The Persistence of Real Exchange Rate and the PPP puzzle

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Abstract

Almost a decade after Rogoff (1996) formulated his celebrated PPP Puzzle, the question of how to explain the high volatility and the simultaneously persistent deviations from equilibrium shown by real exchange rates remains open. As regards the persistence of deviations, various papers using more powerful and sophisticated techniques have obtained half-lives bigger than the 3-5 year "consensus" to which Rogoff refered and, more importantly, have found that the upper bound of the confidence intervals are too high to rule out the failure of PPP.

This paper tries to shed more light on the problems of measuring deviations by using fractional integration models to model more accurately a sample of 22 real exchange rates -the same as in Taylor (2002)- and to avoid any risk of underbias in the estimation of the half-lives. As a first result, we find that, although there are important differences across countries, the memory parameter takes values on the frontier of stationarity, which means that the real exchange rate is a mean reverting process with a high degree of persistence. In terms of half-lives, the classical approach gives a picture of persistence not very different from that of Taylor (2002). However, a more persistent picture is that given by Bayesian approaches, when without any exception, the half lives are higher.

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But now, differently from the results of López et al. (2003, 2004), we can claim -with only 15% probability for the region with permanent shocks- that PPP hold, although the density of half-lives computed with Bayesian techniques shows that the 3-5 year consensus is very unlikely.

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1. INTRODUCTION

Almost a decade after Rogoff (1996) formulated his celebrated PPP Puzzle, the question of how to explain the high volatility and the simultaneously persistent deviations from equilibrium shown by real exchange rates remains open. For this author, the coincident half-lives of 3-5 years found in empirical studies using long-horizon data sets[1], were too lasting to be reconciled with the short-run variability exhibited by real exchange rates, necessarily provoked by nominal factors (hence the Puzzle). As regards the persistence of the deviations these were widely illustrated in the well-known study of Taylor (2002) who, working with a sample of 22 countries and a period of a century, even calculating for detrended series of real exchange rates, found a median half-life of around 4 years. Most importantly, this evidence of a “glacial” speed of reversion[2] has been enlarged in recent efforts to better measure the size of the deviations from PPP.

The most popular persistence measure in the PPP literature, the above mentioned half-life (HL henceforth) of deviations, defined as the number of periods for deviations to be corrected by one half, is now being deeply analyzed. Because it is very sensitive to the estimated coefficient of the autoregressive parameter, the methods of estimating it are being revised. Furthermore, the calculus of HL only gives a point estimator of persistence, offering a poor picture of the path of the shock. So, Murray and Papell (2002) estimate HL based on median unbiased estimations of the autoregressive parameter, providing confidence

[1] The conclusions of Rogoff (1996) come from a set of long-run studies such as Frankel (1986), Abuaf and Jorion (1990), and Lothian and Taylor (1996).

[2] Rogoff (1996).

intervals and impulse response functions. Although the mean value is within the 3-5 year range, their wide intervals are not informative and do not allow us to accept the PPP as a hypothesis of behaviour. Furthermore, the low bounds are too high to be compatible with models based on nominal rigidities. Of special interest in this point is the work of Lopez et al. (2003, 2004), who extend the median unbiased techniques by using the more powerful unit root test GLS of Elliot et al. (1996) and find half-lives of between 8 and 11 years and lower bounds over 3 years for the same sample of countries as Taylor (2002). Moreover, the upper bounds of the confidence intervals in López et al. are so high that they cannot rule out the failure of PPP in the long-run[3]. To sum up, a general conclusion that can be obtained from recent work is that traditional measures of persistence are downwardly biased and the size of puzzle could be even bigger than initially thought.

In this framework, this paper tries to shed further light on the debate by considering an alternative and more flexible statistical approach. The studies cited above are usually based on the too restrictive I(0)-I(1) paradigm determining the reaction of the real exchange rate to a shock. Stationary processes are characterized by autocorrelation functions that decay to zero at an exponential rate, while series containing a unit root have a permanent effect. But some macroeconomic and financial series could react in a different way to shocks (due, for example, to an intertemporal smoothing of consumption) showing no permanent but very lasting mean reversion behaviour.

If this were the case, the standard unit root approach could yield false conclusions about the permanent or non-permanent nature of the shocks, the fractional integration (FI) approach being a more appropriate framework for modelling real exchange rate. In fact, if the real exchange rate follow an I(d) model, the usual finite autoregressive representation will probably not reject the unit roots hypothesis and wrongly conclude that the shocks are permanent. On the other hand, even if we accepted that PPP holds, which would imply the stationarity of the real exchange rate, the persistence would be underestimated because

[3] Other attempts such as Cahin and Mcdermott (2003), Rossi (2004), Caporale et al. (2004) and Murrary and Papell (2005) have obtained similar results.

the I(0) model only considers the short-run dynamics[4].

For this reason, in Section 2 we firstly introduce the basis of the concept of fractional integration and show the results of estimating ARFIMA models with different methods for the real exchange rates of the same 22 countries as Taylor (2002) during the last century [5]. Then in Section 3, to overcome underestimation, impulse response functions and half-lives are estimated in a fractional context by applying classical and Bayesian approaches. In this way, we take into account the uncertainty about the true model and provide density functions of the interest parameter. These allow us to compute exactly the probability of the memory parameter being in each of the regions, classified by their degree of inertia, and to know how likely the 3-5 year consensus is. Finally, Section 4 is devoted to discussing results.

2. FRACTIONAL INTEGRATION IN RER SERIES

As we said earlier, we use the database elaborated by Taylor (2002) for a sample of 22 countries for a period running from 1850 to 1996, although some series start later, and enlarge it to 2000 using the CD-ROM of IMF, *InternationalFinancialStatistics*. The countries included in the study are Argentina (ARG), Australia (AUS), Belgium (BEL), Brazil (BRA), Canada (CAN), Chile (CHL), Denmark (DNK), Finland (FI), France (FR), Germany (DEU), Greece (GRC), Italy (IT), Japan (JPN), Mexico (MEX), Netherlands (NLD), New Zealand (NZL), Norway (NOR), Portugal (POR), Spain (ESP), Sweden (SWE), Switzerland (CHE) and Great Britain (GBR). As well as the different starting dates for some countries, some data are also missing for specific periods such as the World Wars and hyperinflation episodes. The different sample sizes are not a big problem because we are going to work with individual series[6], and the second problem has been solved by interpolation

[4] About the pitfalls of using the integer aproach for measuring persistence, see Gadea and Mayoral (2006).

[5] Examples of FI in favour of PPP are those of Diebold et al. (1991), Cheung and Lai (1993, 2001), Chou and Shih (1997), Dueker and Serletis (2000), Chaung and Lai (2000), Holmes (2000), Achy (2003), Heravi and Patterson (2003), Barkoulas et al. (1998) and Caporale and Gil-Alana (2004).

[6] However, we should consider this point in order to interpret the results suitably, especially in some countries like Greece, whose sample size is relatively small.

using the Tramo-Seats program[7]. The data are annual and include the nominal exchange rate E defined as domestic currency units *per* U.S. dollar and price indices P measured as consumer price deflators and, occasionally, when these are not available, as GDP deflators. Thus, the real exchange rate Q_{it} of country i at time t is calculated as follows:

$$Q_{it} = \frac{P_{it}}{P_{usat}E_{it}}$$

Taking logs we obtain the following expression:

$$q_{it} = p_{it} - p_{usat} - e_{it}$$

where lower case letters denote variables in logs and an increase of q_{it} represents an appreciation of exchange rates in real terms. The series of the real exchange rate for all countries are displayed in Figures 1, 2 and 3.

A preliminary analysis is carried out applying the MZt-GLS unit root tests proposed by Ng and Perron (2001) which are modified forms of Phillips-Perron test [Phillips and Perron (1988)] based on the GLS detrended data as well as, the KPSS of Kwiatkowski et al. (1992) that uses the stationarity of real exchange rate as null hypothesis. Both have been applied to two different specifications that include intercept or intercept and trend. Although there is no consensus, for the majority of countries -as Taylor (2002) found by applying standard Dickey-Fuller tests- we can reject the presence of a unit root in the series and can not reject the null of stationarity and, in some cases, both hypothesis are rejected simultaneously. Although the deterministic trend is significant in some countries, its inclusion does not significantly change the conclusions. In spite of contradictory empirical results, from a theoretical point of view, there is a wide acceptance of the PPP as a long-run rule and, as has been mentioned earlier, recent studies focus on measuring and explaining deviations. so, we devote our attention to elaborating a novel method to measure them starting from the idea of fractional integration.

[7] Gómez and Maravall (1996).

