Announcement Effects on Exchange Rate Movements: Continuity as a Selection Criteria Among the REE*†‡

Mikael Bask§

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Abstract

The aim of this paper is twofold. Firstly, we analyze the announcement effects on exchange rate movements using the basic asset pricing model, where currency trade is partly determined by technical trading in the form of moving averages since it is the most commonly used technique according to questionnaire surveys. Specifically, the announcement and implementation of temporary as well as permanent monetary policy are analyzed, where the exchange rate model developed is summarized in a linear difference equation in current exogenous fundamentals, an infinite number of lags of the endogenous exchange rate and time-t dating of exchange rate expectations. Secondly, since it is shown that least squares learnability is incapable to reduce the infinite number of attainable rational expectations equilibriums, *continuity* is proposed as a selection criteria among the equilibriums, meaning that the parameter for the time-$t-1$ exchange rate should have the limit 0 when there is no technical trading to be economically meaningful. It turns out that there is a unique rational expectations equilibrium that satisfy the continuity criteria, and focusing on this equilibrium, it is shown that the exchange rate is much more sensitive to changes in money supply than when technical trading is absent in currency trade. This result is important since it sheds light on the so-called exchange rate disconnect puzzle in international finance.

1 Introduction

It is a well-known fact that models in economics and finance, in which agents have rational expectations regarding some of the variables in the model, may exhibit a multiplicity of *rational expectations equilibriums* (REE). This is problematic. For instance, without imposing additional restrictions into such a model,

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§ Postal address: Monetary Policy and Research Department, Bank of Finland, P.O. Box 160, FIN-00101 Helsinki, Finland. E-mail addresses: mikael.bask@bof.fi

it is not known in advance which of the REE that the agents will coordinate on. Consequently, the effects of monetary policy changes, to take one example, is not known beforehand. Is it the case that the agents will coordinate on an equilibrium that has undesirable properties, like a very high inflation rate?

One way trying to circumvent the problem with a multiplicity of REE is by simplicity. Therefore, the *minimal state variable* (MSV) solution, suggested by McCallum [7], may be of interest, which is the solution to a linear difference equation that depends linearly on a set of variables such that there does not exist a solution that depends linearly on a smaller set of variables. Of course, there is no guarantee that the agents will coordinate on the MSV solution, but it is often a mathematically tractable solution that is useful as a benchmark when investigating the properties of other REE in the model. Another way to reduce the number of equilibriums that are attainable, is by focusing on the REE that are a possible result of an adaptive learning process for the agents. It can be assumed that the agents' expectations are formed by a correctly specified model, i.e., a model that corresponds to the REE, but without having perfect knowledge about the parameters in the model. However, using past and current values of the variables in the model, the parameters are learned over time since the beliefs are revised as new information is gained. Evans and Honkapohja [3] provide a nice introduction to this literature.

Unfortunately, it is not always the case that adaptive learning as a selection criteria is able to reduce the number of reasonable REE. Therefore, additional tools are necessary to use to find the equilibriums that the agents most likely will coordinate on. One purpose of this paper is to argue that *continuity* should hold for a REE to be economically meaningful, if applicable in the specific context. That is, if the model in focus nests another model, then a root to the general model should have a root to the nested model as its limit. To make the argument more comprehensible, an exchange rate model augmented with technical trading is developed in this paper.

Thus, starting with a basic asset pricing model that consists of two parity conditions, *uncovered interest rate parity* (UIP) and *purchasing power parity* (PPP), as well as equilibrium conditions at the domestic and foreign money markets, and, then, closing the model by assuming that the agents have rational expectations regarding the exchange rate, a linear difference equation for the model is derived. It turns out that the endogenous exchange rate is a function of current exogenous fundamentals and the expected exchange rate in the next time period, based on the information set available at time t. Then, if the model is solved for the current exchange rate, it is a function of current and expected future fundamentals. Moreover, if we ignore rational bubble solutions, the model has a unique REE. Now, what happens if we augment the exchange rate model with agents that use technical trading, or *chartism*, in currency trade? Will it still be the case that there is a unique REE in the model? The answer is no, and it is also demonstrated in the paper that least squares learnability is incapable to select a unique or at most a few REE that are reasonable. The proposed selection criteria, however, will solve the problem with a multiplicity of REE by selecting a unique equilibrium.

Technical trading is introduced into the model in the form of a moving average technique. According to this technique, buying and selling signals are generated by two moving averages of past exchange rates; a short-period moving average and a long-period moving average, where a buy (sell) signal is generated when the short-period moving average rises above (falls below) the long-period moving average. In the model developed, the short-period moving average is the current exchange rate and the long-period moving average is an exponentially weighted moving average of current and past exchange rates. That chartism is used extensively in currency trade is confirmed in several questionnaire surveys made at foreign exchange markets around the world. Examples are Cheung and Chinn [1], who conducted a survey at the U.S. market; Lui and Mole [6], who conducted a survey at the Hong Kong market; Menkhoff [9], who conducted a survey at the German market; Oberlechner [10], who conducted a survey at the markets in Frankfurt, London, Vienna and Zurich; and Taylor and Allen [12], who conducted a survey at the London market. An extensive exploration of the psychology in currency trade may also be found in Oberlechner [11], which is based on surveys conducted at the European and the North American markets.

According to the surveys, the relative importance of technical versus fundamental analysis in the currency market depends on the time horizon in currency trade. For shorter time horizons, more weight is placed on technical analysis, or chartism, while more weight is placed on fundamental analysis for longer horizons. Moreover, moving averages is the most commonly used technical trading technique in currency trade, which motivates why it is introduced into the model developed. Since it is assumed that the long-period moving average is a moving average of current and past exchange rates, it turns out that the endogenous exchange rate is a function of current exogenous fundamentals, an infinite number of lags of the exchange rate as well as the expected exchange rate in the next time period, based on the information set available at time t. Then, assuming that the agents who use fundamental analysis have rational expectations regarding the exchange rate, we can solve the model for the current exchange rate, which, obviously, no longer only is a function of current and expected future fundamentals, but also a function of all past exchange rates.

Therefore, there is an infinite number of REE in the model. Moreover, it is demonstrated in the paper that least squares learnability is incapable to select a unique or at most a few REE that are reasonable. In fact, if we focus on the non-bubble solutions, all REE in the model are least squares learnable. Consequently, it is necessary to find another tool to use to find the economically meaningful equilibriums. This tool is also found in the model developed by observing the behavior of the exchange rate when the time horizon in currency trade is approaching infinity. It turns out that the parameter for the time-$t-1$ exchange rate should have the limit 0, which implies a unique REE. Thus, continuity as a selection criteria among the REE is a powerful criteria since there is only one of an infinite number of REE that are economically meaningful.

Focusing on this unique REE, it is shown that the exchange rate is much more sensitive to changes in money supply than when technical trading is absent in currency trade. For example, when there is a temporary change in money supply, there may be a magnification effect on the exchange rate, meaning that a one percent increase (decrease) in money supply is depreciating (appreciating) the exchange rate with more than one percent. This result is important since it sheds light on the so-called exchange rate disconnect puzzle in international finance. That is, the empirical literature demonstrates that there are often large movements in nominal exchange rates that are apparently unexplained by macroeconomic fundamentals. Frankel and Froot [5] write:

> "[...] the proportion of exchange rate movements that can be explained even after the fact, using contemporaneous macroeconomic variables, is disturbingly low."

Be aware that the magnification effect on the exchange rate is not dependent on any rigidities in the model, like price inertia as in the Dornbusch [2] overshooting model. Also, it is not necessary to view temporary changes in money supply as monetary policy changes. Instead, to make the model even more appealing, it can be viewed as monetary disturbances. Thus, small shocks to the fundamentals may cause a volatile exchange rate.

The model developed is presented and discussed in Section 2, and the formal analysis is carried through in Section 3. Section 4 contains a concluding discussion, and the proofs of two propositions, etc., can be found in the Appendix.

2 Model

The benchmark model is presented in Section 2.1, which is a basic asset pricing model for exchange rate determination. Thereafter, in Section 2.2, the market expectations regarding the exchange rate and the exchange rate expectations formed by fundamental analysis and chartism are formulated and discussed.

2.1 Benchmark model

The benchmark model consists of two parity conditions, UIP and PPP, as well as equilibrium conditions at the domestic and foreign money markets.

2.1.1 UIP

The first parity condition is UIP, which states that the expected change of the exchange rate is equal to the difference between the domestic and foreign interest rates:

$$s^e[t+1] - s[t] = i[t] - i^*[t], \tag{1}$$

where s is the spot nominal exchange rate, and i and i^* are the domestic and foreign nominal interest rates, respectively. Moreover, the exchange rate is defined as the domestic price of the foreign currency, and the superscript e denotes expectations.

