A median approach to spot foreign exchange rates[*]

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Abstract

Modelling as well as forecasting foreign exchange rates has been and is a major theoretical and empirical challenge. Although economic and econometric theory has advanced noticeably, few approaches have generated significant improvements compared to naive methods. This note provides a fresh look at the issue by proposing what is going to be called the median approach to the determination of foreign exchange rates. The standard method based on rational, objective expectations is treated as a special case.

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1   Introduction

The body of literature on spot exchange rate modelling and forecasting is already impressive but nevertheless still growing. It is not only known for its size but also for its many puzzles that have emerged out of it. Many different approaches try to shed light on some of the enigmatic aspects among which are the difficulties to predict spot rates by forward rates (Wang and Jones, 2003) and the hassles in beating the naive random walk hypothesis in forecasting spot rates (Kilian and Taylor, 2001). Despite the strong implications of the rational expectation hypothesis (REH) and efficient market hypothesis (EMH), some consensus has recently emerged, however, about the quality of long–run versus short–run forecasting. Among others Taylor (1995), Rogoff (1995), Faust, Rogers and Wright (2003) and Obstfeld and Rogoff (2000) claim that new theoretical models are capable of better forecasting at longer–run horizons compared to the very short–run. In contrast to their findings, REH and EMH have it that returns to speculative assets in excess of a save alternative are generally unpredictable.

In this note, I investigate spot rates from an expected profit maximisation point of view. In contrast to most of the standard approaches I discuss exchange rate determination and market activity as two indivisible processes. The outcome implies that spot rates are median values of the future spot rates' probability distribution function, *i.e.* they are not systematically linked to the expected value. This has interesting consequences for the modelling of spot rates. In particular, I am going to offer one explanation as to why the evidence on the REH and EMH almost necessarily have to be mixed if not generally inconclusive.

The following section sets the stage for the *representative investor* in the foreign exchange market. It states basic assumptions and the maximisation problem. Next, the market solution for *many investors* are given. Section 3 provides an extensive comparison to the standard REH and EMH results and suggests simple empirical tests. Finally, conclusions are drawn.

2 The median approach

2.1 The median investment

2.1.1 The profit function

Let's assume an investor at the foreign exchange market who acts as an inter–temporal arbitrageuse. She buys (goes long) or sells (goes short) foreign currency if she thinks the spot rate in the future to be higher (lower) than today's: $s_t < s_{t+1}$ ($s_t > s_{t+1}$), where t denotes time. Accordingly, the sum invested, x_t is either positive or negative. Notice that it is explicitly not assumed, that some future *expected value* of the portfolio is maximised, instead, the return on (the marginal) investment is looked at. It cannot be ruled out, however, that both these alternatives could lead to the same decision.

At time t, s_t is known while s_{t+1} is not. Therefore, the investor needs to formulate an expectation about the future spot rate. The expectation generating mechanism be described by a function $M_{t|\mathcal{I}_t}(s_{t+1} \mid \mathcal{I}_t)$ with $\mathcal{I}_t$ representing all the information in the universe available at t. M is a distribution function about all possible values for s_{t+1} and its shape depends on $\mathcal{I}_t$. Of course, $M_{t|\mathcal{I}_t}(s_{t+1} \mid \mathcal{I}_t)$ is again an element of $\mathcal{I}_t$ and hence, $M_{t|\mathcal{I}_t}(s_{t+1} \mid \mathcal{I}_t)$ is in general not identified. However, be $M_{t,i+1|\mathcal{I}_{i+1,t}}(s_{t+1} \mid \mathcal{I}_{i+1,t})$ a function of $M_{t,i|\mathcal{I}_{i,t}}(s_{t+1} \mid \mathcal{I}_{i,t})$ and $\mathcal{I}_{i,t}$ with $\{M_{t,i|\mathcal{I}_{i,t}}(s_{t+1} \mid \mathcal{I}_{i,t}), \mathcal{I}_{i,t}\} \in \mathcal{I}_{i+1,t}$ and consider the sequence $\{M_{t,i|\mathcal{I}_{i,t}}(s_{t+1} \mid \mathcal{I}_{i,t})\}_{i=0}^{N}$ with $\mathcal{I}_{0,t} \in \mathcal{I}_{1,t} \in \mathcal{I}_{2,t} \in \cdots \in \mathcal{I}_{N,t}$, assumption 1 establishes that $\{M_{t,i|\mathcal{I}_{i,t}}(s_{t+1} \mid \mathcal{I}_{i,t})\}_{i=0}^{N}$ is convergent.