TABLE 2.1

Unit roots and stationarity tests

	MZ_t-GLS		KPSS	
	intercept	intercept and trend	intercept	intercept and trend
ARG	$-4.09**$	$-4.14**$	0.11	0.11
AUS	$-2.43**$	-2.70	$0.81**$	$0.15*$
BEL	$-3.37**$	$-4.77**$	$0.74**$	0.06
BRA	$-2.53**$	-2.59	0.16	0.09
CAN	-1.77	-2.04	$0.78**$	$0.21*$
CHL	-1.02	$-3.02*$	$1.06**$	0.19
DNK	-2.00	$-3.36*$	$0.68**$	$0.21*$
FIN	$-5.70**$	$-5.94**$	0.18	0.06
FRA	$-2.33*$	$-4.21**$	$0.89**$	$0.29**$
DEU	$-2.67**$	$-3.61**$	0.39	0.05
GRC	-0.59	-0.81	0.25	$0.15*$
ITA	$-3.99**$	$-4.00**$	0.07	0.06
JPN	0.17	$-3.12*$	$1.14**$	$0.18*$
MEX	$-2.50*$	$-3.52**$	$0.81*$	0.07
NLD	$-2.58**$	$-3.34**$	0.43	$0.17*$
NZL	$-3.35**$	$-3.86**$	$0.53*$	0.02
NOR	$-2.37*$	$-3.87**$	$0.46*$	0.14
PRT	-1.85	-2.89	0.40	$0.23**$
ESP	$-2.87**$	-3.27	0.31	$0.22**$
SWE	$-3.21**$	$-4.59**$	$0.59*$	0.06
CHE	-0.68	$-3.70**$	$0.94**$	0.14
GBR	$-2.58**$	-2.76	0.38	$0.17*$

Notes: **, * Significant at the 1% and 5% level respectively. The lag length has been chosen according to the SBIC criterion in the MZt-GLS and Bartlett's window has been used as a kernel estimator in the PP and KPSS. (bandwidth chosen according to Newey and West (1994)).

Preliminary results show that PPP holds, but the high persistence found in previous studies suggests more subtle parity-reverting dynamics in real exchange rate. Under the integer approach, if a process is I(1), all shocks have a permanent effect while they disappear exponentially when the process is I(0). Less extreme alternatives are the fractionally integrated models, where shocks can be very persistent but not permanent The so-called fractionally integrated (FI) models extend the dichotomy $I(1)$ versus $I(0)$ and, at the same time, are able to account for richer persistence types. In the FI approach, the most popular parametric model is the ARFIMA, independently introduced by Granger and Joyeux (1980) and Hosking (1981). The main advantage of this formulation with respect to the ARIMA is the introduction of a new parameter, d, that models the 'memory' of the process, that is, the medium and long-run impact of shocks on the process. More specifically, y_t is an ARFIMA(p, d, q) if it can be written as,

$$\Phi(L)(1-L)^d y_t = \Theta(L)\varepsilon_t,\ \varepsilon_t \sim i.i.d.\left(0, \sigma_\varepsilon^2\right),$$

where the so-called *memory parameter*, d, determines the integration order of the series and is allowed to take values in the real, as opposed to the integer, set of numbers. The terms $\Phi(L) = 1 - \phi_1 L - ... - \phi_p L^p$ and $\Theta(L) = 1 - \theta_1 L - ... - \theta_q L^q$ represent the autoregressive and moving average polynomials, respectively, with all their roots lying outside the unit circle. While d captures the medium and long-run behavior of the process, $\Phi(L)$ and $\Theta(L)$ model the short-run dynamics.

The bigger the value of d, the more persistent the process. Stationarity and invertibility require $|d| < 1/2$, which can always be achieved by taking a suitable number of (integer) differences. *Short memory* is implied by a value of $d = 0$, where the process is characterized by absolutely summable correlations decaying at an exponential rate. By contrast, *long memory* occurs whenever d belongs to the (0,0.5) interval. Hosking (1981) showed that the correlation function in this case is proportional to k^{2d-1} as $k \to \infty$, that is, it decays at a hyperbolic rather than at an exponential rate. These processes are also characterized by an unbounded spectral density at frequency zero. These facts reflect the slower decay of shocks with respect to the $I(0)$ case. A particularly interesting region for the real exchange rate is the interval $d \in [0.5, 1)$. In this range, shocks are transitory but the impulse response to shocks vanishes so slowly that the variance is not bounded and, therefore, the process is non-stationary in spite of being mean-reverting (as shocks eventually disappear). Shocks have a permanent effect whenever $d \geq 1$.Operationally, a binomial expansion of the operator $(1-L)^d$ is used in order to fractionally differentiate a time series:

$$(1-L)^d = \sum_{i=0}^{\infty} \pi_i \left(d\right) L^i \tag{1}$$

where,

$$\pi_i = \Gamma\left(i-d\right) / \Gamma\left(-d\right) \Gamma\left(i+1\right) \tag{2}$$

and $\Gamma(.)$ denotes the gamma function. When $d = 1$, (1) is just the usual first-differencing filter. For non-integer d, the operator $(1-L)^d$ is an infinite-order lag-operator polynomial with coefficients that decay very slowly. Since the expansion is infinite, a truncation is needed in order to fractionally differentiate a series in practice (see Dolado et al. (2002) for details on the consequences of the truncation).

There are several estimation techniques for FI models. We have considered two semi-parametric methods in the frequency domain, that proposed by Geweke and Porter-Hudak (1983) (GPH) and the Gaussian of Robinson (1995) (GSP); and another two parametric methods in the time domain, the exact maximum likelihood method of Sowell (1992) (EML) and the non-liner least squares of Beran (1994) (NLS). All these methods have specific problems and biases depending, on the first case, of the number of frequencies selected and, in the second, on the specific parametric model. So, although the use of several methods is a guarantee of robustness, we have also computed the d estimation under a Bayesian approach in order to take the model uncertainty into account following, primarily, the work of Koop et al. (1997). The Bayesian approach has the advantage that one may give the same prior probability to different ARFIMA models and, also, to various regions for memory parameter. In fact, we have computed nine models for each country considering the possible combinations of ARFIMA models with $p, q \leq 2$[8]. A uniform density for d in the

[8]A preliminary analysis showed that lags longer than 2 are not significant, a plausible result working with

interval $[0, 1.5]$ has been assumed[9]. So, the method put 1/3 of the prior mass of probability on values of d that imply stationary shocks, non-stationary but mean-reverting shocks and, finally, permanent shocks, corresponding to the intervals [0, 0.5), [0.1,1) and [1, 1.5] respectively. Notice that the extreme cases of $d = 0$and $d = 1$ are embedded in this method as a particular case that will achieve a positive posterior probability. An additional advantage of this approach is that we obtain the density functions of the parameters of interest, in particular the long-memory parameter and the half-lives.

annual data.

[9] 25,000 replications for each ARFIMA specification were computed using the Fortram code provided by Koop et al. (1997).

TABLE 2.2

Estimation of FI(d) models♠

	GPH	GSP	EML	NLS
ARG	0.40 (0.040)	0.27 (0.000)	0.64 (0.000)	0.61 (0.000)
AUS	0.74 (0.284)	0.65 (0.015)	0.50 (0.009)	0.45 (0.035)
BEL	0.46 (0.091)	0.41 (0.060)	0.31 (0.001)	0.34 (0.000)
BRA	0.10 (0.000)	0.61 (0.000)	0.94 (0.041)	0.91 (0.000)
CAN	0.92 (0.759)	0.37 (0.010)	0.48 (0.137)	0.46 (0.032)
CHL	0.44 (0.104)	0.49 (0.004)	0.46 (0.042)	1.00 (0.000)
DNK	0.52 (0.082)	0.46 (0.000)	0.65 (0.002)	0.68 (0.000)
FIN	−0.21 (0.434)	0.05 (0.739)	−0.30 (0.361)	−0.26 (0.398)
FRA	0.18 (0.002)	0.37 (0.000)	0.77 (0.415)	0.68 (0.000)
DEU	0.61 (0.151)	0.67 (0.031)	0.107 (0.000)	0.46 (0.001)
GRC	0.91 (0.857)	0.82 (0.410)	0.94 (0.622)	0.65 (0.000)
ITA	0.33 (0.005)	0.25 (0.000)	0.43 (0.000)	0.39 (0.001)
JPN	0.74 (0.367)	0.59 (0.051)	0.47 (0.117)	1.04 ()
MEX	0.76 (0.410)	0.57 (0.006)	0.50 (0.000)	0.53 (0.000)
NLD	0.72 (0.269)	0.66 (0.020)	0.34 (0.086)	0.41 (0.169)
NZL	0.49 (0.332)	0.31 (0.165)	−0.08 (0.721)	−0.06 (0.762)
NOR	0.59 (0.114)	0.54 (0.001)	0.53 (0.100)	0.42 (0.226)
PRT	0.83 (0.560)	0.69 (0.050)	0.73 (0.100)	0.56 (0.001)
ESP	0.78 (0.415)	0.71 (0.051)	0.56 (0.000)	0.53 (0.000)
SWE	0.38 (0.160)	0.45 (0.003)	0.41 (0.000)	0.46 (0.000)
CHE	0.54 (0.113)	0.51 (0.002)	0.52 (0.000)	0.81 (0.000)
GRB	0.76 (0.346)	0.83 (0.246)	0.67 (0.001)	0.59 (0.000)

♠ Std. dev.in brackets.

