Monetary Policy and Model Uncertainty in a Small Open Economy*

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Abstract

We study the effects of optimized monetary policy in a semi-structural, estimated small open economy in situations where the policymaker has either complete or less than complete confidence in the model being free from misspecification errors. We use the robust control techniques developed in Dennis, Leitemo, and Söderström (2005).

We find that irrespective of the level of confidence to the model, the more or less balanced weights on forward-looking and backward-looking components in the behavioral equations together with a completely forward-looking uncovered interest parity condition, produces large gains in commitment. The central bank may reduce loss by more than 80% by making a commitment.

If the policymaker lacks confidence in the model specification, a robust policymaker mainly fears that the exchange rate and domestic inflation equations are misspecified. Consequently, the robust policy is designed primarily to counteract these types of potential misspecifications. Policy becomes more aggressive towards all shocks. Although the exchange rate equation provides great opportunities for worst-case distortions, the exchange rate channels also provide ample ways in which policy can counteract distortions to the model, especially under policy commitment. The exchange rate channels can hence be viewed as both a curse and a blessing to the policymaker.

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1 Introduction

By its very nature, models are manageable approximations of reality. Modeling is about reducing and simplifying reality by focusing only on the relevant mechanisms that affects the objective of your study. All models are hence misspecified by construction. A good model is however misspecified in such a way that the distortions do not influence the objective variables of the model. Policy analysis has traditionally *by assumption* relied on the model having this property. In other words, the policymaker is assumed to have complete confidence in his or her model. It is our experience that this assumption does not hold in practice. It is rarely the case that the policymaker relies completely on the model - he usually use other sources of information in designing policy. This does not necessarily imply, however, that the policymaker has strong beliefs about *how* the model is relevantly misspecified.

Robust control theory offers methods to deal with policy design without making the assumption should have complete confidence in his model. In fact, it assumes that the policymaker doubts his model, but cannot come up with a probability distribution of the possible misspecification. The policymaker is, however, willing to pay a price in the form of higher loss in order to reduce the impact of possible misspecifications in the model. The less confident the policymaker is about the model, the higher price he is willing to pay for such an insurance.

In this study we discuss how an inflation targeting central bank is affected by model uncertainty under the traditional assumptions about policy loss for an inflation-targeting central bank. The central bank cares about stabilizing both the inflation and output gaps in addition to stabilizing the nominal interest rate. Whereas similar studies of model uncertainty in monetary policy models have been directed at obtaining analytical solutions and study the qualitative implications of model uncertainty (see, Leitemo and Söderström, 2004, 2005, e.g.,), this study makes quantitative assessment of in what way and to what extent policy should be affected. We use the robust control techniques developed in Dennis, Leitemo, and Söderström (2005). These techniques allow misspecifications in both the coefficient of the model and the shock properties of the equations.

We use an empirical version of the New Keynesian Open economy model due to Monacelli (2003) that is specified along the lines suggested by Rudebusch (2002b,a) and estimated in Leitemo (2005) using UK data. Agents are rational and solves optimization problem in order to maximize utility and profits. The model allows for both domestic goods producers and an imported goods sector. Price setting is staggered as in Calvo (1983) in both the domestic and imported goods sector. Consumers are intertemporal utility maximizers across both domestic and imported goods.

In Section 2 we start by presenting the theoretical model and then its empirical counterpart estimated on UK data. In Section 3 we present the methods used in deriving the equilibria when the policymaker and the agents have preferences for model robustness. Section 4 applies these methods in order to derive the policy outcomes for given preferences for robustness in the empirical model of section 2. Section 5 concludes.

2 The rational expectations environment

We study a model in a dynamic general equilibrium setting under the assumptions of price stickiness and monopolistic competition pioneered by (Yun, 1996; Rotemberg and Woodford, 1995, 1997; McCallum and Nelson, 1999). Production is demand determined and there are price contracts as described by Calvo (1983). Clarida, Galí, and Gertler (1999) explore the monetary-policy implications and Woodford (2003) explores several realistic extensions to the basic model along with their policy implications. Clarida, Galí, and Gertler (2001); Clarida, Gali, and Gertler (2002) extends the basic model to the open economy and claims that the model exhibit a form of isomorphism to the closed economy. Monacelli (2003) shows, however, that this property is lost if imported goods are subjected to price stickiness.

In the next section we give a brief presentation of the Monacelli (2003) model

2.1 Domestic households

The household lives forever and expected utility is given by

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\varphi} N_t^{1+\varphi} \right)$$

where

$$C_t \equiv \left[(1-\gamma)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta-1}{\eta}} + \gamma^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}$$

is a Dixit-Stiglitz aggregator of domestically produced and differentiated goods, and imported differentiated good, indexed by $C_{H,t}$ and $C_{H,t}$, respectively, and N is labor hours. The cost-minimizing intratemporal allocation of consumption across different types of goods implies the standard demand functions:

$$\begin{aligned} C_{H,t} &= (1-\gamma)\left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t, \text{ and} \\ C_{F,t} &= \gamma\left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t, \end{aligned}$$

where the consumer price index is given by

$$P_t = \left[(1-\gamma)\, P_{H,t}^{1-\eta} + \gamma P_{F,t}^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{1}$$

The household maximizes lifetime utility subject to standard budget constraint. Under the assumption of complete markets for state-contingent money claims expressed in units of domestic currency, the log-lineared first-order conditions are

$$\begin{aligned} w_t - p_t &= \sigma c_t + \varphi n_t, & (2) \\ c_t &= E_t c_{t+1} - \frac{1}{\sigma}\left(r_t - E_t \pi_{t+1}\right), & (3) \end{aligned}$$

where w_t is nominal wage, n_t is labour hours, r_t is the nominal interest rate and $\pi_t \equiv p_t - p_{t-1}$ is the CPI inflation rate. We get an expression for CPI inflation through log-linearizing equation (1) and taking differences,

$$\pi_t = \pi_{H,t} + \gamma \Delta s_t \tag{4}$$

where $s_t \equiv p_{F,t} - p_{H,t}$ is the log terms of trade. Moreover, since we treat the foreign country as approximately closed, we have that $\pi_t^* = \pi_{F,t}^*$, an equivalence between foreign CPI inflation and foreign price inflation on the foreign good. This result implies a particular relationship between the real exchange rate and the terms of trade,

$$\begin{aligned} q_t &\equiv e_t + p_t^* - p_t \\ &= e_t + p_t^* - \gamma p_{F,t} - (1-\gamma) p_{H,t} \\ &= e_t + p_t^* - \gamma p_{F,t} - (1-\gamma) p_{F,t} - (1-\gamma) p_{H,t} + (1-\gamma) p_{F,t} \\ &= \psi_{F,t} + (1-\gamma) s_t \end{aligned}$$

where e is the nominal exchange rate and

$$\psi_{F,t} \equiv e_t + p_t^* - p_{F,t}$$

is the law-of-one price gap - the deviation of the world price (measured in domestic currency units) from domestic currency price of imported goods.