The parity condition in (1) is based on the assumption that domestic and foreign assets are perfect substitutes, which only holds if there is perfect capital mobility. Since the latter also is assumed, only the slightest difference in expected yields would draw the entire capital into the asset that offers the highest expected yield. Thus, the parity condition in (1) is also an equilibrium condition at the international asset market.

2.1.2 PPP

The second parity condition is PPP, which states that the exchange rate is equal to the difference between the domestic and foreign price levels:

$$s[t] = p[t] - p^*[t], \tag{2}$$

where p and p^* are the domestic and foreign nominal price levels, respectively.

The parity condition in (2) means that the domestic and foreign price levels, expressed in a common currency, are equal to each other. Thus, according to PPP, a relative increase (decrease) in the domestic price level not only means that the domestic price of the foreign currency increases (decreases), it also means that the increase (decrease) in the exchange rate is of such a magnitude that the price levels, expressed in a common currency, are still equal to each other.

2.1.3 Money market equilibrium

Equilibrium at the domestic and foreign money markets hold when real money supply is equal to real money demand:

$$m[t] - p[t] = \alpha y[t] - \beta i[t], \tag{3}$$

and

$$m^*[t] - p^*[t] = \alpha y^*[t] - \beta i^*[t], \tag{4}$$

where m and m^* are the domestic and foreign nominal money supplies, and y and y^* are the domestic and foreign real incomes, respectively. Thus, real money demand increases (decreases) when real income increases (decreases) or the interest rate decreases (increases). Note that we assume that the real income elasticities in (3)-(4) as well as the interest rate (semi-)elasticities in the same equations are equal to each other.

2.2 Expectations formations

After presenting the benchmark model, the expected exchange rate, s^e, will be the focus of interest. In short, we will assume that the agents who use fundamental analysis in currency trade have rational expectations regarding the next time period's exchange rate. Moreover, these agents know that there are (other) agents who use technical trading techniques in currency trade, and they take this into account when forming their exchange rate expectations.

2.2.1 Market expectations

According to questionnaire surveys (see references in Section 1), the relative importance of technical versus fundamental analysis in the currency market depends on the time horizon in currency trade. For shorter time horizons, more weight is placed on technical analysis, or chartism, while more weight is placed on fundamental analysis for longer horizons.

This observation is formulated as

$$s^e[t+1] = \omega(\tau) s_f^e[t+1] + (1 - \omega(\tau)) s_c^e[t+1], \tag{5}$$

where s^e, s_f^e and s_c^e are the market expectations and the expectations formed by fundamental analysis and chartism about the next time period's exchange rate, respectively. Moreover, $\omega(\tau)$ is a weight function that depends on the time horizon, τ, in currency trade:

$$\omega(\tau) = 1 - \exp(-\tau), \tag{6}$$

which is exogenously given in the model.

2.2.2 Fundamental analysis

When fundamental analysis is used in currency trade, it is assumed that the agents have rational expectations regarding the next time period's exchange rate:

$$s_f^e[t+1] = E[s[t+1]], \tag{7}$$

where $E[s[t+1]]$ is equal to the mathematical expectation of $s[t+1]$ based on the information set available at time t, which includes the knowledge of the complete model as well as the realized values of all variables in the model up to and including time t.[1] Thus, because currency trade based on chartism is affecting the exchange rate (as will be clear below), currency trade based on fundamental analysis will take this into account when forming exchange rate expectations.

2.2.3 Chartism

As was mentioned in Section 1, chartism utilizes past exchange rates to detect patterns that are extrapolated into the future. Focusing on past exchange rates is not considered as a shortcoming for agents using any of these technical trading techniques since a primary assumption behind technical analysis is that all relevant information about future exchange rate movements is contained in past movements. Thus, chartism is purely behavioristic in nature and does not examine the underlying reasons of currency traders.

The most commonly used technical trading technique is moving averages (see Lui and Mole [6] and Taylor and Allen [12]). According to this trading technique, buying and selling signals are generated by two moving averages; a short-period moving average and a long-period moving average. Specifically, a buy (sell) signal is generated when the short-period moving average rises above (falls below) the long-period moving average. In its simplest form, the short-period moving average is the current exchange rate and the long-period moving average is an exponentially weighted moving average of current and past exchange rates.

Thus, it is expected that the exchange rate will increase (decrease) when the current exchange rate is larger (smaller) than an exponentially weighted moving average of current and past exchange rates:

$$s_c^e[t+1] - s[t] = \gamma(s[t] - MA[t]). \tag{8}$$

Moreover, the long-period moving average, MA, is formulated as

$$MA[t] = (1 - \exp(-v)) \sum_{j=0}^{\infty} \exp(-jv)\, s[t-j], \tag{9}$$

where the weights given to current and past exchange rates sum up to 1:

$$(1 - \exp(-v)) \sum_{j=0}^{\infty} \exp(-jv) = 1. \tag{10}$$

[1] To make the mathematical notation more compact, $E[s[t+1]]$ is a shortcut for $E[s[t+1] | \mathcal{F}[t]]$, where $\mathcal{F}[t]$ is the information set available at time t.

Note that when $v \to 0$ or $v \to \infty$, the long-period moving average in (9) does not depend at all on past exchange rates. Specifically, for small v, all weights in the long-period moving average get small, including the weight given to the current exchange rate, while for large v, only the weights for past exchange rates get small, but the weight given to the current exchange rate approaches 1.

However, even if past exchange rates do not affect the expected exchange rate, as in these special cases, the market expectations and the expectations formed by fundamental analysis do not coincide. Still, the time horizon in currency trade is not necessarily infinitely long, which means that technical trading affects the exchange rate (as will be clear below).

3 Announcement effects of monetary policy

The complete model is summarized in a linear expectational difference equation, which is stated in Proposition 1 below. Obviously, since both chartism and fundamental analysis are used in currency trade, the current exchange rate is affected by past exchange rates (see the second term at the right-hand side of (11) below) as well as expectational matters (see the third term at the right-hand side of (11) below).

Proposition 1 *The expectational difference equation for the complete model is*

$$s[t] = x_1 f[t] - x_2 \sum_{j=1}^{\infty} \exp(-jv)\, s[t-j] + x_3 E[s[t+1]], \tag{11}$$

where the fundamentals are summarized in

$$f[t] \equiv m[t] - m^*[t] - \alpha(y[t] - y^*[t]), \tag{12}$$

and where

$$\begin{cases} x_1 \equiv \frac{1}{1+\beta(1-\exp(-\tau)-\gamma\exp(-\tau-v))} \\ x_2 \equiv \frac{\beta\gamma\exp(-\tau)(1-\exp(-v))}{1+\beta(1-\exp(-\tau)-\gamma\exp(-\tau-v))} \\ x_3 \equiv \frac{\beta(1-\exp(-\tau))}{1+\beta(1-\exp(-\tau)-\gamma\exp(-\tau-v))} \end{cases}. \tag{13}$$

Two cases are considered in the analysis of the expectational difference equation in (11); a special case in Section 3.1, and the general case in Section 3.2. In Section 3.1, it is assumed that the time horizon in currency trade is infinitely long, which means that currency trade is based only on fundamental analysis. Thus, the complete model reduces to a "traditional" foreign exchange model.

After solving the model in the preamble of each section, it is investigated, in Sections 3.1.1 and 3.2.1, whether the agents adaptively learn the REE in the model via recursive least squares. Thereafter, in Sections 3.1.2 and 3.2.2, the effects on exchange rate movements of the announcement and implementation of temporary as well as permanent monetary policy are analyzed. A main contribution in this paper is found in the latter section, because it is demonstrated therein that there is only one of the infinite number of REE that is economically meaningful.

We will implicitly assume throughout the whole paper that the necessary transversality conditions hold, which means that we rule out rational bubble solutions in the analysis.

3.1 Case: infinitely long time horizon

When the time horizon in currency trade is infinitely long, i.e., when $\tau \to \infty$, (11) reduces to

$$s[t] = \frac{1}{1+\beta} \cdot f[t] + \frac{\beta}{1+\beta} \cdot E[s[t+1]], \tag{14}$$

and a solution to (14) is

$$s[t] = \frac{1}{1+\beta} \cdot \sum_{k=0}^{\infty} \left(\frac{\beta}{1+\beta}\right)^k \cdot E[f[t+k]], \tag{15}$$

which can be confirmed via direct substitution into (14). Before investigating the announcement effects of monetary policy on exchange rate movements, we must examine whether the model in (15) is adaptively learnable.

3.1.1 Adaptive learning

The assumption in (7) is that when fundamental analysis is used in currency trade, the agents have rational expectations in the sense that the expected exchange rate is equal to the mathematical expectation of the exchange rate conditioned on all information available to the currency trader. Thus, since this information not only includes past and current values of the variables in the model, but also a complete knowledge about the structure of the model, rational expectations is a rather strong assumption. This assumption has, therefore, in the more recent literature, been complemented by an analysis of the possible convergence to the REE. Such analysis will also be accomplished in this paper.

