$) = 0.0172				
Max Debt Limit = -0.3684				
Max Borrowing = -0.1652				

benchmark model shows that for the case in which the endowment of the emerging economy follows a persistent Markov process the correlation between domestic interest rates and output is -0.2692 . This value of the correlation is lower than the observed values for emerging economies reported in Neumeyer and Perri (2004)³⁹ but much higher than the value of 0 obtained for an identical economy that trades financially with risk neutral investors. The correlation between domestic interest rates and output found here compares favorably with the existing literature which imposes risk neutral investors: Aguiar and Gopinath (2004) use a quarterly discount rate of 0.8 in a model of an emerging economy that trades financially with risk neutral investors; the economy faces both transitory income shocks and permanent shocks to the growth trend. The authors find a correlation between output and domestic interest rates of -0.11 . Arellano (2003) employs a two sector model of an emerging economy that trades financially with risk neutral investors. Using a quarterly discount rate of 0.89, this paper finds a correlation between output and domestic interest rates of -0.3562 . However, for the same model when considering only a one sector economy and the same discount rate of 0.89, default is not observed at equilibrium. In this case, the correlation of output and domestic interest rate is 0.

The model presented here also performs better than other models in explaining the average sovereign yield spread. The mean for the domestic interest rate is 3.52% which implies an average yield spread in sovereign bonds of 2.52%. The mean spread here corresponds to 34.4% of the mean average EMBI+ stripped yield spread of 7.33% observed for the group of developing countries formed by Argentina, Brazil, Colombia, Ecuador, Egypt, Malaysia, Mexico, Morocco, Nigeria, Panama, Peru, Philippines, South Africa, Turkey, Ukraine and Venezuela during the period 1994-2004. While Aguiar and Gopinath (2004) does not report

³⁹In Neumeyer and Perri (2004), correlations range from -0.38 for Brazil to -0.7 for Korea, with -0.55 for Argentina.

Table 5: Default Probabilities and Investors' Wealth

Default Probabilities as a function of Investors' Wealth									
Wealth	-3.76	-3.4	-3.01	-2.55	-2.04	-1.42	0.29	1.66	4
Def. Prob.	0.161	0.136	0.124	0.12	0.085	0.05	0.0172	0.0172	0.0172

the mean spread, the mean spread obtained here is higher than the 1.6% spread obtained in Arellano (2003).

Regarding the role of investors characteristics, the simulations suggest that the probability of default increases as the investor's wealth falls. For the range in which investors have an initial positive asset position (that is they are not strongly leveraged), the default rate is around 1.72%. This rate is equivalent to an annual default rate of 6.7% or around 11.725 defaults each 175 years. This default rate is higher than the default rate found elsewhere. For example, Reinhart, Rogoff and Savastano (2003) found an average of 5.2 defaults per defaulting country each 175 years. However, even though the results of the current model overestimate the default rate, it is important to highlight that the model exhibits default at equilibrium even when considering relatively high discount rates.

Additionally, according to the results of the benchmark calibration, the probability of default increases when investors' wealth falls. This result is illustrated in Table 5. This result implies that when investors are less wealthy, the maximum safe levels of debt are small enough such that when borrowing increases beyond the safe level and bond prices consequently fall, the potential losses are relatively small in comparison to the benefits.

With respect to levels of debt observed at equilibrium, this model performs better the analogous model of endogenous risk with risk neutral investors and only transitory income shocks; the model also performs better than the model of endogenous default risk with risk neutral agents and stochastic shocks to the growth trend: First, the maximum debt limit for this model corresponds to 36.84% of the GDP, which is larger than the 5% found by Arellano (2003), and the 20% found by Aguiar and Gopinath (2004). Second, the safe level of debts of the model are consistent with the study of Reinhart, Rogoff and Savastano (2003). These authors report that for the countries in the debtor's club (that is, countries which have defaulted in the past, and have in general lower tolerance toward debt), safe levels of debt can be as low as 15%. In the benchmark calibration of the model, safe levels of debt (for the case in which the lender's asset position is positive) are around 14.21% of the GDP. Above this level of debt the economy faces some default risk. As with previous models in the literature that do not consider stochastic shocks to the growth trend (i.e., Arellano (2003)), this model cannot match the positive correlation between output and the

