# Inflation Targeting in Presence of Balassa-Samuelson-type Productivity Shocks

Igor Masten*
University of Ljubljana

December 2005

**Abstract**

This paper develops a two-sector small open-economy dynamic stochastic model with permanent technology shocks and price rigidities that allows for simulation of the differential productivity growth (Balassa-Samuleson-type productivity improvement). The model is calibrated for a typical Acceeding country to see whether the Balassa-Samuelson effect poses a threat to fullfiling the inflation Maastricht criterion when the policy is commited to achieve the inflation objective. In addition, optimal policy is derived for a benchmark parameterization of the model. The results show that productivity growth differential need not generate the inflationary effects that represent significant risks to fullfilling Maastricht criteria in the ERM II.

Keywords: Balassa-Samuelson effect, inflation targeting, ERM II, EMU
JEL codes: E52, E31, F02, F41

## 1 Introduction

New EU member countries will eventually set on a path of entry to the Euro zone. Thus far, only Estonia, Lithuania and Slovenia have entered the ERM II system and chosen a strategy of fast entry to the Euro area. Other countries postponed this decision due to a number of reasons. As one of the main issues in this respect the economic policy debate stresses potential problems with compliance with the Maastricht inflation criterion due to the Balassa-Samuelson effect (BS effect hereafter). Underlying reasoning is the following: The BS effect implies higher average inflation rates and a fast adoption of the Euro would expose a country to the risk of inappropriately low nominal, and thus real, short-term interest rates (ECB, 2004). Ensuing demand and asset price booms could potentially destabilize the economy and impose important welfare costs. This paper shows that a path of low nominal and real interest rates in presence of the BS effect does not act destabilizing on the economy.

*Contact: igor.masten@ef.uni-lj.si I would like to thank Roland Straub, Philip Lane, Giancarlo Corsetti and Arjana Brezigar for their numerous comments and suggestions. I have also benefited from discussions with Ricardo Mestre, Gabriel Fagan and Frank Smets. All errors are authors' sole responsibility.

On the contrary, low interest rates are consistent with the optimal monetary policy response under commitment to the equilibrium process of real exchange rate appreciation.

In recent years there have been many empirical and theoretical contributions analyzing the causes for sizeable appreciations of real exchange rates (see Coricelli and Jazbec, 2001, DeGregorio et al., 1994, Canzoneri et al., 2001 and Egert, 2002 among others). The Balassa-Samuelson effect has often been found as one of the most important causes even though empirical evidence is not conclusive (see Mihaljek and Klau, 2004). Wagner and Hlouskova (2004) find that the BS effect may contribute only about half a percent per annum to the observed inflation differentials between the new EU member countries and the Euro area. In addition, Čihak and Holub (2001), who analyze in detail the required convergence in relative prices to the European level in the next decade, find that much of the adjustment of relative prices (and ensuing inflation differential) cannot be simply attributed to the pure BS effect i.e. pure working of tradable-nontradable productivity growth differential. Adjustment of administrated and regulated prices can account for an important share of excess inflation. Policy implications drawn from the model presented in this paper are valid also in presence of such sources of real appreciation.

The first step in the direction of analyzing alternative monetary policy rules in presence of the BS effect in an explicit modelling framework is the paper by Natalucci and Ravenna (2002). They build a two-sector dynamic stochastic general equilibrium model for a small open economy and calibrate it for the Czech Republic. This allows them to compare fixed exchange rate regime and various specification of Taylor-type rules in presence of differential productivity growth between tradable and non-tradable sector. Their basic conclusion is that the presence of the BS effect causes such a high exchange rate - inflation variability trade-off that it is very unlikely that the new member states will comply with the Maastricht criteria even if the countries sacrifice some growth for this goal. In the model of Natalucci and Ravenna, however, real appreciation in response to their simulation of BS effect is not an equilibrium process. On the contrary, it is a consequence of a large deviation from the equilibrium. It is thus not surprising that they find such implausible effects. The present model constructs real exchange rate appreciation as the equilibrium process and the central bank attempts to stabilize the real exchange rate around the equilibrium path and does not attempt to revert it to the old steady state. Contrary to the findings of Natalucci and Ravenna (2002), the result show that, under the condition that the central bank is able to commit to optimizing the objective function, the BS effect need not impose considerable threats to fulfilling the inflation Maastricht criterion.

This paper essentially builds a simple two-sector small open-economy model with price staggering and non-stationary productivity process. It is calibrated assuming parameter values deemed reasonable for new member countries. The model has four important characteristics. First, I analyze optimal inflation targeting instrument rules under commitment. In addition, following Gali and Monacelli (2005) the optimal objective function for the central bank is derived with the second-order approximation to the utility function under the basic parameterization of the model. Monetary authority thus reacts to shocks optimally. Second, in line with empirical evidence the model assumes that monetary policy has only a lagged effect on inflation (2 quarters) and domestic demand components (1 quarter). This type of specification is motivated by Svensson (2000) and Woodford (2003). Similar motivation for reduced form model equations is used also by Ireland (2003) and Ehrmann and Smets (2003).

Third, the modeling framework allows for market power also in the tradable sector, which has become a standard approach in new open macroeconomics literature. Thus, the law of one price does not automatically hold for tradable goods and deviations from it account for an important part of macroeconomic adjustment. Fourth, the model in this paper allows for permanent sector-specific shocks (hence not only for very persistent), which implies that it enables a proper simulation of the Balassa-Samuelson scenario i.e. as an equilibrium-driving process. This is an important difference with existing literature because when optimizing under perfect foresight the policy maker takes into account that productivity shocks are truly permanent and that sectoral relative prices are nonstationary. Natalucci and Ravenna (2002) construct the Balassa-Samuelson experiment by pushing a stationary process of tradable productivity very far away from equilibrium with a sequence of positive productivity shocks for 40 quarters. This means that at the time when tradable productivity is supposed to reach a new steady state value (in 10 years) is in fact the farthest away from the steady state. The tradable productivity increase is thus not constructed as equilibrium-driving process. In the present case, however, a differential productivity shock induces true convergence to new steady state. It is thus not surprising that results in this paper differ substantially from theirs.

The results shows that the presence of Balassa-Samuelson effect *per se* need not imply a higher inflation rate along the adjustment path. As such, differential productivity growth need not present a threat to fulfilling the Maastricht criteria. It is, however, possible that a tight fluctuation band in the ERM 2 system of exchange rate would require suboptimal responses of monetary policy.

The paper is structured as follows. Section 2 presents the derivation and calibration of the model and presents the design of monetary policy. Section 3 discusses the results and Section 4 concludes.

# 2 The model

## 2.1 Demand side of the economy

In the economy there exists a continuum of differentiated nontradable goods $C^N(i), i \in [0,1]$, tradable home-produced goods $C^H(i), i \in [0,1]$ and tradable imported goods $C^F(i), i \in [1,2]$. The representative consumer maximizes

$$E_{t-1}\sum_{\tau=0}^{\infty}\delta^{\tau}\widetilde{U}\left(\widehat{C}_{t+\tau}, \frac{M_{t+\tau}}{P^C_{t+\tau}}, W\left(l_{t+\tau}\right)\right)$$

subject to a flow budget constraint

$$E_{t-1}\left(P_t^C C_t\right) + E_{t-1}M_t + E_t\left(\Xi_{t,t+1}D_{t+1}\right) \leq E_{t-1}W_t\left(l_t^H + l_t^N\right) + E_{t-1}D_t + M_{t-1} + E_{t-1}T_t$$

Consumers/producers draw their income from employment in tradable and nontradable sectors where they are paid the wage rate $W_t$ that is assumed to be common across sectors due to perfect factor mobility. The assumption of homogeneous labor market and economy-wide wage equalization mechanism with flexible wages makes the model particularly informative about implications productivity growth differential has for optimality of monetary policy as the model exhibits an upper limit for the Balassa-Samuelson effect in the short run. As owners of production technology they also receive residual profits and accrues interest payments on bond holding. All these nominal payoffs in period $t$ of the portfolio held at $t-1$ are denoted by $D_t$. $\Xi_{t,t+1}$ is the stochastic discount factor for nominal payoff. Wealth is also held in money balances $M_t$. It is assumed that demand is predetermined one period.

$\widetilde{U}(\cdot)$ is additively separable in its components. The sub-utility function of consumption incorporates perfect habit formation: $U\left(\widehat{C}_t\right) \equiv \left(1-\frac{1}{\sigma}\right)^{-1}\left(C_t/Z_t^C\right)^{1-\frac{1}{\sigma}}$. More precisely, real consumption $C_t$ enters the utility function stochastically detrended: $\widehat{C}_t \equiv C_t/Z_t^C$. The scaling variable $Z_t^C$ is defined below and ensures a constant steady-state level of utility. $\widehat{C}_t$ is of CES form and consists of corresponding Dixit-Stiglitz aggregators (all components are appropriately stochastically detrended)

$$\widehat{C} = \left[(1-\lambda)^{\frac{1}{\theta}}\,\widehat{C}_T^{\frac{\theta-1}{\theta}} + \lambda^{\frac{1}{\theta}}\widehat{C_N}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \;;\; \widehat{C}_T = \left[(1-\omega)^{\frac{1}{\theta^T}}\,\widehat{C}_H^{\frac{\theta^T-1}{\theta^T}} + \omega^{\frac{1}{\theta^T}}\,\widehat{C}_F^{\frac{\theta^T-1}{\theta^T}}\right]^{\frac{\theta^T}{\theta^T-1}}$$

$$\widehat{C}_N = \left[\int_0^1 \widehat{C}_N(j)^{\frac{\vartheta^N-1}{\vartheta^N}}\,dj\right]^{\frac{\vartheta^N}{\vartheta^N-1}},\; \widehat{C}_H = \left[\int_0^1 \widehat{C}_H(j)^{\frac{\vartheta^T-1}{\vartheta^T}}\,dj\right]^{\frac{\vartheta^T}{\vartheta^T-1}}$$

$$\widehat{C}_F = \left[\int_1^2 \widehat{C}_F(j)^{\frac{\vartheta^T-1}{\vartheta^T}}\,dj\right]^{\frac{\vartheta^T}{\vartheta^T-1}}$$

Elasticities of substitution among tradable and nontradable varieties are denoted by $\vartheta^T > 1$ and $\vartheta^N > 1$ respectively. $\omega$ determines the share of imports in expenditure for tradable goods ($C^T$), while $\lambda$ is the share of nontradable goods in overall domestic consumption expenditure. Elasticity of substitution between nontradable and tradable goods is denoted by $\theta > 0$, the elasticity between foreign and home tradable goods is denoted by $\theta^T > 0$. Functional form of disutiliy of labor takes the form $W(l_t) \equiv \frac{1}{1+\varphi}l_t^{1+\varphi}$, subject to the constraint $l_t = l_t^N + l_t^H$.

Using standard aggregation techniques from cost-minimization first-order conditions we can write the price index of scaled consumption as[1]

$$\widehat{P}_C = \left[(1-\lambda)\,\widehat{P}_T^{1-\theta} + \lambda\widehat{P}_N^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

where

$$\widehat{P}_T = \left[(1-\omega)\,\widehat{P}_H^{1-\theta^T} + \omega\widehat{P}_F^{1-\theta^T}\right]^{\frac{1}{1-\theta^T}},\; \widehat{P}_N = \left[\int_0^1 \widehat{P}_N(j)^{1-\vartheta^N}\,dj\right]^{\frac{1}{1-\vartheta^N}}$$

$$\widehat{P}_H = \left[\int_0^1 \widehat{P}_H(j)^{1-\vartheta^T}\,dj\right]^{\frac{1}{1-\vartheta^T}}$$

[1] Because consumption components are scaled it follows that $\widehat{P}^C = P^C Z^C$. Analogous relations hold for other price indexes.

