

Interest Rates and Currency Prices in a Two-Country Strategic Market Game Model

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Abstract

We present a general equilibrium strategic market game model of an two-country, two-period economy with no uncertainty. In this model, the uncovered interest rate parity, the absolute and the relative purchasing power parity and the Fischer effect may fail at equilibrium, because agents behave strategically and manipulate prices. This result is exclusively due to the finite number of traders in the economy and not from any type of market friction. An international investor may generate free but bounded profit by arbitraging interest rates without equalizing returns across countries.

PRELIMINARY AND INCOMPLETE DRAFT

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1 Introduction

The foreign exchange market is the largest market in terms of volume of trade. Thus it is expected that exchange rates reflect the fundamentals of the economies and that currency markets perform efficiently. Efficiency however is assured when there are no obstacles for speculative trade or arbitrage. In an absolutely frictionless global economy with unrestricted arbitrage activity, the theory of international trade and finance rely on three fundamental conditions to explain the pricing of commodities, currencies and credit, namely the purchasing power parity (PPP), the uncovered interest rate parity (UIRP) and the Fisher effect. When these conditions are not verified by the data, then their failure is an indicator that markets do not behave efficiently and present anomalies. There has been a vast literature and a long debate among economists on the issue of identification of the reasons of failure of these conditions¹. Letting aside econometric issues that deal with the measurement of deviations, various types of market frictions like asymmetric information, restricted participation or transaction costs are expected to be responsible for the failure of PPP, UIRP or the Fischer effect. The purpose of this work is to stress another possible reason for such anomalies, namely imperfect competition.

We model imperfect competition by using a modified version of the Shapley and Shubik [10] strategic market game model which can be found in Postlewaite and Schmeidler [8]². In that model Koutsougeras [5] has shown that the law of one price may fail. Inspired from that work, Papadopoulos [7] has recently shown the possibility of failure of purchasing power parity and exchange rate consistency in an international one period multi-country, multi-commodity economy. In the present model we want to focus on the possible failure of the UIRP and the Fischer effect³. To that end, we construct a two-country, two-period model with a unique commodity available in both national commodity markets and two credit markets one for each currency. We derive the absolute PPP, the relative PPP, the UIRP and the Fisher effect for this imperfectly competitive economy⁴. These conditions turn out to be not necessarily identical to the standard parity relations in a perfectly competitive economy. We then identify individual equilibrium strategies that are not compatible with the validity of PPP, UIRP and the Fisher effect. We show that when the UIRP fails, arbitrage opportunities may provide a free but bounded profit for an agent at equilibrium. The fact that equilibrium profit is bounded is crucial, for if it was unbounded, it would generate infinite profit, hence it could have not been compatible with equilibrium.

The intuition behind the failure of UIRP is the following. Suppose that an agent observes that the interest rate on the new Turkish lira is 20%, the

¹see for instance Goldberd and Knetter [4], Rogoff [9], Froot and Thaler [1].

²See Giraud [3] for an introduction to the literature.

³Froot and Thaler [1] give a lot of importance to the UIRP condition as a test of the efficiency of the currency market (the forward discount bias test).

⁴For the derivation of these conditions in competitive general equilibrium models consult Lucas [6] and Geanakoplos and Tsomocos [2].

interest rate on bulgarian leva is 17,5% and the exchange rate is 0,7 lira/leva today and 0,9 lira/leva tomorrow. Such situation could have never been an equilibrium in a competitive economy - where agents regard interest rates and exchange rates as fixed - since the return on the turkish lira when invested on the bulgarian leva is 22,5% whereas when it is invested on the turkish lira is 20%. An agent could borrow an infinite amount in turkish lira and lend it in bulgarian leva, thus making infinite profit. In our model, such situation could be sustained as equilibrium but with a bounded or even zero profit. If we accept that currency markets are imperfectly competitive, then the trades of agents have non-negligible effects on prices. When the apperent arbitrage opportunity shows up agents are aware that, in their attempt to exploit it, prices will change. In particular, by borrowing in turkish lira the price of lira goes up today but will fall tomorrow and the interest rate on turkish lira will rise. On the other hand, by lending in bulgarian leva the price of leva goes down today but will rise tomorrow and the interest rate on bulgarian leva falls. These price effects may be so strong in imperfectly competitive markets, that will make the apparent 2,5% free rate disappear⁵.

2 The Model

2.1 An Overview

We shall consider the simplest possible general equilibrium model of an international economy with two countries n and k . Each country has its own currency denoted by the letters n, k respectively. Time extends over two periods, $t = 0, 1$. A unique consumption good is available in both countries in both time periods. Let I be the set of agents in the economy consisting of both countries, indexed by $i \in I$. The set I is finite.

In each country there exist a market for the unique commodity and a credit market. The national commodity or credit market accepts and pays only in domestic currency. There also exists one centralised currency market common to both countries. So in the overall economy there exist two national markets for the unique commodity, two national credit markets and an international foreign exchange market.

Agents engage in trade because they desire to transfer consumption across time periods. This is achieved via the credit markets where agents buy or sell promises for the delivery of next period currency. Unbounded short sales are allowed in the credit market.