The results of the classical estimation are displayed in Table 2.2. Several conclusions can be drawn from this table. First, the finding of the long-memory parameter less than one, distant from the unit root, is robust across countries and estimation methods. Secondly, most countries exhibit values of the d parameter in the region of non-stationary but mean reverting response to the shocks. So, this confirms previous findings about the nature of shocks in the real exchange rate, which are transitory but very persistent. Finally, some countries show significant differences in the estimation of the d parameter using different methods. This high uncertainty about the true value of d justifies the use of the Bayesian approach.

TABLE 2.3

BAYESIAN ESTIMATION OF $\mathrm{FI}(d)$♠

	best	mean	weighted
ARG	0.62	0.47	0.64
	(0.31)	(0.23)	(0.32)
AUS	0.51	0.64	0.57
	(0.20)	(0.23)	(0.22)
BEL	0.34	0.44	0.34
	(0.14)	(0.20)	(0.20)
BRA	0.51	0.72	0.72
	()0.28	(0.24)	(0.23)
CAN	0.51	0.70	0.67
	(0.25)	(0.26)	(0.23)
CHL	0.74	0.65	0.69
	(0.10)	(0.24)	(0.19)
DNK	0.51	0.66	0.60
	(0.21)	(0.22)	(0.22)
FIN	0.28	0.40	0.32
	(0.17)	(0.23)	(0.22)
FRA	0.40	0.61	0.52
	(0.21)	(0.23)	(0.27)
DEU	0.52	0.80	0.72
	(0.28)	(0.24)	(0.28)
GRC	1.32	1.05	1.22
	(0.17)	(0.27)	(0.29)
ITA	0.42	0.53	0.44
	(0.20)	(0.81)	(0.29)
JPN	0.68	0.76	0.71
	(0.20)	(0.23)	(0.22)
MEX	0.56	0.59	0.54
	(0.13)	(0.25)	(0.22)
NLD	0.63	0.71	0.64
	(0.20)	(0.22)	(0.22)
NZL	0.58	0.74	0.67
	(0.21)	(0.20)	(0.29)
NOR	0.65	0.67	0.54
	(0.17)	(0.27)	(0.23)
PRT	0.58	0.68	0.63
	(0.21)	(0.21)	(0.20)
ESP	0.40	0.54	0.43
	(0.30)	(0.27)	(0.27)
SWE	0.32	0.55	0.45
	(0.22)	(0.23)	(0.23)
CHE	0.48	0.69	0.61
	(0.24)	(0.23)	(0.24)
GRB	0.49	0.67	0.61
	(0.20)	(0.24)	(0.22)

♠Stand. deviat. in brackets.

Table 2.3 reports the main results of the Bayesian approach. The mean and standard deviations of the d parameter are provided for three types of models, the best, mean and weighted. The best and weighted are selected in accordance with the posterior probability of each ARFIMA specification, and the mean is obtained as the average value of all models. The findings confirm the main conclusions obtained with the classical approach because, for every country, with the exception of Greece, the values of d are less than one and the majority are in the interval of [0.4, 0.6], the frontier of stationarity. But the results also highlight the noticeable variability associated with the estimations of the long-memory parameter, which can be compensated with the analysis of posterior probabilities. For this reason in Figures 4, 5 and 6 we also report the posterior density functions of the d parameter for each country. At the bottom of Figure 6 we display a summary of the values of the d parameter obtained across countries. From the density function of d, we can easily derive the probability that the memory parameter of the real exchange rate appears in each of the regions of interest The results, reported in Figure 7, show that the average probability that $d < 1$ is between 0.82 and 0.86 depending on whether we use a simple average of ARFIMA $p(d < 1)$ models or a weighted average $p * (d < 1)$. The probability of stationary behavior $p(d < 0.5)$ is also computed and displayed in the same figure, obtaining values of 0.33 and 0.36. To sum up, using a broad sample of countries over a century we show that the probability that the real exchange rate is a mean-reverting process with nonpermanent shocks is very high, more than a 80%. Although, the probability that the real exchange rate is a stationary process is around 30%. Consequently, 55% of real exchange rates are situated in a region of mean reverting but very persistent shocks.

3. PERSISTENCE PROPERTIES OF RER

In this section we provide the half-lives, as a measure of the persistence of parity reversion, deriving them from the FI models. This permits a suitable estimation of the inertia of the real exchange rate. The superiority of this approach is clear because ARFIMA formulations are more flexible and are able to characterize non-stationarity without imposing the restriction of permanent shocks as ARIMA specifications do [10]. We use the impulse response functions (IRF) that have the advantage of showing the full path of the shock over time[11]. However, this measure is a vector, not a scalar and, therefore, could be difficult to interpreter. For this reason, and in order to compare our results with previous findings, we calculate the half-life as a scalar measure from the IRF . Starting from the following expression:

$$\Phi(1)(1-L)^d y_t = \Theta(L)\varepsilon_t.$$

the IRF measures "the effect of a change in the innovation ε_t by a unit quantity on the current and subsequent values of y_t" [see Andrews and Chen (1994)] and is given by the coefficients of the polynomial,

$$A(L) = (1-L)^{-d}\Phi(L)^{-1}\Theta(L) = (1-L)^{-d} C(L) = (1-L)^{-d}\left(1 + c_1 L + c_2 L^2 + ...\right),$$

where the parameters c_h come from the Wold representation of the process $(1-L)^d y_t$. It follows that if y_t is $I(0)$, then the $IRF(h)$ is simply given by the corresponding Wold coefficient, c_h. If y_t is $I(1)$, given that $(1-L)^{-1} = \left(1 + L + L^2 + ...\right)$, the $IRF(h)$ associated with y_t can be computed as [see Campbell and Mankiw (1987)],

$$IRF(h) = \sum_{i=0}^{h} c_i. \tag{3}$$

Since the IRF of an $I(1)$ process, or any $FI(d)$ process with $0.5 \leq d \leq 1.5$ is computed by adding up the Wold coefficients c_i of its stationary transformation, $(1-L)y_t$, it is often called the "*cumulative impulse response function*". The effect of a shock in the very long run can be obtained by setting $h = \infty$. If a process is $I(1)$, the $IRF(\infty)$ is given by

$$IRF(\infty) = \sum_{i=0}^{\infty} c_i = C(1) < \infty. \tag{4}$$

[10] A detailed revision of persistence measures in a fractional integration context can be found in Gadea and Mayoral (2005). This paper also discusses the potential pitfalls stemming from applying some popular persistence measures such as the sum of AR coefficients or equivalently, the cumulative impulse response.

[11] For example,15% of the *glacial rate* that Rogoff (1996) describes is based on a scalar measure but doesnot imply a constant rate over time. Really IRF offers a more precise picture of the evolution of shocks.

The above-mentioned expressions are embedded in the general formulation of the $IRF\left(h\right)$ of an ARFIMA(p,d,q) process. This is defined as the h-th coefficient of $A\left(L\right)$. The corresponding coefficients can be computed according to the following formula (see Koop et al. (1997) for details),

$$IRF\left(h\right)=\sum_{i=0}^{h}\pi_i\left(-d\right)J\left(h-i\right), \tag{5}$$

where each $\pi_i\left(-d\right)$ comes from the binomial expansion of $\left(1-L\right)^{-d}$ and is defined in (2) and $J\left(.\right)$ is the standard ARMA(p,q) impulse response, given by

$$J\left(i\right)=\sum_{j=0}^{q}\theta_j f_{i+1-j},$$

with $\theta_0=1$, $f_h=0$ for $h\leq 0$, $f_1=1$ and

$$f_h=-\left(\phi_1 f_{h-1}+...+\phi_p f_{h-p}\right), \qquad \text{for } h\geq 2.$$

Notice that if $d=1$, $\pi_i\left(-1\right)=1$ for all i and, therefore, the traditional IRF for I(1) processes is recovered, i.e., $IRF\left(h\right)=\sum_{i=0}^{h}J\left(h-i\right)$ [see Campbell and Mankiw, (1987)]. The limit behavior of the IRF(h) when $h\to\infty$ depends upon the value of d and verifies

$$IRF\left(\infty\right)=\begin{cases} 0, \text{ if } d<1, \\ \Phi\left(1\right)^{-1}\Theta\left(1\right), \text{ if } d=1, \\ \infty \text{ if } d>1. \end{cases} \tag{6}$$

Expression (6) means that the effect of a shock is transitory for $d<1$, as the long-term impact of any shock is equal to zero. By contrast, shocks are permanent for any $d\geq 1$. If the process contains a unit root ($d=1$), the long-run effect of the shock is bounded away from zero and finite and is given by the sum of the Wold coefficients of its stationary transformation (or alternatively, by $\Phi\left(1\right)^{-1}\Theta\left(1\right)$ if it admits an ARMA representation). Finally, for any $d>1$ the effect of any shock is magnified and the final impact is not bounded.