The expression for the log of aggregate CPI index can be approximated around the steady state with $\widehat{P}^T = \widehat{P}^N = \widehat{P}^F$ as:[2]

$$\widehat{p}^C = (1-\omega)(1-\lambda)\,\widehat{p}^H + \omega(1-\lambda)\,\widehat{p}^F + \lambda\widehat{p}^N$$

The internal price ratio is defined as $Q_t = P_t^N / P_t^H$ and the terms of trade as $S_t = P_t^F / P_t^H$. Their log counterparts are $\widehat{q}_t = \widehat{p}_t^N - \widehat{p}_t^H$ and $\widehat{\mu}_t = \widehat{p}_t^F - \widehat{p}_t^H$ respectively. Both are stochastically scaled (see below), which implies that all hatted variables denote log deviations from their corresponding natural levels. Intertemporal optimization leads to the following first-order condition

$$\delta E_{t-1}\left[\left(\frac{\widehat{C}_{t+1}}{\widehat{C}_t}\right)^{-\frac{1}{\sigma}}\left(\frac{Z_t^C P_t^C}{Z_{t+1}^C P_{t+1}^C}\right)\right] = E_{t-1}\Xi_{t,t+1} \tag{2.1}$$

Taking expectations on both sides and noting that the price of a riskless one-period bond available to consumers is equal to $R_t^{-1} = E_{t-1}\{\Xi_{t,t+1}\}$ this can be rearranged to obtain

$$\delta R_t E_{t-1}\left[\left(\frac{\widehat{C}_{t+1}}{\widehat{C}_t}\right)^{-\frac{1}{\sigma}}\left(\frac{Z_t^C P_t^C}{Z_{t+1}^C P_{t+1}^C}\right)\right] = 1$$

Log-linearization of this intertemporal optimality condition yields

$$\widehat{c}_{t+1/t} = \widehat{c}_{t+2/t} - \sigma\left(i_{t+1/t} - \pi_{t+2/t}^C - \Delta z_{t+2/t}^C\right) \tag{2.2}$$

$c_t$ denotes log of aggregate real domestic consumption and $r_t \equiv i_t - \pi_{t+1/t}^C$ is the real (CPI based) interest rate expressed as deviation from long-run mean real interest rate $(-\log\delta)$. $\Delta z_{t+1/t}^C$ is the source of variations in the natural real interest rate. Note that $\pi_t^C$ is usual inflation rate and not the change in the price index of stochastically detrended consumption. $t$ dated rational expectations are generically denoted as $E_t x_{t+s} \equiv x_{t+s/t}$. The nominal interest rate $i_t$ is the instrument of the central bank. Because it is additionally assumed that the utility function is separable in its components I omit the the first-order condition that characterizes optimal holdings of money balances from characterization of the equilibrium. The logs of demand functions for domestically and foreign produced tradable goods and nontradable goods are

$$\begin{aligned}\widehat{c}_t^i &= \widehat{c}_t^T - \theta^T\left(\widehat{p}_t^i - \widehat{p}_t^T\right),\ i \in \{H, F\}\\ \widehat{c}_t^N &= \widehat{c}_t - \theta\left(\widehat{p}_t^N - \widehat{p}_t^C\right)\end{aligned}$$

[2] In case of the Cobb-Douglas form of subutilities ($\theta^T = \theta = 1$) the relation is exact. Moreover, it holds also for the usual price index of non-scaled consumption. This parameterization is taken as a baseline in the paramaterization of the model and enables also to derive the appropriate approximation to the representative consumer's utility function. This rather restrictive assumption is not uncommon, however, in the literature. Cobb-Douglas intratemporal utility in consumption has been used by Parrado and Velasco (2002), Obstfeld and Rogoff (1998), Corsetti and Pesenti (2001), Lubik (2000) and in the final parameterization of the model also Svensson (2000). Also in these cases the reasons are of technical nature since it allows to impose balanced trade conditions and find close form solutions in welfare analysis. See also Gali and Monacelli (2002) who combine this intratemporal specification with log intertemporal utility in their welfare analysis in closed form.

Domestic demand for domestically produced traded goods is related to overall tradable consumption through

$$\widehat{c}_t^H = \widehat{c}_t^T + \theta^T \omega \widehat{\mu}_t$$

where I have used the following relation: $\widehat{p}_t^H - \widehat{p}_t^T = -\omega \widehat{\mu}_t$. By plugging $\widehat{p}_t^T - \widehat{p}_t^C = -\lambda \left( \widehat{q}_t - \omega \widehat{\mu}_t \right)$ into $\widehat{c}_t^T = \widehat{c}_t - \theta \left( \widehat{p}_t^T - \widehat{p}_t^C \right) = \widehat{c}_t + \theta \lambda \left( \widehat{q}_t - \omega \widehat{\mu}_t \right)$ we obtain

$$\widehat{c}_t^H = \widehat{c}_t + \theta \lambda \widehat{q}_t + \left( \theta^T - \theta \lambda \right) \omega \widehat{\mu}_t \tag{2.3}$$

By analogy, the expected change in (log) demand for nontradable goods is

$$\widehat{c}_t^N = \widehat{c}_t + \theta \left( \omega \left( 1 - \lambda \right) \widehat{\mu}_t - \left( 1 - \lambda \right) \widehat{q}_t \right) \tag{2.4}$$

where the expansion $\widehat{p}^N - \widehat{p}_t^C = \left( 1 - \lambda \right) \widehat{q}_t - \omega \left( 1 - \lambda \right) \widehat{\mu}_t$ was used. Using these decompositions and inserting them to (2.2) yields the following expressions

$$\begin{aligned}
\widehat{c}_{t+1/t}^H &= \widehat{c}_{t+2/t}^H - \sigma \left( i_{t+1/t} - \pi_{t+2/t}^H - \Delta z_{t+2/t}^H \right) - \omega \left[ \theta^T - \theta \lambda - \sigma \left( 1 - \lambda \right) \right] \Delta \widehat{\mu}_{t+2/t} && (2.5) \\
& - \left( \theta - \sigma \right) \lambda \Delta \widehat{q}_{t+2/t} \\
\widehat{c}_{t+1/t}^N &= \widehat{c}_{t+2/t}^N - \sigma \left( i_{t+1/t} - \pi_{t+2/t}^N - \Delta z_{t+2/t}^N \right) - \left( \theta - \sigma \right) \left( 1 - \lambda \right) \omega \Delta \widehat{\mu}_{t+2/1} && (2.6) \\
& + \left( \theta - \sigma \right) \left( 1 - \lambda \right) \Delta \widehat{q}_{t+2/t}
\end{aligned}$$

The model assumes complete financial markets that enable perfect risk sharing. Thus, a condition analogous to (2.1) must hold also for the world economy

$$\delta E_{t-1} \left[ \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\frac{1}{\sigma}} \left( \frac{e_t P_t^*}{e_{t+1} P_{t+1}^*} \right) \right] = E_{t-1} \Xi_{t,t+1} \tag{2.7}$$

It is assumed that foreign output is also appropriately stochastically detrended without introducing new notation. Alternatively and without loss of generality we could assume that scaling variables for variables of home economy are expressed in terms of deviations from the foreign scaling variable. Combining (2.1) and (2.7) it follows that (omitting expectations operator for simplicity)

$$\widehat{C}_t = \Psi C_t^* \left( Q_t^C / Z_t^C \right)^\sigma = \chi C_t^* \left[ \widehat{Q}_t^{-\lambda} \widehat{S}_t^{1-(1-\lambda)\omega} \right]^\sigma \tag{2.8}$$

This expression holds for all $t$. $\Psi$ is a constant that depends on initial conditions (see Gali and Monacelli, 2005 for detailed treatment). $Q_t^C \equiv e_t P_t^* / P_t^C$ is CPI-based real exchange rate. Log-linearization of (2.8) yields the exact relation (up to a constant)

$$\widehat{c}_t = c_t^* + \sigma \left[ \left( 1 - \left( 1 - \lambda \right) \omega \right) \widehat{\mu}_t - \lambda \widehat{q}_t \right] \tag{2.9}$$

For future reference it is useful to combine (2.9) and (2.3) to express the previous expression in terms of consumption of home tradable goods

$$\widehat{c}_t^H = c_t^* + \widetilde{\gamma}_\mu \widehat{\mu}_t + \widetilde{\gamma}_q \widehat{q}_t \tag{2.10}$$

where $\widetilde{\gamma}_q = (\theta - \sigma)\lambda$ and $\widetilde{\gamma}_\mu = \sigma(1 - (1-\lambda)\omega) + \left(\theta^T - \theta\lambda\right)\omega$. Foreign demand for home traded variety is $c^{*H}(j) = \left(\frac{P_t^H(j)}{P_t^H}\right)^{-\vartheta^T} \omega^* \left(\frac{P_t^H}{P_t^*}\right)^{-\theta^T} C_t^*$, where $\omega^*$ represents the share of home goods in foreign consumption. Since home economy is a small open economy (small $\omega^*$) we can use the following simplifications: $C_t^* = Y_t^{*d}$ and $P_t^* = P_t^F$. Plugging the previous expression into $C_t^{*H} = \left[\int_0^1 c^{*H}(j)^{\frac{\vartheta^T-1}{\vartheta^T}} dj\right]^{\frac{\vartheta^T}{\vartheta^T-1}}$ and log-linearizing we obtain (up to a constant):

$$\widehat{c}_t^{*H} = c_t^* + \theta^* \widehat{\mu}_t = y_t^* + \theta^T \widehat{\mu}_t$$

$y_t^*$ is foreign output. Total (log) demand for nontradable goods (appropriatelly stochastically detrended) is $y_t^N = \hat{c}_t^N$. The definition of total demand for home tradable goods is (again stochastically detrended)

$$\begin{aligned}
Y_t^H &= \left[\int_0^1 \left(\hat{c}^H(j) + c^{*H}(j)\right)^{\frac{\vartheta^T-1}{\vartheta^T}} dj\right]^{\frac{\vartheta^T}{\vartheta^T-1}}. \\
&= (1-\omega)(1-\lambda)\widehat{C}_t \widehat{Q}_t^{\theta\lambda} S_t^{\left(\theta^T - \theta\lambda\right)\omega} + \omega^* S_t^{\theta^T} C_t^* \\
&= (1-\omega)(1-\lambda)\chi Y_t^* \left[\widehat{Q}_t^{-\lambda} \widehat{S}_t^{1-(1-\lambda)\omega}\right]^\sigma \widehat{Q}_t^{\theta\lambda} S_t^{\left(\theta^T-\theta\lambda\right)\omega} + \omega^* S_t^{\theta^T} Y_t^* \\
&= \chi Y_t^* S_t^{\theta^T} \left[(1-\omega)(1-\lambda)\widehat{Q}_t^{\widetilde{\gamma}_q} \widehat{S}_t^{\widetilde{\gamma}_\mu - \theta^T} + 1 - (1-\omega)(1-\lambda)\right]
\end{aligned}$$

The third equality makes use of (2.8), while the last equality uses the condition $\frac{\omega^*}{\chi} = 1 - (1-\omega)(1-\lambda)$ that is required to ensure balanced trade in the steady state[3]. Up to first-order approximation we obtain

$$y_t^H = y_t^* + \gamma_q \widehat{q}_t + \gamma_\mu \widehat{\mu}_t \tag{2.11}$$

where $\gamma_q = (1-\omega)(1-\lambda)\widetilde{\gamma}_q$ and $\gamma_\mu = [1 - (1-\omega)(1-\lambda)]\theta^T + (1-\omega)(1-\lambda)\widetilde{\gamma}_\mu$. Note that under the assumption $\sigma = \theta^T = \theta = 1$ this expression simplifies to

$$y_t^H = y_t^* + \widehat{\mu}_t \tag{2.12}$$

From (2.10) it also follows that

$$y_t^H = \widehat{c}_t^H + \overline{\gamma}_q \widehat{q}_t + \overline{\gamma}_\mu \widehat{\mu}_t \tag{2.13}$$

where $\overline{\gamma}_q = [(1-\omega)(1-\lambda) - 1]\widetilde{\gamma}_q$ and $\overline{\gamma}_\mu = [(1-\omega)(1-\lambda) - 1]\left(\widetilde{\gamma}_\mu - \theta^T\right)$. Collecting the results we obtain the sector-specific IS curves. Market clearing in the nontradable goods market requires $y_t^N = \widehat{c}_t^N$, which leads to the IS curve for the nontradable sector

[3]The proof of existence of unique steady state of stochastically detrended variables of the economy given the stated restriction on intial conditions is just an extension of the proof in Gali and Monacelli (2002). Repeating its construction in the paper would not yield any new insights.