2.2 Domestic producers

There is a continuum of domestic firms, indexed over the unit interval, operating in a setting of monopolistic competition. The production technology is given by

$$y_t = z_t + n_t$$

where z_t is log labour productivity. The marginal costs measured in units of domestic good is then given as

$$mc_t = (w_t - p_{H,t}) - z_t. \tag{5}$$

The firms are subject to Calvo pricing and receives a signal to adjust prices in each period with probability θ_H. Gali and Monacelli (2004) show that this leads to inflation following the forward-looking Phillips curve

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \lambda_H mc_t, \tag{6}$$

where $\lambda_H = \frac{(1-\theta_H)(1-\beta\theta_H)}{\theta_H}$.

2.3 Imported goods sector

We assume that the imported goods sector consists of several firms that channel different foreign goods into the domestic economy without spending resources in doing so. Domestic pricing is subject to Calvo pricing and firm receive a signal to adjust prices with probability θ_F. Monacelli shows that the firms adjusts prices in such a way that imported goods prices follows the forward-looking process:

$$\pi_{F,t} = \beta E_t \pi_{F,t+1} + \lambda_F \psi_{F,t},$$

where $\lambda_F = \frac{(1-\theta_F)(1-\beta\theta_F)}{\theta_F}$. Imported goods prices rises as long as the foreign prices exceeds the domestic price on imported goods.

2.4 International risk sharing and marginal costs

Under complete international risk sharing, Gali and Monacelli (2004) show that the real exchange rate varies with relative consumption, so that

$$\begin{aligned} c_t &= c_t^* + \frac{1}{\sigma} q_t \\ &= c_t^* + \frac{1}{\sigma} \left[(1-\gamma) s_t + \psi_{F,t} \right]. \end{aligned} \tag{7}$$

Under complete international asset markets, the uncovered interest parity condition holds

$$e_t = E_t e_{t+1} - r_t + r_t^*.$$

By using equation (5) in combination with equation (2) and (7), we can show that marginal costs satisfies

$$mc_t = \varphi y_t - (1+\varphi) z_t + \sigma y_t^* + s_t + \psi_{F,t} \tag{8}$$

which also constitutes the labor market equilibrium. Monacelli shows that marginal costs may be written as

$$mc_t = \left(\varphi + \frac{\sigma}{\omega_s} \right) x_t + \left(1 - \frac{\omega_\psi}{\omega_s} \right) \psi_{F,t} \tag{9}$$

where $x_t \equiv y_t - \bar{y}_t^n$ is the output gap and $\bar{y}_t^n = \left(\frac{\omega_s (1+\varphi)}{\sigma + \varphi \omega_s} \right) z_t + \left(\frac{\sigma(1-\omega_s)}{\sigma + \varphi \omega_s} \right) y_t^*$ is the flexible price level of output. By using equation (9) in (6), the domestic inflation equation becomes

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa_x x_t + \kappa_\psi \psi_{F,t}, \tag{10}$$

where $\kappa_x \equiv \lambda_H \left(\varphi + \frac{\sigma}{\omega_s} \right)$ and $\kappa_\psi \equiv \lambda_H \left(1 - \frac{\omega_\psi}{\omega_s} \right)$.

2.5 Demand

Monacelli (2003) shows that using market clearing conditions with (7), and making use of the output gap definition and equation (4), equation (3) can be written as

$$x_t = E_t x_{t+1} - \frac{\omega_s}{\sigma} \left(r_t - E_t \pi_{H,t+1} - r\bar{r}_t \right) + \Gamma_x E_t \Delta \psi_{F,t+1} \tag{11}$$

where $\Gamma_x \equiv \left(\frac{\gamma(1-\gamma)(\sigma\eta - 1)}{\sigma} \right)$ and $r\bar{r}_t \equiv \sigma \left(\frac{\varphi(\omega_s - 1)}{\sigma + \varphi\omega_s} \right) E_t \Delta y_{t+1}^* - \left(\frac{\sigma(1-\rho)(1+\varphi)}{\sigma + \varphi \omega_s} \right) z_t$ is the natural rate of interest.

The model is summarized by equations (10), (2.3), (11) and (2.4).

2.6 The empirical model

Although the above theoretical framework gives a canonical representation of private-sector behavior in an economy where goods prices are subject to stickiness, the framework abstracts from possible information and implementation lags that may give rise to gradual adjustment in the real world. Such inertial responses may be rationalized and explained by agents using rule-of-thumb pricing (Christiano et al., 2005), and consumers being subject to habit formation (Fuhrer, 2000).

We adopt the empirical specification of the Monacelli (2003) model as estimated by Leitemo (2005). He follows Rudebusch (2002a,b) in allowing data to influence the structure of leads and lags in the economy and add an assumed white-noise error term to each estimated equation. The estimation strategy chosen implies starting out with a general lag structure with four lags of the endogenous variable and then reduce the structure by eliminating insignificant lags, starting with the least significant one. However, insignificant lags that are important for the residuals to be relatively free of serial correlation are not eliminated. As in Rudebusch (2002b), Leitemo (2005) use the expected annual inflation over the next year to represent the forward component of inflation in the Phillips curve, assuming that prices, on average, are changed once a year. The decisions are subject to a one quarter implementation lag. Moreover, all equations are estimated with a (non-reported) intercept term. The model is estimated on UK data obtained from either the national accounts or the IMF and OECD databases.

The Phillips curve for domestic inflation is estimated as

$$\pi_{t+1}^{H} \quad = \quad \mu_H E_t \bar{\pi}_{t+4}^{H} + (1-\mu_H)\sum_{j=0}^{3}\alpha_j \pi_{t-j}^{H} + \kappa_x E_t x_{t+1} + \kappa_\psi E_t \psi_{t+1}^{F} + \varepsilon_{t+1}, \qquad (12)$$

where $\pi_t^H \equiv 4(p_t^H - p_{t-1}^H)$ is the quarterly percentage increase in the GDP deflator measured as an annual rate, and $\bar{\pi}_t \equiv \frac{1}{4}\sum_{j=0}^{3}\pi_{t-j}$ is the four-quarter inflation rate. The estimation period is $1980Q1 - 2001Q4$ and the model is estimated by GMM. We impose dynamic homogeneity, i.e., $\sum_{j=0}^{3}\alpha_j = 1$. The LOP gap has been computed according to equation (**??**), using a detrended effective real exchange rate and terms of trade.[1] The terms of trade were derived as the percentage deviation between the imported goods prices and the domestic price level. The share of imported goods in the consumer basket is set at $\gamma = 0.25$.[2] The output gap is detrended log GDP. Expected future inflation,

[1]All detrending was performed using a Hodrick-Prescott filter with the smoothing parameter set at 1600.

[2]This corresponds to the value used in Batini and Haldane (1999) and is reasonable for a small open economy.

$E_t\bar{\pi}^H_{t+4}$, was instrumented using eight lags of the quarterly domestic inflation rate, fours lags of the deviations from the law of one price, the output gap, the UK 3-month interest rate, the US federal funds rate and the OECD output gap. The preferred equation[3] is given by

$$\begin{aligned}\pi^H_{t+1} \;=\;& \underset{(0.08)}{0.58} E_t\bar{\pi}^H_{t+4} + \underset{(-)}{0.42}\left(-\underset{(0.16)}{0.39}\pi^H_t + \underset{(0.06)}{0.22}\pi^H_{t-1} + \underset{(0.11)}{0.72}\pi^H_{t-2} + \underset{(-)}{0.45}\pi^H_{t-3}\right) \\ & + \underset{(0.13)}{0.28} E_t x_{t+1} + \underset{(0.06)}{0.04} E_t\psi^F_{t+1} + \varepsilon_{t+1} \\ & \bar{R}^2 = 0.67 \qquad \sigma = 0.02 \qquad DW = 1.60.\end{aligned} \tag{13}$$

We note that the data prefers a slightly higher weight on the forward component as compared to the backward component. The proposition that the weights are equal cannot be rejected, however. Moreover, we find endogenous inflation persistence with a considerable lag length. Both the output gap and the LOP gap estimates have the sign that should be expected from theory.