Currency is purely inside money and has no intrinsic value, however it is desirable due to cash-in-advance constraints and serves as store of value when invested. Agents need currency to buy the unique commodity from either or both of the two countries in each period. Since promises must be denominated

⁵The possibility of equilibria in strategic market games at which apparent arbitrage opportunities exist, but disappear as soon as agents try to exploit them, were first demonstrated in Koutsougeras [5].

in certain currency, they also need currency to buy credit in period 0 and honor their promises in period 1. For example, currency n (or k) may be obtained by monetizing part or all of their endowment in the n (resp. k) commodity market, or by selling k for n (resp. n for k) currency in the foreign exchange market or by short selling n (resp. k) currency in the credit market. Although short selling is unbounded agents are required to honor their promises, i.e. satisfy their budget constraints.⁶

2.2 The Commodity Markets

In period t , an agent chooses how much of his physical endowment to send for sale in the trading post of country n or k . Let q_{nt}^i (q_{kt}^i) be the offer of individual $i \in I$ of the unique commodity to the trading post of country n (resp. k) in period 0. In exchange to his offer, an agent receives $q_{nt}^i p_{nt}$ ($q_{kt}^i p_{kt}$) units of n (resp k) currency, where p_{nt} (p_{kt}) is price of the commodity in period t in country n (resp. k). Commodity offers are "put on the table" hence they cannot exceed endowments

$$q_{nt}^i + q_{kt}^i \leq e_t^i \text{ for } t = 0, 1.$$

An individual $i \in I$, may purchase the unique commodity either from country k or n or both. Each national commodity market accepts only domestic currency, hence bids must be denominated in the appropriate currency. Let b_{nt}^i (b_{kt}^i) be the bid of individual $i \in I$, in n (resp. k) currency, to the trading post of country n (resp. k) in period t . In exchange to his bid, an agent receives b_{nt}^i/p_{nt} (resp. b_{kt}^i/p_{kt}) units of the commodity.

Price formation is according to the following mechanism. Trading posts collect individual signals from the agents in the economy. These signals are buy and sell orders for quantities of commodities, currencies or credit. Signals are then aggregated to express total demand and supply. The market price is such that demand is equal to supply, and is by definition market clearing. Due to the finite number of agents in the economy, market prices of commodities, as well as exchange rates and interest rates, will not be taken as fixed by agents. Agents are aware of the price formation process and take into account the effect of their strategies on prices, thus they know that bidding more for a commodity or offering less increases the price of the commodity.

Given $(b_{nt}^i, b_{kt}^i, q_{nt}^i, q_{kt}^i)_{i \in I}$, the price of the commodity in country n at time t is given according to the following rule:

$$p_{nt} = \frac{\sum_{i \in I} b_{nt}^i}{\sum_{i \in I} q_{nt}^i}, t = 0, 1. \quad (1)$$

The price of the commodity in country k is

$$p_{kt} = \frac{\sum_{i \in I} b_{kt}^i}{\sum_{i \in I} q_{kt}^i}, t = 0, 1. \quad (2)$$

⁶It is beyond the scope of this work to deal with equilibria with default, the interested reader may consult , Geanakoplos Dubey [],Geanakoplos Tsomocos [].

We may then define the rate of inflation, for country n , as

$$\pi_n = \frac{p_{n1} - p_{n0}}{p_{n0}} \quad (3)$$

and for country k as

$$\pi_k = \frac{p_{k1} - p_{k0}}{p_{k0}}. \quad (4)$$

2.3 The Currency Market

A centralised currency market is available in both time periods. Let c_{knt}^i be the offer of k currency in exchange for n currency or equivalently the bid for n currency in period t . Similarly c_{nkt}^i is the bid for k currency or the offer of n currency. Given $(c_{knt}^i, c_{nkt}^i)_{i \in I}$, the k/n exchange rate is endogenously determined by

$$\varepsilon_{nkt} = \frac{\sum_{i \in I} c_{knt}^i}{\sum_{i \in I} c_{nkt}^i} \equiv \frac{1}{\varepsilon_{knt}}. \quad (5)$$

In exchange to his bid c_{knt}^i (c_{nkt}^i), an individual receives $c_{knt}^i \varepsilon_{knt}$ (resp. $c_{nkt}^i \varepsilon_{nkt}$) units of country's n (resp. k) currency. Exchange rates are not regarded as fixed by the agents.

Given ε_{nk0} and ε_{nk1} we may define

$$E_n = \frac{\varepsilon_{nk1} - \varepsilon_{nk0}}{\varepsilon_{nk0}} \quad (6)$$

as the rate of change of currency n and

$$E_k = \frac{\varepsilon_{kn1} - \varepsilon_{kn0}}{\varepsilon_{kn0}} \quad (7)$$

as the rate of change of currency k . Of course

$$1 + E_n = \frac{1}{1 + E_k} \quad (8)$$

2.4 The Credit Market

Currency can be carried over from one time period to another through the credit market. Agents may deposit k or n currency at $t = 0$, and receive interest payments at $t = 1$. They may also borrow any unlimited amount of currency at $t = 0$ provided they pay back their debts at $t = 1$.⁷

Let d_{n0}^i be the bid (deposit) of agent i , a quantity of currency n , sent to the market at $t = 0$ for the purchase of next's period n currency. In exchange to his bid the agent receives $(1 + r_n)d_{n0}^i$ units of n currency at $t = 1$.

⁷We may say that there exist two national banks accessible by all agents which however accept deposits only in national currency or equivalently that there exists a unique international bank that offers separate deposit accounts in multiple currencies.

Let d_{n1}^i be the amount of currency n that an agent promises at $t = 0$ to deliver at $t = 1$. In exchange to his promise the agent receives $d_{n1}^i/(1 + r_n)$ at $t = 0$. Alternatively the agent is borrowing $d_{n1}^i/(1 + r_n)$ units of currency n at $t = 0$ and pays back d_{n1}^i at $t = 1$.⁸ An agent is a net borrower if $d_{n1}^i/(1 + r_n) > d_{n0}^i$.