$$y^N_{t+1/t} = y^N_{t+2/t} - \sigma\left(i_{t+1/t} - \pi^N_{t+2/t} - \Delta z^N_{t+2/t}\right) - \widetilde{\beta}^N_\mu \Delta\widehat{\mu}_{t+2/t} + \widetilde{\beta}^N_q \Delta\widehat{q}_{t+2/t}$$

where $\widetilde{\beta}^N_\mu = (\theta-\sigma)(1-\lambda)\omega$ and $\widetilde{\beta}^N_q = (\theta-\sigma)(1-\lambda)$. Collecting (2.5) and (2.13) the corresponding expression for the tradable sector follows

$$y^H_{t+1/t} = y^H_{t+2/t} - \sigma\left(i_{t+1/t} - \pi^H_{t+2/t} - \Delta z^H_{t+2/t}\right) - \widetilde{\beta}^H_\mu \Delta\widehat{\mu}_{t+2/t} - \widetilde{\beta}^H_q \Delta\widehat{q}_{t+2/t}$$

where $\widetilde{\beta}^H_\mu = \omega\left[\theta^T - \theta\lambda - \sigma(1-\lambda)\right] + \overline{\gamma}_\mu$and $\widetilde{\beta}^H_q = \left(\theta^T - \sigma\right)\lambda + \overline{\gamma}_q$. Note that all variables are expressed in terms of deviations from their natural levels. This means that $y^i_t$ represent corresponding output gaps. As discussed in Ireland (2003) the assumption of partial adjustment in demand can fit better the data. Theoretically it can be justified with adjustment costs or habit persistence (see e.g. Giannoni and Woodford, 2003 for a formal derivation).

$$\begin{aligned}
y^H_{t+1} &= \beta_y y^H_t + \left(1-\beta_y\right) y^H_{t+1/t} \\
&= \beta_y y^H_t + \left(1-\beta_y\right) y^H_{t+2/t} - \beta^H_r\left(i_{t+1/t} - \pi^H_{t+2/t} - \Delta z^H_{t+2/t}\right) \qquad (2.14)\\
&\quad -\beta^H_\mu \Delta\widehat{\mu}_{t+2/t} - \beta^H_q \Delta\widehat{q}_{t+2/t} + \eta^{d\,H}_{t+1} \\
y^N_{t+1} &= \beta_y y^N_t + \left(1-\beta_y\right) y^N_{t+2/t} - \beta^N_r\left(i_{t+1/t} - \pi^N_{t+2/t} - \Delta z^N_{t+2/t}\right) \qquad (2.15)\\
&\quad -\beta^N_\mu \Delta\widehat{\mu}_{t+2/t} + \beta^N_q \Delta\widehat{q}_{t+2/t} + \eta^{d\,N}_{t+1}
\end{aligned}$$

where an zero-mean i.i.d. demand shock $\eta^{d\,i}_{t+1}$ has been added and where $\beta^i_r = \left(1-\beta_y\right)\sigma$, $\beta^i_q = \left(1-\beta_y\right)\widetilde{\beta}^i_q$ and $\beta^H_\mu = \left(1-\beta_y\right)\widetilde{\beta}^i_\mu$, $i \in \{H, N\}$.

### 2.2 Supply side of the economy

In goods market equilibrium in each sector firms face the following demand schedules for their differentiated products

$$Y^N(j) = Y^N_t\left(\frac{P_t(j)}{P^N_t}\right)^{-\vartheta^N} \quad \text{and } Y^H(j) = \left(C^H_t + C^{*H}_t\right)\left(\frac{p_t(j)}{P^H_t}\right)^{-\vartheta^T}$$

$Y^i_t, i \in \{N, H\}$ are aggregate demands for goods of sector $i$, $P_t(j)$ is a price for domestic good $j$, and $P^i_t, i \in \{N, H\}$ are defined above. Own price elasticity of substitution between varieties $\vartheta^i$ is allowed to differ across sectors. The model incorporates staggered price adjustment in the spirit of Calvo (1983), i.e. a firm chooses a new price $\widetilde{P}_t$ in period $t$ with probability $1-\alpha$, and keeps the same price as in previous period with probability $\alpha$. In the calibration procedure $\alpha$ is assumed to be the same in both sectors, however, we could easiliy allow it to differ. The optimization problem of a firm setting a new price in period $t$ for $i \in \{N, H\}$ it the following:

$$\max_{\widetilde{P}^i_t} E_{t-2}\left\{\sum_{\tau=0}^{\infty} \alpha^\tau \widetilde{\Lambda}_{t+\tau}\Xi_{t,t+\tau}\left[\widetilde{P}^i_t - \frac{\left(1-\tau^i\right)W_{t+\tau}}{A^i_{t+\tau}}\right] Y^d_{t+\tau}\left(\frac{\widetilde{P}^i_t}{P^i_{t+\tau}}\right)^{-\vartheta^i}\right\}$$

where $\widetilde{\Lambda}_t$ is the marginal utility of consumer/producer's nominal wealth in period $t$ and. $\chi^i$ is sector-specific employment subsidy whose role is to neutralize the monopolistic distortion in efficient level of output. It is assumed that pricing decisions are predetermined two periods in advance in order to yield a more realistic inflation dynamics in the model. This specification has been advocated by Svensson (2000) and Woodford (2003) to yield the dynamics consistent with empirical evidence on the effect of monetary policy on inflation. Demand is appropriately stochastically detrended.

Firms operate the following production function:

$$Y_t^i = A_t^i l_t^i$$

where $A_t^i$ is a sector-specific productivity parameter. It is assumed that there exist an exogenously specified (negative) externality operating through costs of production, which implies the following form of the total cost function:

$$TC_t^i = W_t l_t^i \chi_t$$

If $\chi_t = 0$ for all $t$ the model exhibits a standard specification of the cost function for which it holds that an increase in $A_t^i$ reduces real unit costs of production. The problem with this specification is that in an open economy permanent improvement in tradable productivity reduces the relative price of home tradable goods relative to the world economy and thus worsens the terms of trade. Consequently the real exchange rate need not appreciate, which is inconsistent with the empirical evidence for new EU member countries presented in Figure 1.[4] Contrary to the case of ratio of nontradable to tradable prices that exhibits a pronounced upward trend for all countries, the terms of trade do not exhibit a similary pronounced upward trend. Significant worsening of the terms is observed only for Latvia in the initial years, but after 1997 worsening disappears. To capture this empirical regularity we let the exogenous shifter of the cost function $\chi_t$ be correlated with aggregate level of productivity, i.e. $\chi_t = A_t^{\gamma_a}$. With a properly parameterized $\gamma_a > 0$, we can simulate the type of productivity improvement that leads to the use of more expensive (also imported) inputs required to produce higher-quality goods, and use of more skilled labor in the production that is on average paid a higher real wage. What is important is that the externality spills over to other sectors of economy whenever the change in productivity in one sector changes also aggregate productivity. With a suitable choice of $\gamma_a$ we can obtain the equilibrium dynamics that is consistent with the basic feature of the Balassa-Samuelson effect: increase in nontradable to tradable price ratio while the terms of trade do not worsen without resorting to the implausible assumption of perfect competition in the tradale sector. As a result, optimal monetary policy can be studied in the situation where a permanent shock to tradable productivity results in appreciation of the real exchange rate. Such a specification of production function can in a very simple and stylized way fit better with the developments in the new member countries, because we can observe from the data that the catching-up process has spurred an upward adjustment of overall price levels with contemporaneous increase in real wages.

[4]I would like to thank Martin Wagner and Jaroslava Hlouskova for providing the data used to construct the price ratios presented in Figure 1.

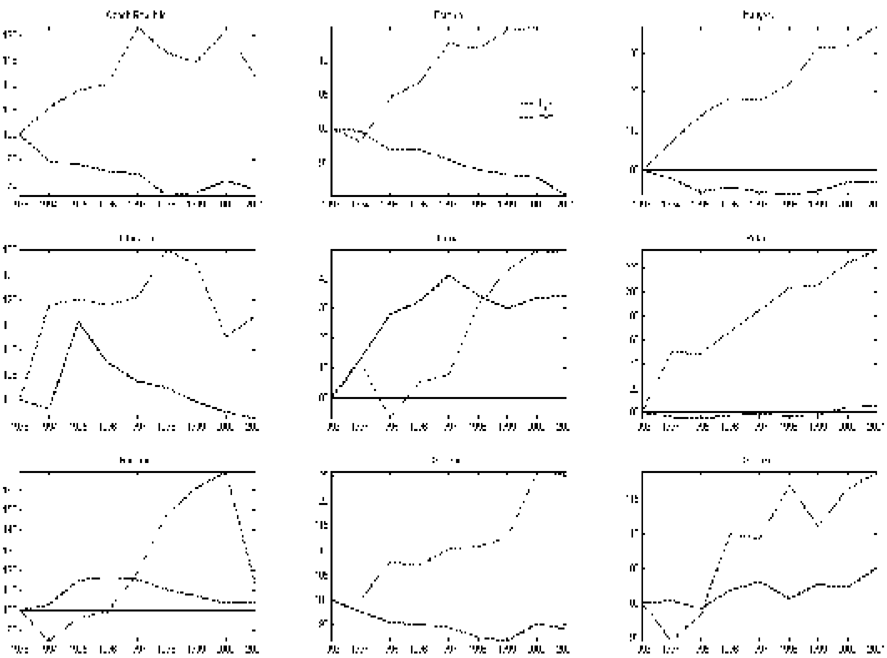

Figure 1: Terms of trade (ToT - dashed line) and internal price ratios (Pn/Pt - doted line)

The log of technology parameter is specified to follow a AR(2) process with a unit root and a positive drift term[5]

$$a_t^i = \ln A + (1+\gamma^n)\, a_{t-1}^i - \gamma^n a_{t-2}^i + \eta_{t+1}^{n\,i}, \quad i \in \{H, N\}$$

$\eta_{t+1}^{n\,i}$ is a zero-mean i.i.d. productivity shock and $0 \leq \gamma^n < 1$. This specification allows for a simulation of permanent productivity increases of the type where a certain permanent productivity shock at time $t$ continues to cumulate the level of productivity also in the future and only gradually reaches the new steady state.

The first-order condition of firms' optimization problem is

$$E_{t-2}\left\{\sum_{\tau=0}^{\infty}\alpha^{\tau}\widetilde{\Lambda}_{t+\tau}\Xi_{t,t+\tau}\left[\frac{\widetilde{P}_t^i}{P_{t+\tau}^i}-\frac{\vartheta^i}{\vartheta^i-1}\frac{\left(1-\tau^i\right)W_{t+\tau}}{A_{t+\tau}^i P_{t+\tau}^i}\right]P_{t+\tau}^i Y_{t+\tau}^{d\,i}\left(\frac{\widetilde{P}_t^i}{P_{t+\tau}^i}\right)^{-\vartheta^i}\right\}=0 \quad (2.16)$$

where $\left(\vartheta^i / \left(\vartheta^i - 1\right)\right)$ measures the mark-up over marginal cost. In a symmetric equilibrium share $1-\alpha$ of producers will set a new price in period $t$, and share $\alpha$ will keep the price at the previous level.

[5]The drift term is assumed to be equal to the corresponding term in the exogenous process of foreign productivity.