Imported goods price inflation is estimated according to the form

$$\pi^F_{t+1} \;=\; \mu_F E_t\bar{\pi}^F_{t+4} + (1-\mu_F)\sum_{j=0}^{3}\varkappa_j\pi^F_{t-j} + \omega_\psi E_t\psi^F_{t+1} + v_{t+1}, \tag{14}$$

where $\pi^F_t \equiv 4(p^F_t - p^F_{t-1})$ is quarterly imported goods price inflation measured as an annual rate, and $\bar{\pi}^F_t \equiv \frac{1}{4}\sum_{j=0}^{3}\pi^F_{t-j}$ is the four-quarter imported goods inflation rate. The model was estimated over the period $1980Q1 - 2001Q4$ using GMM. The instruments are eight lags of imported goods price inflation and four lags of the LOP gap, the output gap, the UK 3-month interest rate, the US federal funds rate and the OECD output gap. The preferred model is then given as

$$\begin{aligned}\pi^F_{t+1} \;=\;& \underset{(0.05)}{0.78} E_t\bar{\pi}^F_{t+4} + \underset{(-)}{0.22}\left(\underset{(0.19)}{1.11}\pi^F_t - \underset{(-)}{0.11}\pi^F_{t-3}\right) + \underset{(0.10)}{0.56} E_t\psi^F_{t+1} + v_{t+1} \\ & \bar{R}^2 = 0.46 \qquad \sigma = 0.06 \qquad DW = 1.92.\end{aligned} \tag{15}$$

Imported goods inflation seems to be following a significantly more forward-looking process than domestic inflation, and there is considerably less endogenous inflation persistence. This is evidence consistent with a smaller share of price setters that follows backward-looking pricing rules in this sector. We do note, however, that the goodness

[3]The standard error of the estimates is stated in parenthesis under the estimate, $\bar{R}^2$ states the percentage variation in the endogenous variable that is explained by the explanatory variables, σ is the standard error of the residuals and DW is the Durbin-Watson statistics for autocorrelation in the residuals.

of fit of the equation is considerably worse as compared to that in the case of domestic inflation, thereby suggesting that the equation might not pick up all factors influencing imported goods prices.

The output gap is estimated as

$$\begin{aligned} x_{t+1} &= \mu_x E_t x_{t+2} + (1-\mu_x)\left(\eta x_t + (1-\eta)x_{t-1}\right) - \chi(r_t - E_t\bar{\pi}^H_{t+3}) \\ &\quad + \varsigma E_t \Delta\psi^F_{t+1} + \phi E_t \Delta y^*_{t+1} + u_{t+1}, \end{aligned} \tag{16}$$

where y^* is foreign output approximated by the OECD output gap. The future expected output gap was instrumented using four lags of quarterly domestic inflation rate, the LOP gap, the output gap, the UK 3-month interest rate, the US federal funds rate and the growth rate of OECD GDP. The preferred equation is given by

$$\begin{aligned} x_{t+1} &= \underset{(0.04)}{0.53} E_t x_{t+2} + \underset{(-)}{0.47}\left(\underset{(0.08)}{1.36} x_t - \underset{(-)}{0.36} x_{t-1}\right) - \underset{(0.01)}{0.07}(r_t - E_t\bar{\pi}^H_{t+3}) \\ &\quad + \underset{(0.01)}{0.11} E_t \Delta\psi^F_{t+1} + \underset{(0.07)}{0.25} E_t \Delta y^*_{t+1} + u_{t+1}. \\ &\quad \bar{R}^2 = 0.90 \quad \sigma = 0.004 \quad DW = 2.05, \end{aligned} \tag{17}$$

The parameters in the output gap equation are more precisely estimated and the goodness of fit is greater than in the equations for inflation. The parameters have the correct sign; in particular the parameters in front of the real interest rate and the change in the LOP gap have small standard errors. The forward and backward-components are of similar size, and endogenous persistence has a shorter lag length than inflation.

The uncovered interest parity condition was estimated assuming that the unobserved risk-free foreign real interest rate can be approximated with an autoregressive process. Unconstrained estimation yields

$$q_t = \underset{(0.001)}{0.997} E_t q_{t+1} - \underset{(0.58)}{0.965}(r_{q,t} - E_t \pi_{q,t+1}) + rr^*_{q,t} \tag{18}$$

$$rr^*_{q,t} = \underset{(0.07)}{0.34} rr^*_{q,t-1} + w_t \tag{19}$$

$$\bar{R}^2 = 0.87 \quad \sigma = 0.034 \quad DW = 2.15,$$

where $r_{q,t} \equiv \frac{1}{4} r_t$, $\pi_{q,t} \equiv \frac{1}{4}\pi_t$ and $rr^*_{q,t} \equiv \frac{1}{4} rr^*_t$ are the UK 3-month interest rate, the quarterly CPI inflation rate, the foreign real interest rate, respectively; all measured as quarterly rates. The instruments are four lags of the real effective exchange rate, the UK 3-month interest rate, the US federal funds rate, the quarterly CPI inflation rate and the OECD output gap. The residuals were found to be well modelled by an AR(1) process as additional lags were insignificant. Although imprecisely estimated, the interest rate term has a coefficient almost equal to the theoretical expected value of unity. Similarly, the coefficient on the forward exchange rate term is also almost unity, as expected from theory. By constraining the coefficients to unity, the preferred model is given by

$$\begin{aligned} q_t &= E_t q_{t+1} - (r_{q,t} - E_t \pi_{q,t+1}) + rr^*_{q,t} && (20) \\ rr^*_{q,t} &= \underset{(0.07)}{0.50} rr^*_{q,t-1} + \underset{(0.06)}{0.19} rr^*_{q,t-2} + \underset{(0.04)}{0.11} rr^*_{q,t-3} + w_t && (21) \\ & \bar{R}^2 = 0.85 \quad \sigma = 0.037 \quad DW = 2.12, \end{aligned}$$

and the foreign real interest rate is best approximated by an AR(3) process.

Finally, the OECD output growth is modelled according to an autoregressive process as

$$\begin{aligned} \Delta y^*_t &= \underset{(0.07)}{0.51} \Delta y^*_{t-1} + \xi_t && (22) \\ & \bar{R}^2 = 0.25 \quad \sigma = 0.005 \quad DW = 2.14. \end{aligned}$$

3 Policy objectives, robustness and constraints

In this section, we closely follow Dennis, Leitemo, and Söderström (2005). In that paper, we derive the structural approach to robust control who provides a method of constructing robust policy rules in forward-looking in which the misspecification is associate with each structural equations. The method builds on the solution methods developed by Dennis (2005) and work on robust control by Leitemo and Söderström (2004, 2005).