It will be assumed that expectations are formed by a correctly specified model, i.e., a model that corresponds to the REE, but without having perfect knowledge about the parameters in the model. However, using past and current values of the variables in the model, the parameters are learned over time since the beliefs are revised as new information is gained. Thus, one may think of the agents in the model that use fundamental analysis, that they act as econometricians who adaptively learn the parameters in the model. Specifically, it will be investigated whether the model is characterized by least squares learnability. However, since expectational stability, i.e., E-stability, implies least squares learnability (see, e.g., McCallum [8]), the focus in the analysis will be on E-stability. This is because the latter concept is easier to handle mathematically.

According to the foreign exchange model in (15), the current exchange rate is

$$s[t] = \frac{1}{1+\beta} \cdot \sum_{k=0}^{\infty} \left(\frac{\beta}{1+\beta}\right)^k \cdot f^e[t+k], \tag{16}$$

where f^e is the expected value of the fundamentals that may differ from the mathematically expected value of the fundamentals.

Turning to the learning environment, it is assumed that the agents (or, the econometricians) know the functional form of the foreign exchange model in (16), but without having perfect knowledge about the parameters. Thus, the *perceived law of motion* (PLM) of the exchange rate is

$$s[t] = \sum_{k=0}^{\infty} \beta_k f^e [t+k], \tag{17}$$

where $\{\beta_k\}_{k=0}^{\infty}$ are the parameters that are estimated by the agents. Consequently, if these parameters differ from the corresponding parameters in (16), the agents have non-rational expectations.

The appropriate forecast of the next time period's exchange rate, $s^e[t+1]$, is, according to (17),

$$s^e[t+1] = \sum_{k=0}^{\infty} \beta_k f^e [t+k+1], \tag{18}$$

which is substituted into

$$\begin{aligned} s[t] &= \frac{1}{1+\beta} \cdot f[t] + \frac{\beta}{1+\beta} \cdot s^e[t+1] \\ &= \frac{1}{1+\beta} \cdot f[t] + \frac{\beta}{1+\beta} \cdot \sum_{k=0}^{\infty} \beta_k f^e [t+k+1]. \end{aligned} \tag{19}$$

The first row in (19) is the expectational difference equation in (14) allowing for non-rational expectations, and the second row in (19) is the *actual law of motion* (ALM) of the exchange rate.

The question is now whether the parameters in the PLM will converge to the parameters in the ALM, i.e., if the foreign exchange model in (15) is characterized by E-stability? To investigate this, note that there is a mapping, $M_f : \mathbb{R}^{\infty} \to \mathbb{R}^{\infty}$, from the parameters in the PLM to the parameters in the ALM:

$$M_f \begin{pmatrix} \beta_0 \\ \beta_k \end{pmatrix} = \begin{pmatrix} \frac{1}{1+\beta} \\ \frac{\beta}{1+\beta} \cdot \beta_{k-1} \end{pmatrix}, \tag{20}$$

where $k \in \mathbb{N}^+$. Then, consider the differential equation

$$\begin{aligned} \frac{d}{d\tau_a} \begin{pmatrix} \beta_0 \\ \beta_k \end{pmatrix} &= M_f \begin{pmatrix} \beta_0 \\ \beta_k \end{pmatrix} - \begin{pmatrix} \beta_0 \\ \beta_k \end{pmatrix} \\ &= \begin{pmatrix} \frac{1}{1+\beta} \\ \frac{\beta}{1+\beta} \cdot \beta_{k-1} \end{pmatrix} - \begin{pmatrix} \beta_0 \\ \beta_k \end{pmatrix}, \end{aligned} \tag{21}$$

where $k \in \mathbb{N}^+$ and τ_a is "artificial" time. According to Evans and Honkapohja [3] (and references therein), the REE is E-stable if the REE is locally asymptotically stable under (21). Thus, since, according to (21),

$$\frac{d\left(\frac{d\beta_k}{d\tau_a}\right)}{d\beta_k} = -1 < 0, \tag{22}$$

where $k \in \mathbb{N}$, the foreign exchange model in (15) is characterized by E-stability, which means that the model also is characterized by least squares learnability.

3.1.2 Monetary policy: announcement and implementation

Since the foreign exchange model in (15) is adaptively learnable, it makes sense to use this model to investigate the effects on exchange rate movements of monetary policy changes. Note that we interpret monetary policy changes as changes in a monetary aggregate, which in this paper is the domestic money supply.

To begin with, the effect on the exchange rate of a *temporary* change in the domestic money supply at time $t = t_0$ is[2]

$$0 < \left.\frac{ds\,[t_0]}{dm\,[t_0]}\right|_{\text{temporary}}^{\text{fundamentalist}} = \frac{1}{1+\beta} < 1, \tag{23}$$

which means that the exchange rate is depreciating (appreciating) less than the size of the increase (decrease) in the domestic money supply. However, if real money demand is not affected by a change in the interest rate, i.e., if $\beta = 0$, the multiplier in (23) is equal to 1. On the other hand, the more sensitive real money demand is to a change in the interest rate, the smaller is the effect on the exchange rate. In the next time period, which is the period after the temporary change in money supply, the exchange rate will return to the level it had before the change in monetary policy.

If we turn to the case of a *permanent* change in the domestic money supply, which takes place at time $t = t_0$, the effect on the exchange rate is

$$\left.\frac{ds\,[t_0]}{dm\,[t_0]}\right|_{\text{permanent}}^{\text{fundamentalist}} = 1, \tag{24}$$

which means that the effect is one-to-one, i.e., a one percent increase (decrease) in the domestic money supply is depreciating (appreciating) the exchange rate with one percent.

Then, what is the effect on the exchange rate of a future monetary policy change that is announced today? Let us start with an *announced temporary* change in the domestic money supply at time $t = t_0$ that will take place $a \geq 1$ time periods from the announcement:

$$0 < \left.\frac{ds\,[t_0]}{dm\,[t_0+a]}\right|_{\text{temporary}}^{\text{fundamentalist}} = \frac{1}{1+\beta}\left(\frac{\beta}{1+\beta}\right)^{a} < 1, \tag{25}$$

which means that the exchange rate is depreciating (appreciating) even less than when the change in monetary policy is implemented the same time period as it is announced, as is the case in (23).

The exchange rate is, of course, not only affected at time $t = t_0$, but in all time periods from the announcement until the temporary change in monetary policy is actually implemented. Generally, if $0 \leq t_1 \leq a - 1$ is the number of time periods after the announcement, the multiplier is

$$0 < \left.\frac{ds\,[t_0+t_1]}{dm\,[t_0+a]}\right|_{\text{temporary}}^{\text{fundamentalist}} = \frac{1}{1+\beta}\left(\frac{\beta}{1+\beta}\right)^{a-t_1} < 1. \tag{26}$$

[2] Derivations of most of the equations in this section can be found in the Appendix.

Thus, as the time evolves after the announcement of the new monetary policy, the effect on the exchange rate will be larger and larger. In the end, when the new policy is actually implemented, the effect is described by (23). Thereafter, in the time period after the change in monetary policy, the exchange rate will return to the level it had before the announcement of the new policy.

If we now turn to the case of an *announced permanent* change in the domestic money supply, that is announced at time $t = t_0$ and implemented at time $t = t_0 + a$, the immediate effect on the exchange rate is

$$0 < \left. \frac{ds\,[t_0]}{dm\,[t_0 + a]} \right|_{\text{permanent}}^{\text{fundamentalist}} = 1 - \frac{1}{1+\beta} \cdot \sum_{k=0}^{a-1} \left(\frac{\beta}{1+\beta} \right)^k < 1, \tag{27}$$

which is a smaller multiplier than the one-to-one multiplier in (24). In fact, the more distant in the future the announced monetary policy will be implemented, the smaller is the immediate effect on the exchange rate.

Again, as in the case with an announced temporary change in monetary policy, the exchange rate is affected in all time periods from the announcement until the permanent change in monetary policy is actually implemented. Generally, if $0 \leq t_1 \leq a - 1$ is the number of time periods after the announcement, the multiplier is

$$0 < \left. \frac{ds\,[t_0 + t_1]}{dm\,[t_0 + a]} \right|_{\text{permanent}}^{\text{fundamentalist}} = 1 - \frac{1}{1+\beta} \cdot \sum_{k=0}^{a-1-t_1} \left(\frac{\beta}{1+\beta} \right)^k < 1. \tag{28}$$

Thus, as the time evolves after the announcement of the new monetary policy, the effect on the exchange rate will be larger and larger. In the end, when the new policy is actually implemented, the effect is one-to-one (see (24)).