From the IRF, the half life (HL), defined as the number of periods that a shock needs to vanish by 50 percent, can be easily calculated as,

$$IRF(HL) = 0.5$$

The above measure has been calculated through the classical and Bayesian estimations of ARFIMA models. Notice that, for the first case we need a parametric formulation of the ARFIMA model and have used the MLE obtained by the Sowel method. By using the Bayesian approach we achieve a more precise picture of the half-lives because the method allows us compute the full posterior density. The results are reported in Table 3.1, the first column for the classical approach and the others for the three models obtained through Bayesian techniques. In order to clarify the findings we have simplified the values of over ten years which, based on previous results, can be considered a superior limit which is not compatible with any nominal model and casts doubt about the long-run PPP [12]. In addition, Figures 8, 9 and 10 display the detailed evolution of the IRF for a horizon of twenty years. Because point estimations of HL are not conclusive, we have computed their probabilities based on the posterior density of IRF. More specifically, we have calculated the probability that $HL < k$ as the $p(I(k)) < 0.5$ for k=3, 5 and 10, values corresponding to the so-called interval of Rogoff and the upper limit fixed by us. Several interesting conclusions can be drawn from the inspection of Figure 11, which reports the main results. The probability of half-lives being inferior to 3 years is only of 14% on average. With the exception of Argentina, Belgium, Finland, New Zealand and Mexico, it is always less than 30%. This means that models based on nominal rigidities are unlikely in our sample. The probability of HL inferior to 5 years, the upper bound of Rogoff is around 30%, as a result of which 3-5 "consensus" of Rogoff only has a probability of 15%. Finally, the probability of HL being inferior to 10 years, and consequently bigger than 10 years, is around 50%. In short, our results point to very persistent deviations of the real exchange rate from its equilibrium level, with a 50% probability of them being superior to 10 years.

[12] See for example Lothian and Taylor (1996), Taylor (2002), Taylor (2003). Other works, such as Murray and Papell (2005), obtained, even higher values.

TABLE 3.1

ESTIMATION OF HALF LIVE♠

	Classical	Bayesian		
		best	mean	weighted
ARG	2.8	> 10	4.0	> 10
AUS	5.8	9.3	8.6	8.7
BEL	2.5	2.8	4.7	3.6
BRA	> 10	> 10	9.9	> 10
CAN	8.5	> 10	> 10	> 10
CHL	2.7	7.4	6.1	6.9
DNK	7.9	9.9	9.4	9.4
FIN	1.7	2.2	3.2	2.2
FRA	2.3	4.8	5.9	5.1
DEU	8.7	> 10	> 10	> 10
GRC	> 10	> 10	> 10	> 10
ITA	3.8	7.4	6.7	6.1
JPN	7.9	> 10	6.0	5.8
MEX	2.8	3.4	4.3	4.0
NLD	7.3	> 10	> 10	> 10
NZL	0.9	> 10	> 10	> 10
NOR	5.7	> 10	> 10	> 10
PRT	> 10	> 10	> 10	> 10
ESP	8.9	2.5	5.8	4.2
SWE	3.6	4.8	6.7	6.2
CHE	6.9	> 10	> 10	> 10
GRB	7.9	6.9	7.9	8.0

♠.

4. CONCLUSIONS

Accepting that the PPP is a long-run rule for real exchange rate behaviour, some empirical work has recently focused on measuring the size of deviations from equilibrium. Various papers using more powerful and sophisticated techniques have obtained half-lives bigger than the 3-5 year "consensus" of Rogoff and, more importantly, have found that the upper bound of the confidence intervals are too high to rule out the failure of PPP.

This paper tries to shed more light on the problems of measuring deviations by using fractional integration models to capture more accurately the dynamics of real exchange rates and to avoid any risk of underbias in the estimation of the half-lives. As a first result, we find robust evidence of long-memory in real exchange rates and, although there are important differences across countries, the memory parameter takes values on the frontier of stationarity, which means that the real exchange rate is a mean reverting process with a high degree of persistence. In terms of half-lives, the classical approach gives a picture of persistence not very different from that of Taylor (2002), whose database we use. However, a more persistent picture is that given by Bayesian approaches. In this case, without any exception, the half lives of 22 countries are higher than those of Taylor (2002). But now, differently from the results of López et al. (2003, 2004), we can claim -with only 15% probability for the region with permanent shocks- that PPP hold, although the density of half-lives computed with Bayesian techniques shows that the 3-5 year consensus is very unlikely, with a probability around 15%, and the probability of half-lives being more than five years is around 70%.