The model of the previous section, denoted *the reference model*, can be set up in second-order structural form as

$$\mathbf{A}_0 \mathbf{y}_t = \mathbf{A}_1 \mathbf{y}_{t-1} + \mathbf{A}_2 \mathrm{E}_t \mathbf{y}_{t+1} + \mathbf{A}_3 \mathbf{u}_t + \mathbf{A}_4 \varepsilon_t,$$

where $\mathbf{y}_t$ is the vector of endogenous variables, $\mathbf{u}_t$ is the vector of policy instrument(s), ε_t is a vector of innovations, and $\mathbf{A}_0$, $\mathbf{A}_1$, $\mathbf{A}_2$, $\mathbf{A}_3$, $\mathbf{A}_4$, and $\mathbf{A}_4$ are matrices with dimensions conformable with $\mathbf{y}_t$, $\mathbf{u}_t$, and ε_t that contain the structural parameters. The matrix $\mathbf{A}_0$ is assumed to be nonsingular and the elements of $\mathbf{A}_4$ are determined to ensure that the shocks are distributed according to $\varepsilon_t \sim iid\,[0, \mathbf{I}_s]$. The dating on the variables is such that any variable that enters $\mathbf{y}_{t-1}$ is known by the beginning of period t; by construction, then, the variables in $\mathbf{y}_{t-1}$ are predetermined.

With the reference model written in second-order structural form, private agents and the policymaker acknowledge their concern for misspecification by surrounding their reference model with a class of models of the form

$$\mathbf{A}_0\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1} + \mathbf{A}_2\mathrm{E}_t\mathbf{y}_{t+1} + \mathbf{A}_3\mathbf{u}_t + \mathbf{A}_4\left(\mathbf{v}_t + \varepsilon_t\right), \tag{23}$$

where $\mathbf{v}_t$ denotes the vector of model misspecifications and equation (23) represents the "distorted" model. The model misspecification, $\mathbf{v}_t$, is intertemporally constrained to satisfy a fixed budget:

$$\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\mathbf{v}_t'\mathbf{v}_t \leq \omega, \tag{24}$$

where $\omega \in [0, \overline{\omega})$ represents the evil agent's total budget for misspecification.

The policy objective function is given by

$$\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\left[\mathbf{y}_t'\mathbf{W}\mathbf{y}_t + \mathbf{u}_t'\mathbf{Q}\mathbf{u}_t\right], \tag{25}$$

where $\mathbf{W}$ and $\mathbf{Q}$ are matrices containing policy weights and are symmetric positive-semi-definite, and symmetric positive-definite, respectively.

The problem of minimizing equation (25) with respect to $\{\mathbf{u}_t\}_0^\infty$ and maximizing with respect to $\{\mathbf{v}_t\}_0^\infty$ subject to equations (23) and (24) can be replaced with an equivalent multiplier problem in which

$$\mathrm{E}_0\sum_{t=0}^{\infty}\beta^t\left[\mathbf{y}_t'\mathbf{W}\mathbf{y}_t + \mathbf{u}_t'\mathbf{Q}\mathbf{u}_t - \theta\mathbf{v}_t'\mathbf{v}_t\right], \tag{26}$$

is minimized with respect to $\{\mathbf{u}_t\}_0^\infty$ and maximized with respect to $\{\mathbf{v}_t\}_0^\infty$, subject to equation (23). For the model of section 2.6, the vector of distortions specifies to

$$\mathbf{v_t} = \left[\begin{array}{ccccc} v_t^H & v_t^F & v_t^x & v_t^q & v_t^{\Delta y} \end{array}\right]$$

where the elements associate with equations (13), (15), (17), (20) and (22), respectively. The multiplier $\theta \in [\underline{\theta}, \infty)$ is inversely related to the budget for misspecification, ω.

The solution to this problem makes $\mathbf{u}_t$ and $\mathbf{v}_t$ functions of $\mathbf{y_{t-1}}$ and $\varepsilon_\mathbf{t}$. By solving equation (23) with these values for the instrument and distortions, produces the outcome for $\mathbf{y_t}, \mathbf{u_t}$ and $\mathbf{v_t}$ in the worst-case equilibrium.

By retaining private sector expectations formation and monetary policy rules $(\mathbf{u_t})$ as functions of $\mathbf{y_{t-1}}$ and ε_t formed in the worst-case equilibrium, but setting $\mathbf{v_t} = \mathbf{0}$ and solving equation (23) with these values, the solution describes the approximating equilibrium. In this equilibrium, the private sector and monetary policymaker fear the distortions and forms expectations and policy respectively under the assumption that $\mathbf{v_t}$ takes the values as in the worst-case equilibrium. By designing expectations and policy in this way, the policymaker and the private sector becomes robust to the situation they fear the most. This is described in details and with greater mathematical rigor in Dennis, Leitemo, and Söderström (2005) for both the commitment and discretion cases. We refer the reader to this paper for more information.

So far θ, which is inversely related to the budget of the evil agent, is taken as a free parameter. Anderson, Hansen, and Sargent (2003) describe the concept of a detection-error probability and introduce it as a tool of calibrating θ. A detection-error probability is the probability that an econometrician observing equilibrium outcomes would make an incorrect inference about whether the approximating equilibrium or the worst-case equilibrium generated the data. A smaller θ allows for greater distortions to the model and for a given reference model, it is easier for the econometrician to differentiate between the worst-case and approximating equilibria. In this study, as in Dennis, Leitemo, and Söderström (2005), we calibrate θ so as to produce a detection-error probability of 15%. This allows distortions to be of a reasonable magnitude, not to large so that we should expected empirical modeling to have picked them up in modeling, and not too small so that robustness strategy does not protect sufficiently against possible misspecification.

4 Policy analysis

In this section we apply the robust control methodology to describe the different equilibria. The policy objective function is specified as being that of inflation targeting, i.e., the central bank has preferences for minimization of the inflation, output and interest rate gap. Hence, social loss (25) and (26) specify to

$$E_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda x_t^2 + \nu i_t^2], \quad \text{and} \tag{27}$$

$$E_0 \sum_{t=0}^{\infty} \beta^t [\pi_t^2 + \lambda x_t^2 + \nu i_t^2 + \theta \mathbf{v}_t' \mathbf{v}_t], \tag{28}$$

respectively, where we set $\lambda = 1$, $\nu = 0.1$ and $\beta = 0.99$. We first study the case where agents have complete confidence in the model and thus have no preferences for robustness ($\theta = \infty$), and characterize the discretion and commitment equilibria. In which case $\lim_{\theta \to \infty} \theta \mathbf{v}_t' \mathbf{v}_t = 0$ and the objective functions (27) and (28) are identical.

We then introduce a preference for robustness and discuss the worst-case and approximating equilibria for both discretion and commitment. We choose to set θ to the value that produces a detection error probability of 15%.

4.1 The rational expectations equilibrium

The rational expectations equilibrium is characterized by both the private and central bank correctly having full confidence in the model as describing the other agents behavioral equation. Hence, there are no reasons to be robust towards alternative descriptions of behavior.

Table 1 shows the variance of key variables for policy under discretion and commitment in rows one and four respectively. The empirical model produces relatively high volatility in all variables. The estimation period of the model includes periods with relatively high volatility, with shocks to the equations having large variance. Since the purpose here, however, is to compare the relative policy performance under different policies and confidence in the model, the high average volatility is less of a concern.