The log-linear approximation of the first-order condition is performed around the perfect foresight, zero-inflation, balanced trade equilibrium. In particular, I allow for bounded fluctuations in $\left\{\widehat{Y}_t^{d\ i}, \Pi_t, \widetilde{\Pi}_t, \widetilde{\Lambda}_t, \frac{W_t}{A_t^i P_t^i}\right\}$, $i = \{N, H\}$ around the steady state $\left\{\overline{Y}, 1, 1, \widetilde{\Lambda}, \frac{\vartheta^i - 1}{(1-\chi^i)\vartheta^i}\right\}$. In the treatment of $\widetilde{\Lambda}_t$ I follow Woodford (1996), who argues that pricing decisions of consumer/producers can make only an infinitesimal contribution to hers budget constraint. Thus $\widetilde{\Lambda}_t$ can be for simplicity treated as constant. Details of log-linear approximation are given in Appendix A. It leads to the following expression for the sector-specific Phillips curves[6]

$$\pi^i_{t/t-2} = \delta \pi^i_{t+1/t-2} + \phi^i z^i_{t/t-2} \tag{2.17}$$

where

$$\phi^i = \frac{\left(1 - \alpha^i\right)\left(1 - \alpha^i \delta\right)}{\alpha^i \left(1 + \varphi \vartheta^i\right)}$$

and $x^i$ are defined as follows

$$\begin{aligned} x_t^H &= \gamma_y y_t^H + \gamma_\mu^H \widehat{\mu}_t + \gamma_q^H \widehat{q}_t \\ x_t^N &= \gamma_y y_t^N + \gamma_\mu^N \widehat{\mu}_t - \gamma_q^N \widehat{q}_t \end{aligned}$$

where $y_t^i = y_t^{d\ i} - y_t^{n\ i}$, $\widehat{\mu}_t = \mu_t - \mu_t^n$, $\widehat{q}_t = q_t - q_t^n$, and $\gamma_y = \left(\varphi + \frac{1}{\sigma}\right)$, $\gamma_q^H = \lambda\left[1 - \frac{\theta}{\sigma}\right] - \frac{\overline{\gamma}_q}{\sigma}$, $\gamma_\mu^H = \omega\left[(1-\lambda) + \frac{1}{\sigma}\left(\theta\lambda - \theta^T\right)\right] - \frac{\overline{\gamma}_\mu}{\sigma}$, $\gamma_\mu^N = \left(1 - \frac{\theta}{\sigma}\right)(1-\lambda)\omega$ and $\gamma_q^N = \left(1 - \frac{\theta}{\sigma}\right)(1-\lambda)$. The Appendix reports the derivation of the natural levels of output, the internal price ratio and the terms of trade. They are defined in the following way (all relations hold up to a constant)

$$y_t^{n\ N} = a_t^N \tag{2.18a}$$

$$y_t^{n\ H} = a_t^H + \frac{1}{1+\varphi}\left[\overline{\gamma}_q q_t^n + \overline{\gamma}_\mu \mu_t^n\right] \tag{2.18b}$$

$$\mu_t^n = \frac{1}{\alpha_\mu}\left(y_t^{n\ H} - y_t^* - \alpha_q q_t^n\right) \tag{2.19}$$

$$q_t^n = a_t^H - a_t^N \tag{2.20}$$

Equations (2.18) - (2.20) uniquely determine the natural levels of sectoral outputs and natural levels of the internal price ratio and terms of trade as the function of sectoral productivity levels and foreign output. These expressions allow us also to determine $Z_t^N$ (trivially from (2.18)) and $Z_t^H$ from (2.10), (2.19) and (2.20).

Following Svensson (2000) it is assumed that general type of inertia and /or adjustment cost result in a simple partial adjustment of the inflation rate. It can be justified also by assuming partial indexation of prices with past sector-specific inflation rate by those monopolistic price setters that do not get the signal for optimal price adjustment in current period

[6] All small-case letters denote generically $x \equiv d \log X$.

(Smets and Wouters, 2001). Together with the assumption that inflation is predetermined two periods in advance, we obtain

$$\pi^i_{t+2/t} = \alpha_\pi \pi^i_{t+1/t} + (1-\alpha_\pi)\left[\delta \pi^i_{t+3/t} + \phi^i x^i_{t+2/t}\right]$$

where quantitites $x^i_t,\ i \in \{H, N\}$ are specified in the Appendix A. The assumption of inflation being predetermined two periods allows the expectations of the instrument setting $i_{t+1/t}$ to affect inflation. Note that $\pi^i_{t+2} = \pi^i_{t+2/t+1} + \varepsilon^i_{t+2},\ i \in \{N, H\}$, where $\varepsilon^i_{t+2}$ is a sector-specific cost-push shock. The cost-push shock is added to the model ad hoc. It could be formally included in the model by assuming that the elasticity of demand parameter $\vartheta^i$ is time varying and follows a stochastic stationary process (Ireland, 2003). This is, however, non-essential for the analysis in this paper. Defining $\alpha^i_y = (1-\alpha_\pi)\,\phi^i\gamma_y$, $\alpha^i_\mu = (1-\alpha_\pi)\,\phi^i\gamma^i_\mu$ and $\alpha^i_q = (1-\alpha_\pi)\,\phi^i\gamma^i_q$, $i \in \{N, H\}$ leads us to

$$\pi^H_{t+2} = \alpha_\pi \pi^H_{t+1/t} + (1-\alpha_\pi)\,\delta\pi^H_{t+3/t} + \alpha^H_y y^H_{t+2/t} + a^H_\mu \widehat{\mu}_{t+2/t} + \alpha^H_q \widehat{q}_{t+2/t} + \varepsilon^H_{t+2} \quad (2.21)$$

$$\pi^N_{t+2} = \alpha_\pi \pi^N_{t+1/t} + (1-\alpha_\pi)\,\delta\pi^N_{t+3/t} + \alpha^N_y y^N_{t+2/t} + a^N_\mu \widehat{\mu}_{t+2/t} - \alpha^N_q \widehat{q}_{t+2/t} + \varepsilon^N_{t+2} \quad (2.22)$$

which is the final specification of sector-specific Phillips curves of the model. In contrast to Phillips curve in the closed economy we can note that inflation stabilization requires also the stabilization of sectoral relative price gaps. This makes inflation stabilization in an open economy inherently more difficult because the stabilization of the general price level must allow also for relative price adjustment. This feature is even more important in the case considered in this paper. Permanent sector-specific productivity shocks induce a permanent change to natural level of the internal price ration and terms of trade. The central bank that fails to acknowledge that the BS effect causes an equilibrium appreciation of the real exchange rate also fails to achieve price stability.

### 2.3 Exogenous processes

Foreign variables are exogenously given to the small open economy. In particular, foreign inflation and output are assumed to follow stationary AR(1) processes

$$\pi^*_t = \gamma^*_\pi \pi^*_{t-1} + \varepsilon^*_t \quad (2.23)$$

$$y^*_t = \gamma^*_y y^*_{t-1} + \eta^*_t \quad (2.24)$$

where $\varepsilon^*_t$ and $\eta^*_t$ are white-noise processes. Furthermore, it is assumed that foreign consumer prices are equal to prices of foreign tradable goods. Foreign central bank follows a Taylor-type rule of the form

$$i^*_t = f^*_\pi \pi^*_t + f^*_y y^*_t + \xi^*_{i,t} \quad (2.25)$$

where $\xi^*_{i,t}$ is a zero-mean i.i.d. foreign monetary policy shock.

### 2.4 Monetary policy

There are two potential sources of average inflation bias in monetary policy in this model. The first is the presence of market power in both tradable and nontradable sector; the second is the possibility of monetary policy to influence the terms of trade in presence of sticky prices in a way that is favorable to domestic consumers (see Corsetti and Pesenti, 2001). Following Rotemberg and Woodford (1999) it is assumed that the government distributes the lump-sum taxes to domestic producers in the form of an employment subsidy that eliminates the monopolistic distortion in steady state and thus the average inflation bias.[7] The employment subsidies $\tau^N$ and $\tau^H$ are set so as to satisfy (complete derivation can be found in the Appendix)

$$\begin{aligned} \frac{\vartheta^N - 1}{\vartheta^N} &= \left(1 - \tau^N\right)\lambda \\ \frac{\vartheta^H - 1}{\vartheta^H} &= \left(1 - \tau^H\right)(1-\lambda)(1-\omega) \end{aligned}$$

The model assumes that spending is predetermined one period and pricing decisions are predetermined two periods. This implies that current interest changes do not affect spending nor inflation. Moreover, any unanticipated movements in the interest rate do not have a role in stabilizing output or inflation. With even the slightest preference for interest rate smoothing optimal monetary policy would completely eliminate any unforecastable interest rate variation. This implies that in the present model discretionary decision making cannot improve the output - inflation variability trade-off in any period. The solution method for the rational expectation equilibrium considered in the paper is thus the solution under commitment that requires the following optimal interest rate setting

$$i_t = E_{t-1} i_t$$

In other words, in each period $t$ the central bank acts so as to achieve the value of its instrument that has decided to set one period in advance. The path of the nominal exchange rate is linked to the interest rate through the uncovered interest rate parity relation. Complete international financial markets equalize the returns of foreign and domestic riskless bonds in terms of domestic currency

$$E_t \Xi_{t,t+1} R_t = R_t^* \frac{E_{t+1}}{E_t}$$

and log-linearization of this condition leads to the uncovered interest rate parity condition

$$i_t = i_t^* + \Delta e_{t+1/t} \tag{2.26}$$

Expected change in the terms-of-trade is then linked to the domestic and foreign nominal interest rates and domestic and foreign inflation through the real interest parity condition

$$\Delta \mu_{t+1/t} = i_t - i_t^* + \pi_{t+1/t}^* - \pi_{t+1/t}^H \tag{2.27}$$

[7] It is also assummed that the same occurs in the foreign economy.

The central bank uses its instrument to minimize the following objective function

$$J_t = E_t \sum_{\tau=0}^{\infty} \delta^{\tau} L_{t+\tau}$$

The period $t$ component of the loss function is

$$L_t = \mu_\pi^C \left(\pi_t^C\right)^2 + \mu_\pi^H \left(\pi_t^H\right)^2 + \mu_\pi^N \left(\pi_t^N\right)^2 + k_y^H \left(y_t^H\right)^2 + k_y^N \left(y_t^N\right)^2$$

with all weights non-negative. Optimal weights are derived (see the Appendix for details) only for the special case $\sigma = \theta^T = \theta = 1$, derivation for the general case are left for future research. Optimal weights in the special case are as follows

$$\begin{aligned}
\mu_\pi^H &= \frac{(1-\lambda)(1-\omega)\vartheta^H \alpha^H}{(1-\alpha^H)(1-\delta\alpha^H)} \\
\mu_\pi^N &= \frac{\lambda \vartheta^N \alpha^N}{(1-\alpha^N)(1-\delta\alpha^N)} \\
k_y^H &= (1-\lambda)(1-\omega)(1+\varphi) \\
k_y^N &= \lambda(1+\varphi)
\end{aligned}$$

(and $\mu_\pi^C = 0$). The same weights are for simplicity used also for the parameterization of the model that deviates from $\sigma = \theta^T = \theta = 1$. Monetary policy in such a case is not optimal. However, note that the focus of the paper is to determine whether the BS effects can lead to violation of the Maastricht inflation criterion when the policy the central bank essentially tries to target that criterion i.e. the inflation, given the commitment to enter the Euro area. In such a case using sub-optimal weights can still lead to valid conclusions as it should not be surprising to find that membership in the ERM II and the existence of Maastricht criteria constrain the monetary policy to act suboptimally.

## 2.5 Calibration of the model

Basic structural parameters of the model are chosen in accordance with the practice in the literature calibrating small open-economy models. The baseline specification of the model is able to reproduce the standard deviation of CPI inflation reported by Natalucci and Ravenna (2002) and Laxton and Pesenti (2003) for the case of the Czech Republic.

The paper considers two sets of values for the parameters $\sigma$, $\theta^T$ and $\theta$. Under the baseline scenario the papers considers the optimal welfare criterion as the objective function of the central bank. Validity of a simple second-order approximation to the utility function requires to set the intertemporal elasticity of substitution $\sigma$ to unity (which implies $U\left(\widehat{C}_t\right) = \ln \widehat{C}_t$) and $\theta^T = \theta = 1$ (CES form for aggregate consumption collapses to Cobb-Douglas form). As a robustness check the model is parameterized also with empirically more plausible values. In this alternative scenario $\sigma$ is set to 0.33, $\theta^T$ to 1.5 and $\theta$ to 0.5, which is in line with the literature. Laxton and Pesenti (2003) consider just a slightly different specification with both $\theta^T$and $\theta$ set to 1.1 and $\sigma = 0.33$.[8] Smets and Wouters (2002) set $\theta^T$ to 1.5, which is also the choice of Natalucci and Ravenna (2002). The latter authors set $\theta$ to 0.5 in line with Stockman and Tesar (1995). The parameter determining the disutility of work $\varphi$ has been set to 3, the choice of Gali and Monacelli (2005). It implies a labor supply elasticity of 0.33.