Given a profile of strategies in the credit market $(d_{n0}^i, d_{n1}^i, d_{k0}^i, d_{k1}^i)_{i \in I}$, the interest rate on currency n is endogenously determined by

$$1 + r_n = \frac{\sum_{i \in I} d_{n1}^i}{\sum_{i \in I} d_{n0}^i} \quad (9)$$

2.5 Agents

Let $e^i = (e_0^i, e_1^i) \in \mathfrak{R}_{++}^2$, be the endowment of the unique consumption good of individual i in period 0 and 1. A consumption bundle is $x = (x_0, x_1) \in \mathfrak{R}_+^2$. Preferences are represented by a utility function over consumption bundles $u^i(x)$, $u^i : \mathfrak{R}_+^2 \rightarrow \mathfrak{R}$. We assume that the utility function is twice continuously differentiable, strictly concave and that the indifference curves passing through the endowment do not intersect the axes. The overall economy is defined as $\mathcal{E} = \{(\mathfrak{R}_+^2, u^i, e^i) : i \in I\}$.

A strategy s^i for agent i consists of a list of actions in the currency market $(c_{nk0}^i, c_{kn0}^i, c_{nk1}^i, c_{kn1}^i)$, the commodity markets $(b_{n0}^i, b_{k0}^i, q_{n0}^i, q_{k0}^i, b_{n1}^i, b_{k1}^i, q_{n1}^i, q_{k1}^i)$ and the credit market $(d_{n0}^i, d_{k0}^i, d_{n1}^i, d_{k1}^i)$, that is

$$s^i = (c_{nk0}^i, c_{kn0}^i, c_{nk1}^i, c_{kn1}^i; b_{n0}^i, b_{k0}^i, q_{n0}^i, q_{k0}^i, b_{n1}^i, b_{k1}^i, q_{n1}^i, q_{k1}^i; d_{n0}^i, d_{k0}^i, d_{n1}^i, d_{k1}^i).$$

The strategy set of agent $i \in I$ is then given by

$$S^i = \{s^i \in \mathfrak{R}_+^{16} : q_{nt}^i + q_{kt}^i \leq e_t^i \text{ for } t = 0, 1\}$$

Each consumer faces two monetary budget constraints per period, one for each currency, so for $t = 0$,

$$b_{n0}^i + c_{nk0}^i + d_{n0}^i \leq q_{n0}^i p_{n0} + c_{kn0}^i \varepsilon_{kn0} + \frac{d_{n1}^i}{1 + r_n}, \quad (10)$$

$$b_{k0}^i + c_{kn0}^i + d_{k0}^i \leq q_{k0}^i p_{k0} + c_{nk0}^i \varepsilon_{nk0} + \frac{d_{k1}^i}{1 + r_k} \quad (11)$$

and for $t = 1$,

$$b_{n1}^i + c_{nk1}^i + d_{n1}^i \leq q_{n1}^i p_{n1} + c_{kn1}^i \varepsilon_{kn1} + d_{n0}^i (1 + r_n), \quad (12)$$

$$b_{k1}^i + c_{kn1}^i + d_{k1}^i \leq q_{k1}^i p_{k1} + c_{nk1}^i \varepsilon_{nk1} + d_{k0}^i (1 + r_k). \quad (13)$$

⁸ Alternatively, these loans may be viewed as zero coupon bonds that promise to deliver 1 unit of either k or n currency $t = 1$. Each agent is allowed to issue as many bonds as he likes at $t = 0$, however he is not allowed to go bankrupt at $t = 1$. An agent is purchasing $(1 + r_n)d_{n0}^i$ units of bond for d_{n0}^i at $t = 0$ and receives $(1 + r_n)d_{n0}^i$ next period. Equivalently the agent is issuing an amount of d_{n1}^i zero coupon bonds, receives $d_{n1}^i/(1 + r_n)$ today and pays d_{n1}^i tomorrow.

The right hand side of these inequalities represent currency receipts and the left hand side currency expenditures. In the right hand side of (10) we have the money receipts from sale of commodity, $q_{n0}^i p_{n0}$, the proceeds of the sale of k currency, $c_{kn0}^i \varepsilon_{n0}^k$, and the amount of n currency borrowed, $d_{n1}^i / (1 + r_n)$. In the left hand side, these receipts are used to finance his bids for the commodity in the n country, b_{n0}^i , or the purchase of k currency, c_{nk0}^i , or they may be deposited, d_{n0}^i . Similarly we read (11). Once we are at $t = 1$, the agent receives interest payment on his 0 period deposit, $d_{n0}^i (1 + r_n)$, and has to pay back his loan, d_{n1}^i . Both commodity and currency markets reopen and agents may exchange commodity and currency as in period 0. An agent is bankrupt if in some period he spends more currency than that he collects.

The allocation rule is the following. Take a strategy profile $\{s^i \in S^i : i \in I\}$. Individual's i consumption of the unique commodity for $t = 0, 1$ is given by

$$x_t^i = \begin{cases} e_t^i - q_{nt}^i - q_{kt}^i + \frac{b_{nt}^i}{p_{nt}} + \frac{b_{kt}^i}{p_{kt}}, & \text{if (10) to (13) are satisfied} \\ e_t^i - q_{nt}^i - q_{kt}^i & \text{otherwise.} \end{cases} \quad (14)$$

We use capital letters to denote aggregates. Given a strategy profile $\{s^i \in S^i : i \in I\}$, define

$$Q_{nt} = \sum_{i \in I} q_{nt}^i, B_{nt} = \sum_{i \in I} b_{nt}^i, C_{nkt} = \sum_{i \in I} c_{nkt}^i, D_{nt} = \sum_{i \in I} d_{nt}^i,$$

where Q_{nt} is the aggregate commodity offer in country's n trading post at time t , B_{nt} is the aggregate bid in country's n commodity trading post at time t , C_{nkt} is the total quantity of currency n sent in exchange for currency k , $D_{n0} = \sum_{i \in I} d_{n0}^i$ is the total amount of currency n bid at $t = 0$ for the purchase of next's period n currency (or simply deposited at $t = 0$) and $D_{n1} = \sum_{i \in I} d_{n1}^i$ the total amount of currency n promised at $t = 0$ for delivery at $t = 1$. In an analogous manner we may define the k aggregates $Q_{kt}, B_{kt}, C_{knt}, D_{kt}$.