Table 1

Unconditional variance in percent and loss.[a]

Parameters	**Variables**							**Distortions**					**Loss**
θ	π	x	π^H	π^F	r	Δq	Ψ	v^H	v^F	v^x	v^q	$v^{\Delta y}$	$L(1,.1)*10^4$
Discretion													
Rational expectations equilibrium													
–	31.48	27.97	10.74	480.74	95.24	591.12	840.94	–	–	–	–	–	66.24
Worst-case equilibrium													
0.074	41.59	36.35	12.64	625.58	132.00	789.84	1135.68	7.68	1.39	0.62	161.82	0.50	87.39
Approximating equilibrium													
0.074	35.59	29.25	10.59	544.07	105.50	800.53	969.23	–	–	–	–	–	72.58
Commitment													
Rational expectations equilibrium													
–	6.44	3.62	10.18	152.55	9.95	89.38	76.24	–	–	–	–	–	10.61
Worst-case equilibrium													
0.016	8.49	7.28	11.97	221.44	14.24	121.01	110.07	116.14	10.99	7.28	87.37	0.56	16.31
Approximating equilibrium													
0.016	7.35	6.07	10.72	185.70	12.33	119.29	101.96	–	–	–	–	–	13.98

[a] The table shows the variance in percent of key variables together with the distortions in the structural equations. Information shown for RE, worst-case and approximating equilibria under both discretion and commitment. θ is chosen so as to produce a detection error probability of 15%.

We find that the value of commitment is large in the model. Loss can be reduces by more than 80% by optimal commitment. The reason for the importance of commitment in the model can be understood in terms of the form of the structural equations in the model. The structural equations for both domestic inflation, imported goods inflation and output have approximately balanced weights on the backward-looking and forward-looking components. The processes are thus highly persistent or history-dependent. The impact of history on the variable is reinforced due to the variables reliance on the expected future values: Due to the high degree of persistence, also the future values will be expected to depend greatly on the past. However, if the central bank can commit to future policies, policy can be designed to influence agents expectations about the future to counteract the high degree of persistence. Figures A1 – A10 in the appendix show the impulse response functions to the five different shocks of the model. These impulse responses illustrate that the policymaker achieves the counteraction by responding less vigorously with the interest rate at the time of the impact of the shock but rather in a more persistent manner. This prolonged response often creates expected future "over-stabilization or "overshooting of key variables, i.e., expectations of partly reversals of the impact of shocks. The forward components in the model thus contribute to improved stability if a policy commitment is possible.

The results indicate that under commitment, the policymaker realizes the commitment potential through the use of the completely forward-looking exchange rate channel. By an appropriate commitment, the improved stability of the real exchange rate does not only stabilizes the LOP gap and imported inflation better, but counteracts the influence of higher volatility in output on domestic inflation. Under commitment, the interest rate is $\frac{1}{3}$ as volatile compared to its volatility under discretion.

4.2 The worst-case distortions and equilibrium

We now introduce a lack of confidence in the model. In accordance with the description above, we choose the confidence level in accordance with the detection error probability. We allow an evil agent to choose the distortions ($\mathbf{v}_t$) in such a way that they maximize policy loss. The policymaker chooses policy so as to minimize the impact of these distortions.

The distribution of the distortions tells us in which equation a distortion have the most impact on policy loss. This will normally depend on whether we consider a commitment or a discretion equilibrium. Rows 2 and 5 in Table 1 show the variance of these distortions for discretion and commitment respectively. There are at least two important observations to be made.

First, the variance of the distortions are of a larger magnitude in the commitment case. In the commitment case, it is (for given distortions) more difficult to detect misspecification to the model. This allows greater distortions for the same detection-error probability of 15%. The ability of the evil agent to commit to future distortions allows the possible distortions to appear when they have a greater impact on policy loss. Distortions may therefore be less volatile but still have a greater potential impact on volatility and policy loss, although the impact may be reduced by the commitment of the policymaker. This is well illustrated by the impulse response figures in the appendix. In the discretionary policy equilibrium, the distortions are generally proportional to the contemporary RE state of the economy, whereas in the commitment equilibrium, the distortions are generally more outstretched and disconnected from the contemporary RE state. This often leads to distortions having a more persistent effect on the variables under commitment, leading target variables to be more greatly affected. The loss in the worst-case equilibrium is 47% and 35% higher than in the RE equilibrium under commitment and discretion, respectively.

A second important observation from Table 1 is the relative magnitudes of the distortions. Although under both discretion and commitment, the evil agent put most most emphasis on distorting the exchange rate and domestic inflation equations, there are important differences.

Under discretion, the standard deviation of the distortion to the exchange rate equation is more than four times greater than the second largest distortion which is to the domestic inflation equation. The distortions to the remaining equations are negligible compared to these two distortions. This result tells us that weak confidence in the exchange rate (UIP) equation and domestic inflation equations are most problematic to successful monetary policy. From a modeling perspective, the exchange rate is an elusive variable. The views regarding the potential for exchange rate modeling and forecasting has not changed markedly since the pessimistic results reported by Meese and Rogoff (1983). There is hence an match between worst-case specification of distortions and an empirical modelers perspective of degree of misspecifications. This match is unfortunate from the perspective of monetary policy in the open economy. A similar, but arguably less serious match regards the worst-case distortion and an empirical modeler's confidence in the domestic inflation equation. There is a large literature that discusses the specification of the Phillips curve[4], i.e. degree of forward-looking specifications, what the driving variable(s) is, what is the best empirical representation of the driving variable etc. Having (correctly) high confidence in these equations seems to be important for the policy outcome.

Under commitment, the results are similar but distortions affect the domestic inflation equation to a larger extent. The reduced emphasis on distorting the exchange rate

[4]References to be added.

equation could possibly reflect the increased ability of the policymaker to counteract distortions to this equation through an appropriate commitment. The imported inflation and demand equations are also more subject to distortions in the commitment equilibrium.

What are the reason for why the evil agent distorts in particular the domestic inflation and exchange rate equations? The exchange rate influences the LOP gap which directly influences all target variables. The exchange rate channels thus represent potential ways distortions can affect policy loss. This is a necessary condition for a successful distortion of an equilibrium. It is not sufficient though. The reason is that if the policymaker can easily counter the effect of the distortions on the target variables, then such distortions will not be costly. Successful distortions introduces policy trade-offs for the policymaker. In equilibrium, the policymaker cannot stabilize the effects of the distortions on one of the target variable without creating higher variability in one of the other target variables. This is important insights in understanding the problematic implication of the distortions to the UIP condition. In the theoretical model, price setters set prices to reflect average future marginal costs. Consequently, domestic and imported goods inflation depend on the expected future sum of the level of the LOP gaps. Aggregate domestic demand on the other hand depends on the expected current LOP gap as consumers can easily do expenditure switching between foreign and domestic goods. The empirical model have the same effects, however, they have more gradual impact on the objective variables. Since the exchange rate channel has asymmetric effects on output and inflation, the exchange rate channel offer possibilities in which inflation and output can both be increased. A third reason for why the UIP condition is distorted is the high degree of shocks to the (risk-premium corrected) foreign interest rate. As the robust control problem is put up, it provides the evil agent a place to "hide" larger distortions in high residual variance.