ASSUMPTION 1. *There exists a finite N with $M_{t,N|\mathcal{I}_{N,t}}(s_{t+1} \mid \mathcal{I}_{N,t}) = M_{t,N-1|\mathcal{I}_{N-1,t}}(s_{t+1} \mid \mathcal{I}_{N-1,t})$, $\mathcal{I}_{N,t} = \mathcal{I}_t$ and hence, $M_{t,i|\mathcal{I}_{i,t}}(s_{t+1} \mid \mathcal{I}_{i,t})$ is convergent in i. The function $M_{t,N|\mathcal{I}_{N,t}}(s_{t+1} \mid \mathcal{I}_{N,t})$ is denoted $M_t(s_{t+1})$.*

Assumption 1 ensures that the investor truly uses all necessary information for the investment decision. This decision is made about the amount of x_t to invest which will be based on the following expected profit function, $\pi(x_t)$,

$$\pi(x_t) = x_t \int_{s_t}^{\infty} M_t(s_{t+1}) ds_{t+1} - x_t \int_0^{s_t} M_t(s_{t+1}) ds_{t+1} \tag{1}$$

One may abbreviate $p_2(s_t, x_t) = \int_{s_t}^{\infty} M_t(s_{t+1})ds_{t+1}$ and $p_1(s_t, x) = \int_0^{s_t} M_t(s_{t+1})ds_{t+1}$ noticing that $p_1 + p_2 = 1$ and that they both are functions of x_t since, for example, the x_t has an impact on both, the shape of M and on s_t. The latter is a simple consequence of the law of demand and supply while the first arises from the fact that the distribution function depends on the individual perception of the risk which changes with the net foreign exchange position. Institutional investors will more likely influence $p_{(.)}$ via s_t in due course of their investment, while small private investors are more likely to readily change their perception of the risk involved.

The first argument needs no further justification, for the second I try to give an illustrative example. Imagine an investor who believes the spot rate to be higher next period. Without the second argument she would invest not only until all her wealth is in foreign exchange but she would even pile up infinite debt in order to buy even more of it. It is easy to guess that she would stop investing somewhat earlier since too high a net foreign exchange position appears too risky. Even very large players, like big investment companies or commercial banks know some kind of rules that prevent themselves from building up portfolios with just one single asset. In general, holding s_t constant the absolute change of $p_{(.)}$ due to a certain amount of investment will be related to the investor's risk aversion. It is important to keep in mind that upon acting the investor changes herself. This is different from assuming constancy in the environment and in the investor's preferences as is more commonly done. In the logic of the model, this behaviour is therefore reflected in assumption 2.

ASSUMPTION 2. *The function $M_t(s_{t+1})$ is continuously twice differentiable with respect to x_t and $p_2' \equiv \partial p_2/\partial x < 0$, $p_1' \equiv \partial p_1/\partial x$, $\forall x_t$.*

Thus, there is going to be a one-to-one mapping in the price-quantity space as is always the case in the demand-supply scheme of market economics.

2.1.2 The investment rule

It is now straightforward to mimic the investor's calculation by maximising the expected profit of an investment.

$$\max_{x_t} \pi(x_t) : \frac{\partial \pi}{\partial x} = x_t(p_2' - p_1') + p_2 - p_1 \tag{2}$$