Let $B_{nt}^{-i} = B_{nt} - b_{nt}^i$ denote all the bids sent for the unique commodity in country's n trading post except that of player i . The superscript $-i$ means "except player i ". Similarly we define $Q_{nt}^{-i}, C_{nkt}^{-i}, D_{nt}^{-i}, Q_{kt}^{-i}, B_{kt}^{-i}, C_{knt}^{-i}, D_{kt}^{-i}$.

The problem of agent i is then

$$\begin{aligned} \max_{s^i \in S^i} & u^i(x^i(s^i)); (B_{nt}^{-i}, Q_{nt}^{-i}, C_{nkt}^{-i}, D_{nt}^{-i}, B_{kt}^{-i}, Q_{kt}^{-i}, C_{knt}^{-i}, D_{kt}^{-i})_{t \in T} \quad (15) \\ & \text{subject to (10) to (13)} \end{aligned}$$

In period 0 agents choose their strategies once for both periods, taking as given the strategies of all the players for all periods for all markets.

The market game of this economy Γ , consists of a set of players I , their strategy sets S^i , the outcomes x^i , and the payoffs $u^i(x^i)$. A Nash equilibrium (NE) for Γ is a profile $\{s^i \in S^i : i \in I\}$ such that $(s^i) \in \arg \max u(x^i)$ and (10) to (13) are satisfied with equality for every $i \in I$.

Proposition 1 *At a Nash equilibrium of the game commodity, currency and credit markets clear in all periods.*

Proof. Simply sum over all $i \in I$ the budget constraints (10) to (13) which are satisfied with equality at a Nash equilibrium. ■

3 Equilibrium Prices, Exchange Rates and Interest Rates

In this section we derive the three fundamental relations in international economics, namely the uncovered interest rate parity (UIRP), the absolute (APPP) and relative purchasing power parity (RPPP) and the Fischer effect (FE).

3.1 The Uncovered Interest Rate Parity

Proposition 2 *At a Nash equilibrium of the game Γ the appreciation (or depreciation) rate of currency n is given by*

$$(1 + E_n)^2 = \left(\frac{1 + r_k}{1 + r_n} \right)^2 W_1 \quad (16)$$

where

$$W_1 = \frac{D_{n1}^{-i} D_{k0}^{-i} C_{nk0}^{-i} C_{kn1}^{-i}}{D_{n0}^{-i} D_{k1}^{-i} C_{kn0}^{-i} C_{nk1}^{-i}}.$$

Proof. In the appendix. ■

3.2 The Absolute and Relative Purchasing Power Parity

Proposition 3 (Absolute PPP) *At a Nash equilibrium of the game Γ the prices of the unique commodity between country k and n within the same time period are related according to the following condition*

$$\left(\frac{p_{nt}}{p_{kt}} \right)^2 = (\varepsilon_{knt})^2 W_2 \quad (17)$$

where

$$W_2 = \frac{C_{knt}^{-i} B_{nt}^{-i} Q_{kt}^{-i}}{C_{nkt}^{-i} Q_{nt}^{-i} B_{kt}^{-i}}.$$

Proof. In the appendix. ■

Proposition 4 (Relative PPP) *At a Nash equilibrium of the game Γ inflation and the rate of change of the exchange rate between country k and n are related according to the following condition*

$$\left(\frac{1 + \pi_k}{1 + \pi_n} \right)^2 = (1 + E_n)^2 W_3 \quad (18)$$

where

$$W_3 = \frac{C_{kn0}^{-i} B_{n0}^{-i} Q_{k0}^{-i} C_{nk1}^{-i} Q_{n1}^{-i} B_{k1}^{-i}}{C_{nk0}^{-i} Q_{n0}^{-i} B_{k0}^{-i} C_{kn1}^{-i} B_{n1}^{-i} Q_{k1}^{-i}}.$$

Proof. In the appendix. ■

3.3 The Fischer Effect

Proposition 5 *At a Nash equilibrium of the game Γ the interest rates and inflation rates between country k and n are related according to the following condition*

$$\left(\frac{1 + \pi_k}{1 + \pi_n}\right)^2 = \left(\frac{1 + r_k}{1 + r_n}\right)^2 W_4 \quad (19)$$

where

$$W_4 = \frac{C_{kn0}^{-i} B_{n0}^{-i} Q_{k0}^{-i} C_{nk1}^{-i} Q_{n1}^{-i} B_{k1}^{-i}}{C_{nk0}^{-i} Q_{n0}^{-i} B_{k0}^{-i} C_{kn1}^{-i} B_{n1}^{-i} Q_{k1}^{-i}}.$$

Proof. In the appendix. ■

Notice that at a Nash equilibrium the W terms are the same for all agents. It is evident that if $W_1 = W_2 = W_3 = W_4 = 1$, then (16),(17),(18) and (19) reduce to the standard equilibrium conditions of a competitive economy.

4 Characterization of Equilibria

It is well known that the autarkic equilibrium where all markets are closed is always a Nash equilibrium of the game Γ . Here we will be interested only at interior Nash equilibria, that is equilibria where all markets are endogenously open. A market is open when there exist at least one bid and offer sent to that market.

On the other hand it is possible that at a Nash equilibrium $W_j \neq 1, j = 1, 2, 3, 4$. In that case, the uncovered interest rate parity, the absolute and relative PPP and the Fischer effect fail altogether at equilibrium allowing for arbitrage opportunities. In fact, it is possible to identify budget feasible strategy configurations that generate positive profit that require no initial wealth⁹.

We will present strategy profiles in terms of individual net trades.