The foreign inflation equation is also subject to high residual variance and has a direct impact on the target variable. Nevertheless, the distortions to this equation is of a comparable small magnitude. The reason for this is that the distortions can be responded to in a relatively costless way by an exchange rate movements induced by small interest rate movement. The exchange rate has a strong impact on this process (through the LOP) gap and the needed exchange rate movements (and interest rate movement) induces only smaller changes to the other objective variables.

Distortions to domestic inflation equations is more problematic to the policymaker, since they primarily require counteractions in terms of opposite movements of the output gap. Although the exchange rate channel through the LOP gap provides a secondary channel for counteraction, this channel has only weak effects on domestic inflation and would therefore require substantial movements in the exchange rate, which would anyhow greatly affect the output gap.

4.3 Robust policy and the approximating equilibrium

The simulation results reported in Table 1 on the approximating equilibrium suggest that policy is more aggressive under a robust policy. The policymaker insures to avoid the bad policy outcomes by having a more aggressive stance. From the inspection of the impulse response functions in Figures (A1) – (A10), we see that interest rate volatility increases for all types of shocks.

The insurance against model misspecification creates increased volatility in all target variables. Interestingly, increased imported goods inflation rate contributes to increased CPI inflation volatility, domestic inflation remains almost unaffected. This suggests that the robust policymaker uses the exchange rate channel extensively in order to counteract the effects of potential distortions to the economy. As a reflection of this, the volatility in the rate of change in the real exchange rate is higher than in the RE equilibrium.

Compared to the RE equilibrium, loss is 10% and 26% greater in the approximating equilibrium for discretion and commitment, respectively. The slightly higher number in the commitment case, reflect greater difficulty in distinguishing between the worst-case and approximating models. This requires stronger responses to the potential greater distortions. We find that the optimal insurance against misspecifications under commitment and discretion, leads to different effects on inflation and output. Under commitment, output is more affected than inflation, whereas under discretion, inflation is the variable more affected.

5 Concluding remarks: What have we learned?

The analysis suggests that in the open economy, there may be great advantages in making policy commitments as loss can be reduces substantially. We subscribe this to the half and half weights on backward- and forward-looking components in the structural equation together with an entirely forward-looking UIP condition. These results are independent of the degree of robustness against model uncertainty preferred by the policymaker. Introducing a desire for robustness, we reach two main conclusions.

First, we find that monetary policy is *primarily sensitive to exchange rate and domestic inflation equation* distortions. Distortions to the other equations are of minor importance for the policy outcome. Since especially exchange rate model uncertainty is perceived to be high from an empirical modeling perspective, the sensitivity of the monetary policy outcome to exchange rate model uncertainty is a major challenge to monetary policy.

The robust policymaker's optimal *response to model uncertainty is to use the exchange rate channel extensively* in order to insure against possible distortions. Due to the importance of the exchange rate for the policy outcome, the exchange rate channels provide ways of counteracting the influence of the distortions. The presence of exchange rate channels in the open economy are both a blessing and a curse.

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Appendix

A Extra Figures

The figures below show the impulse response functions to the five different shocks in the rational expectations (RE) equilibrium, worst-case equilibrium and the approximating equilibrium. These impulses are shown for both discretion and commitment policy.

Figure A1

Impulse responses due to a domestic inflation shock under commitment policy.

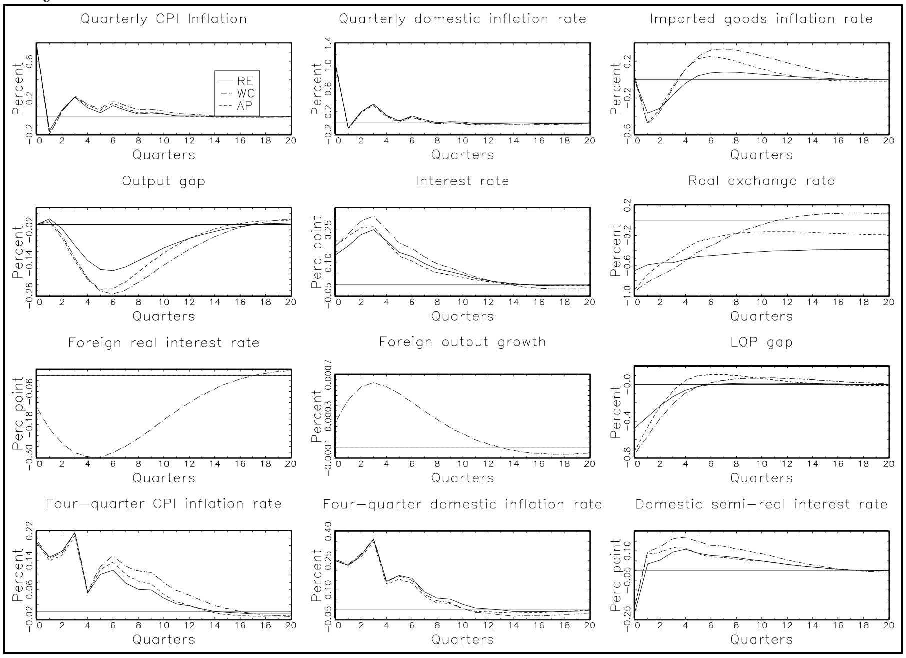

Figure A2

Impulse responses due to a domestic inflation shock under discretionary policy.

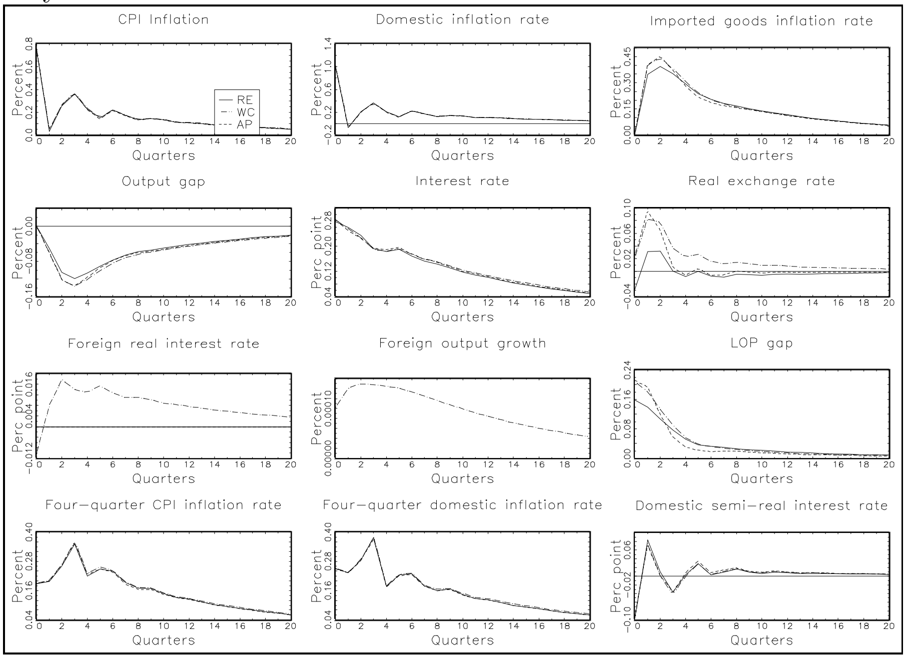

Figure A3

Impulse responses due to an imported goods inflation shock under commitment policy.