If we compare the adjustments of the exchange rate after an *announced temporary* change and an *announced permanent* change in the domestic money supply, the effect is always larger when the new monetary policy is permanent:

$$\left. \frac{ds\,[t_0 + t_1]}{dm\,[t_0 + a]} \right|_{\text{permanent}}^{\text{fundamentalist}} > \left. \frac{ds\,[t_0 + t_1]}{dm\,[t_0 + a]} \right|_{\text{temporary}}^{\text{fundamentalist}}. \tag{29}$$

To sum up the findings in this section, we have analyzed the effects on the exchange rate of an announced change in monetary policy that later on is implemented. Specifically, we have analyzed the effects of an announced temporary change as well as an announced permanent change in monetary policy, where these changes are in a monetary aggregate.

Not surprisingly, the immediate effect on the exchange rate is larger when the new monetary policy is permanent than when it is temporary. Moreover, a permanent policy, compared to a temporary policy, has a larger effect on the exchange rate when the implementation of the new policy takes place at a later date than the announcement, and irrespective if the new policy is temporary or permanent, the effect on the exchange rate will be larger and larger as the time evolves until the new policy is actually implemented.

Turning to the specific magnitudes of the exchange rate effects, there is a one-to-one effect when a permanent change in monetary policy is implemented, i.e., a one percent increase (decrease) in the domestic money supply is depreciating (appreciating) the exchange rate with one percent. When the change in monetary policy is temporary, the size of the exchange rate effect depends negatively on the sensitivity of real money demand to a change in the interest rate. It is, of course, also possible to derive how the monetary policy multipliers are affected by changes in the structural parameters, but to save space in the paper, we disregard from these derivations.

After investigating the behavior of the foreign exchange model when the time horizon in currency trade is infinitely long, we will turn to the general case in which both chartism and fundamental analysis are affecting the exchange rate. Thus, we will turn to the main section of this paper.

3.2 General case

To begin with, the focus in this section will be on the expectational difference equation

$$s[t] = x_1 f[t] - x_2 \sum_{j=1}^{j_{\max}} \exp(-jv)\, s[t-j] + x_3 E[s[t+1]], \tag{30}$$

where $j_{\max}$ is large, and not on (11). Of course, in the limit when $j_{\max} \to \infty$, (30) coincides with (11).

The aim is here to determine a solution of a similar form as when the time horizon in currency trade is infinitely long (see (15)), i.e., we will determine a solution in which also the expected fundamentals in all future time periods are part of the solution, and not only current fundamentals. Be aware of the fact that the MSV solution is not appropriate to analyze the announcement effects on exchange rate movements since expected future fundamentals are not part of this solution. Instead, a suggested solution to (30) is

$$s[t] = \sum_{j=1}^{j_{\max}} \beta_j s[t-j] + \sum_{k=0}^{k_{\max}} \beta_{j_{\max}+1+k} E[f[t+k]], \tag{31}$$

where $\{\beta_j\}_{j=1}^{j_{\max}+1+k_{\max}}$ are parameters to be determined, and where $k_{\max}$ is large. Assuming that the solution in (31) is correct, determine the rationally formed forecast of the next time period's exchange rate, substitute this forecast into the expectational difference equation in (30), and solve the resulting equation for $s[t]$:

$$\begin{aligned} s[t] \quad = \quad & \frac{1}{1-\beta_1 x_3} \cdot \sum_{j=1}^{j_{\max}-1} \left(\beta_{j+1} x_3 - x_2 \exp(-jv)\right) s[t-j] - \\ & \frac{x_2 \exp(-j_{\max} v)}{1-\beta_1 x_3} \cdot s[t-j_{\max}] + \frac{x_1}{1-\beta_1 x_3} \cdot f[t] + \\ & \frac{x_3}{1-\beta_1 x_3} \cdot \sum_{k=0}^{k_{\max}} \beta_{j_{\max}+1+k} E[f[t+k+1]]. \end{aligned} \tag{32}$$

Then, the solution to the following equation system determines the parameters in (31):

$$\begin{cases} \beta_{j_0} = \frac{\beta_{j_0+1}x_3 - x_2 \exp(-j_0 v)}{1-\beta_1 x_3} \\ \beta_{j_{\max}} = -\frac{x_2 \exp(-j_{\max} v)}{1-\beta_1 x_3} \\ \beta_{j_{\max}+1} = \frac{x_1}{1-\beta_1 x_3} \\ \beta_{j_1} = \frac{\beta_{j_1-1}x_3}{1-\beta_1 x_3} \end{cases}, \tag{33}$$

where $j_0 \in \{1, ..., j_{\max} - 1\}$ and $j_1 \in \{j_{\max} + 2, ..., j_{\max} + 1 + k_{\max}\}$.

If the equation system in (33) is partly solved via recursion, a solution to (30) is

$$s[t] = \sum_{j=1}^{j_{\max}} \beta_j s[t-j] + \frac{x_1}{1-\beta_1 x_3} \cdot \sum_{k=0}^{k_{\max}} x_3^k E[f[t+k]], \tag{34}$$

or, when $k_{\max} \to \infty$,

$$s[t] = \sum_{j=1}^{j_{\max}} \beta_j s[t-j] + \frac{x_1}{1-\beta_1 x_3} \cdot \sum_{k=0}^{\infty} x_3^k E[f[t+k]]. \tag{35}$$

Of course, we can also solve for β_j, $j \in \{1, ..., j_{\max}\}$, in (35). However, since we will not make use of the terms that include past exchange rates in the analysis below, we skip all these derivations, except the derivation of β_1.

Obviously, (35) is not easy to analyze since, according to Proposition 2 below, there are $j_{\max} + 1$ roots to the equation that determines β_1, which is a parameter that is part of the second term at the right-hand side of the equation. Moreover, since all these solutions for β_1 are adaptively learnable, which will be shown below, the problem of multiplicity of REE remains.

Proposition 2 *β_1 satisfy the following equation:*

$$\beta_1 = -x_2 \sum_{j=1}^{j_{\max}} \frac{x_3^{j-1} \exp(-jv)}{(1-\beta_1 x_3)^j}, \tag{36}$$

which has $j_{\max} + 1$ roots, but

$$\beta_1 \neq \frac{1}{x_3}. \tag{37}$$

Note that when the time horizon in currency trade is infinitely long in (36), $\beta_1 = 0$ since $x_2|_{\tau\to\infty} = 0$. According to (33), this also implies that all parameters for the lagged exchange rates in (35) vanish. Moreover, since $x_1|_{\tau\to\infty} = \frac{1}{1+\beta}$ and $x_3|_{\tau\to\infty} = \frac{\beta}{1+\beta}$, the second term at the right-hand side of (35) reduces to the term at the right-hand side of (15). Certainly, this should also be the case since the analysis in Section 3.1 is a special case of the analysis in this section.

3.2.1 Adaptive learning

Then, is the solution in (35) characterized by least squares learnability, as was claimed above? To answer this question, we will investigate whether the solution is E-stable since this implies least squares learnability.

If $E[f[\cdot]]$ is replaced by $f^e[\cdot]$ in (30)-(32), i.e., if we allow for non-rational expectations in these equations, note that the suggested solution in (31) is the PLM of the exchange rate, where $\{\beta_j\}_{j=1}^{j_{\max}+1+k_{\max}}$ are the parameters that are estimated by the agents, and that (32) is the ALM of the exchange rate. Moreover, note that there is a mapping, $M : \mathbb{R}^{j_{\max}+1+k_{\max}} \to \mathbb{R}^{j_{\max}+1+k_{\max}}$, from the parameters in the PLM to the parameters in the ALM:

$$M\begin{pmatrix} \beta_{j_0} \\ \beta_{j_{\max}} \\ \beta_{j_{\max}+1} \\ \beta_{j_1} \end{pmatrix} = \begin{pmatrix} \frac{\beta_{j_0+1}x_3 - x_2\exp(-j_0 v)}{1-\beta_1 x_3} \\ -\frac{x_2\exp(-j_{\max} v)}{1-\beta_1 x_3} \\ \frac{x_1}{1-\beta_1 x_3} \\ \frac{\beta_{j_1-1}x_3}{1-\beta_1 x_3} \end{pmatrix}, \tag{38}$$

where $j_0 \in \{1, ..., j_{\max}-1\}$ and $j_1 \in \{j_{\max}+2, ..., j_{\max}+1+k_{\max}\}$. Then, consider the differential equation

$$\begin{aligned} \frac{d}{d\tau_a}\begin{pmatrix} \beta_{j_0} \\ \beta_{j_{\max}} \\ \beta_{j_{\max}+1} \\ \beta_{j_1} \end{pmatrix} &= M\begin{pmatrix} \beta_{j_0} \\ \beta_{j_{\max}} \\ \beta_{j_{\max}+1} \\ \beta_{j_1} \end{pmatrix} - \begin{pmatrix} \beta_{j_0} \\ \beta_{j_{\max}} \\ \beta_{j_{\max}+1} \\ \beta_{j_1} \end{pmatrix} \\ &= \begin{pmatrix} \frac{\beta_{j_0+1}x_3 - x_2\exp(-j_0 v)}{1-\beta_1 x_3} \\ -\frac{x_2\exp(-j_{\max} v)}{1-\beta_1 x_3} \\ \frac{x_1}{1-\beta_1 x_3} \\ \frac{\beta_{j_1-1}x_3}{1-\beta_1 x_3} \end{pmatrix} - \begin{pmatrix} \beta_{j_0} \\ \beta_{j_{\max}} \\ \beta_{j_{\max}+1} \\ \beta_{j_1} \end{pmatrix}, \end{aligned} \tag{39}$$

where $j_0 \in \{1, ..., j_{\max}-1\}$ and $j_1 \in \{j_{\max}+2, ..., j_{\max}+1+k_{\max}\}$. Thus, since all investigated REE are locally asymptotically stable under (39),

$$\frac{d\left(\frac{d\beta_j}{d\tau_a}\right)}{d\beta_j} = -1 < 0, \tag{40}$$

where $j \in \{1, ..., j_{\max}+1+k_{\max}\}$, the foreign exchange model in (35) is characterized by E-stability, which means that the model also is characterized by least squares learnability.