The net trade of an agent in any market is defined as quantity purchased minus quantity sold. Let

$$z_{nt}^i = \frac{b_{nt}^i}{p_{nt}} - q_{nt}^i \quad (20)$$

be the net trade of individual $i \in I$ for commodity n at t . We define z_{kt}^i similarly. Let

$$\xi_{nt}^i = c_{knt}^i \varepsilon_{knt} - c_{nkt}^i \quad (21)$$

be the net trade of individual i for country's n currency and $\xi_{kt}^i = c_{nkt}^i \varepsilon_{nkt} - c_{knt}^i$ for country's k currency at t . Obviously,

$$\xi_{nt}^i = -\xi_{kt}^i \varepsilon_{knt}. \quad (22)$$

Let

$$\zeta_{n0}^i = \frac{d_{n1}^i}{(1 + r_n)} - d_{n0}^i \quad (23)$$

⁹Of course without short sales, this could not have been possible.

be the net trade of agent i in the credit market in n currency at $t = 0$ and let

$$\zeta_{n1}^i = d_{n0}^i(1 + r_n) - d_{n1}^i. \quad (24)$$

Then

$$\zeta_{n1}^i = -\zeta_{n0}^i(1 + r_n). \quad (25)$$

Proposition 6 *If at a Nash Equilibrium of the game an agent is a net borrower and a first period net buyer of one country's currency and a net lender and a second period net seller of the other country's currency then the Uncovered Interest Rate Parity Condition fails i.e.*

$$1 + E_n \neq \frac{1 + r_k}{1 + r_n}$$

Proof. In the appendix. ■

Proposition 7 *If at a Nash Equilibrium of the game an agent is a net buyer of one country's commodity and currency and a net seller of the other country's commodity then the Absolute Purchasing Power Parity condition fails, i.e.*

$$p_{nt} \neq \varepsilon_{knt} p_{kt}$$

Proof. In the appendix. ■

Proposition 8 *If at a Nash Equilibrium of the game an agent is performing the type of trades described in proposition (7) in the first period but with opposite sign in the second period then the Relative Purchasing Power Parity condition fails i.e.*

$$\frac{1 + \pi_k}{1 + \pi_n} \neq 1 + E_n$$

Proof. In the appendix. ■

Corollary 9 *If at a Nash Equilibrium of the game an agent is performing the following type of trades*

$$\zeta_{n0}^i \geq 0, \zeta_{k0}^i \leq 0, \xi_{n0}^i \leq 0, \xi_{n1}^i \geq 0, z_{n0}^i \leq 0, z_{k0}^i \geq 0, z_{n1}^i \geq 0, z_{k1}^i \leq 0$$

with ζ^i, ξ^i, z^i not all zero, then the UIRP, APPP, RPPP conditions fail, i.e

$$\begin{aligned} 1 + E_n &< \frac{1 + r_k}{1 + r_n}, \\ p_{n0} &> \varepsilon_{kn0} p_{k0}, \\ p_{n1} &< \varepsilon_{kn1} p_{k1}, \\ 1 + E_n &< \frac{1 + \pi_k}{1 + \pi_n}. \end{aligned}$$

Proof. It follows from propositions (6),(7) and (8). ■

Lemma 10 *If at a Nash equilibrium an agent plays the following strategy*

$$\begin{aligned} d_{n1}^i &> 0, d_{n0}^i = 0, c_{nk0}^i = \frac{d_{n1}^i}{(1+r_n)}, c_{kn0}^i = 0, \\ d_{k0}^i &= c_{nk0}^i \varepsilon_{nk0}, d_{k1}^i = 0, c_{kn1}^i = d_{k0}^i (1+r_k), c_{nk1}^i = 0 \end{aligned}$$

he obtains a free arbitrage profit in the second period equal to

$$d_{n1}^i \left(\varepsilon_{kn1} \varepsilon_{nk0} \frac{(1+r_k)}{(1+r_n)} - 1 \right) > 0.$$

Proof. In the appendix. ■

Lemma 11 *The arbitrage profit is bounded*

Proof. In the appendix. ■

5 Conclusion

In this work we have presented a simple general equilibrium model of an imperfectly competitive international economy without any type of frictions or obstacles to arbitrage. In that model goods, currency and credit markets are interdependent and exchange rates, commodity prices and interest rates are determined endogenously. We have derived the fundamental relations in international trade and finance, that is the absolute and relative PPP, UIRP and the Fisher Effect. We characterised equilibria where these relations fail and identified individual strategies that generate non-negative but bounded and free profit from arbitrage.

This work suggests that imperfect competition may be an additional reason for the failure of UIRP and the Fischer effect and it is an open question whether it could be tested empirically...(incomplete)

6 Appendix

Proof of Proposition 2.

The Langrangean of player i maximization problem (15) is

$$\begin{aligned} L^i &= u^i(x^i(c_{nkt}^i, c_{knt}^i; b_{nt}^i, b_{kt}^i, q_{nt}^i, q_{kt}^i; d_{nt}^i, d_{kt}^i)_{t \in T}; (B_{nt}^{-i}, Q_{nt}^{-i}, C_{nkt}^{-i}, D_{nt}^{-i}, B_{kt}^{-i}, Q_{kt}^{-i}, C_{knt}^{-i}, D_{kt}^{-i})_{t \in T})) \\ &\quad + \lambda_{n0}^i \left(q_{n0}^i p_{n0} + c_{kn0}^i \varepsilon_{kn0} + \frac{d_{n1}^i}{1+r_n} - b_{n0}^i - c_{nk0}^i - d_{n0}^i \right) \\ &\quad + \lambda_{k0}^i \left(q_{k0}^i p_{k0} + c_{nk0}^i \varepsilon_{nk0} + \frac{d_{k1}^i}{1+r_k} - b_{k0}^i - c_{kn0}^i - d_{k0}^i \right) \\ &\quad + \lambda_{n1}^i \left(q_{n1}^i p_{n1} + c_{kn1}^i \varepsilon_{kn1} + d_{n0}^i (1+r_n) - b_{n1}^i - c_{nk1}^i - d_{n1}^i \right) \\ &\quad + \lambda_{k1}^i \left(q_{k1}^i p_{k1} + c_{nk1}^i \varepsilon_{nk1} + d_{k0}^i (1+r_k) - b_{k1}^i - c_{kn1}^i - d_{k1}^i \right) \end{aligned}$$