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FIGURES

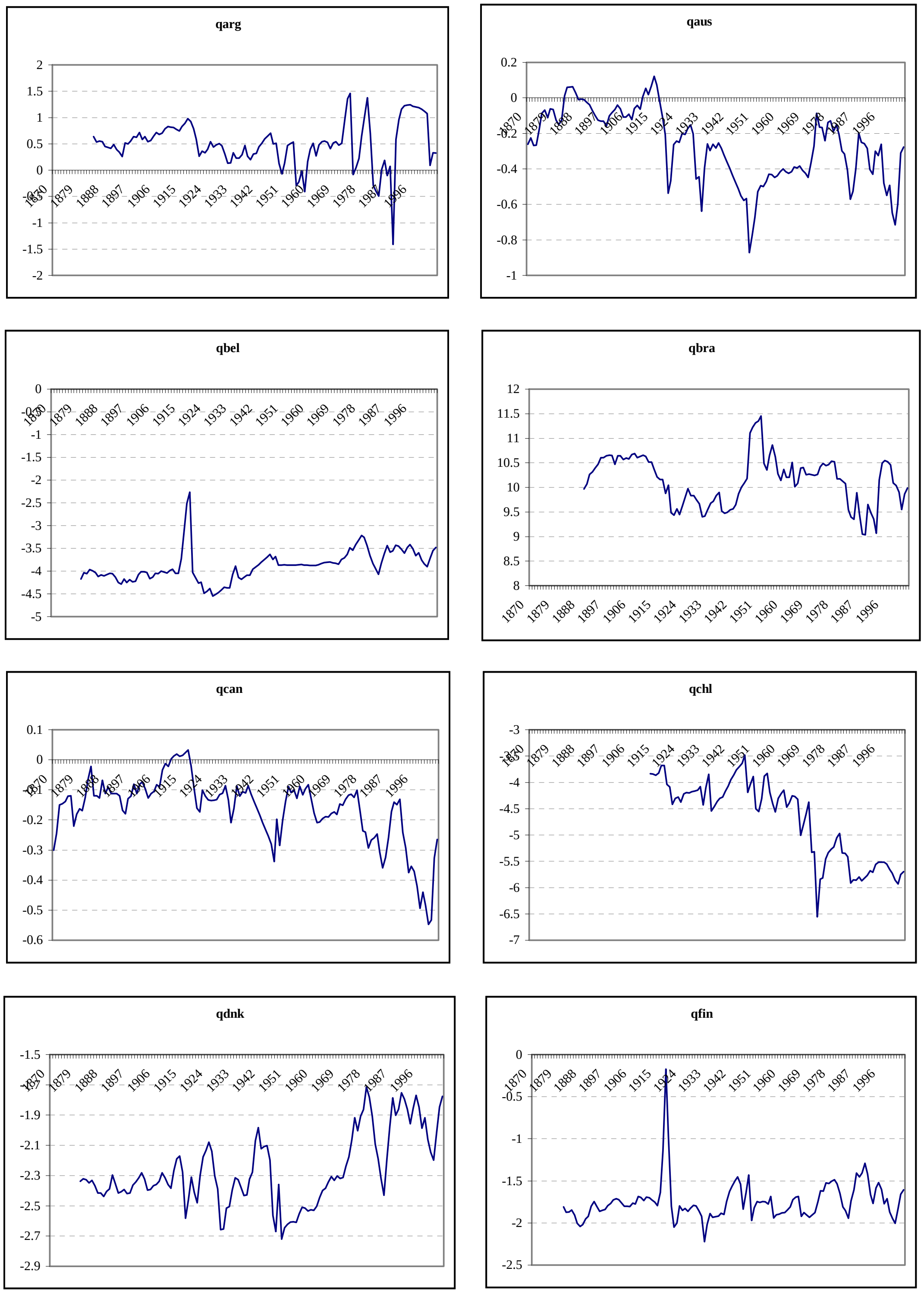


Fig. 1. Evolution of Real Exchange Rates during a century

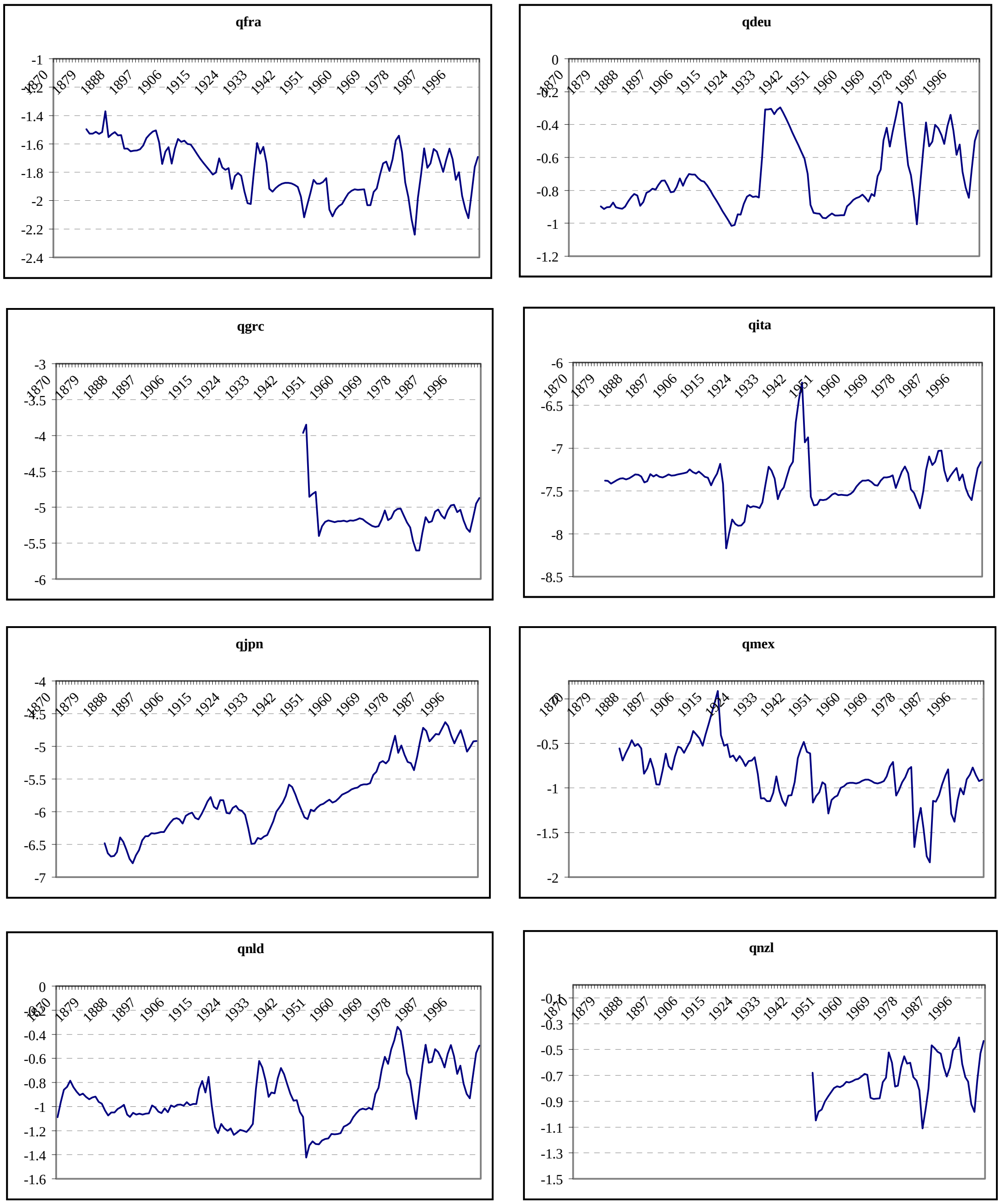


Fig. 2. Evolution of Real Exchange Rates during a century