capital account when considering relatively standard parameters for the volatility of the income process and discount rates. The intuition for this result is explained in detail in Aguiar and Gopinath (2004), where the output and capital account correlation *is* matched. Following Aguiar and Gopinath, the lack of a match in the current paper depends on the large steepness of the price function that results from considering only transitory income shocks. For the emerging economy, when only transitory income shocks are considered, the difference between the value of autarky and the value of maintaining the contract in the neighborhood of the indifference point (i.e., the point for which $V^C(s) = V^A(s)$) changes very little in response to income shocks⁴⁰. In the current paper, given the level of wealth of international investors, the decision of default is not too sensitive to the realization of income shocks but instead highly sensitive to the quantity borrowed. These interactions result in a very steep equilibrium price function.

Given the steep equilibrium price function, in order to have a counter-cyclical domestic interest rate with transitory income shocks, it must hold that the economy is borrowing more when income is low than when income is high. But more borrowing for low levels of income implies a counter-cyclical current account. This result is counterfactual. However, as in Aguiar and Gopinath (2004), it is reasonable to expect that this model might account for both a counter-cyclical domestic interest rate and a pro-cyclical capital account by considering an endowment process with a permanent component.

4.4 Sensitivity Analysis

The following exercise compares the business cycles statistics for four economies with a discount rate of 0.89. The first economy is an economy trading with risk averse investors who have a risk aversion parameter of 0.5. The second economy trades with investors having a risk aversion parameter of 1. The third economy trades with investors having a risk aversion parameter of 0.5, and additionally a higher world interest rate 1.225% (instead of 1%). Finally, the last economy is a two sector economy which trades with risk neutral

⁴⁰First, when the income process is highly persistent and a negative shock occurs, the persistence of the process ensures that the consumption path will be relatively smooth even in the absence of access to credit markets. Second, when the income process is i.i.d., a current negative shock does not imply a very large change in the expected lifetime utility of having access versus not having access to credit markets. Therefore as discussed by Aguiar and Gopinath (2004), at both extremes of the spectrum, the decision to default is not too sensitive to the particular realization of the transitory income shock. In this case, the level of outstanding debt is the main determinant of default.

Table 6: Sensitivity Analysis (Quarterly)

	BM	$\gamma_L = 1$	$r^f = 1.225\%$	Risk Neutral [†]
Mean (r)	0.0288	0.0694	0.0432	0.016
Std. Dev.(r)	0.0241	0.1367	0.13	0.0107
Corr.(r, y)	-0.4364	-0.5295	-0.4521	-0.3562
Corr.(r, W)	-0.0451	-0.0826	-0.093	0
Mean. (c)	1	1	1.001	n/a
Std. Dev. (c)	0.0844	0.0887	0.0833	0.0904
Corr. (c, y)	0.9708	0.9831	0.9805	0.9741
Corr. (c, W)	0.0737	0.1297	0.1182	0
Mean(c^L)	0.0565	0.0588	0.0564	n/a
Std.(c^L)	0.0122	0.0181	0.019	n/a
Corr.(c^L, y)	-0.1256	-0.3747	-0.314	n/a
Corr.(c^L, W)	0.6064	0.2378	0.3944	n/a
Mean Net Flows—($b - b'q$)	0	0	0	n/a
Std. Dev.—($b - b'q$)	0.0226	0.0148	0.0166	0.0044
Corr. ($-(b - b'q), y$)	-0.5018	-0.3384	-0.3748	-0.0832
Corr.($-(b - b'q), W$)	0.0218	0.0085	0.0203	n/a
Mean. Default Probability ($W > 0$)	0.111	0.389	0.3332	0.0012
Mean Max Debt Limit ($W > 0$)	-0.0816	-0.0796	-0.0784	-0.165
Benchmark model(BM) $\beta = 0.89, \gamma_L = 0.5, r^f = 1\%$				

[†] Source: Arellano (2003).

investors. This last economy is the economy studied in Arellano (2003)⁴¹.