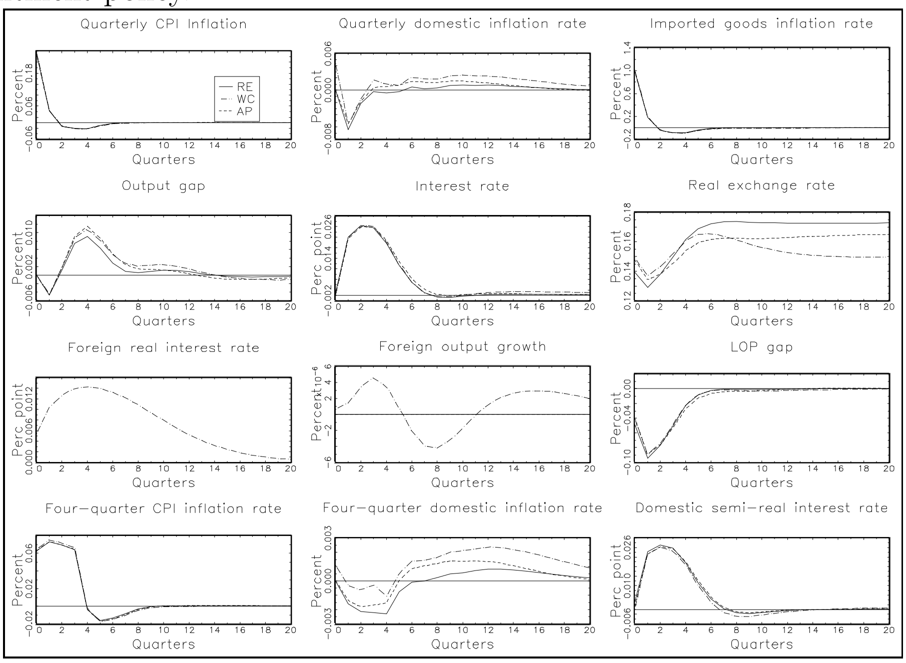

Figure A4

Impulse responses due to an imported goods inflation shock under discretionary policy.

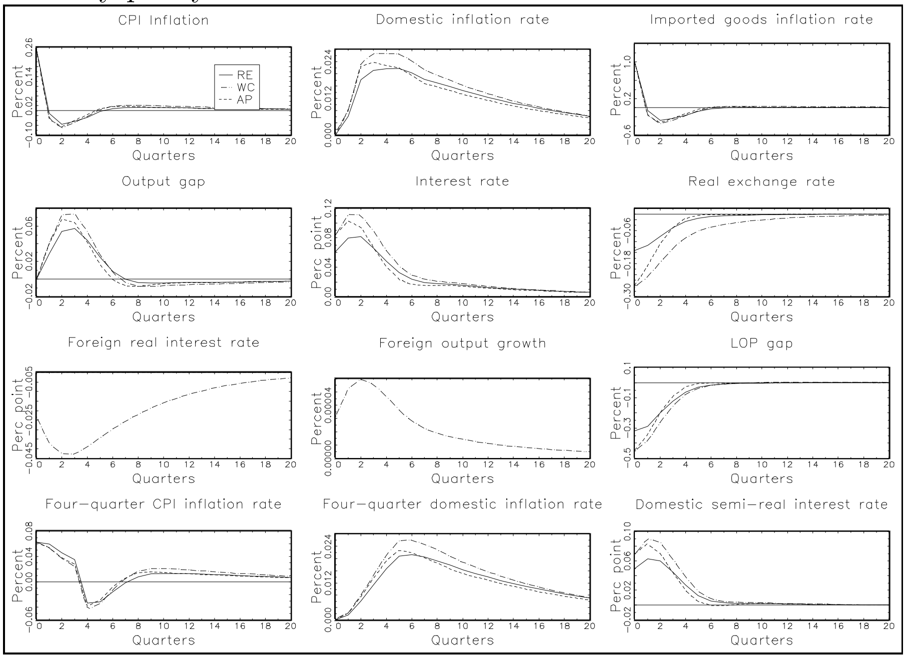

Figure A5

Impulse responses due to a domestic demand shock under commitment policy.

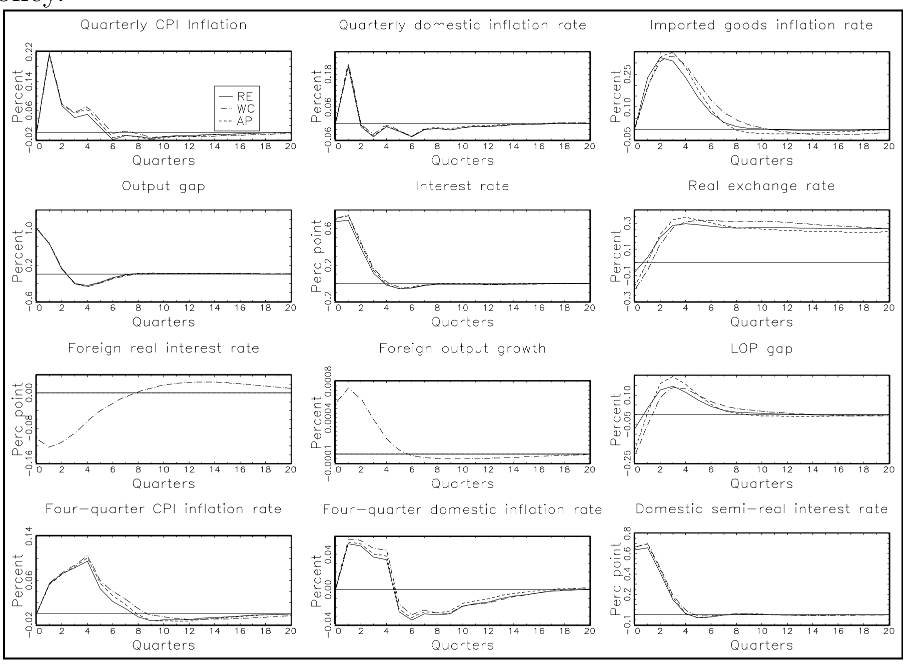

Figure A6

Impulse responses due to a domestic demand shock under discretionary policy.