The first order condition can thus be given as

$$p_1(s_t, x_t) = p_2(s_t, x_t) \tag{3}$$

because $\frac{\partial p_2'}{\partial x} - \frac{\partial p_1'}{\partial x} = 0$. The second derivative of $\pi(x_t)$ is zero, hence (3) is an inflexion point of the profit function. Note that $\pi(s_t, x_t = 0) = \pi(s_t, x_t) \mid_{p_1 = p_2} = 0$ and therefore expected profits have a lower bound of zero. Put differently, there are always profit opportunities unless (3) holds. In this sense, (3) is the profit maximisation strategy's optimal point. This strategy is simply to buy (or sell) foreign exchange on the spot until the spot rate equals the future spot rate's median value. Denoting the median of the future spot rate's distribution function by $m_{\mathcal{I}_t}(s_{t+1})$ one can summarise

$$s_t = m_{\mathcal{I}_t}(s_{t+1}). \tag{4}$$

That means that today's and tomorrow's spot rates are not related by the mean of the future spot rate's probability distribution but by its median. This is an important result since – for one thing – it allows to separate the roles of expected and median values. In an extreme case, the mean of tomorrow's spot rate probability distribution need not even exist in order to determine the price of foreign exchange. The existence of the median, however, is a far less restrictive assumption.

2.2 The median investor

Equation (3) establishes the individual investor's decision about buying or selling in the foreign exchange market. As every purchase or sell requires a counterpart, this result alone does not imply a market solution. Such a solution is described in the following.

Assume a finite number $J \geq 1$ of *distinct* investors $j = 1, 2, \ldots, J$, who individually form beliefs about the future spot rate. They offer and demand foreign exchange according to the aforementioned rule. Demand and supply coincide under the following conditions.

DEFINITION 1 (Market clearing and equilibrium price). *The market clears if*

$$\sum_{i=1}^{J} x_{t,i} = 0.$$

The market is in equilibrium if

$$p_2^{(i)}(s_t, x_{t,i}) = p_1^{(i)}(s_t, x_{t,i}) \, \forall i = 1, 2, \ldots, J.$$

The price s_t is the equilibrium price.

Using this definition the market solution can be derived.

PROPOSITION 1 (Median investor outcome). *Under assumptions 1 and 2 there always exists a unique set $S := \{x_{t,1}, x_{t,2}, \ldots, x_{t,J}; s_t\}$ that clears the market.*

Proof. By assumption 2, using obvious notation and definition 1 the system of $J + 1$ equations

$$
\begin{aligned}
m_{\mathcal{I}_t}^1(s_{t+1}) &= s_t \\
m_{\mathcal{I}_t}^2(s_{t+1}) &= s_t \\
\vdots \quad &\quad \vdots \; \vdots \\
m_{\mathcal{I}_t}^J(s_{t+1}) &= s_t \\
x_1 + x_2 + \cdots + x_J &= 0
\end{aligned}
$$

can be solved for the $J + 1$ unknowns in S. Suppose now there are two solutions, say S and S^1. These two sets must differ in either all components or in none because the mapping of x_t on s_t is unique for all individuals. Then according to assumption 1, if $s_t > s_t^1$ it follows $x_{t,j} > x_{t,j}^1$, $\forall j$. Thus $\sum_{j=1}^J x_{t,j} > \sum_{j=1}^J x_{t,j}^1$ and only one of the two sets can be a solution. Hence the market solution is feasible and unique. $\qquad\square$

To illustrate the principle, assume that $p_1^{(j)}(s_t) - p_2^{(j)}(s_t) = \delta$ and δ close to zero. The solution for $x_{t,j}$ would then be equivalent to solving $x_{t,j}\frac{\partial p_1^{(j)}(s_t)}{\partial x} = \delta$. In general, since $p_{(\cdot)}$ is a one–to–one mapping of x_t and s_t there exist a number of algorithms to solve this system of equations even if it was non-linear (see *e.g.* Lütkepohl, 1993, section 7.3.2).

Thus, the market price is s_t and for each investor $s_t = m_{\mathcal{I}_t}^{(j)}(s_{t+1})$. Moreover, proposition 1 implies that upon investing each investor changes the market. This result can be given a structural interpretation by noting that the outcome is a result of each investors' optimising behaviour where all other investor's actions, it ı.e. bids and asks are taken into consideration. In the end, however, everybody assumes the same 'opinion' as their medians coincide. Therefore, this mechanism describes the coordinated and efficient use of all available information on the

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market. An interesting case of proposition 1 is $J = 1$. It follows $x_{t,1} = 0$ and hence no transaction takes place. Nevertheless, the spot rate is defined. However, as no transaction takes place, the 'true' spot rate cannot be observed, instead the last period's spot rate will feature in the statistics. This situation can be seen as if *all* agents expect the same future spot rate, have the same probability distribution in mind including identical attitudes toward risk and accordingly want to either go short or long. The latter makes sure that either no foreign exchange is supplied or no foreign exchange is demanded. It is therefore a matter of taste to call the situation a break down of the market or not.