3.2.2 Monetary policy: announcement and implementation

To be able to derive the specific magnitudes of the exchange rate effects, after a change in monetary policy, it is necessary to know the value of β_1. However, since there are $j_{\max}+1$ roots to (36) that determine β_1, it is not easy to perform such a task. Therefore, we will begin the analysis by investigating a special case, namely, when past exchange rates do not affect the expectations formed by chartism, which hereafter is called a *degenerated technical trading technique*. Note that $j_{\max}$ is still large when investigating this case.

A degenerated technical trading technique is investigated in the seminal paper by Frankel and Froot [4] in the form of a random walk model. Specifically, chartists are introduced into a model with portfolio managers and fundamentalists, even though the authors do not distinguish between temporary and permanent monetary policy as well as the announcement and implementation of the same policy. Furthermore, it is the portfolio managers who trade in currencies in their model, and they form exchange rate expectations as a weighted average of the chartists' and fundamentalists' expectations. Thus, the difference between the setup in this paper and the setup in Frankel and Froot [4] is only semantic since the market expectations and the portfolio managers' expectations are the same.

After analyzing a degenerated technical trading technique, we will investigate the case when only the most recent exchange rate of past rates is affecting the current exchange rate, i.e., we will set $j_{\max} = 1$. There are two reasons for this. Firstly, this analysis is an easy way to demonstrate the idea behind the proposed selection criteria that a root to (36) should satisfy a certain limit to be economically meaningful. Secondly, as will be obvious when solving the model numerically, there is a very small difference between the monetary policy multipliers when only the most recent exchange rate matters and when, for example, the four most recent exchange rates matter in the technical trading technique, i.e., when $j_{\max} = 1$ and $j_{\max} = 4$, respectively. Consequently, the analysis when $j_{\max} = 1$ is a good approximation of the general setting when $j_{\max}$ is large (when $j_{\max} \to \infty$).

A degenerated technical trading technique The most simple way to investigate the case of a degenerated technical trading technique, i.e., a technique in which past exchange rates do not affect the expected exchange rate, is to set $\gamma = 0$ in (8), which means that an unchanged exchange rate is expected in the next time period:

$$s_c^e [t+1] = s[t]. \tag{41}$$

Thus, having the analysis in Frankel and Froot [4] in mind, (41) is a "random walk" model in a deterministic setting, even if it is a contradiction in terms. Moreover, note that (41) does not mean that technical trading is absent in the currency market. Still, depending on the specific time horizon in currency trade, chartism affects the exchange rate, as we will demonstrate now.

Note that since $x_2|_{\gamma=0} = 0$, all $j_{\max} + 1$ roots to (36) are $\beta_1 = 0$. Consequently, the solution in (35) reduces to

$$s[t] = x_1|_{\gamma=0} \sum_{k=0}^{\infty} x_3|_{\gamma=0}^{k} E[f[t+k]], \tag{42}$$

since, according to the two first equations in (33), $\beta_j = 0$, $j \in \{1, ..., j_{\max}\}$, when $\beta_1 = 0$.

Now, if we compare the solution in (42) with the solution in (15), where the time horizon in currency trade is infinitely long, it is clear that it is the relative magnitudes of $\frac{1}{1+\beta}$ and $x_1|_{\gamma=0}$ as well as of $\frac{\beta}{1+\beta}$ and $x_3|_{\gamma=0}$ that determine whether the exchange rate effect is smaller or larger in the presence of chartism than when only fundamental analysis is used in currency trade. Therefore, since

$$0 < \frac{1}{1+\beta} \leq x_1|_{\gamma=0} = \frac{1}{1+\beta(1-\exp(-\tau))} \leq 1, \tag{43}$$

the effect on the exchange rate of a *temporary* change in the domestic money supply at time $t = t_0$ is larger when a degenerated technical trading technique is also used in currency trade:

$$\left.\frac{ds\,[t_0]}{dm\,[t_0]}\right|_{\text{temporary}}^{\text{fundamentalist}} \leq \left.\frac{ds\,[t_0]}{dm\,[t_0]}\right|_{\text{temporary}}^{\gamma=0} = x_1|_{\gamma=0}\,. \tag{44}$$

Moreover, the exchange rate effect is larger, the shorter the time horizon in currency trade is. In the limiting case of only chartism in currency trade, the effect is one-to-one. In the period after the temporary change in money supply, the exchange rate will return to the level it had before the change in monetary policy.

However, since chartism obviously weakens the link between exchange rates and expected future fundamentals,

$$1 > \frac{\beta}{1+\beta} \geq x_3|_{\gamma=0} = \frac{\beta\,(1-\exp(-\tau))}{1+\beta\,(1-\exp(-\tau))} \geq 0, \tag{45}$$

the aforementioned result does not necessarily mean that the exchange rate effects of all kinds of monetary policy changes are larger than when only fundamental analysis is used in currency trade. For example, in the case of a *permanent* change in the domestic money supply at time $t = t_0$, both multipliers are equal to 1:[3]

$$\left.\frac{ds\,[t_0]}{dm\,[t_0]}\right|_{\text{permanent}}^{\gamma=0} = \left.\frac{ds\,[t_0]}{dm\,[t_0]}\right|_{\text{permanent}}^{\text{fundamentalist}} = 1. \tag{46}$$

Then, if we pose the same question as we did in the analysis in Section 3.1.2, what is the effect on the exchange rate of a future monetary policy change that is announced today? Let us start with an *announced temporary* change in the domestic money supply at time $t = t_0$ that will take place $a \geq 1$ time periods from the announcement:

$$0 < \left.\frac{ds\,[t_0]}{dm\,[t_0+a]}\right|_{\text{temporary}}^{\gamma=0} = x_1|_{\gamma=0}\; x_3|_{\gamma=0}^{a} < 1, \tag{47}$$

which means that the exchange rate is depreciating (appreciating) even less than when the change in monetary policy is implemented the same time period as it is announced, as is the case in (44).

Of course, as the time evolves after the announcement of the change in monetary policy, the effect on the exchange rate will be larger and larger:

$$0 < \left.\frac{ds\,[t_0+t_1]}{dm\,[t_0+a]}\right|_{\text{temporary}}^{\gamma=0} = x_1|_{\gamma=0}\; x_3|_{\gamma=0}^{a-t_1} < 1, \tag{48}$$

where $0 \leq t_1 \leq a-1$ is the number of time periods after the announcement. In the end, when the new policy is actually implemented, the effect is described by (44). Thereafter, in the time period after the change in monetary policy, the exchange rate will return to the level it had before the announcement of the new policy. In the limiting case of only chartism in currency trade, there are no announcement effects on exchange rate movements. This is also obvious since

[3] Derivations of some of the equations in this section can be found in the Appendix, which are similar as the derivations of the equations in Section 3.1.2.

chartism is backward-looking by nature, and, consequently, is not affected by the announcement of future policy changes.

Turning to the case of an *announced permanent* change in the domestic money supply, that is announced at time $t = t_0$ and implemented at time $t = t_0 + a$, the immediate effect on the exchange rate is

$$0 < \left. \frac{ds\left[t_0\right]}{dm\left[t_0+a\right]} \right|_{\text{permanent}}^{\gamma=0} = 1 - x_1|_{\gamma=0} \sum_{k=0}^{a-1} x_3|_{\gamma=0}^{k} < 1, \tag{49}$$

which is a smaller multiplier than the one-to-one multiplier in (46). In fact, the more distant in the future the announced monetary policy will be implemented, the smaller is the immediate effect on the exchange rate.