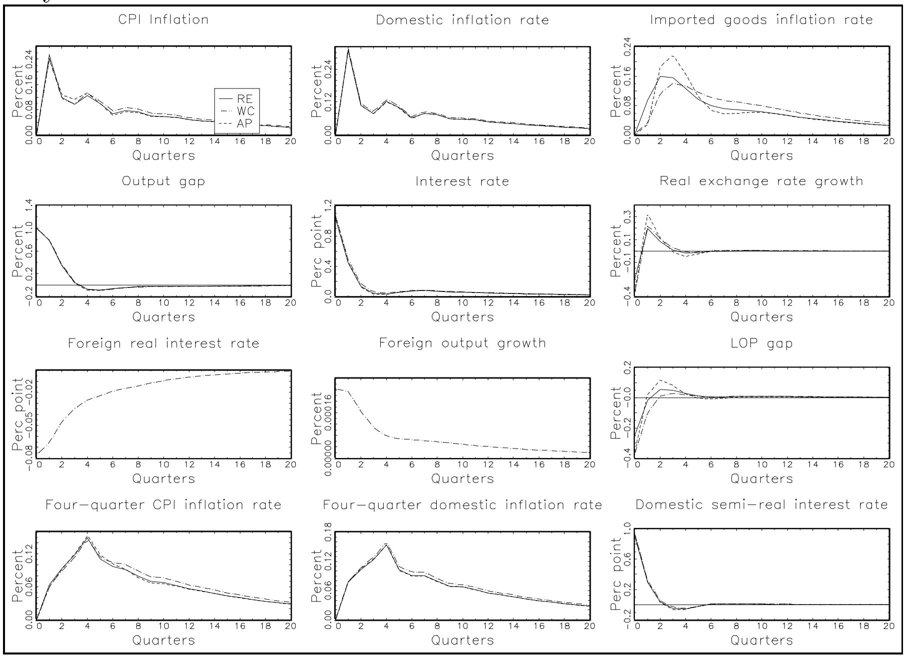

Figure A7

Impulse responses due to a risk premium shock under commitment policy.

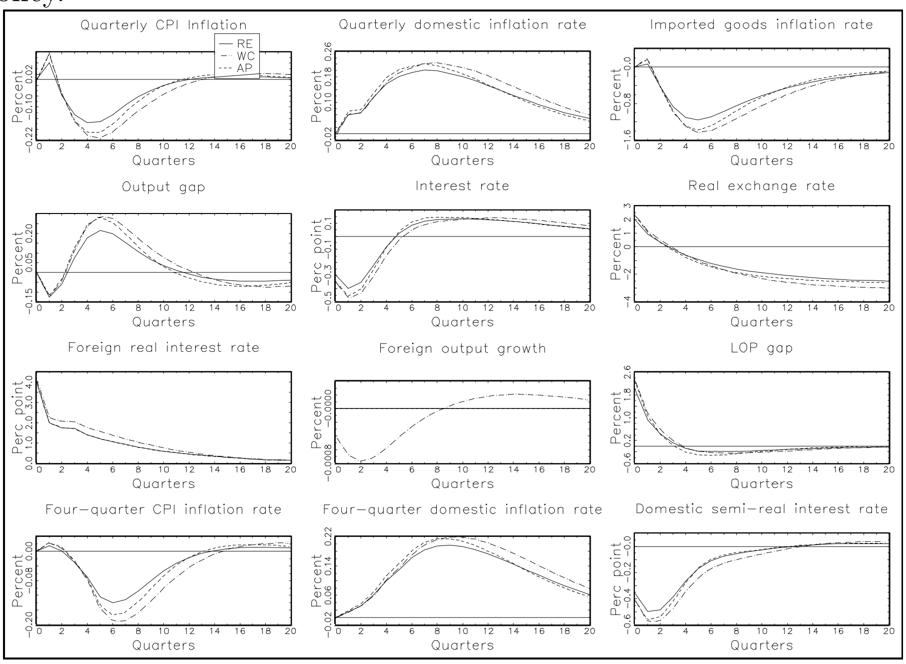

Figure A8

Impulse responses due to a risk premium shock under discretionary policy.

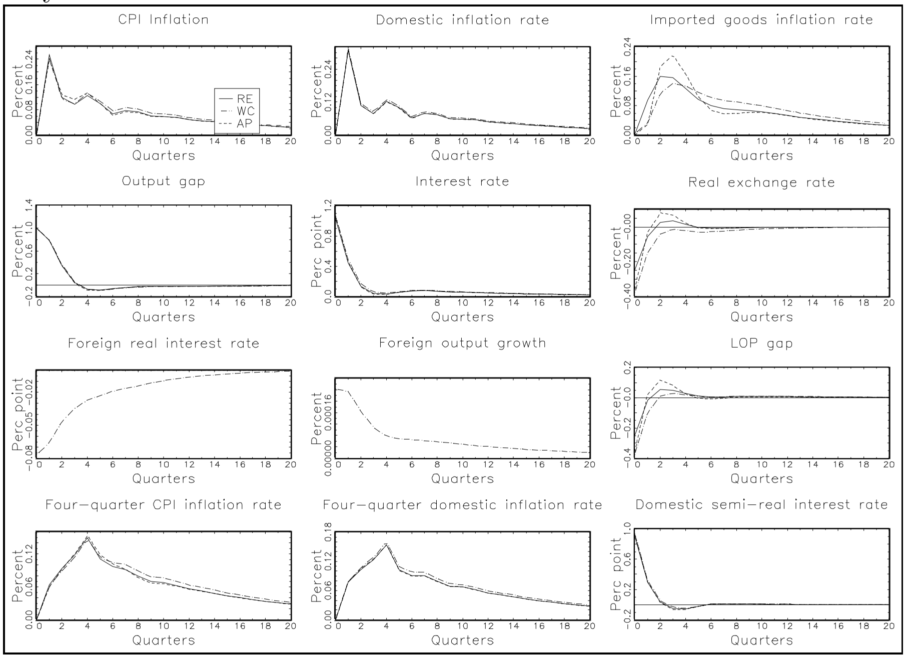

Figure A9

Impulse responses due to a foreign output growth shock under commitment policy.

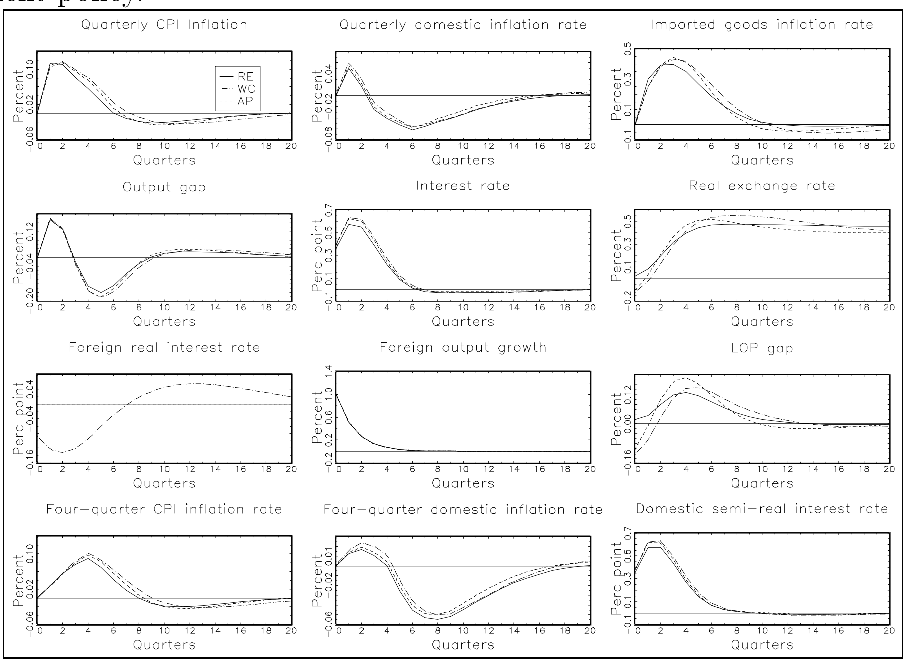

Figure A10

Impulse responses due to a foreign output growth shock under discretionary policy.