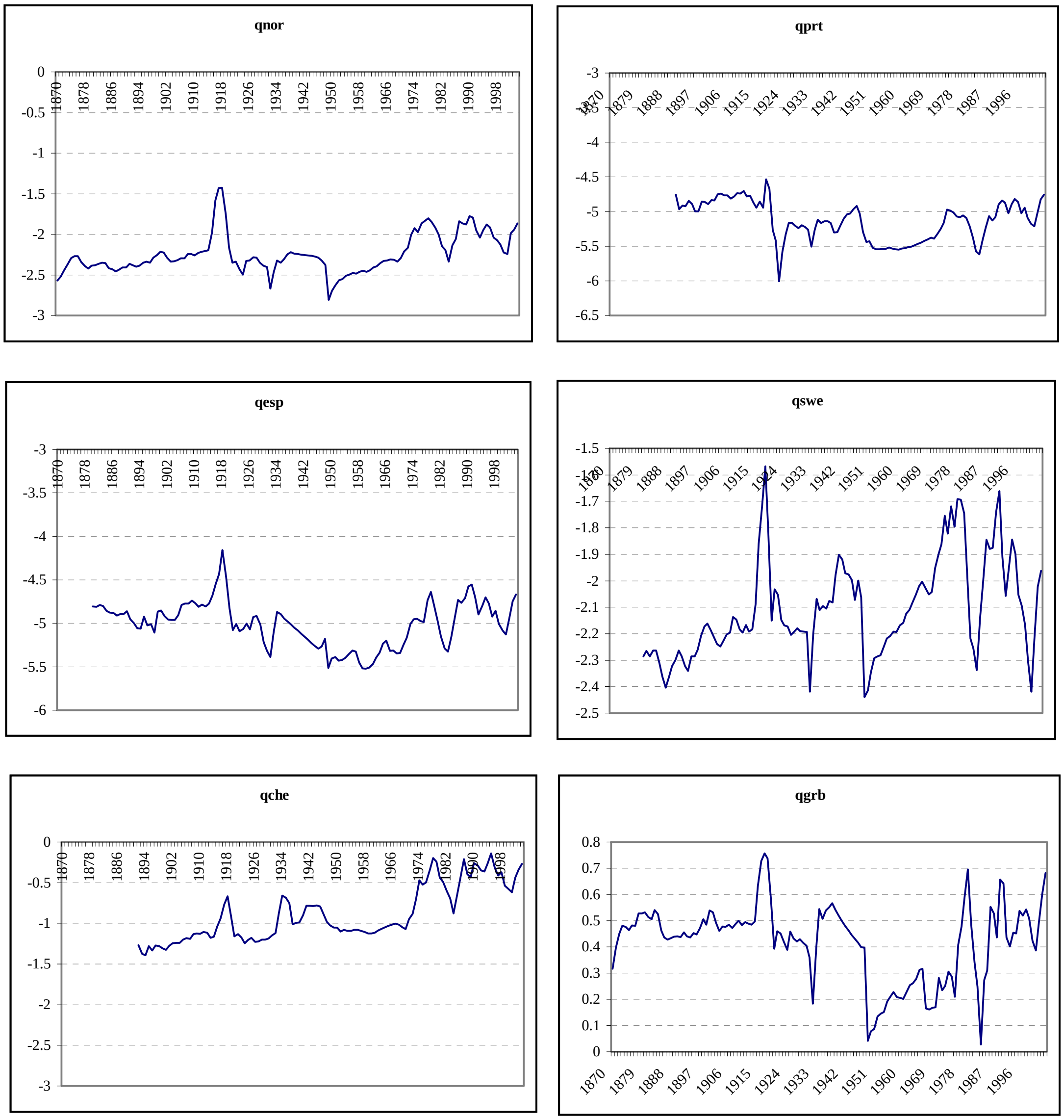


FIG. 3. Evolution of Real Exchange Rates during a century

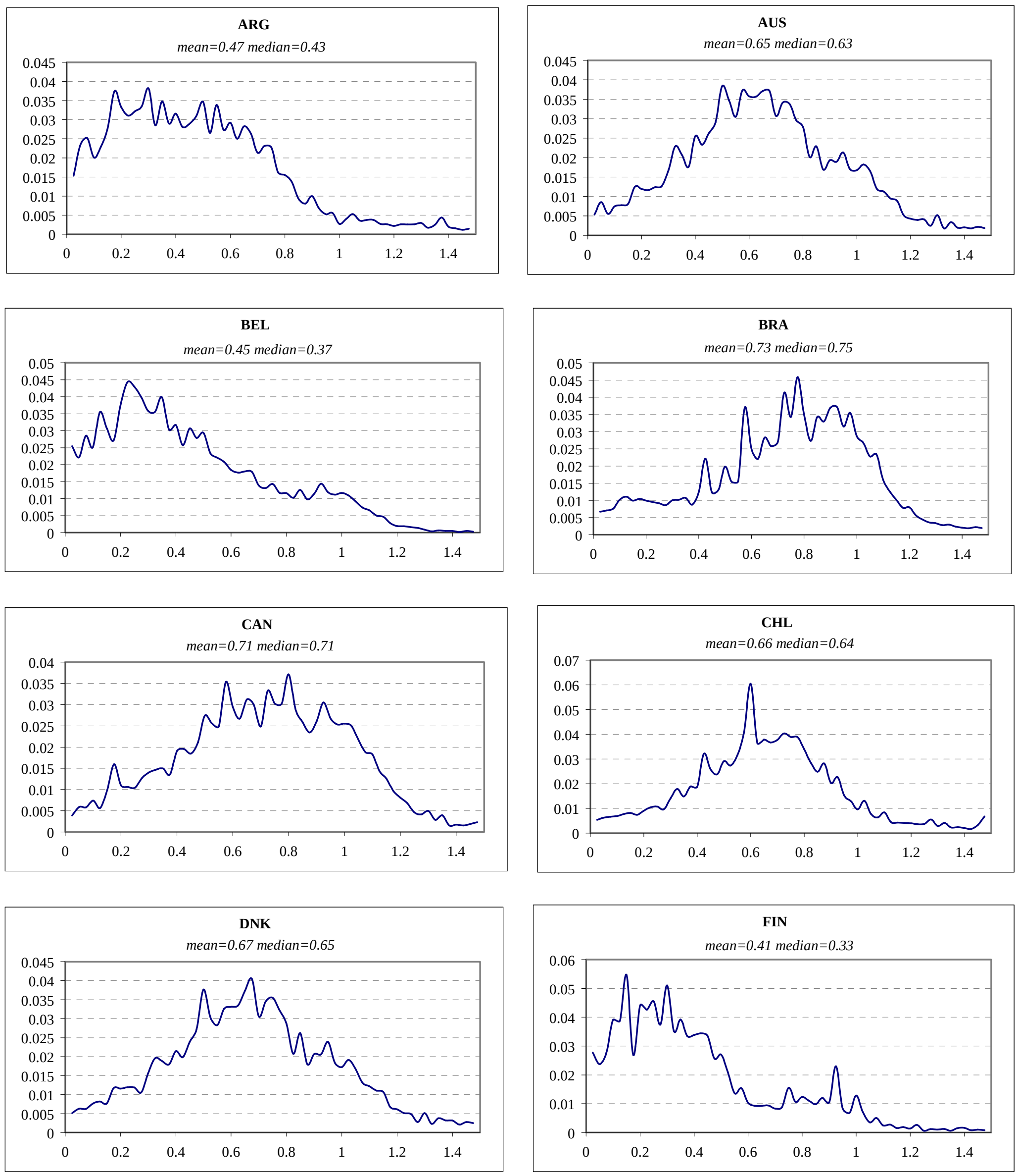


FIG. 4. Posterior density function of d

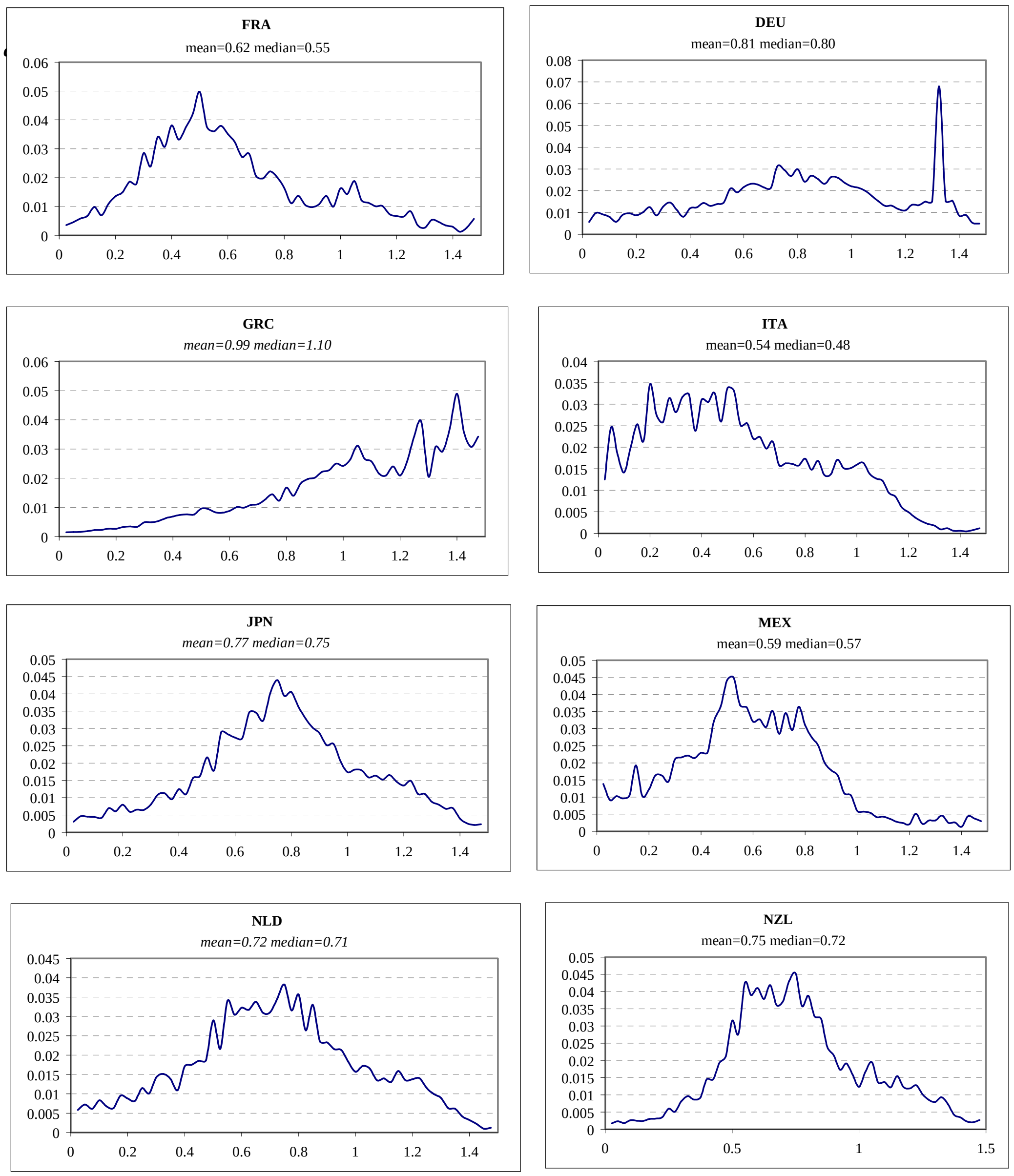
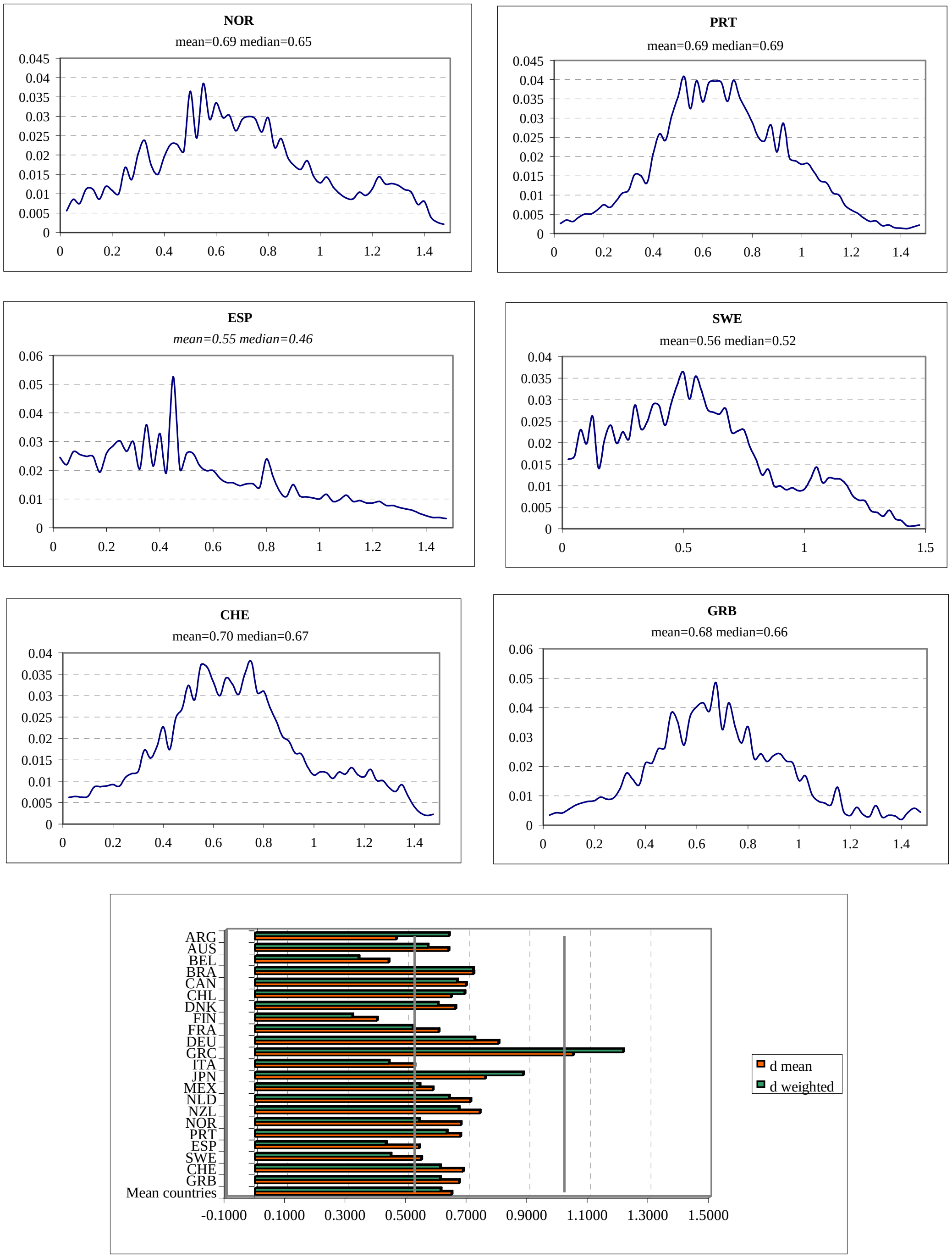