The elasticities of substitution among differentiated products are chosen as to yield sensible steady state markups, which should fall within 1.1 and 1.4 (see Laxton and Pesenti, 2003). The elasticity of substitution for nontradable varieties $\vartheta^N$ is set to 6. Its counterpart in the nontradable sector $\vartheta^H$ is set to 10, reflecting a lower market power in the tradable sectors of open economies. This is still very different from the assumption of perfect competition in the tradable sector used in Natalucci and Ravenna (2002) or Calvo et al. (2002). As a consequence, terms of trade can change after a shock. The share of nontradables goods in consumption index $\lambda$ has been set to 0.45. This is very close to the nontradable shares in the new EU member states as calculated by Wagner and Hlouskova (2004). The share of domestically produced tradable goods in tradable consumption $\omega$ is assumed to be 0.5, which is in line with the choice of Natalucci and Ravenna(2002) and also corresponds to average consumption shares in GDP for the Czech Republic. The discount factor $\delta$ is 0.99.

On the supply side of the model the Calvo parameters are set to 0.5 in both sectors in the baseline. This implies a mean price quotation length of two quarters, which can be seen as realistic in small open economies. In the sensitivity analysis the case of a longer price quotation length is considered with $\alpha = 0.75$. This affects the variability of output gaps and inflation but it does not affect the basic conclusions of the paper as regards the inflationary consequences of the BS effect with optimal policy under commitment and associated path of the nominal exchange rate.

For foreign variables it is assumed that they follow stationary AR(1) processes. The autoregressive parameters for foreign inflation and foreign output is set to 0.8, which follows Svensson (2000). The foreign interest rate follows a Taylor rule with coefficient 1.5 on inflation and 0.5 on output.

For the coefficients on lagged inflation in the Phillips curves $\alpha_\pi$ and lagged output gap in the IS curves $\beta_y$ there is less evidence in the calibration for small open economies. Both parameters were set to 0.5, very close to the values estimated by Smets (2000) for the Euro area.

The most important parameter to calibrate was the persistence parameter for growth of productivity. The process should generate a large and gradual increase in productivity corresponding to the driving force of the catching-up process. For this reason I set $\gamma^{n\,H} = \gamma^{n\,N} = 0.95$. An initial shock of 1.7% percent to tradable productivity generates a roughly 30% increase in a ten-year period. Average yearly growth rate of tradable productivity is within a plausible range for Acceding countries and close to the increase considered by Natalucci and Ravenna (2002) (2.65%). This is a situation of a large increase in productivity induced by a single shock, such that it can be argued that the impulse responses presented below present responses of the economy in a situation of a very large increase in relative tradable-nontradable productivity.

The calibration of standard deviation of exogenous shocks of the model was less important for the purposes of this paper, because the focus is on generating a differential productivity

[8] It has been checked that varying the elasticity of intertemporal substitution on the interval $[0.33, 1]$ does not change the conclusions of the paper.

growth of desirable size. They have been calibrated in a way that the model replicates the above-mentioned variability of inflation. The variance of productivity shocks is 0.25 in both sectors, for their demand counterparts it is 0.5. The variance of the remaining shocks: domestic cost-push shocks, foreign price, foreign output and the foreign interest rate shocks has been set to 0.2.

# 3 Effects of Permanent Sector-Specific Productivity Shocks

The model is solved for the rational expectations equilibrium under commitment using Soderlind's method (1999).[9] Figures 1-5 present the impulse responses to the tradable productivity shock of the size described in the previous section. Two types of model specification are considered. The first is the case with $\gamma_a = 0$, a classic productivity shock that leads to a decrease in prices of home tradable goods and worsening of terms of trade. The second is the case with $\gamma_a > 0$. In such a case the increase of tradable to nontradable productivity leads to appreciation of the internal price ratio as in the classic case, but with a suitable parameterization of $\gamma_a$ does not lead to worsening of the terms of trade. This allows to analyze the realistic situation new EU member countries are confronted with: catching-up process accompanied with real exchange rate appreciation. For each of the two specifications of the production technology the responses to the productivity shock are plotted for the baseline specification of the model: $\sigma = \theta^T = \theta = 1$ and the alternative where the parameters have been set to $\sigma = 0.33$, $\theta^T = 1.5$ and $\theta = 0.5$.

Figure 1 presents the impulse responses in case of a classic productivity shock and optimal monetary policy for the baseline model specification. The optimal response requires the central bank to increase the nominal interest rate that raises also the real interest rate. In principle the bank in such a case mimics the deviation in the natural real interest rate that also increases as can be seen from (2.14) and (2.15) accompanied by the fact that $\Delta z^H$ increases. Consequently also the nominal exchange rate increases, which is also why the CPI inflation rate deviates positively from the steady state value, but not so dramatically that it would represent a significant risk to fulfilling the inflation Maastricht criterion. As mentioned above such productivity improvement in the tradable sector leads to appreciation of the nontradable - tradable price ratio; however, worsening of the terms of trade overweights the effect and leads to depreciation of the real exchange rate.

Similar conclusions are reached also by considering a model specification that does not use the assumption $\sigma = \theta^T = \theta = 1$ (see Figure 4 in the Appendix). In such a case the proposed policy is no longer optimal from the social welfare point of view, but the results are nevertheless reported for comparability. In this case the exchange rate channel of monetary policy transmission becomes operational. The responses are qualitatively similar to the benchmark. A slightly sharper initial response of the interest rate with faster decline in subsequent periods results in slightly smaller nominal and real depreciation. It thus holds also for this case that worsening of the terms of trade largely offsets the appreciation of the

[9] A rigid fixed exchange rate regime that corresponds to a credible adoption of foreign currency results in a solution with multiple equilibria. The policy instrument is in all periods set at the level of the foreign policy rate and hence for a small open economy exogenously given. The indeterminacy result follows from a direct application of the results in Woodford (2003).

internal price ratio.[10] The response of CPI inflation rate is comparable to the baseline.

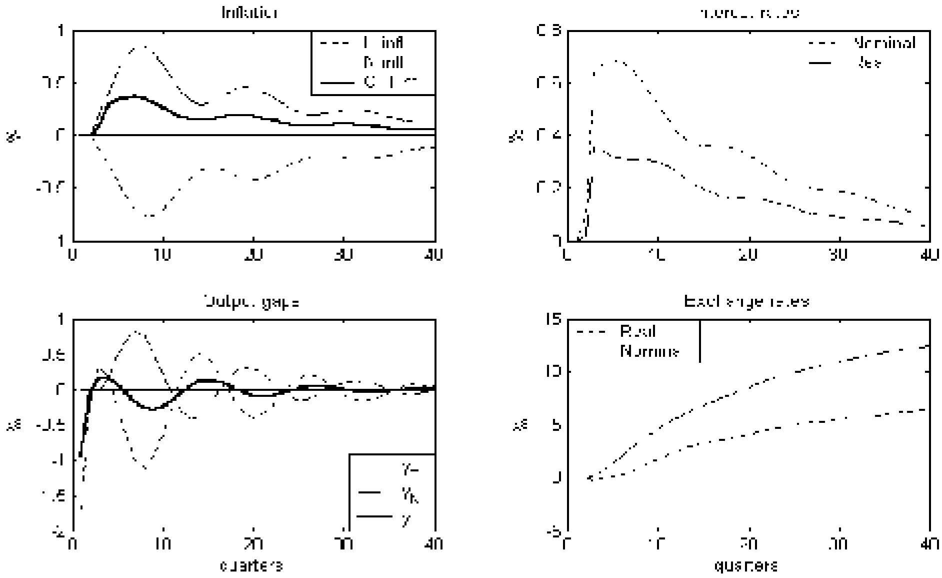

Figure 2: Classic productivity shock - optimal monetary policy with baseline model specification

A different conclusion about optimal response of monetary policy emerges when the production technology is specified with $\gamma_a > 0$.[11] It follows from (2.5) and (2.6) that in this case the aggregate (CPI-based) natural real interest rate temporarily falls. From real interest rate parity follows that this is what we should expect in a country experiencing real exchange rate appreciation. As we can observe from Figure 2, the real exchange rate in such a case indeed appreciates and the optimal response of monetary policy is to lower the nominal interest rate along the adjustment path. As a consequence the nominal exchange rate falls and the currency appreciates also in nominal terms. Even though in the home tradable sector there is no initial pressure to decrease prices *per se,* nominal appreciation induces a negative response of home tradable inflation. This response, coupled with nominal appreciation, implies also that under optimal policy CPI inflation rate does not deviate significantly from zero.

Similar conclusions emerge also if we consider alternative model specification, but we keep the objective function of the central bank the same as in the baseline (see Figure 3). The responses of variables are similar both in terms of the response of the interest rate and nominal and real appreciation. Also, the variability of the output gap is smaller because of the exchange rate channel of monetary transmission that is additionally available to monetary

[10] Note that a different response of the real exchange rate comapred to the baseline model specification should not be surprising. As follows from equations (2.18) - (2.20) that the paths of natural levels of home tradable output and the terms of trade change.

[11] $\gamma_a$ is set to $1/(1-\lambda)$. This offsets the effect of differential productivity growth on the terms of trade in the baseline specification of the model.

policy in such a case.

As a simple robustness check I consider also the case of classic flexible CPI inflation targeting where the parameters in the central banks objective function are set to $k_\pi^C = 1$ and $k_y^H = k_y^N = 0.2$. From Figure 5 in the Appendix we observe that the findings about optimal response of the nominal interest rate and the nominal exchange rate do not change also in this case.

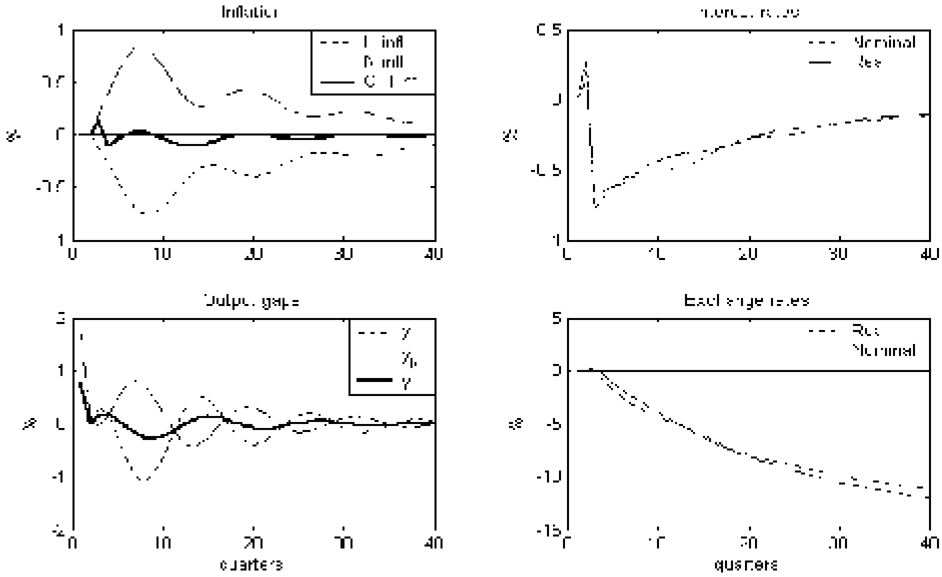

Figure 3: Balassa-Samuelson productivity shock - optimal monetary policy with baseline model specification

The resuls presented here lead to policy implications that differ substantially from the conclusions of Natalucci and Ravenna (2002). The reason lies in a different modelling approach. This paper constructs the Balassa-Samuelson shock as an equilibrium driving process and under perfect foresight the policymakers take this into account and do not attempt to revert this process. Resulting inflation and nominal exchange rate dynamics are not so dramatic to surely prevent the new EU member states to fulfill the inflation Maastricht criterion and go through a successful membership in the ERM II. In addition, the paper shows that the policy implications about the optimal response to the BS shock relies heaviliy on how we specify permanent tradable productivity improvement. If the productivity improvement leads to worsening of the terms of trade completely different implications follows relative to the case where the terms of trade do not worsen and thus do not offest the appreciating effect of the internal price ratio on the real exchange rate. In the new EU member states the data reveal a catching-up process accompanied by real exchange rate appreciation. In such a case, the CPI-based natural real interest rate is lower along the adjustment path, implying that a path of lower nominal interest rates is consistent with the optimal monetary response under commitment to differential productivity growth. This finding does not support the view that a drop of interest rates in the case of fast process of Euro adoption could lead to