Table 6 compares these four economies. This comparison suggests that the probability of observing default in the model increases when the assumption of risk neutrality of investors is relaxed. Specifically, for a discount rate of 0.89, while the model with risk neutral investors barely generates a positive probability of default, the model with risk averse investors produces (excessively) high default probability.

The comparison in this table also suggests that the probability of observing default increases when the world interest rate increases. This result suggests that an increase in the interest rate increases the cost of borrowing to a degree that strongly reduces the opportunities for the economy to smooth consumption by maintaining participation in credit markets. Indeed, the higher is the world interest rate, the higher is the correlation between consumption and income, and the lower is the correlation between consumption and wealth of the investors.

Finally this table shows that increasing the risk aversion of investors tends to increase

⁴¹There is one difference between the economy studied in Arellano (2003) and the economy studied here. When the economy studied in Arellano (2003) defaults, the economy is not excluded from credit markets permanently, but may reenter markets after some time.

the probability of observing default at equilibrium.⁴² Increasing risk aversion of investors also makes consumption and the domestic interest rate of the economy relatively more sensitive to changes in the wealth of international investors.

5 Conclusion

This paper presents a stochastic dynamic general equilibrium model of default risk that endogenizes the role of external factors in the determination of small open economies' incentives to default, sovereign bond prices, capital flows and default episodes.

The empirical literature on international finance presents evidence that points out to a very relevant role for investors' characteristics—risk aversion and wealth—in the determination of sovereign credit spreads and capital flows to emerging economies. The model in this paper is the first model with endogenous default risk that can account for these empirical findings. By relaxing the assumption of risk neutrality on the side of international investors and assuming that the preferences of these agents exhibit absolute decreasing risk aversion, this model generates a link between international investors' characteristics and emerging economies' sovereign credit markets.

Therefore, the contribution of the paper is twofold. First, the paper qualitatively and quantitatively characterizes the role of investors' characteristics in the determination of small open economies' optimal plans when international credit contracts cannot be enforced. Second, the paper presents a theoretical framework that can be extended to a multi-country setup to study endogenous financial links across countries with common investors. This extension can explain contagion in financial markets.⁴³

Regarding the role of investors' characteristics, the analytical results of this model establish that default risk increases with investors' risk aversion and decreases with investors' wealth. Investors' characteristics have the opposite effect on capital flows. Capital flows decrease with investors' risk aversion and increase with investors' wealth. As a consequence, credit limits are tighter when investors are more risk averse or less wealthy.

Quantitatively, in several dimensions the model developed here outperforms previous models of endogenous default risk. The model performs better at explaining sovereign yield

⁴²This result must be qualified, in the sense that the values considered in here for the risk parameters of the investors are relatively small—0.5 and 1. Since it is not clear that the relation between equilibrium default and risk aversion is monotonic, this result may not hold for other values of γ^L .

⁴³This extension of the model is analyzed in the second chapter of this dissertation.

spreads levels and equilibrium debt levels. And the model is able to replicate the counter-cyclical behavior of domestic interest rates. In general, in order to generate default at equilibrium, previous models of endogenous default risk require a very low time invariant discount rate on the part of the emerging economy. However, the present model can generate default at equilibrium with much higher values for the discount rate. As a consequence, in comparison with models with risk neutral investors, the present model supports higher levels of debt at equilibrium—a maximum of 37% of the GDP vis. 20% for Aguiar and Gopinath (2004), and 5% for Arellano (2003). Additionally, because risk averse investors require a risk premium in order to take default risk, the present model is able to explain a larger proportion of sovereign yield spreads than previous models in the literature—31.5% of sovereign spreads vis. a 20% of these spreads for Arellano (2003)).

With respect to the role of external factors in determining sovereign spreads and capital inflows to emerging economies, quantitative results of the model are consistent with the empirical evidence: First, this model exhibits the expected negative correlation between investors' wealth and sovereign spreads. Second, the correlation between investors' risk appetite (given by $-\frac{v_{c^L c^L}(c^L)}{v_{c^L}(c^L)} = \frac{\gamma^L}{c^L}$) and sovereign spreads has the expected negative sign.