Borrowing from the public choice literature (Downs, 1957) I suggest to call the the market price the median investor result. This is because the spot price is defined by the price that complies with part one of the market clearing condition and hence by the investor who offers or demands the pivotal investment $x_{t,j}$.

This median principle implies among other things that the notion of 'bears' and 'bulls' in the market is sometimes used abusively (Branch, 2004). This is because the question who is a 'bear' and who is a 'bull' will be answered only in due course of market clearing. Only *after* the announcement of s_t, it is clear who is who. There cannot be such thing as an *a priori* allocation of 'bear' and 'bull' characteristics.

2.2.1   A generalisation

So far I have considered some s_{t+1} as if the future spot rate's was the only concern of the investor. However, one could look at some $s^*_{t+1} := g(s_{t+1}, z_t)$ where z_t reflects all other factors that might be of interest to the investor. For example, the (expected future) interest rate differential between the two currencies, the difference in prices or inflation rates and so on and so forth could enter z_t. The analysis would nevertheless be applicable as long as the investor's (net) gain is given by $s^*_{t+1} - s_t$ and hence an (adjusted) profit function as in (1) can be defined.

Furthermore, as economists are highly aware of utility maximisation $g()$ could as well be thought of as a mapping of utility on s_{t+1}.

3 Comparison to the REH solution

The standard rational expectation approach to exchange rate forecasting is closely related to the efficient market hypothesis and can be summarised along the lines of Timmermann and Granger (2004), for example. Be Q_{t+1} a "pricing kernel" that represents a discount factor due to the degree of risk aversion (or affinity) of an investor. Further, fp_{t+1} is a known return on an alternative investment, then the REH asset pricing rule is

$$s_t = E\left[Q_{t+1}\frac{s_{t+1}}{1 + fp_{t+1}} \mid I_t\right]. \tag{5}$$

Timmermann and Granger (2004) emphasise that (5) is a single moment condition. This evaluation does not change even if more sophisticated versions are considered. Such variants may take care of transaction and information costs, trending variables, trading restrictions, further alternative investment opportunities, and so on.

Under REH, all investors use the *same* functional form for expectation formation, *i.e.* the same probability distribution, the same set of exogenous, or state variables and the same set of information. In other words, there exists an 'objective' probability distribution for $\left[Q_{t+1}\frac{s_{t+1}}{1+fp_{t+1}} \mid I_t\right]$ that all agents either know (simple version of REH) or are able to discover (rational learning, see Pesaran, 1987). The existence of such an 'objective' probability distribution is crucial and difficult to maintain, or as Branch (2004) has it:

> ...even econometricians must approximate the true structure of the economy.
> Given the inability of econometricians to estimate the economic model perfectly, it
> is unrealistic to expect agents to have such ability. Branch (2004, p.593)

Examples where, like in Branch (2004) this assumption is not made are rather rare. The median approach is another one of those.

In principle, the median approach can be regarded a general version of the standard REH approach. To see this one might consider $s_{t+1}^* = \frac{s_{t+1}}{1+fp_{t+1}}$, for example. In fact, combining (5) and (4) gives rise to

$$m_{\mathcal{I}_t}(s_{t+1}^*) = E\left[Q_{t+1}\frac{s_{t+1}}{1 + fp_{t+1}} \mid I_t\right] \tag{6}$$

where the fact is accounted for that the probability distribution of s_{t+1}^* in the median approach fully reflects all issues related to the perception of risk. Thus, median and standard approach

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lead to the same result if Q_{t+1} is chosen appropriately. That is, the introduction of a possibly time varying risk premium reflects nothing but the necessary adjustment of $\frac{s_{t+1}}{1+fp_{t+1}}$ such that the mean value of the newly created variable coincides with the median of the underlying variables' probability distribution. Consequently, when henceforth the asterisk on s_{t+1} is omitted, it should be kept in mind that the results are not affected.