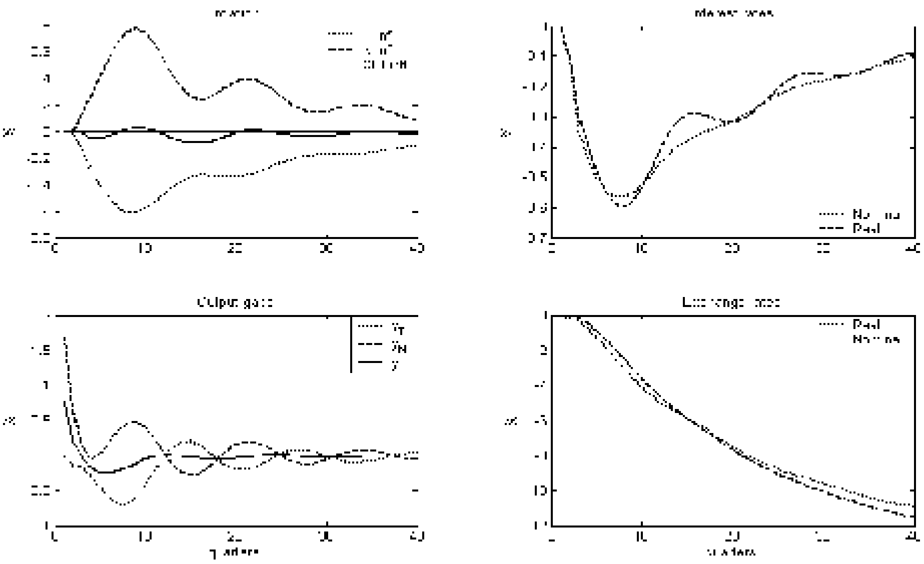

Figure 4: Balassa-Samuelson productivity shock - alternative model specification

overheating of the economy due to the Balassa-Samuelson effect.[12]

# 4 Conclusion

The main objectives of the paper are first to analyze the inflationary consequences of the Balassa-Samuelson effect in a simple New Keynesian dynamic stochastic general equilibrium model with optimal monetary policy, and second, to check whether the optimal responses are in line with the institutional constraints ERM II system imposes on new EU member countries during the run-up to EMU.

The model presented in the paper is rather simple and does not model explicitly complex dynamics in the labor market and investment activity. However, it is realistically calibrated and sensitivity analysis revealed that variation in model's main parameters do not change the conclusion. In addition, it is worth repeating that the productivity shock considered in the paper represents a rather large differential productivity increase generated by a single shock. In answer to the first question the analysis shows that the BS effect need not cause inflation differentials that would impose a significant threat to fulfilling the Maastricht inflation criterion. Of course, an important caveat is that the central bank must be able to pursue the goal of price stability under commitment. Discretionary behavior, real exchange rate targeting or imprudent fiscal policies, among other reasons, may lead to a different situation. However, the cause of potentially larger inflation differential in such a case does not lie directly in the presence of the Balassa-Samuelson effect.

[12] Of course, other reasons, such as considerable fiscal and trade imbalances, could support the validity of such a view.

One of the important reasons policymakers have stressed against fast adoption of the Euro is inappropriately low nominal (and real) interest rates within the ERM II systems that could potentially backfire demand and asset booms and ultimately lead to high inflation. One of the causes for such risks is also related to the Balassa-Samuelson effect (ECB, 2004). This reasoning is correct only if the productivity growth differential leads to significant worsening of the terms of trade that largely offsets the effect of increased nontradable - tradable price ratio on the real exchange rate. In such a case ERM II system would be inappropriate because it does not allow for an upward adjustment of the central parity. If, on the other hand, differential productivity growth does not lead to the offsetting effect of the terms of trade, different policy implications follows. Empirically the Balassa-Samuelson leads to real appreciation and such a scenario has been simulated within the model by considering a simple alternative specification of production technology. Even though in such a case the increase in tradable -nontradable productivity ratio does not lead to deflationary pressures in the tradable sector, pursuing the goal of price stability that is shown to be optimal at least under the baseline specification of the model does not result in positive inflation differential with respect to the rest of the world. The optimal response of the policy is to keep the nominal interest rates low along the adjustment path. This corroborates the fact that in a country facing real appreciation also the natural real interest rate is lower along the adjustment path. Because it is optimal for the nominal exchange rate to fall this does not represent an inconsistency with the institutional framework of the ERM II: a downward adjustment of the parity is formally allowed prior to the adoption of the Euro.

# 5 Appendix A: Derivation of the Phillips curves

It is useful first to find the expressions for real marginal cost in each sector. This will allow us to determine the natural levels of sectoral output, the internal price ratio and the terms of trade. Starting from the optimality condition for intratemporal allocation of leisure and consumption $W_t/\widehat{P}_t^C = \widehat{C}_t^{\frac{1}{\sigma}} l_t(j)^{\varphi}$ note that it can be written as

$$\frac{W_t}{\widehat{P}_t^i} = \frac{\widehat{P}_t^C}{\widehat{P}_t^i} \widehat{C}_t^{\frac{1}{\sigma}} \left(A_t^i\right)^{-\varphi} \left(Y_{t+\tau}^{d\,i}(j)\right)^{\varphi}$$

where $Y_t^{d\,i}$ denotes the real demand to distinguish it from $Y_t^i$ that denotes the output gap. Then the expression for the real marginal costs in the nontradable sector follows

$$\begin{aligned}
\frac{W_t \chi_t}{A_t^N P_t^N} &= \frac{Z_t^C P_t^C}{P_t^N} \widehat{C}_t^{\frac{1}{\sigma}} \left(A_t^N\right)^{-(1+\varphi)} \left(Y_t^{d\,N} \left(\frac{\widetilde{P}_t^N}{P_t^N}\right)^{-\vartheta^N}\right)^{\varphi} \\
&= \left(\frac{1}{\lambda}\right)^{\frac{1}{\sigma}} \left(\frac{\widehat{P}_t^C}{\widehat{P}_t^N}\right)^{1-\frac{\theta}{\sigma}} \chi_t Z_t^N \left(Y_t^{d\,N}\right)^{\varphi} \left(Y_t^N\right)^{\frac{1}{\sigma}} \left(A_t^N\right)^{-(1+\varphi)} \left(\frac{\widetilde{P}_t^N}{P_t^N}\right)^{-\vartheta^N \varphi}
\end{aligned}$$

For the process of loglinear approximation of firms' first-order condition it is useful to define an exact log-linear relationship (up to a constant)[13]

$$\begin{aligned}
x_t^N &\equiv \ln \chi_t + z_t^N + \varphi y_t^{d\,N} + \frac{1}{\sigma} y_t^N - (1+\varphi)\, a_t^N - \left(1 - \frac{\theta}{\sigma}\right)(1-\lambda)\left(\widehat{q}_t - \omega \widehat{\mu}_t\right) \\
&= \left(\varphi + \frac{1}{\sigma}\right) y_t^N - \left(1 - \frac{\theta}{\sigma}\right)(1-\lambda)\left(\widehat{q}_t - \omega \widehat{\mu}_t\right)
\end{aligned}$$

where the second equality follows from the fact that $y_t^{n\ N} = a_t^N - \frac{1}{1+\varphi} \ln \chi_t$ (see below). I have also used the fact that $\log Z_t^N \equiv z_t^N = y_t^{n\ N}$ i.e. the log of natural level of output in nontradable sector. $y_t^N$ denotes the corresponding output gap. For the tradable sector the corresponding expression is

$$\begin{aligned}
\frac{W_t \chi_t}{A_t^H P_t^H} &= \frac{Z_t^C P_t^C}{Z_t^H P_t^H} \chi_t Z_t^H \widehat{C}_t^{\frac{1}{\sigma}} \left(A_t^H\right)^{-(1+\varphi)} \left(Y_{t+\tau}^{d\,H} \left(\frac{\widetilde{P}_t^H}{P_t^H}\right)^{-\vartheta^H}\right)^{\varphi} \\
&= \left(\frac{1}{(1-\lambda)(1-\omega)}\right)^{\frac{1}{\sigma}} \frac{\widehat{P}_t^C}{\widehat{P}_t^H} \left(\frac{\widehat{P}_t^C}{\widehat{P}_t^T}\right)^{-\frac{\theta}{\sigma}} \left(\frac{\widehat{P}_t^T}{\widehat{P}_t^H}\right)^{-\frac{\theta^T}{\sigma}} \chi_t Z_t^H \left(\widehat{C}_t^H\right)^{\frac{1}{\sigma}} \\
&\quad \times \left(A_t^H\right)^{-(1+\varphi)} \left(Y_t^{d\,H} \left(\frac{\widetilde{P}_t^H}{P_t^H}\right)^{-\vartheta^H}\right)^{\varphi}
\end{aligned}$$

[13] This is essentially the deviation of log real marginal cost from the steady state value without $\left(\widetilde{P}_t^N / P_t^N\right)^{-\vartheta^N \varphi}$.

As for the nontradable sector we can define a similar log-linear relation[14]

$$
\begin{aligned}
x_t^H &\equiv \ln \chi_t + z_t^H + \varphi y_t^{d\,H} + \frac{1}{\sigma}\widehat{c}_t^H - (1+\varphi)\, a_t^H + \omega\left[(1-\lambda) + \frac{\theta\lambda}{\sigma} - \frac{\theta^T}{\sigma}\right]\widehat{\mu}_t + \lambda\left[1 - \frac{\theta}{\sigma}\right]\widehat{q}_t \\
&= y_t^{n\,H} - \overline{\gamma}_q q_t^n - \overline{\gamma}_\mu \mu_t^n + \varphi y_t^{d\,H} + \frac{1}{\sigma}\left(y_t^H - \overline{\gamma}_q \widehat{q}_t - \overline{\gamma}_\mu \widehat{\mu}_t\right) - (1+\varphi)\, a_t^H \\
&\quad + \omega\left[(1-\lambda) + \frac{1}{\sigma}\left(\theta\lambda - \theta^T\right)\right]\widehat{\mu}_t + \lambda\left[1 - \frac{\theta}{\sigma}\right]\widehat{q}_t \\
&= \left(\varphi + \frac{1}{\sigma}\right) y_t^H + \left\{\omega\left[(1-\lambda) + \frac{\theta\lambda}{\sigma} - \frac{\theta^T}{\sigma}\right] - \frac{\overline{\gamma}_\mu}{\sigma}\right\}\widehat{\mu}_t + \left\{\lambda\left[1 - \frac{\theta}{\sigma}\right] - \frac{\overline{\gamma}_q}{\sigma}\right\}\widehat{q}_t
\end{aligned}
$$

where the second line uses relation (2.13) that is consistent with balanced-trade equilibrium.

The steady-state value of the log real marginal cost $mc_t^i$ is $\xi^i \equiv -\log \frac{\vartheta^i\left(\vartheta^i-1\right)^{-1}}{(1-\tau^i)}$. By imposing the constant mark-up conditions for all $t$ i.e. in the equilibrium with flexible prices the natural levels of output in each sector follow

$$
\begin{aligned}
y_t^{n\,H} &= a_t^H + \frac{1}{1+\varphi}\left[\overline{\gamma}_q q_t^n + \overline{\gamma}_\mu \mu_t^n - \ln \chi^H + \log \frac{\vartheta^H\left(\vartheta^H - 1\right)^{-1}}{\left(1-\tau^H\right)(1-\omega)^{\frac{1}{\sigma}}(1-\lambda)^{\frac{1}{\sigma}}}\right] \\
y_t^{n\,N} &= a_t^N - \frac{1}{1+\varphi}\ln \chi^N + \frac{1}{1+\varphi}\log \frac{\vartheta^N\left(\vartheta^N - 1\right)^{-1}}{\left(1-\tau^N\right)\lambda^{\frac{1}{\sigma}}}
\end{aligned}
$$

Furthermore, from firms' optimal pricing conditions in flexible-price equilibrium and by imposing balanced trade equilibrium we can obtain the natural levels of terms of trade and internal price ratio

$$
\begin{aligned}
\mu_t^n &= \frac{1}{\gamma_\mu}\left(y_t^{n\,H} - y_t^* - \gamma_q q_t^n\right) \\
q_t^n &= a_t^H - a_t^N
\end{aligned}
$$

where the expression for the natural terms of trade follows from (2.11). Both relations hold up to a constant.