The first order conditions reduce to the following equations

$$(p_{nt})^2 = \frac{1}{\lambda_{nt}^i} \frac{\partial u}{\partial x_t^i} \frac{B_{nt}^{-i}}{Q_{nt}^{-i}}, t = 0, 1, \quad (26)$$

$$(p_{kt})^2 = \frac{1}{\lambda_{kt}^i} \frac{\partial u}{\partial x_t^i} \frac{B_{kt}^{-i}}{Q_{kt}^{-i}}, t = 0, 1 \quad (27)$$

and

$$(\varepsilon_{knt})^2 = \frac{\lambda_{kt}^i}{\lambda_{nt}^i} \frac{C_{nkt}^{-i}}{C_{knt}^{-i}}, t = 0, 1 \quad (28)$$

$$(1 + r_n)^2 = \frac{\lambda_{n0}^i}{\lambda_{n1}^i} \frac{D_{n1}^{-i}}{D_{n0}^{-i}} \quad (29)$$

$$(1 + r_k)^2 = \frac{\lambda_{k0}^i}{\lambda_{k1}^i} \frac{D_{k1}^{-i}}{D_{k0}^{-i}} \quad (30)$$

Combining (28), (29) and (30) we obtain (16). Since the above conditions must hold for every agent at equilibrium then W_1 is the same for every $i \in I$.

Proof of Proposition 3 Dividing (26) by (27) we have

$$\frac{(p_{nt})^2}{(p_{kt})^2} = \frac{\lambda_{kt}^i}{\lambda_{nt}^i} \frac{B_{nt}^{-i}}{Q_{nt}^{-i}} \frac{Q_{kt}^{-i}}{B_{kt}^{-i}}$$

and then using (28) we obtain (17)

Proof of Proposition 4 From (26) we have

$$\frac{(p_{n1})^2}{(p_{n0})^2} = \frac{\lambda_{n0}^i}{\lambda_{n1}^i} \frac{\partial u}{\partial x_1^i} \left(\frac{\partial u}{\partial x_0^i} \right)^{-1} \frac{B_{n1}^{-i}}{Q_{n1}^{-i}} \frac{Q_{n0}^{-i}}{B_{n0}^{-i}} \quad (31)$$

and from (27)

$$\frac{(p_{k1})^2}{(p_{k0})^2} = \frac{\lambda_{k0}^i}{\lambda_{k1}^i} \frac{\partial u}{\partial x_1^i} \left(\frac{\partial u}{\partial x_0^i} \right)^{-1} \frac{B_{k1}^{-i}}{Q_{k1}^{-i}} \frac{Q_{k0}^{-i}}{B_{k0}^{-i}} \quad (32)$$

Dividing (31) by (32) we have

$$\left(\frac{p_{n1}/p_{n0}}{p_{k1}/p_{k0}} \right)^2 = \frac{\lambda_{n0}^i}{\lambda_{n1}^i} \frac{\lambda_{k1}^i}{\lambda_{k0}^i} \frac{B_{n1}^{-i}}{Q_{n1}^{-i}} \frac{Q_{n0}^{-i}}{B_{n0}^{-i}} \frac{B_{k0}^{-i}}{Q_{k0}^{-i}} \frac{Q_{k1}^{-i}}{B_{k1}^{-i}} \quad (33)$$

and since

$$\frac{\lambda_{n0}^i}{\lambda_{n1}^i} \frac{\lambda_{k1}^i}{\lambda_{k0}^i} = \left(\frac{\varepsilon_{kn1}}{\varepsilon_{kn0}} \right)^2 \frac{C_{nk1}^{-i}}{C_{kn1}^{-i}} \frac{C_{nk0}^{-i}}{C_{kn0}^{-i}}$$

we obtain the analogue of the relative purchasing power parity for our economy (18).

$$\left(\frac{p_{n1}/p_{n0}}{p_{k1}/p_{k0}} \right)^2 = \left(\frac{\varepsilon_{kn1}}{\varepsilon_{kn0}} \right)^2 \frac{C_{nk1}^{-i}}{C_{kn1}^{-i}} \frac{C_{nk0}^{-i}}{C_{kn0}^{-i}} \frac{B_{n1}^{-i}}{Q_{n1}^{-i}} \frac{Q_{n0}^{-i}}{B_{n0}^{-i}} \frac{B_{k0}^{-i}}{Q_{k0}^{-i}} \frac{Q_{k1}^{-i}}{B_{k1}^{-i}}$$

Proof of Proposition 5 Using (29) and (30) we have

$$\frac{\lambda_{n0}^i \lambda_{k1}^i}{\lambda_{n1}^i \lambda_{k0}^i} = \left(\frac{1+r_n}{1+r_k} \right)^2 \frac{D_{n0}^{-i} D_{k1}^{-i}}{D_{n1}^{-i} D_{k0}^{-i}}$$

which we plug in (33) to obtain (19).