Again, if $0 \leq t_1 \leq a-1$ is the number of time periods after the announcement of a permanent change in monetary policy, the multiplier is

$$0 < \left. \frac{ds\left[t_0+t_1\right]}{dm\left[t_0+a\right]} \right|_{\text{permanent}}^{\gamma=0} = 1 - x_1|_{\gamma=0} \sum_{k=0}^{a-1-t_1} x_3|_{\gamma=0}^{k} < 1. \tag{50}$$

Of course, the exchange rate is affected in all time periods from the announcement until the permanent change in monetary policy, and this effect will be larger and larger as the time evolves. In the end, when the new policy is actually implemented, the effect is one-to-one (see (46)). As previously noted in the case of a temporary change in monetary policy, there are no announcement effects on exchange rate movements when there is only chartism in currency trade since technical trading techniques are backward-looking by nature.

If we compare the adjustments of the exchange rate after an *announced temporary* change and an *announced permanent* change in the domestic money supply, the effect is always larger when the new monetary policy is permanent:

$$\left. \frac{ds\left[t_0+t_1\right]}{dm\left[t_0+a\right]} \right|_{\text{permanent}}^{\gamma=0} > \left. \frac{ds\left[t_0+t_1\right]}{dm\left[t_0+a\right]} \right|_{\text{temporary}}^{\gamma=0}. \tag{51}$$

A general message in this section is that when the time horizon in currency trade is shorter, which means that chartism has a larger influence on currency trade, the smaller is the difference between temporary and permanent monetary policies. This is also natural since chartism weakens the link between exchange rates and expected future fundamentals. In the limiting case when only chartism is used in currency trade, this link no longer exists, which also means that there are no announcement effects on exchange rate movements. Again, to save space in the paper, we do not derive how the monetary policy multipliers are affected by changes in the structural parameters as well as other multiplier relationships.

Let us now turn to another special case of the foreign exchange model in (35), namely, when $j_{\max} = 1$, before we try to deduce the behavior of the exchange rate in the general setting when $j_{\max}$ is large (when $j_{\max} \to \infty$).

Only the most recent exchange rate matters When $j_{\max} = 1$, the solution in (35) reduces to

$$s\left[t\right] = \beta_1 s\left[t-1\right] + \frac{x_1}{1-\beta_1 x_3} \cdot \sum_{k=0}^{\infty} x_3^k E\left[f\left[t+k\right]\right], \tag{52}$$

where β_1 solves, according to (36),

$$\beta_1 = -\frac{x_2 \exp(-v)}{1 - \beta_1 x_3}, \tag{53}$$

with the solutions

$$\beta_1 = \frac{1}{2x_3} \pm \sqrt{\frac{1}{4x_3^2} + \frac{x_2 \exp(-v)}{x_3}}, \tag{54}$$

which means, because $\frac{x_2}{x_3}$ is positive when the time horizon in currency trade is finite, that one root is positive while the other root is negative. Thus, in the case when only the most recent exchange rate of past rates is affecting the current exchange rate, there are two REE that also are learnable.

Starting with the case of a very long but finite time horizon in currency trade, it is clear that the two roots to (53) are close to 0 and $\frac{1}{x_3}$, respectively, since $\frac{x_2}{x_3}$ gets very small. Thus, having the second root in mind, the term $\frac{x_1}{1-\beta_1 x_3}$ at the right-hand side of (52) is, in absolute value, very large. In fact, in the limit when $\beta_1 \to \frac{1}{x_3}$, the impact of current fundamentals on the current exchange rate becomes infinite. Then, turning to the first root, which, obviously, from an economic point of view is more reliable, the terms $\frac{x_1}{1-\beta_1 x_3}$ and x_3 at the right-hand side of (52) are close to $x_1|_{\tau \text{ large}}$ and $x_3|_{\tau \text{ large}}$, respectively, and the analysis in Section 3.1.2 is a good approximation of the behavior of the exchange rate under different monetary policies.

Continuing with the case of a very short time horizon in currency trade, the two roots to (53) are very large in absolute value since $\frac{x_2}{x_3}$ gets very large. Thus, the term $\frac{x_1}{1-\beta_1 x_3}$ is close to 0, as is also the term x_3, which means that current as well as future expected fundamentals have almost no effect on the current exchange rate. This result is, of course, natural since chartism weakens the link between exchange rates and fundamentals. Note that there is a difference in this case compared to when a degenerated technical trading technique is used in currency trade. In the latter case, chartism weakens the link between exchange rates and expected future fundamentals, whereas, in this case, chartism also weakens the link between exchange rates and current fundamentals.

If we summarize what will turn out to be an important finding, there are two roots to (53) that determine β_1. Thus, there are two REE in the model that also are learnable. Moreover, which is clear from (54), the magnitudes of the two roots depend on the time horizon in currency trade:

$$\beta_1 = \beta_1(\tau). \tag{55}$$

However, as was demonstrated above, there is only one root with the property that

$$\lim_{\tau \to \infty} \beta_1(\tau) = 0, \tag{56}$$

which we hereafter denote β_1^0. This property is important since, if it does not hold for a root, the complete model would not reduce to the model investigated in Section 3.1 in a continuous manner when $\tau \to \infty$. Loosely speaking, there would be a discontinuity in the size of a monetary policy multiplier when going from an infinite to a finite time horizon in currency trade. However, if *continuity* is used as a selection criteria among the REE, there is only one equilibrium that is economically meaningful. Thus, since least squares learnability is incapable to reduce the number of attainable REE, as was shown in Section 3.2.1, continuity is a useful selection criteria for this task.

The general setting Now, is it the case that continuity can be utilized as a successful selection criteria in the general setting when $j_{\max}$ is large (when $j_{\max} \to \infty$)? In fact, this turns out to be the case in the model developed in this paper. To see this, let us focus on (36) that determines β_1. Even if it is true that (36) has $j_{\max}+1$ roots, implying that there are $j_{\max}+1$ REE in the model, if we ignore rational bubble solutions, there is only one root β_1^0. Thus, all other $j_{\max}$ roots imply a discontinuity in the size of a monetary policy multiplier when going from an infinite to a finite time horizon in currency trade, which is not reliable from an economic point of view.

Consequently, if we use continuity as a selection criteria among the REE, the problem is to solve (36) for the single root β_1^0 with the property in (56). Obviously, this is not an easy task since, in general, algebraic equations of degree five or higher are not solvable analytically. Besides, even if it would be possible to derive a solution for the economically interesting root, there is no guarantee that the solution is easy to handle in, for example, a comparison of the relative sizes of monetary policy multipliers. Therefore, we will in this paper solve (36) numerically for β_1^0.[4]

In Figures 1-8 below, we have solved numerically for the single root with the property in (56), where $j_{\max} \in \{1, ..., 4\}$ in (36), to determine the size of the monetary policy multiplier of a *temporary* change in the domestic money supply at time $t = t_0$:[5]

$$\left.\frac{ds\left[t_0\right]}{dm\left[t_0\right]}\right|_{\text{temporary}} = \frac{x_1}{1-\beta_1^0 x_3}. \tag{57}$$

In Figure 1, graphs of the multiplier in (57) when $j_{\max} = 1$ and $j_{\max} = 4$, respectively, are shown, corresponding to the cases when only the most recent exchange rate as well as the four most recent exchange rates matter in the technical trading rule. Moreover, the graph of the temporary monetary policy multiplier in (44) when a degenerated technical trading technique is used in currency trade is shown. As is clear by visual inspection of Figure 1, the exchange rate effect of a temporary monetary policy is larger when moving averages are used in currency trade than when a degenerated technical trading technique is used. This relationship holds for all reliable parameter values that we have investigated, i.e., the parameter values that give rise to positive monetary policy multipliers. Moreover, for all reliable parameter values, the exchange rate effect depends inversely on the time horizon in currency trade. The parameter values in Figure 1 are $\beta = 1$, $\gamma = 1$ and $v = 1$.[6]

[4] MATLAB routines for this purpose are available on request from the author.

[5] The derivations of the equations in this section are similar as the derivations of the equations in Section 3.1.2 and when a degenerated technical trading technique is used in currency trade. See the Appendix for these derivations.

[6] In all figures in the paper, the share of currency trade that is guided by technical analysis when $\tau = 0.05$, $\tau = 1.5$ and $\tau = 3$ is 95.1 percent, 22.3 percent and 5.0 percent, respectively. These numbers should be compared to Taylor and Allen [12], who found that 90 percent of the currency traders at the London market placed some weight on technical analysis at the intraday to one week horizon.

[Figure 1 about here.]

The weight parameter in the long-period moving average in (9) is, in Figure 2, decreased from $v = 1$ to $v = 0.2$. Recall that for small v, all weights in the long-period moving average get small, including the weight given to the current exchange rate, which, according to the graphs, means that the exchange rate effect of a temporary monetary policy is even larger when moving averages are used in currency trade. Obviously, the graph of the monetary policy multiplier when a degenerated technical trading rule is used is unaffected since the multiplier in (44) is independent of v. At the other extreme, when $v \to \infty$, only the weights given to past exchange rates in the long-period moving average get small, but the weight given to the current exchange rate approaches 1. As a consequence, the technical trading technique in (8) reduces to (41), which means that the graphs of the three monetary policy multipliers overlap each other. The case $v = 2$ is shown in Figure 3.