The first most important hypothesis within REH is that the probability distributions across agents are identical since they all are copies of the true, 'objective' distribution. In the median approach we would find $M_t^{(j)}(s_{t+1}^*) = M_t^{(i)}(s_{t+1}^*) = M_t(s_{t+1}^*)$, $\forall j \neq i$ and hence $J = 1$. Therefore, the REH solution is a special case. I leave it to the standard REH to identify the quantities of foreign exchange traded and sideline this issue for the moment.

I now look at the possibility to obtain an objective probability distribution for s_{t+1}. Objectivity is obtained when the subject does not play a role, that is J does not affect the market outcome. Such a situation is commonly characterised as the presence of small, negligible investors, or as atomised markets and so on. In the median approach the first distinction has to be made between a probability distribution conditional on $x_t^j = 0$, $\forall j$ and $x_t^j \neq 0$. In the first case, I suppose that there may exist a probability distribution over all $m_{\mathcal{I}}^j(s_{t+1} \mid x_t^j = 0)$, $\forall j$. In this case we would necessarily find $s_t = s_{t+1}$ and spot price determination would not be an issue. In other words, the only interesting question is whether there is a probability distribution for $m_{\mathcal{I}}^j(s_{t+1})$ in proposition 1 that is independent of J. The answer is no and will be justified next.

Given the median model, all information sets, all individual probability distributions and proposition 1, then, s_t can be calculated. I define

$$\mu^{(J-1)} \quad := \quad \frac{1}{J-1} \sum_{i=1}^{J-1} m_{\mathcal{I}}^i$$

The $\mu^{(J)}$ and $s_t^{(J)}$ are defined accordingly. Being the Jth investor the pre-condition for objectivity would therefore be

$$\mu^{(J)} = \mu^{(J-1)} \tag{7}$$

That means that the individual medians converge to a fixed number, such that if the pool of investors was growing, the observed medians would converge to a stationary number.

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The answer to the question whether there is a J for which condition (7) hold is, however, negative. Notice first that by proposition 1 $s_t^{(J)} - s_t^{(J-1)} = \gamma \neq 0$. Then write

$$\begin{aligned} \mu^{(J)} &= \frac{1}{J-1}\sum_{i=1}^{J-1} s_t^{(J-1)} - \frac{1}{J(J-1)}\sum_{i=1}^{J-1} s_t^{(J-1)} + \frac{1}{J}s_t^{(J-1)} + \frac{1}{J}\sum_{i=1}^{J}\gamma \\ &= \mu^{(J-1)} + \frac{1}{J}s_t^{(J-1)} - \frac{1}{J}\mu^{(J-1)} + \frac{J\gamma}{J} \end{aligned}$$

to see that only the two middle terms disappear for large J. But $\lim_{J\to\infty} \frac{J\gamma}{J} = \gamma$ and therefore a limiting value for $\mu^{(J)}$, $J \to \infty$ does not exist. Hence, s_t is nonstationary in J and an objective distribution probability does not exist. Notice also that as J increases, the variance of s_t increases at the same rate. That could be an explanation why the volatility of spot market rates with many participants is much larger than the volatility of forward rates with far less investors.

Observe furthermore, regardless of whether we abstain from taking an active part in the market or not, we could make perfect predictions about s_t. As soon however, as we would like to take advantage of this knowledge, our business opportunities would be gone because we would become part of the market clearing mechanism. Put differently, the only way to beat the market, is to not strive for profits.

Another common feature of the REH and the median approach is that both can be linked to EMH. For the median approach the argument can be cast as the determination of the value or price of the perfect forecast model (pfm). Suppose therefore that the pfm exists. Perfection can be described as a probability: $Prob(s_t = s_t^{pfm}) = 1, \forall t$ where s_t^{pfm} is the forecast of s_t generated by the pfm. I now ponder two possibilities. Either the model is used for *describing* the economy or its is used for *earning money* and it thus potentially has a price.