Proof of Proposition 6 At a Nash equilibrium currency and credit markets clear

$$\sum_{i \in I} \xi_{nt}^i = 0 \Rightarrow \xi_{nt}^i = - \sum_{j \neq i \in I} \xi_{nt}^j = - \sum_{j \neq i \in I} (c_{knt}^j \varepsilon_{knt} - c_{nkt}^j) = -C_{knt}^{-i} \varepsilon_{knt} + C_{nkt}^{-i},$$

$$\sum_{i \in I} \zeta_{n0}^i = 0 \Rightarrow \zeta_{n0}^i = - \sum_{j \neq i \in I}^j \zeta_{n0}^j = - \sum_{j \neq i \in I} \left(\frac{d_{n1}^j}{(1+r_n)} - d_{n0}^j \right) = -\frac{D_{n1}^{-i}}{(1+r_n)} + D_{n0}^{-i}$$

$$\sum_{i \in I} \zeta_{n1}^i = 0 \Rightarrow \zeta_{n1}^i = - \sum_{j \neq i \in I}^j \zeta_{n1}^j = - \sum_{j \neq i \in I} (d_{n0}^j (1+r_n) - d_{n1}^j) = -D_{n0}^{-i} (1+r_n) + D_{n1}^{-i}$$

so we may relate the sum of all players equilibrium strategies in a market with the individual net trade,

$$C_{knt}^{-i} = (C_{nkt}^{-i} - \xi_{nt}^i) \varepsilon_{nkt}, \quad (34)$$

$$D_{n1}^{-i} = (D_{n0}^{-i} - \zeta_{n0}^i) (1+r_n) \quad (35)$$

$$D_{n0}^{-i} = (D_{n1}^{-i} - \zeta_{n1}^i) (1+r_n)^{-1} \quad (36)$$

Then using (34), (35) and (36) the term W_1 can be written as

$$W_1 = \left(\frac{(D_{n0}^{-i} - \zeta_{n0}^i) (1+r_n)}{D_{n0}^{-i}} \frac{D_{k0}^{-i}}{(D_{k0}^{-i} - \zeta_{k0}^i) (1+r_k)} \frac{C_{nk0}^{-i}}{(C_{nk0}^{-i} - \xi_{n0}^i) \varepsilon_{nk0}} \frac{(C_{nk1}^{-i} - \xi_{n1}^i) \varepsilon_{nk1}}{C_{nk1}^{-i}} \right)$$

so (16) becomes

$$(1 + E_n) = \left(\frac{1+r_k}{1+r_n} \right) \left(\frac{(D_{n0}^{-i} - \zeta_{n0}^i)}{D_{n0}^{-i}} \frac{D_{k0}^{-i}}{(D_{k0}^{-i} - \zeta_{k0}^i)} \frac{C_{nk0}^{-i}}{(C_{nk0}^{-i} - \xi_{n0}^i)} \frac{(C_{nk1}^{-i} - \xi_{n1}^i)}{C_{nk1}^{-i}} \right). \quad (37)$$

When an agent is a net borrower and a first period net buyer of one country's currency and a net lender and a second period net seller of the other country's currency then either $\zeta_{n0}^i > 0, \xi_{n0}^i > 0, \zeta_{k0}^i < 0, \xi_{n1}^i < 0$ or $\zeta_{n0}^i < 0, \xi_{n0}^i < 0, \zeta_{k0}^i > 0, \xi_{n1}^i > 0$. For such net trades all terms in the right hand side parenthesis of (37) are less than one or greater than one respectively. So if $\zeta_{n0}^i > 0, \xi_{n0}^i > 0, \zeta_{k0}^i < 0, \xi_{n1}^i < 0$, then

$$(1 + E_n) < \left(\frac{1+r_k}{1+r_n} \right)$$

and if $\zeta_{n0}^i < 0, \xi_{n0}^i < 0, \zeta_{k0}^i > 0, \xi_{n1}^i > 0$,

$$(1 + E_n) > \left(\frac{1+r_k}{1+r_n} \right)$$

Proof of Proposition 7 From the market clearing condition in the commodity market at a Nash equilibrium, we have

$$\sum_{i \in I} z_{nt}^i = 0 \Rightarrow z_{nt}^i = - \sum_{j \neq i \in I} z_{nt}^j = - \sum_{j \neq i \in I} \left(\frac{b_{nt}^j}{p_{nt}} - q_{nt}^j \right) = Q_{nt}^{-i} - \frac{B_{nt}^{-i}}{p_{nt}}$$

Solving for B_{nt}^{-i} , we obtain

$$B_{nt}^{-i} = (Q_{nt}^{-i} - z_{nt}^i) p_{nt}. \quad (38)$$

then W_{2t} becomes

$$W_{2t} = \left(\frac{(C_{nkt}^{-i} - \xi_{nt}^i) \varepsilon_{nkt}}{C_{nkt}^{-i}} \frac{(Q_{nt}^{-i} - z_{nt}^i) p_{nt}}{Q_{nt}^{-i}} \frac{Q_{kt}^{-i}}{(Q_{kt}^{-i} - z_{kt}^i) p_{kt}} \right)$$

and (17) is written

$$\frac{p_{nt}}{p_{kt}} = \varepsilon_{knt} \left(\frac{(C_{nkt}^{-i} - \xi_{nt}^i) (Q_{nt}^{-i} - z_{nt}^i)}{C_{nkt}^{-i} Q_{nt}^{-i}} \frac{Q_{kt}^{-i}}{(Q_{kt}^{-i} - z_{kt}^i)} \right) \quad (39)$$

If at a Nash Equilibrium of the game an agent is a net buyer of one country's commodity and currency and a net seller of the other country's commodity then either $z_{nt}^i > 0, z_{kt}^i < 0, \xi_{nt}^i > 0$ or $z_{nt}^i < 0, z_{kt}^i > 0, \xi_{nt}^i < 0$. Then the term

$$\frac{(C_{nkt}^{-i} - \xi_{nt}^i) (Q_{nt}^{-i} - z_{nt}^i)}{C_{nkt}^{-i} Q_{nt}^{-i}} \frac{Q_{kt}^{-i}}{(Q_{kt}^{-i} - z_{kt}^i)}$$

in (39) is greater than or less than one respectively so either $p_{nt} > p_{kt} \varepsilon_{knt}$ or $p_{nt} < p_{kt} \varepsilon_{knt}$