[Figures 2-3 about here.]

In Figure 4, the sensitivity of real money demand to a change in the interest rate is decreased from $\beta = 1$ to $\beta = 0.2$, which means that the exchange rate effect of a temporary monetary policy is not affected so much by a change in the time horizon in currency trade, and this is true irrespective of the technical trading technique used. When $\beta = 0$, the graphs of the three monetary policy multipliers overlap each other at the multiplier size 1. On the other hand, when real money demand is very sensitive to a change in the interest rate, i.e., when β is large, a change in the time horizon in currency trade has a large impact on the monetary policy multiplier in (57). The case $\beta = 2$ is shown in Figure 5. Note that the sizes of the monetary policy multipliers in Figures 4-5, when the share of currency trade that is guided by chartism is very small, are approximately given by the multiplier in (23).

[Figures 4-5 about here.]

The parameter that determines the adjustment speed of the exchange rate according to the moving averages technique in (8) is, in Figure 6, decreased from $\gamma = 1$ to $\gamma = 0.2$. This means that the graphs of the monetary policy multipliers in (57) is approaching the monetary policy multiplier when a degenerated technical trading technique is used in currency trade since this multiplier is defined by $\gamma = 0$. Consequently, the graph of the latter multiplier is unaffected by a slower adjustment speed since it is independent of γ. At the other extreme, when $\gamma \to \infty$, the faster adjustment speed means that a change in the money supply has a large exchange rate effect. This is also confirmed in Figure 7, where the case $\gamma = 2$ is shown.

[Figures 6-7 about here.]

It has not been mentioned until now, but what is clear by visual inspection of Figures 1-7, is that the two graphs in each figure, where moving averages are used as the technical trading rule, are almost overlapping. Thus, the exchange rate effect of a change in the domestic money supply seems to be the same irrespective of the number of past exchange rates in the long-period moving average in (9).

A closer examination of this matter reveals that the exchange rate effect, in fact, decreases when $j_{\max}$ increases, given the time horizon in currency trade. However, which is demonstrated in Figure 8, this change in the exchange rate effect becomes smaller and smaller suggesting that when $j_{\max} \to \infty$, the graph of the temporary monetary policy multiplier still almost overlap the graph of the same multiplier when $j_{\max} = 1$.

[Figure 8 about here.]

In Figures 9-13 below, we have solved numerically for the single root with the property in (56), where $j_{\max} \in \{1, ..., 4\}$ in (36), to determine the size of the monetary policy multiplier of a *permanent* change in the domestic money supply at time $t = t_0$:

$$\left.\frac{ds[t_0]}{dm[t_0]}\right|_{\text{permanent}} = \frac{x_1}{\left(1 - \beta_1^0 x_3\right)(1 - x_3)}, \tag{58}$$

where $|x_3| < 1$ since, otherwise, $\sum_{k=0}^{\infty} x_3^k \neq \frac{1}{1-x_3}$ (see the second term at the right-hand side of (35)), and the monetary policy multiplier is of infinite size.

Obviously, the graph of the permanent monetary policy multiplier in (46), when a degenerated technical trading technique is used in currency trade, is horizontal at the multiplier size 1 in all figures. In Figure 9, graphs of the monetary policy multiplier in (58) when $j_{\max} = 1$ and $j_{\max} = 4$, respectively, are shown, and the parameter values are $\beta = 1$, $\gamma = 1$ and $v = 1$, which is the same values as in Figure 1. Clearly, there is a magnification effect at all finite time horizons in currency trade, meaning that a one percent increase (decrease) in the domestic money supply is depreciating (appreciating) the exchange rate with more than one percent. Moreover, for all reliable parameter values, the exchange rate effect depends inversely on the time horizon in currency trade.

[Figure 9 about here.]

The parameter values in Figures 10-12 are the same as in Figures 2, 5 and 7, respectively, and if we compare the graphs of the monetary policy multiplier in (58), in the corresponding figures, it is clear that the exchange rate effect is even larger when the change in monetary policy is permanent than when it is temporary, given the time horizon in currency trade. There is also a magnification effect at all finite time horizons in currency trade, and, moreover, the permanent monetary policy multiplier in (58) is affected by changes in the parameters β, γ and v in the same qualitative way as the temporary monetary policy multiplier in (57). Finally, the graph of the permanent monetary policy multiplier is not affected so much when the number of past exchange rates in the long-period moving average in (9) is increased, as is shown in Figure 13. Recall a similar result when the change in the domestic money supply is temporary.

[Figure 10-13 about here.]

Then, what is the announcement effect on the exchange rate of a future monetary policy change? If we start with an *announced temporary* change in the domestic money supply at time $t = t_0$ that will take place $a \geq 1$ time periods from the announcement, the multiplier is

$$\left.\frac{ds[t_0]}{dm[t_0 + a]}\right|_{\text{temporary}} = \frac{x_1 x_3^a}{1 - \beta_1^0 x_3}. \tag{59}$$

The corresponding multiplier for an *announced permanent* change in monetary policy is

$$\left.\frac{ds\left[t_0\right]}{dm\left[t_0+a\right]}\right|_{\text{permanent}} = 1 - \frac{x_1}{1-\beta_1^0 x_3} \cdot \sum_{k=0}^{a-1} x_3^a. \tag{60}$$

Moreover, as the time evolves after the announcement of the change in monetary policy, the effect on the exchange rate will be larger and larger:

$$\left.\frac{ds\left[t_0+t_1\right]}{dm\left[t_0+a\right]}\right|_{\text{temporary}} = \frac{x_1 x_3^{a-t_1}}{1-\beta_1^0 x_3}, \tag{61}$$

and

$$\left.\frac{ds\left[t_0+t_1\right]}{dm\left[t_0+a\right]}\right|_{\text{permanent}} = 1 - \frac{x_1}{1-\beta_1^0 x_3} \cdot \sum_{k=0}^{a-1-t_1} x_3^a, \tag{62}$$

respectively, where $0 \leq t_1 \leq a-1$ is the number of time periods after the announcement. In the end, when the new policy is actually implemented, the effect is described by (57)-(58), respectively. Thereafter, in the next time period, and in the case of a temporary change in monetary policy, the exchange rate will return to the level it had before the announcement of the new policy. Note again that in the limiting case of only chartism in currency trade, there are no announcement effects on exchange rate movements.

4 Concluding discussion

One important lesson from the analysis in this paper is the fact that the solution

$$s\left[t\right] = \beta_1^0 s\left[t-1\right] + \frac{x_1}{1-\beta_1^0 x_3} \cdot \sum_{k=0}^{\infty} x_3^k E\left[f\left[t+k\right]\right], \tag{63}$$

if we substitute $\beta_1 = \beta_1^0$ into (52), is a good approximation of the foreign exchange model developed. This is because the exchange rate in the previous time period has a first-order effect on the current exchange rate, if there is a change in the fundamentals like in the domestic money supply, while other past exchange rates have a second-order effect, a third-order effect, and so on, on the current exchange rate. Therefore, when analyzing the announcement effects on exchange rate movements, there is a minor difference between the complete model in (30) when $j_{\max}$ is large (when $j_{\max} \to \infty$) and the approximation of the same model when $j_{\max} = 1$,

$$s\left[t\right] = x_1 f\left[t\right] - x_2 \exp\left(-v\right) s\left[t-j\right] + x_3 E\left[s\left[t+1\right]\right]. \tag{64}$$

A mathematical advantage of the model in (64) is that it is easy to derive an explicit solution for β_1^0 in (63).

Recall that there are no rigidities in the model, like price inertia as in the Dornbusch [2] overshooting model, since it is an asset pricing model that consists of two parity conditions, UIP and PPP, as well as equilibrium conditions at the domestic and foreign money markets that all hold continuously. Still, there may be a magnification effect on the exchange rate when there is a change in monetary policy, meaning that a one percent increase (decrease) in the domestic money supply is depreciating (appreciating) the exchange rate with more than one percent, and this is because currency trade is partly determined by technical trading. A similarity between the Dornbusch [2] model and the model in this paper is that the fundamentalists (which is the only trader type in Dornbusch [2]) have rational expectations regarding the exchange rate. The incorporation of technical trading into the model is motivated by the fact that these trading techniques are used extensively at foreign exchange markets around the world.

The possibility of a magnification effect on exchange rate movements is an important result since it sheds light on the so-called exchange rate disconnect puzzle in international finance, where the quote in the introduction by Frankel and Froot [5] illustrates the puzzle. That is, the observed volatility of exchange rates is rarely associated with volatile macroeconomic fundamentals, meaning that there is a high conditional volatility of exchange rates. Moreover, since it is not necessary to interpret the temporary changes in the domestic money supply as monetary policy changes, but, instead, as disturbances to money supply, the model developed in this paper is an interesting contribution to the debate on the exchange rate disconnect puzzle. In short, technical trading in the currency market is a sufficient condition for a high conditional volatility of exchange rates.