If it used for descriptive purposes, it is fine. If, however, it is going to be used for the alternative, it will result in the destruction ob both, the model *and* the market: First note that a pre-condition for earning money would be that the model predicts $s_t^{pfm} - s_{t+k}^{pfm} = \delta \neq 0$, for some $|k| > 1$. Second the profit maximising investment would be: $\max_{x} \delta x$ which implies that the optimal $x = \pm\infty$ depending on the sign of δ. Thus the pfm would lead to the investment of an infinite sum which means that the whole market is absorbed by the pfm investor. The implication is straightforward: the price of the currency, s_t jumps to s_{t+k}, the profit opportunity vanishes and hence $Prob(s_t = s_t^{pfm})$ drops below one. In other words, upon acting along the lines of the pfm changes the pfm. However, instead of investing an infinite sum, the pfm investor

may be tempted to restrict herself to a finite amount, such that the impact on s_t goes unnoticed at large. Consequently, the potential benefit of the pfm is at least limited by the maximum amount that would not affect the price, denoted $\underline{x}$. However, even this strategy will not work, at least not repeatedly. Following assumption 1 $n > 0$ $(J - 1 \geq n)$ fellow investors will notice that there is one strategy that generates sure profits. In order to also participate they are going to mimic this successful strategy, and soon the price is again moving towards the zero profit line as $(n + 1)\underline{x} \gg \underline{x}$. Moreover as n increases not only falls $Prob(s_t = s_t^{pfm})$ to zero, but also J decreases. In fact, J approaches one. In sum, applying the perfect forecast model destroys both, the model and the market.

The price of the pfm is therefore zero and the efficient market hypothesis also holds in the median approach. Furthermore, in order to prevent a pfm which is made for purely descriptive purposes from annihilation the researcher better hides the model. Thus, either way, even though a perfect forecast model may exist, it is hardly ever to surface.

The results of this section thus imply that the true pfm are more likely to be a very dynamic herd constantly changing its leader rather than featuring one bright star all the time. It is therefore imperative for econometricians and macro-modellers alike to expect frequent adjustments of their models to be the rule rather than an exception.

The main result of this section has also been reached by Timmermann and Granger (2004) which gives rise to another similarity with the REH and the median approach. Timmerman and Granger point out that asset price forecasting under the REH need to be seen in relation to the inevitable uncertainty of model selection. Due to the ever revolving model selection process, the 'objective' probability distribution can hardly be observed and asset price are inherently going to be nonstationary.

3.1 Caveats

The median approach mainly rests on a single behavioural assumption. This assumption is profit maximisation. It cannot, however, be taken for granted at all times that foreign exchange is only bought for this purpose. The most frequent exception certainly is the demand for service and goods transactions. For example, Europeans travelling the United States and vice versa pay their hotel bills with their credit cards hardly ever noticing the foreign exchange markets. To them the

world might look as if $s_t = s_{t+1}$ at least during their holidays. The decision about the holiday destination and hence about the expected exchange rate is taken some time in advance and has therefore little to do with the liquid, fast clearing market dynamics referred to in the paper. On the other hand, this kind of transaction mainly affects the currency, but not the foreign exchange markets, which are two different, although not independent facilities. Likewise, small to medium enterprises with comparatively low trading volume outside their own currency can be regarded active on the currency market. To the extent their non-profit maximising activities spill over to the foreign exchange market knowledgeable foreign exchange market investors may make a sure profit. Of course, the larger those profits the larger the incentives of the firms and individuals to manage their foreign exchange more carefully. No wonder therefore, that big international companies run their own foreign exchange management departments.

The second, maybe more important class of investors without profit intentions are certainly central banks. Though it is not clear what the true intentions are – *e.g.* Beine, Laurent and Christelle (2003) argue in favour of exchange rate smoothing while Fischer (2003) concedes profit objectives – it will be assumed in the following. In contrast to the previously mentioned agents, central banks cannot be regarded negligible. Note, while proposition 1 implies that save profits are not available, it also implies that it is possible to change the price in a predictable manner. Therefore, systematic profit making is not an option, but exchange rate management is, although it may become a bit expensive. In fact, since every bid or ask serves as a signal to fellow investors, a central bank may be mistaken as such and it may move the spot rate in the desired direction. It could be investigated whether this intervention possibility was the more effective the less other investors are aware of the non-profit seeking nature of the corresponding signals. The apparent ex-ante secrecy under which central banks intervene in the foreign exchange market may at least be a hint that covered interventions are regarded more efficient.