Due to Calvo-type price staggering the price indexes in both sectors obey

$$
P_t^i = \left[\alpha^i P_{t-1}^{i\;1-\vartheta^i} + \left(1-\alpha^i\right)\widetilde{P}_{t-1}^{i\;1-\vartheta^i}\right]^{\frac{1}{1-\vartheta^i}}
$$

Using the definitions: $\widetilde{\Pi}_t^i \equiv \widetilde{P}_t^i / P_t^i, \Pi_t^i \equiv P_{t+1}^i / P_t^i$ this can be more conveniently written as

$$
\begin{aligned}
\Pi_t^i &= \left[\alpha^i + \left(1-\alpha^i\right)\Pi_t^{i\;1-\vartheta^i}\widetilde{\Pi}_{t-1}^{i\;1-\vartheta^i}\right]^{\frac{1}{1-\vartheta^i}} = \\
&= \alpha^{i\,\frac{1}{1-\vartheta^i}}\left[1 - \left(1-\alpha^i\right)\widetilde{\Pi}_{t-1}^{i\;1-\vartheta^i}\right]^{\frac{1}{\vartheta^i - 1}}
\end{aligned}
$$

[14] Note that with $\sigma = 1$ and $\theta^T = \theta^N = 1$ the last two expressions simplify to $z_t^N = (1+\varphi)\left(y_t^{d\,N} - a_t^N\right)$ and $z_t^H = (1+\varphi)\left(y_t^{d\,H} - a_t^H\right)$ such that we can observe that the exchange rate channel is switched off.

The first-order condition (2.16) can with this notation be can written in terms of stationary variables as

$$E_{t-2}\left\{\sum_{\tau=0}^{\infty}(\alpha\delta)^{\tau}\,\widetilde{\Lambda}_{t+\tau}\left[\frac{\widetilde{\Pi}_{t}^{i}}{\prod_{s=1}^{\tau}\Pi_{t+s}^{i}}-\frac{\vartheta^{i}}{\vartheta^{i}-1}\frac{\left(1-\chi^{i}\right)W_{t+\tau}}{A_{t+\tau}^{i}P_{t+\tau}^{i}}\right]\frac{\widehat{P}_{t+\tau}^{i}\widehat{Y}_{t+\tau}^{d\,i}}{\widehat{P}_{t+\tau}^{C}\widehat{C}_{t+\tau}^{\frac{1}{\sigma}}}\left(\frac{\widetilde{P}_{t}^{i}}{P_{t+\tau}^{i}}\right)^{-\vartheta^{i}}\right\}=0$$

The log-linearized first-order condition is

$$E_{t-2}\left\{\sum_{\tau=0}^{\infty}\alpha^{\tau}\delta^{\tau}\left[\left(1+\varphi\vartheta^{i}\right)\left(\widetilde{\pi}_{t}^{i}-\sum_{s=1}^{\tau}\pi_{t+s}^{i}\right)-x_{t}^{i}\right]\right\}=0$$

with all components summable. After some manipulation we can obtain the following expression for the inflation rate of newly-set prices:

$$E_{t-1}\widetilde{\pi}_{t}^{i}=E_{t-2}\left\{\alpha\delta\pi_{t+1}^{i}+\frac{1-\alpha\delta}{1+\varphi\vartheta^{i}}x_{t}^{i}\right\}+\alpha\delta E_{t-2}\widetilde{\pi}_{t+1}^{i}$$

where $z_t^i$ are defined above. Then we can use the relation $\pi_t^i=\frac{1-\alpha}{\alpha}\widetilde{\pi}_t^i$ $i\in\{N,T\}$ to eliminate $\widetilde{\pi}_t^i$ and get

$$\pi_{t/t-2}^{i}=\delta\pi_{t+1/t-2}^{i}+\phi^{i}x_{t/t-2}^{i}$$

where

$$\phi^{i}=\frac{\left(1-\alpha^{i}\right)\left(1-\alpha^{i}\delta\right)}{\alpha^{i}\left(1+\varphi\vartheta^{i}\right)}$$

This is expression (2.17) in the text. In terms of output gaps and deviations of relative sector prices from their natural levels we have

$$\begin{aligned}\pi_{t/t-2}^{H}&=\delta\pi_{t+1/t-2}^{H}+\phi^{H}\left(\gamma_{y}y_{t/t-2}^{H}+\gamma_{\mu}^{H}\widehat{\mu}_{t/t-2}+\gamma_{q}^{H}\widehat{q}_{t/t-2}\right)\\\pi_{t/t-2}^{N}&=\delta\pi_{t+1/t-2}^{N}+\phi^{N}\left(\gamma_{y}y_{t/t-2}^{N}+\gamma_{\mu}^{N}\widehat{\mu}_{t/t-2}-\gamma_{q}^{N}\widehat{q}_{t/t-2}\right)\end{aligned}$$

where the coefficients are given in the main text.

# 6 Appendix B: Derivation of the Welfare Loss Function

Derivation of the welfare loss function essentially follows Gali and Monacelli (2005) taking into account the specific setting of the present model. The presence of monopolistic power and associated deadweight losses imply a lower equilibrium level of output than is socially desirable. For the monetary policy this distorts the incentives and induces an average inflation bias. In the open economy monetary faces an additional channel that distorts its incentives. In combination with price stickiness there is a possibility to influence the terms of trade in a way that is beneficial to domestic economy. Following Rotemberg and Woodford (1999) it is assumed that the government distributes the lump-sum taxes to domestic producers in the form of an employment subsidy that eliminates the monopolistic distortion in steady state and thus the average inflation bias. It is also assumed that the same occurs in the foreign economy. In the derivation of the loss function I assume particular values of model parameters $\sigma = \theta^T = \theta = 1$ that offers a simple second-order approximation to the utility of the representative agent.

To achieve the optimal allocation the social planner maximizes $U\left(\widehat{C}_t\right) - W(l_t)$ subject to the constraints: $Y_t^H = A_t^H l_t^H$, $Y_t^N = A_t^N l_t^N$, $l_t = l_t^H + l_t^N$ and

$$C_t = \lambda^{-\lambda} \left(Y_t^N\right)^{\lambda} \left(Y_t^H\right)^{(1-\lambda)(1-\omega)} \left(\chi Y_t^*\right)^{(1-\lambda)\omega}.$$

Note that the last expression is an exact expression under the specific parameterization of the model and follows from noting that

$$\begin{aligned}
\widehat{C}_t^T &= (1-\lambda)\,\chi Y_t^* \widehat{S}_t^{1-\omega} \\
\widehat{C}_t^H &= (1-\lambda)(1-\omega)\,\chi Y_t^* \widehat{S}_t \\
Y_t^H &= \chi Y_t^* \widehat{S}_t
\end{aligned}$$

Combining the first and third expression leads to

$$\begin{aligned}
\widehat{C}_t^T &= (1-\lambda)\,\chi Y_t^* \widehat{S}_t^{1-\omega} \\
&= (1-\lambda)\,\chi Y_t^* \left(\frac{Y_t^H}{\chi Y_t^*}\right)^{1-\omega} \\
&= (1-\lambda)\left(\chi Y_t^*\right)^{\omega} \left(Y_t^H\right)^{1-\omega}
\end{aligned}$$

The first-order conditions for the optimal allocation in the small open economy are

$$\begin{aligned}
\lambda U_C \frac{C_t}{Y_t^N} - W_l \frac{1}{A_t^N} &= 0 \\
(1-\lambda)(1-\omega)\, U_C \frac{C_t}{Y_t^H} - W_l \frac{1}{A_t^H} &= 0
\end{aligned}$$

$$W_l l_t^N = \lambda U_C C_t \tag{B.1}$$

$$W_l l_t^H = (1-\lambda)(1-\omega)\, U_C C_t \tag{B.2}$$

Under assumed preferences the first-order conditions imply (besides balanced trade for all $t$) also constant employment $l^N = \lambda^{\frac{1}{1+\varphi}}$, $l^H = \left[(1-\lambda)(1-\omega)\right]^{\frac{1}{1+\varphi}}$. Below the following equilibrium relations will be used

$$\begin{aligned}
\overline{W}_l \bar{l}_t^N &= \lambda \\
\overline{W}_l \bar{l}_t^H &= (1-\lambda)(1-\omega)
\end{aligned}$$

It also hold that optimal allocation satisfies

$$
\begin{aligned}
\frac{\vartheta^N - 1}{\vartheta^N} &= \overline{MC}^N \\
&= \left(1 - \tau^N\right)\left(\overline{N}^N\right)^{1+\varphi} \\
&= \left(1 - \tau^N\right)\lambda
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\vartheta^H - 1}{\vartheta^H} &= \overline{MC}^H \\
&= \left(1 - \tau^H\right)\left(\overline{N}^H\right)^{1+\varphi} \\
&= \left(1 - \tau^H\right)(1-\lambda)(1-\omega)
\end{aligned}
$$

From the above expressions it follows that by setting $\frac{\vartheta^N - 1}{\vartheta^N} = \left(1 - \tau^N\right)\lambda$ and $\frac{\vartheta^H - 1}{\vartheta^H} = \left(1 - \tau^H\right)(1-\lambda)(1-\omega)$ the government ensures the optimal allocation in flexible price equilibrium and removes the average inflation bias.

Next I proceed with second-order approximation of the representative consumer's utility function. $U\left(\widehat{C}_t\right)$ can be approximated as follows

$$
\begin{aligned}
U\left(\widehat{C}_t\right) &= \overline{U} + \ln \widehat{C}_t + O\left(\left\|\Delta a_t, \eta_t^d, \varepsilon_t^p\right\|^3\right) \\
&= \lambda y_t^N + (1-\lambda)(1-\omega)\, y_t^H + t.i.p. + O\left(\left\|\Delta a_t, \eta_t^d, \varepsilon_t^p\right\|^3\right)
\end{aligned}
$$

where I have used the general expression for second-order approximation of percentage deviation from steady state of generic variable $X_t$ from steady state in terms of log deviations $x_t$:

$$
\frac{X_t - X}{X} = x_t + \frac{1}{2}x_t^2 + O\left(\left\|\Delta a_t, \eta_t^d, \varepsilon_t^p\right\|^3\right)
$$

The same expansion is used also below. $W(l_t)$ is approximated as follows

$$
\begin{aligned}
W(l_t) &= \overline{W} + \sum_{i=H,N}\left[\overline{W}_l \overline{l}^i \left(\frac{l_t^i - l^i}{l^i}\right) + \frac{1}{2}\overline{W}_{ll}\left(\overline{l}^i\right)^2\left(\frac{l_t^i - l^i}{l^i}\right)^2\right] + O\left(\left\|\Delta a_t, \eta_t^d, \varepsilon_t^p\right\|^3\right) \\
&= \overline{W} + \sum_{i=H,N}\left[\overline{W}_l \overline{l}^i \left(\frac{l_t^i - l^i}{l^i}\right) + \frac{1}{2}\varphi \overline{W}_l \overline{l}^i \left(\frac{l_t^i - l^i}{l^i}\right)^2\right] + O\left(\left\|\Delta a_t, \eta_t^d, \varepsilon_t^p\right\|^3\right) \\
&= \overline{W} + \overline{W}_l \sum_{i=H,N}\left[\overline{l}^i \left(\frac{l_t^i - l^i}{l^i}\right) + \frac{1}{2}\varphi \overline{l}^i \left(\frac{l_t^i - l^i}{l^i}\right)^2\right] + O\left(\left\|\Delta a_t, \eta_t^d, \varepsilon_t^p\right\|^3\right) \\
&= \overline{W} + \overline{W}_l \sum_{i=H,N}\left[\overline{l}^i \widehat{l}_t^i + \frac{1}{2}(1+\varphi)\,\overline{l}^i \left(\widehat{l}_t^i\right)^2\right] + O\left(\left\|\Delta a_t, \eta_t^d, \varepsilon_t^p\right\|^3\right)
\end{aligned}
$$