While the model improves on explaining the behavior of prices and quantities with respect to models of the same type that don't consider investor's characteristics, the model is still far from perfectly matching the behavior of these variables. That is, the maximum level of debt supported at equilibrium is only around 37% of the GDP, which is much lower than the 70% average reached by countries at the verge of default reported in Reinhart, Rogoff, and Savastano (2003). The present model could also improve upon the relationship between the current account and output. Currently the model generates a positive correlation which is not consistent with the data. However, it is reasonable to expect that if permanent shocks to the endowment are considered (as in Aguiar and Gopinath (2004)), the model will be able to account for a positive correlation between current account and output.

The model presented here opens the door for creditworthiness of a country to be explained by factors additional to the country's own fundamentals. This more general framework can shed light on a multitude of policy questions: the optimal degree of diversification of international portfolios; the appropriateness of capital controls to exclude volatile short-term flows; the role of the IMF in preventing crises; the impact of term-structure on debt markets; and the transmission of crises from debt markets to equity markets. While these questions remain to be explored, a clear message emerges from the current analysis. The consideration of risk adverse lenders goes a long way toward explaining sovereign bond spreads and the behavior of borrowers and lenders in emerging markets.

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Appendix

Proposition 1 *For any state of the world s , the incentives to default that the emerging economy faces are stronger in a world with a more risk averse representative investor than in a world with a less risk averse representative lender*

Proof. The investor's value function can be written as

$$V^L = E \sum_{t=\tau}^{\infty} \beta^{t-\tau} v \left(X + \theta_t^{TB} - q^f \theta_{t+1}^{TB} + D_t [\theta_t - q_t \theta_{t+1}] \right).$$

Assuming an interior solution for the allocation to the emerging economy's asset

$$\phi(\theta') = ED_t \{ -qv_c(c_L(\theta')) + \beta v_c(c'_L(\theta')) d' \} = 0.$$

If the periodic utility of the international investor is of the CRRA type and $\gamma_1^L < \gamma_2^L$, then it exists a concave function $\psi(\cdot)$ such that $v_2(c; \gamma_2^L) = \psi(v_1(c; \gamma_2^L))$. If θ'_1 is the optimal allocation when $\gamma^L = \gamma_1^L$, and θ'_2 is the optimal allocation when $\gamma^L = \gamma_2^L$ then it holds that

$$\begin{aligned} \phi_1(\theta'_1) &= ED \{ -qv_{1,c}(c_L(\theta'_1)) + \beta v_{1,c}(c'_L(\theta'_1)) d' \} = 0. \\ \phi_2(\theta'_2) &= ED \{ -qv_{2,c}(c_L(\theta'_2)) + \beta v_{2,c}(c'_L(\theta'_2)) d' \} = 0. \end{aligned}$$

Using $v_2(c; \gamma_2^L) = \psi(v_1(c; \gamma_2^L))$ it is possible to define

$$\phi_2(\theta'_1) = ED \psi' [v_1(\theta'_1)] \{ -qv_{1,c}(c_L(\theta'_1)) + \beta v_{1,c}(c'_L(\theta'_1)) d' \} < 0.$$

The last inequality comes from the fact that $\psi'(\cdot)$ is positive and decreasing. The inclusion of this function in the previous equation implies that $\phi_2(\theta'_1)$ is lower than $\phi_2(\theta'_2)$ because $\psi'(\cdot)$ gives little weight to the realizations of $d' = 1$, and high weight to the realizations of $d' = 0$. Therefore

$$\phi_2(\theta'_2) > \phi_2(\theta'_1).$$

The concavity of $V^L(\cdot)$ implies that given q and the risk of default (represented by the expected realizations of d') $\phi(\theta')$ is a decreasing function, and as consequence

$$\theta'_2 < \theta'_1$$

which in equilibrium implies $b'_2 < b'_1$.