The second caveat comes from the fact that the major assumption builds on the difference between today's and tomorrow's spot rate. If there was no, it is held, there is no transaction. This might not be quite true, as has already been argued. In fact, in this particular situation the trade volume is not identified and can therefore be anything between zero and infinity. On the other hand if $s_t = s_{t+1}$ was credible, then the two currencies were just like a currency union and spot rate determination would not be an issue.

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3.2 Putting the theory to a test

Testing REH and EMH are not easy tasks. Pesaran (1987) has already noticed that the impossibility to directly observe expectations invokes indirect tests of the REH. These, however

> can be carried out only conditional on the behavioural model .... This means
> that conclusions concerning the expectations process will not be invariant to the
> choice of the underlying behavioural model. Pesaran (1987, p.22)

Therefore, Pesaran goes on, the REH is 'immunized' against possible falsification by the 'inherently unobservable nature of expectations' (Pesaran, 1987, ibid.).

The case is not much different for the median approach. I may nevertheless propose three directions of research. First, I notice that a REH derivative is also present in the median model. In contrast to the standard REH with an objective probability distribution for s_{t+1}, it suggests a generic probability distribution under proposition 1. Conditioning on $x_{t,j} = 0$, however, gives rise to a possibly stationary probability distribution also in the median approach. For the latter predicts zero turnover at the foreign exchange market and constant prices under this condition, a simple test of whether the median approach can be reduced to its special case, standard REH is to look at the trade volumes. A regular observation is, that volumes are not zero. There are, however times, when they are, e.g. between two ticks. During that time span, the standard REH suffices.

Second, perfect knowledge of all market features allows to correctly forecast spot rates even though this is certainly very difficult. At the same time making profits out of this knowledge should be impossible. Therefore, the median model would be falsified if one can show that both is possible simultaneously: forecasting spot rates *and* making profits. Note this does not mean to relax oneself in an as-if-position. It implies to actually invest in the foreign exchange market. If the researcher or her sponsor shies away from this research path, she might collect individual actual investment data instead. Note again, it is not possible to consider some average value. This however, incurs the next problem. How to detect a superior investment strategy that would defy the EMH? For one thing, it is not sufficient to notice that someone invested profitably. It has to be proven, that the *individual* success was *systematically better* than the rest of the market. That is, repeated gains have to be observed. As a first shot one might

look at the track record of investors. A rational investor in an efficient market would have a fifty-fifty chance of success. Investing twice, the chance of having been successful both times is .25 only, and so on and so forth. Unfortunately, the probability of winning all the time is never going to be zero. Therefore, a comparison has to be made to the other investors' performances. Here yet another problem arises because under REH/EMH the individual observations are not independent of one another. I leave it to future research to hopefully find a way out of these hassles.

Finally, in contrast to the objective probability distribution approach where it is assumed that investors refer to the same expectation generating mechanism, the median approach supposes J distinct investors. Therefore, as J increases, the variance of the foreign exchange price varies too. This is not the case within the standard approach since the more investors are active, the more information should be available about the expectation mechanism. By the law of the large number the foreign exchange price should thus oscillate around its expected value with an ever smaller variance as J becomes larger.

4   Summary and conclusions

This paper suggests a model for the determination of spot rates which emphasises the role of the median of the probability distribution for the price of foreign exchange. The according model is called median approach. It is based on a profit maximisation assumption and provides structural explanations for the frequent finding of nonstationary exchange rate data. The nonstationarity has two dimension, one is time, the other the number of market investors.

The median approach can be regarded a generalisation of the standard rational expectation hypothesis approach. Next to being less restrictive the new model can be regarded simpler. It is hunted, however, by similar problems when it comes to empirical tests of the model.

Among other things the new method implies that forecasting exchange rates is, in principle, feasible, but a good forecast model should be hidden to make sure its quality is preserved. Furthermore, foreign exchange market investors should detect investors who are not interested in profit making for making profits themselves.

Finally, it might be worthwhile to investigate whether the present approach can be gener-

alised to other asset prices.

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