The second line uses $\overline{W}_{ll}\left(\overline{l}^i\right)^2=\varphi\overline{W}_l\overline{l}^i$ and the last $\frac{l_t^i-l^i}{l^i}=\widehat{l}_t^i+\frac{1}{2}\left(\widehat{l}_t^i\right)^2+O\left(\left\|\Delta a_t,\eta_t^d,\varepsilon_t^p\right\|^3\right)$ where $\widehat{l}_t^i=\ln\left(l_t^i/\overline{l}^i\right)$. We can also note that $\widehat{l}_t^i=\ln\left(\frac{y_t^i}{A_t^i}\right)+\ln\int_0^1\frac{P_t^i(j)}{P_t^i}dj=y_t^i+\frac{\vartheta^i}{2}var_jp_t^i\left(j\right)+O\left(\left\|\Delta a_t,\eta_t^d,\varepsilon_t^p\right\|^3\right)$ (see Gali and Monacelli, 2005 for a proof). Using equations (B.1) and (B.2) we can write

$$
\begin{aligned}
U\left(\widehat{C}_t\right)-W\left(l_t\right) &= -\frac{\lambda}{2}\left[\vartheta^N var_jp_t^N\left(j\right)+\left(1+\varphi\right)\right]\left(y_t^N\right)^2 \\
&\quad -\frac{\left(1-\lambda\right)\left(1-\omega\right)}{2}\left[\vartheta^H var_jp_t^H\left(j\right)+\left(1+\varphi\right)\right]\left(y_t^H\right)^2 \\
&\quad +t.i.p.+O\left(\left\|\Delta a_t,\eta_t^d,\varepsilon_t^p\right\|^3\right)
\end{aligned}
$$

Furthermore, following Woodford (2003) we have

$$
\sum_{\tau=0}^{\infty}\delta^\tau var_jp_t^i\left(j\right)=\frac{\alpha^i}{\left(1-\alpha^i\right)\left(1-\delta\alpha^i\right)}\sum_{\tau=0}^{\infty}\delta^\tau\left(\pi_{t+\tau}^i\right)^2
$$

Collecting the results leads to

$$
\begin{aligned}
V_t &= \sum_{\tau=0}^{\infty}\delta^\tau\left(U\left(\widehat{C}_t\right)-W\left(l_t\right)\right)= \\
&= -\frac{1}{2}\sum_{\tau=0}^{\infty}\delta^\tau\left[\begin{array}{c}\frac{\lambda\vartheta^N\alpha^N}{\left(1-\alpha^N\right)\left(1-\delta\alpha^N\right)}\left(\pi_{t+\tau}^N\right)^2+\lambda\left(1+\varphi\right)\left(y_t^N\right)^2 \\ +\frac{\left(1-\lambda\right)\left(1-\omega\right)\vartheta^H\alpha^H}{\left(1-\alpha^H\right)\left(1-\delta\alpha^H\right)}\left(\pi_{t+\tau}^H\right)^2+\left(1-\lambda\right)\left(1-\omega\right)\left(1+\varphi\right)\left(y_t^H\right)^2\end{array}\right] \\
&\quad +t.i.p.+O\left(\left\|\Delta a_t,\eta_t^d,\varepsilon_t^p\right\|^3\right)
\end{aligned}
$$

## References


[1] Backus D. K., and Smith G. W. (1993), "Consumption and Real Exchange Rates in Dynamic Economies with Non-traded Goods", Journal of International Economics 35, 297-316.

[2] Calvo, G. (1983), "Staggered Prices in a Utility Maximizing Framework", Journal of Monetary Economics 12, 383-398.

[3] Buiter, W. and Grafe, C. (2002), "Anchor, Float or Abandon Ship: Exchane Rate Regimes for Accession Countries", CEPR Discussion Paper No. 3184.

[4] Calvo, G., Celasun, O. and Kumhof, M. (2002), "Nominal Exchange Rate Anchoring Under Inflation Inertia", IMF Working Paper 02/30.

[5] Canzoneri, M., Valles, J. and Vinals, J. (1996), "Do Exchange Rates Move to Address International Macroeconomic Imbalances?", CEPR Discussion Papers No. 1498.

[6] Coricelli, F. and Jazbec, B. (2001), "Real Exchange Rate Dynamics in Transition Countries", CEPR Discussion Paper No. 2869

[7] Coricelli F., Jazbec, B. and Masten I. (2004), "Exchange Rate Pas-Through in Aceeding Countries: the Role of Exchange Rate Regimes", EUI Working Paper ECO 2004/16.

[8] Corsetti, G. and Pesenti, P. (2001), "Welfare and Macroeconomic Interdependence", Quarterly Journal of Economics vol. CXVI, iss. 2, 421-446.

[9] Corsetti, G., Dedola, L. and Leduc, S. (2003), " International Risk Sharing and the Transmission of Productivity Shocks", mimeo, Yale University.

[10] Čihak, M. nad Holub, T. (2001), "Convergence in Relative Price and Inflation in Central and Eastern Europe", IMF Working Paper 01/124.

[11] De Gregorio, J., Giovannini, A. and Wolf H. C. (1994), "International Evidence in Tradeables and Nontradeables Inflation", European Economic Review, vol. 38, 1225-44.

[12] Egert, B. (2002), "Estimating the Impact of the Balassa-Samuelson effect on Inflation and the Real Exchange Rate During the Transition", Economic Systems 26, 1-26.

[13] Ehrmann, M. and Smets, F. (2003), "Uncertain potential output: implications for monetary policy", Journal of Economic Dynamics and Control, Vol 27/9, pp 1611-1638.

[14] Gali, J. and Monacelli, T. (2005), "Monetary Policy and Exchange Rate Volatility in a Small Open Economy.", Review of Economic Studies, 72/3, p. 707.

[15] Giannoni, M. and Woodford, M. (2003), " Optimal Inflation targeting Rules", in B.S. Bernanke and M. Wooodford, eds., Inflation Targeting, Chicago: University of Chicago Press, forthcoming.

[16] Ireland, P. (2003), "Technology Shocks in the New Keynesian Model", mimeo, Boston College.

[17] Laxton, D. and Pesenti, P. (2003), "Monetary Rules for Small, Open, Emerging Economies", Journal of Monetary Economics, 50, 1109 - 1146.

[18] Lubik, A. T. (2003), "Industrial Structure and Monetary Policy in a Small Open Economy", mimeo, The Johns Hopkins University.

[19] Mihaljek, D and Klau M. (2004), "The Balassa-Samuelson Effect in Central Europe: a Disaggregated Analysis", mimeo, Bank for International Settlements.

[20] Natalucci, F. M. and Ravenna, F. (2002), " The Road to Adopting the Euro: Monetary Policy and Exchange rate Regimes in EU Candidate Countries", Board of Governors of the Federal Reserve System, International Finance Discussion Papers.

[21] Obstfeld, M. and Rogoff, K. (1999), "New Directions for Stochastic Open Economy Models", Journal of International Economics, vol. 50, no. 1, 117-153.

[22] Parrado, E. and Velasco, A. (2002), "Optimal Interest Rate Policy in a Small Open Economy", NBER Working Paper 8721.

[23] Rotemberg, J., Woodford, M. (1998), "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy: Expanded version", National Bureau of Economic Research Technical Working Paper 0233.

[24] Tille, C. (2002), "How Valuable is Exchange Rate Flexibility? Optimal Monetary Policy Under Sectoral Shocks", Federal Bank of New York Staff Report 147.

[25] Soderlind, P. (1999), "Solution and Estimation of RE Macromodels with Optimal Policy", European Economic Review 41, 1111-1146.

[26] Smets, F. (2000), "What Horizon for Price Stability", ECB Working Paper 24.

[27] Smets, F. and Wouters, R. (2002), "Openness, imperfect exchange rate pass-through and monetary policy", Journal of Monetary Economics 49, 947-981.

[28] Svensson, L. E. O (2000), "Open-Economy Inflation Targeting", Journal of International Economics, 50, 155-183.

[29] Svensson, L. E. O. and Woodfrod, M. (2002), " Implementing Optimal Policy through Inflation-Forecast Targeting", mimeo, Princeton University.

[30] Wagner, M. and Hlouskova, J. (2004), "What's the Story with this Balassa-Samuelson Effect in the CEECs?", *mimeo.*

[31] Woodford, M. (1996), "Control of the Public Debt: A Requirement for Price Stability", NBER Working Paper No. 5684.

[32] Woodford, M. (2003), Interest and Prices, Princeton University Press, New Jersey.

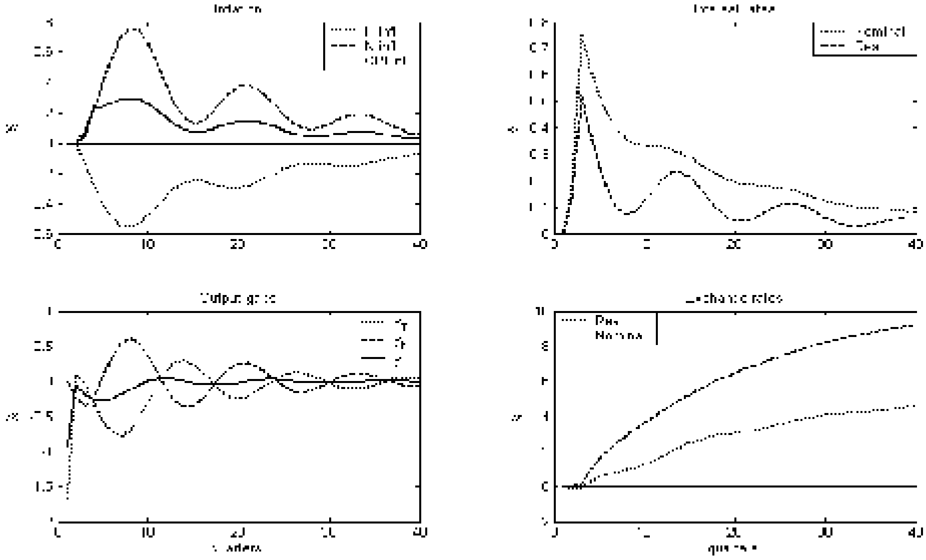

Figure 5: Classic productivity shock - alternative model specification

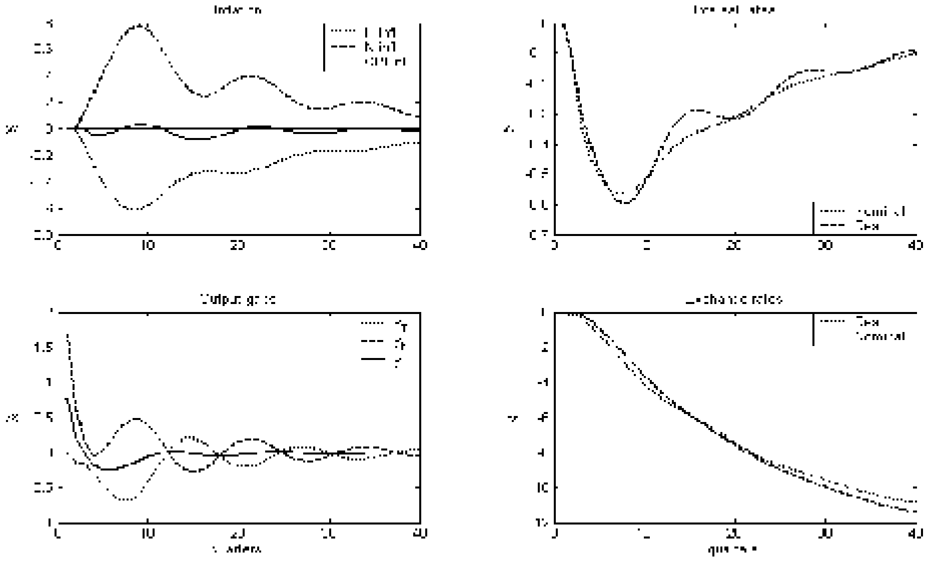

Figure 6: CPI inflation targeting - alternative model specification