Then for any state of the world s and taking as given q and the risk of default (δ), a higher degree of risk aversion of the investor would result in this agent allocating a lower

proportion of her portfolio to the economy's sovereign bonds. Therefore, when the investor is less risk averse there are financial contracts that are available to the emerging economy that are not when the investor is more risk averse. Consequently given q and δ

$$V_1^C(s; \gamma_1^L) \geq V_2^C(s; \gamma_2^L)$$

Because the utility of autarky for the emerging economy does not depend on the investor's risk aversion, it is clear that if for some state of the world s default is optimal if $\gamma^L = \gamma_1^L$, then for the same state of the world default would be optimal if $\gamma^L = \gamma_2^L$. Additionally, because incentives to default would be higher whenever $\gamma^L = \gamma_2^L$ than if $\gamma^L = \gamma_1^L$ this implies that at equilibrium $\delta(s, b'; \gamma_2^L) > \delta(s, b'; \gamma_1^L)$, and therefore $q(s, b'; \gamma_2^L) < q(s, b'; \gamma_1^L)$. Then, unambiguously for all states of the world the emerging economy faces stronger incentives to default the more risk averse is the investor. ■

Proposition 2 *Default sets are shrinking in assets of the representative investor. For all $W_1 < W_2$, if default is optimal for b in some states y , given W_2 then default will be optimal for b for the same states y , given W_1 therefore $D(b | W_2) \subseteq D(b | W_1)$*

Proof. Proof: From (??) if $W_1 < W_2$ then for any given q and taking as given the level of default risk

$$b'_2 < b'_1.$$

This inequality holds because decreasing absolute risk aversion implies that $v(X + W_1 - q^f \theta_{t+1}^{TB} - D_t q_t \theta_{t+1})$ is a concave transformation of $v(X + W_2 - q^f \theta_{t+1}^{TB} - D_t q_t \theta_{t+1})$ (see Proposition 6.C.3 of Mas-Collel and Whinston), so if θ'_1 is the optimal allocation when $W = W_1$, and θ'_2 is the optimal allocation when $W = W_2$, and defining $v_1(\theta_{1,t+1}) = v(X + W_1 - q^f \theta_{t+1}^{TB} - D_t q_t \theta_{1,t+1})$, and $v_2(\theta_{2,t+1}) = v(X + W_2 - q^f \theta_{t+1}^{TB} - D_t q_t \theta_{2,t+1})$ then

$$\begin{aligned} \phi_1(\theta'_1) &= ED \{ -qv_{1,c}(c_L(\theta'_1)) + \beta v_{1,c}(c'_L(\theta'_1)) d' \} = 0. \\ \phi_2(\theta'_2) &= ED \{ -qv_{2,c}(c_L(\theta'_2)) + \beta v_{2,c}(c'_L(\theta'_2)) d' \} = 0. \end{aligned}$$

and because $v_1(\theta_{t+1}) = \psi(v_2(\theta_{t+1}))$

$$\phi_1(\theta'_2) = ED \psi' [v_2(\theta'_2)] \{ -qv_{2,c}(c_L(\theta'_2)) + \beta v_{2,c}(c'_L(\theta'_2)) d' \} < 0.$$

The previous inequality comes from the fact that $\psi'(\cdot)$ is positive and decreasing. The inclusion of this function in the previous equation implies that $\phi_1(\theta'_2)$ is lower than $\phi_1(\theta'_1)$

because $\psi'(\cdot)$ gives little weight to the realizations of $d' = 1$, and high weight to the realizations of $d' = 0$. Therefore

$$\phi_1(\theta'_2) < \phi_1(\theta'_1).$$

The concavity of $V^L(\cdot)$ implies that given q and the risk of default (represented by the expected realizations of d') $\phi(\theta')$ is a decreasing function, and as consequence

$$\theta'_2 > \theta'_1$$

which in equilibrium implies $b'_2 < b'_1$.