FIG. 5. Posterior density function of d



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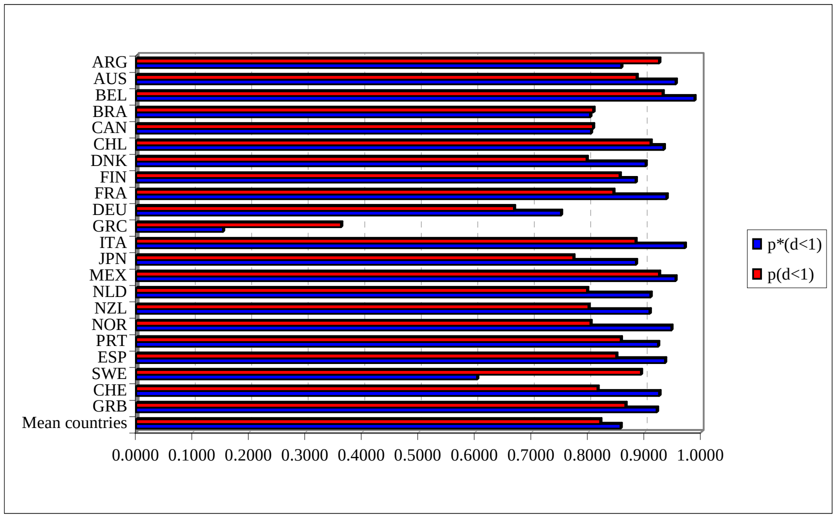