Proof of Proposition 8 We follow the same line of proof as for proposition above. We have two possible configurations of net trades here, either $z_{n0}^i > 0, z_{k0}^i < 0, \xi_{n0}^i > 0, z_{n1}^i < 0, z_{k1}^i > 0, \xi_{n1}^i < 0$ or $z_{n0}^i < 0, z_{k0}^i > 0, \xi_{n0}^i < 0, z_{n1}^i > 0, z_{k1}^i < 0, \xi_{n1}^i > 0$. Then $W_3 =$

$$\left(\frac{(C_{nk0}^{-i} - \xi_{n0}^i) \varepsilon_{nk0}}{C_{nk0}^{-i}} \frac{(Q_{n0}^{-i} - z_{n0}^i) p_{n0}}{Q_{n0}^{-i}} \frac{Q_{k0}^{-i}}{(Q_{k0}^{-i} - z_{k0}^i) p_{k0}} \frac{C_{nk1}^{-i}}{(C_{nk1}^{-i} - \xi_{n1}^i) \varepsilon_{nk1}} \frac{Q_{n1}^{-i}}{(Q_{n1}^{-i} - z_{n1}^i) p_{n1}} \frac{(Q_{k1}^{-i} - z_{k1}^i) p_{k1}}{Q_{k1}^{-i}} \right)$$

and (18) reduces to

$$\frac{1 + \pi_k}{1 + \pi_n} = (1 + E_n) \left(\frac{(C_{nk0}^{-i} - \xi_{n0}^i) (Q_{n0}^{-i} - z_{n0}^i)}{C_{nk0}^{-i} Q_{n0}^{-i}} \frac{Q_{k0}^{-i}}{(Q_{k0}^{-i} - z_{k0}^i)} \frac{C_{nk1}^{-i}}{(C_{nk1}^{-i} - \xi_{n1}^i)} \frac{Q_{n1}^{-i}}{(Q_{n1}^{-i} - z_{n1}^i)} \frac{(Q_{k1}^{-i} - z_{k1}^i)}{Q_{k1}^{-i}} \right).$$

Then if $z_{n0}^i > 0, z_{k0}^i < 0, \xi_{n0}^i > 0, z_{n1}^i < 0, z_{k1}^i > 0, \xi_{n1}^i < 0$, then

$$\frac{1 + \pi_k}{1 + \pi_n} < (1 + E_n)$$

and if $z_{n0}^i < 0, z_{k0}^i > 0, \xi_{n0}^i < 0, \xi_{n1}^i > 0, z_{k1}^i < 0, \xi_{n1}^i > 0$, then

$$\frac{1 + \pi_k}{1 + \pi_n} > (1 + E_n)$$

Proof of Lemma 10 This type of strategy results to the following net trades of agent i , $\zeta_{n0}^i > 0, \xi_{n0}^i > 0, \zeta_{k0}^i < 0, \xi_{n1}^i < 0$, for which, according to proposition (6), the UIRP fails and in particular

$$(1 + E_n) < \frac{1 + r_k}{1 + r_n}. \quad (40)$$

or

$$\varepsilon_{kn0}\varepsilon_{nk1}(1 + r_n) < 1 + r_k$$

which means that the return of a deposit on currency k is greater than the return of a deposit on currency n . The agent borrows in the cheap currency n an amount equal to $d_{n1}^i/(1 + r_n)$ at $t = 0$ and commits himself to deliver d_{n1}^i at $t = 1$. He then converts the borrowed money to currency k , $c_{nk0}^i = d_{n1}^i/(1 + r_n)$ in order to invest $d_{k0}^i = (d_{n1}^i/(1 + r_n))\varepsilon_{nk0}$ units with the higher interest deposit. In period $t = 1$, his receipt from the k currency deposit is

$$d_{k0}^i(1 + r_k) = \frac{d_{n1}^i}{(1 + r_n)}\varepsilon_{nk0}(1 + r_k)$$

which is converted to n currency,

$$c_{kn1}^i = \frac{d_{n1}^i}{(1 + r_n)}\varepsilon_{nk0}(1 + r_k),$$

in order to fulfill his obligation to repay his debt equal to d_{n1}^i . In exchange to c_{kn1}^i the agent receives

$$\frac{d_{n1}^i}{(1 + r_n)}\varepsilon_{nk0}(1 + r_k)\varepsilon_{kn1}$$

units of k currency. From (40) we have that

$$\varepsilon_{kn1}\varepsilon_{nk0}\frac{(1 + r_k)}{(1 + r_n)} > 1$$

so his period 1 receipt is higher than his debt

$$\frac{d_{n1}^i}{(1 + r_n)}\varepsilon_{nk0}(1 + r_k)\varepsilon_{kn1} > d_{n1}^i$$

and the agent obtains a strictly positive profit equal to

$$d_{n1}^i \left(\varepsilon_{kn1}\varepsilon_{nk0}\frac{(1 + r_k)}{(1 + r_n)} - 1 \right) > 0$$

This profit is free because it requires no initial wealth in any period.

Proof of Lemma 11 We have to show that there exists $M \in R_+$ such that

$$d_{n1}^i \left(\varepsilon_{kn1} \varepsilon_{nk0} \frac{(1+r_k)}{(1+r_n)} - 1 \right) = d_{n1}^i \left(\frac{C_{nk1}}{C_{kn1}} \frac{C_{kn0}}{C_{nk0}} \frac{D_{k1}}{D_{k0}} \frac{D_{n0}}{D_{n1}} - 1 \right) < M$$

To be completed...

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