Because the representative agent of the emerging economy is able to observe the optimal response function of the investors, when W increases this agent modifies her actions to get the best available contract under this state of the world which is given by

$$\{q(s_2; b'(s_2)), b'(s_2)\}$$

(The representative agent of the emerging economy chooses $b'(s_2)$ knowing that the collective action of the investors implies that the equilibrium price of the sovereign bonds is $q(s_2; b'(s_2))$). Then for any given level of bond prices the emerging economy is able to borrow at least as much when the wealth of the investors is W_2 , as it would be able when the wealth of those investors is W_1 . Because the representative agent of the emerging economy chooses $\{q(s_2; b'(s_2)), b'(s_2)\}$ even when the financial contract that is optimal when the state of the world is s_1 (that is, $\{q(s_1; b'(s_1)), b'(s_1)\}$) is available to the emerging economy when the state of the world is s_2 , it is clear that because the representative agent is maximizing

$$V^c(s_2) > V^c(s_1)$$

Given that when the state of the world is s_2 default is the optimal choice, it must hold

$$V^A(y) > V^c(s_2) > V^c(s_1)$$

which implies that if default is optimal for b in some states y , given W_2 , then default is optimal for the same states given W_1 . ■

Proposition 3 *Default sets are shrinking in emerging economy's assets. For all $b_1 < b_2$, if default is optimal for b_2 in some states y , given θ^{TB} then default will be optimal for b_1 for the same states y , given W therefore $D(b_2 | W) \subseteq D(b_1 | W)$.*

Proof. Proof: If $b_1 < b_2$ then

$$\begin{aligned}
& u(y + b_2 - q(s_2; b'(s_2)) b'(s_2)) + \beta E \left[V(b'(s_2), y', W'(s_2)) \right] \\
& > u(y + b_2 - q(\tilde{s}_1; b'(\tilde{s}_1)) b'(\tilde{s}_1)) + \beta E \left[V(b'(\tilde{s}_1), y', W'(\tilde{s}_1)) \right] \\
& > u(y + b_1 - q(\tilde{s}_1; b'(\tilde{s}_1)) b'(\tilde{s}_1)) + \beta E \left[V(b'(\tilde{s}_1), y', W'(\tilde{s}_1)) \right] > \\
& > u(y + b_1 - q(s_1; b'(s_1)) b'(s_1)) + \beta E \left[V(b'(s_1), y', W'(s_1)) \right]
\end{aligned}$$

where $s_1 = (b_1, W, y)$, $s_2 = (b_2, W, y)$, and $\tilde{s}_1 = (b_1, \tilde{W}, y)$, where $\tilde{W} = \theta^{TB} + (-b_1) > \theta^{TB} + (-b_2) = W$.

The first inequality holds because given b_2 the economy could choose to borrow $\tilde{b}'(s_2) = b'(\tilde{s}_1)$, to obtain a bond price $\tilde{q}(s_2; b'(\tilde{s}_1))$, such that $\tilde{q}(s_2; b'(\tilde{s}_1)) = q(s_1; b'(\tilde{s}_1))$ but chooses not to. That is, the emerging economy always could choose to transfer $T = b_2 - b_1$ to the investors, so that the economy cash flow would be $y + b_1$ instead of $y + b_2$, and the investors' wealth would be \tilde{W} instead of W . The third inequality holds as a consequence of the result in proposition ??.

Because for b_2 default is optimal, it must hold that

$$V^A(y) > V^C(s_2) > V^C(s_1)$$

which implies that for b_1 default is also optimal in the same states y . ■

Proposition 4 *If the endowment process is i.i.d., default incentives are stronger the lower the endowment. For all $y_1 < y_2$ if $y_2 \in D(b | W)$ then $y_1 \in D(b | W)$.*

Proof. Because $y_2 \in D(b | W)$ then $V^A(y_2) \geq V^C(s_2)$, where

$$V^C(s_2) = u(y_2 + b - q(s_2; b'(s_2)) b'(s_2)) + \beta E \left[V(b'(s_2), y', W'(s_2)) \right]$$