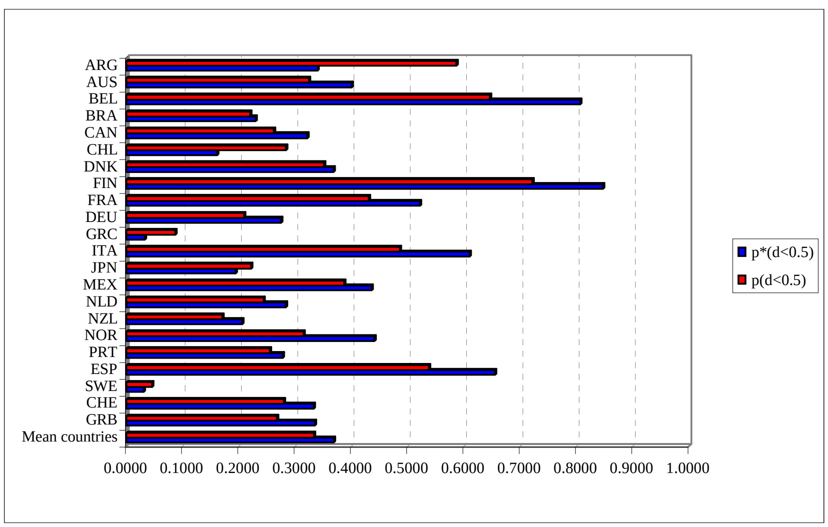

Fig. 7. Posterior probabilities of d

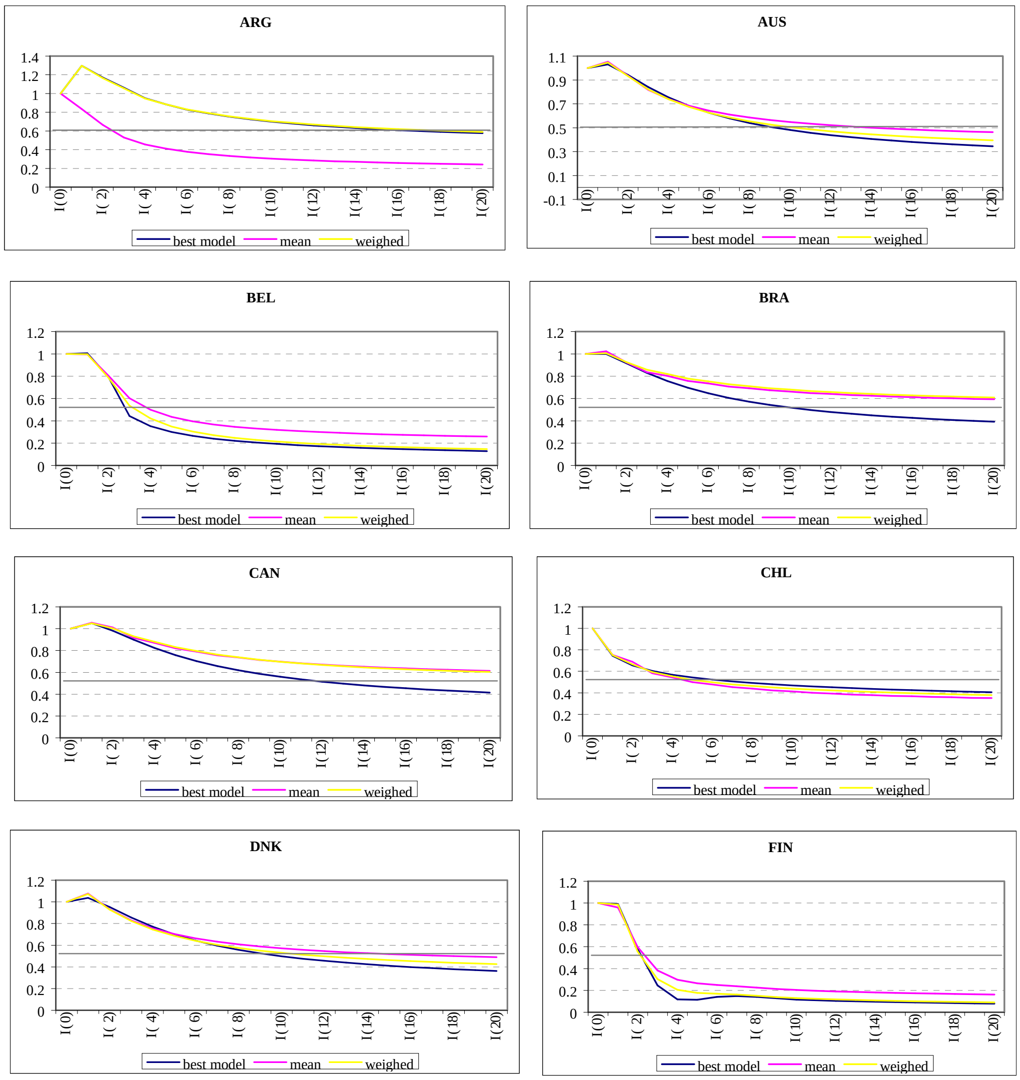

Fig. 8. Impulse response functions

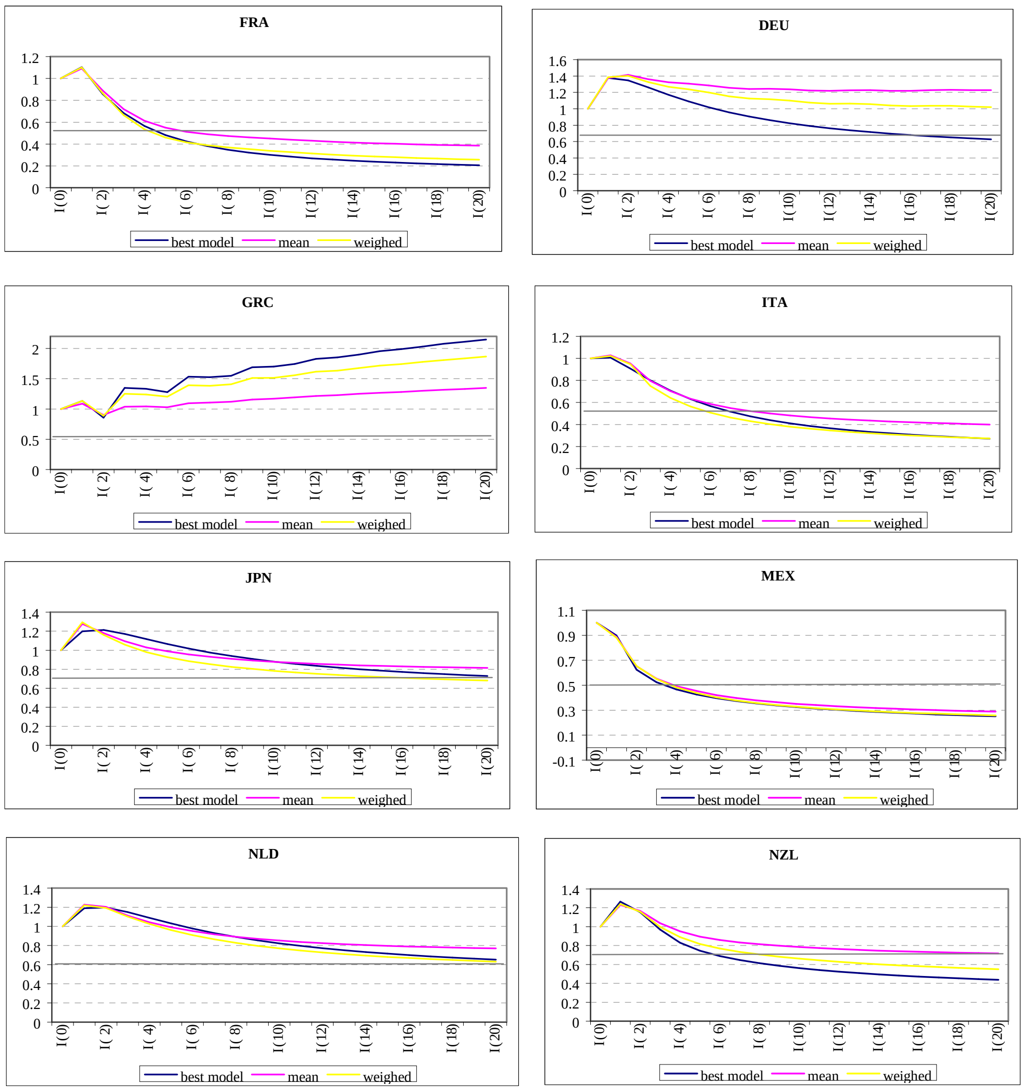

FIG. 9. Impulse response functions

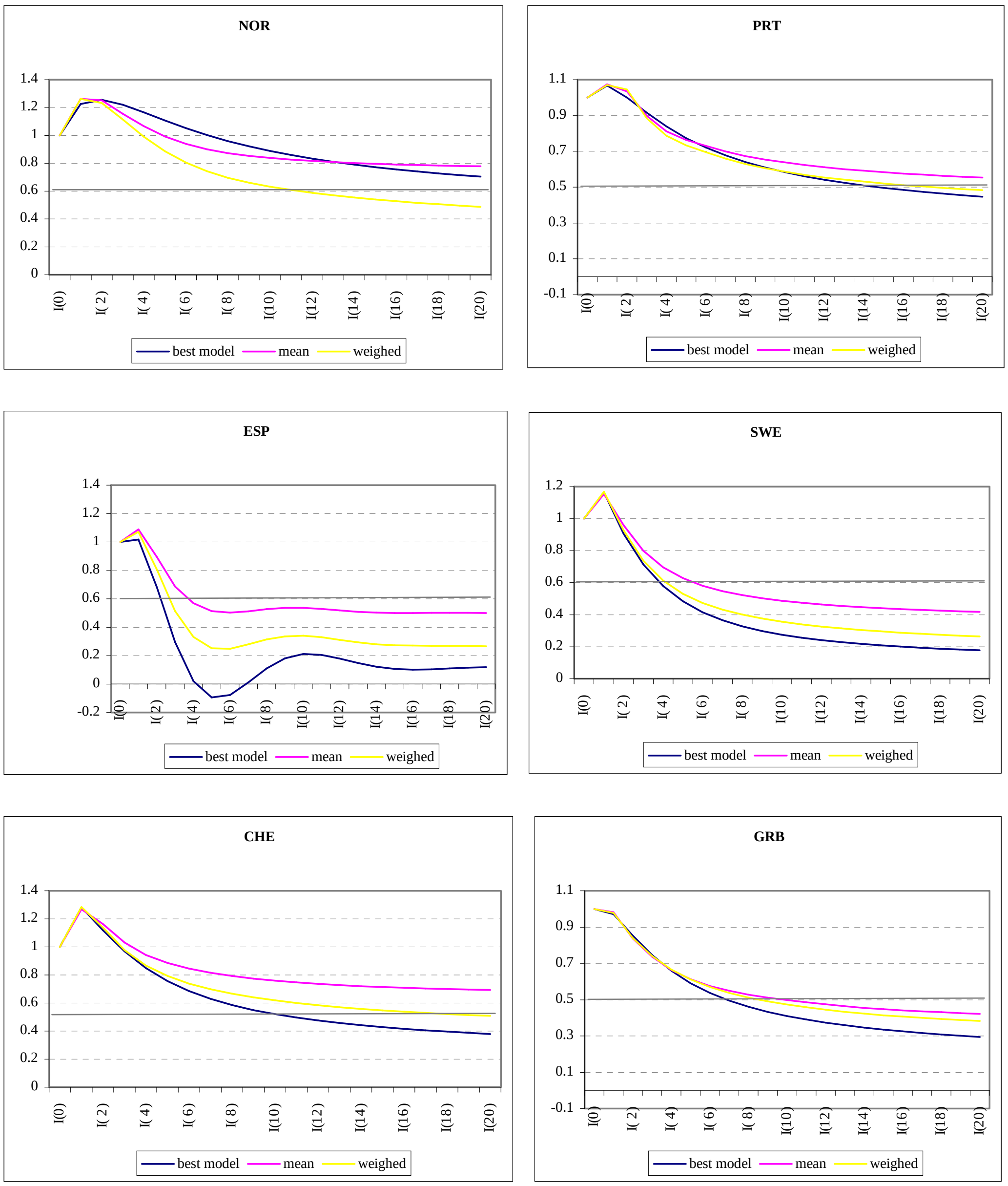

FIG. 10. Impulse response functions

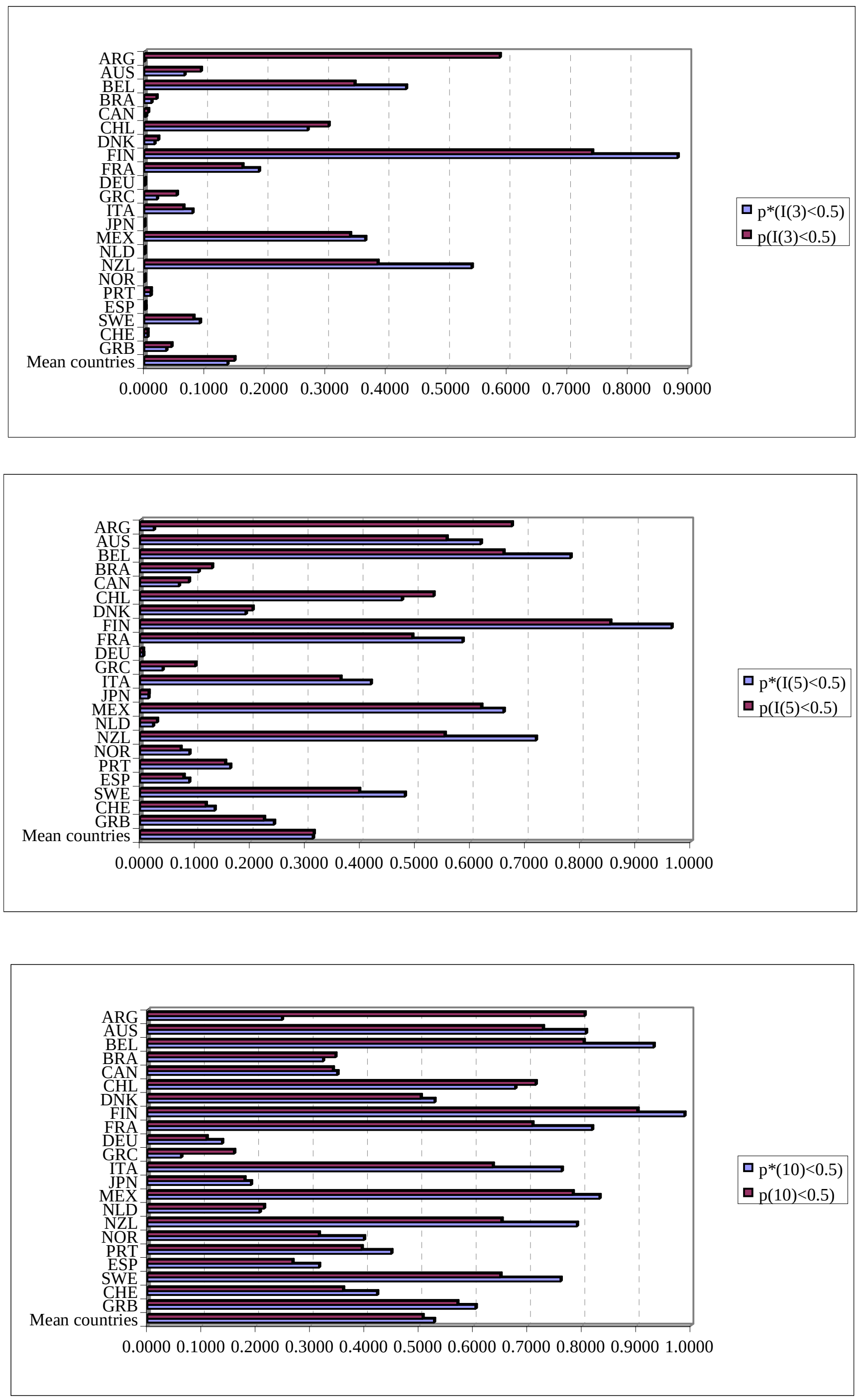

Fig. 11. Probabilities of half life