A sufficient condition to have $y_1 \in D(b | W)$ is to have

$$0 \geq V^C(s_2) - V^A(y_2) > V^C(s_1) - V^A(y_1) \quad (\text{A-1})$$

In state s_2 the representative agent of the emerging economy can choose to destroy a part $T = y_2 - y_1$ of her endowment. If that were the case the state of the world would be $\tilde{s}(b, y_2 - T, W)$ which is simply s_1 , therefore in this case the economy would optimally enter the contract $\{b'(\tilde{s}), q(\tilde{s}; b'(\tilde{s}))\}$ which corresponds to $\{b'(s_1), q(s_1; b'(s_1))\}$. Therefore the

contract $\{b'(s_1), q(s_1; b'(s_1))\}$ is in the emerging economy's possibility set when the state of the world is given by s_2 . Utility maximization implies

$$V^C(s_2) \geq u(y_2 + b - q(s_1; b'(s_1))b'(s_1)) + \beta E \left[V(b'(s_1), y', W'(s_1)) \right]$$

If

$$\begin{aligned} u(y_2 + b - q(s_1; b'(s_1))b'(s_1)) + \beta E \left[V(b'(s_1), y', W'(s_1)) \right] &= V^C(s_1) \\ &> V^A(y_2) - V^A(y_1) \end{aligned} \quad (\text{A-2})$$

by transitivity (A-1) holds. For the *i.i.d.* case (A-2) holds if, and only if

$$u(y_2 + b - q(s_1; b'(s_1))b'(s_1)) - u(y_1 + b - q(s_1; b'(s_1))b'(s_1)) > V^A(y_2) - V^A(y_1) \quad (\text{A-3})$$

Because the contract $\{b'(s_1), q(s_1; b'(s_1))\}$ belongs to the possibility set for the emerging economy when the state of the world is s_2 , and since for this state of the world the economy finds it optimal to default (i.e., $y_2 \in D(b | W)$), it must hold $b - q(s_1; b'(s_1))b'(s_1) < 0$. Given that $u(\cdot)$ is increasing in y , and strictly concave (A-3) holds, and therefore $y_1 \in D(b | W)$. ■

Proposition 5 *Given $\bar{b}(W'(s))$, where $\bar{b}(W'(s))$ is –as defined before– the maximum level of borrowing of the emerging economy for which default risk is zero given the current state of the world s , default at equilibrium is a possible outcome of the time series of this model if for $b' = \bar{b}(W'(s))$*

$$\frac{\partial c}{\partial \bar{b}(W'(s))} = -\frac{\partial (q\bar{b}(W'(s)))}{\partial \bar{b}(W'(s))} < 0$$

In other words, default can be an equilibrium outcome if for $b' = \bar{b}(W'(s))$, it holds $\frac{\partial (q\bar{b}(W'(s)))}{\partial \bar{b}(W'(s))} > 0$, so that by increasing its borrowing, the emerging economy is able to increase its consumption.

Proof. For default to be a possibility it must be true that the economy finds it optimal to borrow beyond the maximum safe level of debt, so that for some $b'(W'(s)) < \bar{b}(W'(s))$ such that $\delta(y, b', W'(s)) > \delta(y, \bar{b}, W'(s)) = 0$

$$\begin{aligned} u(y + b - q(s; b'(W'(s)))b'(W'(s))) + \beta E \left[V(b(W'(s))', y', W'(s)) \right] \\ > u(y + b - q(s; \bar{b}(W'(s)))\bar{b}(W'(s))) + \beta E \left[V(\bar{b}(W'(s)), y', W'(s)) \right]. \end{aligned} \quad (\text{A-4})$$

But by Proposition 3

$$\beta E \left[V(b(W'(s))', y', W'(s)) \right] < \beta E \left[V(\bar{b}(W'(s)), y', W'(s)) \right].$$

Therefore, in order for (A-4) to hold, it must be true that

$$u(y + b - q(s; b'(W'(s)))b'(W'(s))) > u(y + b - q(s; \bar{b}(W'(s)))\bar{b}(W'(s))) \quad (\text{A-5})$$

which implies that

$$\frac{\partial c}{\partial \bar{b}(W'(s))} = -\frac{\partial (q\bar{b}(W'(s)))}{\partial \bar{b}(W'(s))} < 0.$$

■