Last but not least, a principal aim of this paper has been to demonstrate how one can reduce a large number of reasonable REE in a model by using continuity as a selection criteria among the equilibriums. Specifically, if the model in focus nests another model, then a root to the general model should have a root to the nested model as its limit to be economically meaningful. At a first sight, it may seem that this criteria has a limited applicability. However, having in mind the large and growing literature on heterogeneous agents in economics and finance, we believe the contrary to be true. Thus, in many cases, it is possible to shrink a heterogenous agents model to one or several homogenous agents models, i.e., one model for each type of agent, and use the continuity criteria to find the REE that are economically meaningful. Focusing on the model in this paper, the continuity criteria was successful since it was able to isolate a unique REE. It is our belief that this also is possible in most other heterogenous agents models in which one group of traders have rational expectations regarding some of the variables in the model. Of course, it is a matter of future research to investigate this claim.

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Appendix

Derivations of some of the equations in Section 3.1.2 In the derivations below, we are making use of appropriate differentiations of (15) as well as, except in the derivation of (29), a one-to-one relationship between a change in the fundamentals and a change in the domestic money supply that follows from (12). *(23):*

$$ds\left[t_0\right] = \frac{1}{1+\beta} \cdot df\left[t_0\right] = \frac{1}{1+\beta} \cdot dm\left[t_0\right]. \tag{65}$$

(24):

$$\begin{aligned} ds\left[t_0\right] &= \frac{1}{1+\beta} \cdot \sum_{k=0}^{\infty} \left(\frac{\beta}{1+\beta}\right)^k \cdot E\left[df\left[t_0+k\right]\right] \qquad (66) \\ &= \frac{1}{1+\beta} \cdot df\left[t_0\right] \sum_{k=0}^{\infty} \left(\frac{\beta}{1+\beta}\right)^k \\ &= \frac{1}{1+\beta} \cdot df\left[t_0\right] \cdot \frac{1}{1-\frac{\beta}{1+\beta}} = df\left[t_0\right] = dm\left[t_0\right], \end{aligned}$$

since $df[t_0 + k] = df[t_0]$, $\forall k \geq 0$. *(25):*

$$ds[t_0] = \frac{1}{1+\beta}\left(\frac{\beta}{1+\beta}\right)^a \cdot E[df[t_0+a]] = \frac{1}{1+\beta}\left(\frac{\beta}{1+\beta}\right)^a \cdot dm[t_0+a], \quad (67)$$

since the change in monetary policy is announced, $E[dm[t_0 + a]] = dm[t_0 + a]$. *(27):*

$$\begin{aligned}
ds[t_0] &= \frac{1}{1+\beta}\cdot\sum_{k=a}^{\infty}\left(\frac{\beta}{1+\beta}\right)^k \cdot E[df[t_0+k]] \qquad (68)\\
&= \frac{1}{1+\beta}\cdot df[t_0+a]\sum_{k=a}^{\infty}\left(\frac{\beta}{1+\beta}\right)^k\\
&= \frac{1}{1+\beta}\cdot df[t_0+a]\cdot\left(\sum_{k=0}^{\infty}\left(\frac{\beta}{1+\beta}\right)^k - \sum_{k=0}^{a-1}\left(\frac{\beta}{1+\beta}\right)^k\right)\\
&= \frac{1}{1+\beta}\cdot df[t_0+a]\cdot\left(\frac{1}{1-\frac{\beta}{1+\beta}} - \sum_{k=0}^{a-1}\left(\frac{\beta}{1+\beta}\right)^k\right)\\
&= \left(1-\frac{1}{1+\beta}\cdot\sum_{k=0}^{a-1}\left(\frac{\beta}{1+\beta}\right)^k\right)\cdot df[t_0+a]\\
&= \left(1-\frac{1}{1+\beta}\cdot\sum_{k=0}^{a-1}\left(\frac{\beta}{1+\beta}\right)^k\right)\cdot dm[t_0+a],
\end{aligned}$$

since $df[t_0 + k] = df[t_0 + a]$, $\forall k \geq a$, and that the change in monetary policy is announced, $E[dm[t_0 + k]] = dm[t_0 + k]$, $\forall k \geq a$. *(29):*

$$\begin{aligned}
&\left.\frac{ds[t_0+t_1]}{dm[t_0+a]}\right|_{\text{permanent}}^{\text{fundamentalist}} \qquad (69)\\
&= 1-\frac{1}{1+\beta}\cdot\sum_{k=0}^{a-1-t_1}\left(\frac{\beta}{1+\beta}\right)^k\\
&= 1-\frac{1}{1+\beta}\cdot\sum_{k=0}^{\infty}\left(\frac{\beta}{1+\beta}\right)^k + \frac{1}{1+\beta}\cdot\sum_{k=a-t_1}^{\infty}\left(\frac{\beta}{1+\beta}\right)^k\\
&= 1-\frac{1}{1+\beta}\cdot\frac{1}{1-\frac{\beta}{1+\beta}} + \frac{1}{1+\beta}\cdot\sum_{k=a-t_1}^{\infty}\left(\frac{\beta}{1+\beta}\right)^k\\
&= \frac{1}{1+\beta}\cdot\sum_{k=a-t_1}^{\infty}\left(\frac{\beta}{1+\beta}\right)^k\\
&= \frac{1}{1+\beta}\left(\frac{\beta}{1+\beta}\right)^{a-t_1} + \frac{1}{1+\beta}\cdot\sum_{k=a-t_1+1}^{\infty}\left(\frac{\beta}{1+\beta}\right)^k\\
&> \frac{1}{1+\beta}\left(\frac{\beta}{1+\beta}\right)^{a-t_1} = \left.\frac{ds[t_0+t_1]}{dm[t_0+a]}\right|_{\text{temporary}}^{\text{fundamentalist}}.
\end{aligned}$$

Derivations of some of the equations in Section 3.2.2 In the derivations below, we are making use of appropriate differentiations of (42) as well as a one-to-one relationship between a change in the fundamentals and a change in the domestic money supply that follows from (12). *(46):*

$$ds[t_0] = x_1|_{\gamma=0} \, df[t_0] \sum_{k=0}^{\infty} x_3|_{\gamma=0}^{k} = x_1|_{\gamma=0} \, df[t_0] \cdot \frac{1}{1 - x_3|_{\gamma=0}} \tag{70}$$

$$= \frac{\frac{1}{1+\beta(1-\exp(-\tau))}}{1 - \frac{\beta(1-\exp(-\tau))}{1+\beta(1-\exp(-\tau))}} \cdot df[t_0] = dm[t_0],$$

since $df[t_0 + k] = df[t_0]$, $\forall k \geq 0$. *(49):*

$$ds[t_0] = x_1|_{\gamma=0} \, df[t_0 + a] \sum_{k=a}^{\infty} x_3|_{\gamma=0}^{k} \tag{71}$$

$$= x_1|_{\gamma=0} \, df[t_0 + a] \cdot \left(\sum_{k=0}^{\infty} x_3|_{\gamma=0}^{k} - \sum_{k=0}^{a-1} x_3|_{\gamma=0}^{k} \right)$$

$$= x_1|_{\gamma=0} \, df[t_0 + a] \cdot \left(\frac{1}{1 - x_3|_{\gamma=0}} - \sum_{k=0}^{a-1} x_3|_{\gamma=0}^{k} \right)$$

$$= \left(\frac{\frac{1}{1+\beta(1-\exp(-\tau))}}{1 - \frac{\beta(1-\exp(-\tau))}{1+\beta(1-\exp(-\tau))}} - x_1|_{\gamma=0} \sum_{k=0}^{a-1} x_3|_{\gamma=0}^{k} \right) \cdot df[t_0 + a]$$

$$= \left(1 - x_1|_{\gamma=0} \sum_{k=0}^{a-1} x_3|_{\gamma=0}^{k} \right) \cdot dm[t_0 + a],$$

since $df[t_0 + k] = df[t_0 + a]$, $\forall k \geq a$, and that the change in monetary policy is announced, $E[dm[t_0 + k]] = dm[t_0 + k]$, $\forall k \geq a$. *(51):*

$$\left. \frac{ds[t_0 + t_1]}{dm[t_0 + a]} \right|_{\text{permanent}}^{\gamma=0} \tag{72}$$

$$= 1 - x_1|_{\gamma=0} \sum_{k=0}^{\infty} x_3|_{\gamma=0}^{k} + x_1|_{\gamma=0} \sum_{k=a-t_1}^{\infty} x_3|_{\gamma=0}^{k}$$

$$= 1 - \frac{\frac{1}{1+\beta(1-\exp(-\tau))}}{1 - \frac{\beta(1-\exp(-\tau))}{1+\beta(1-\exp(-\tau))}} + x_1|_{\gamma=0} \sum_{k=a-t_1}^{\infty} x_3|_{\gamma=0}^{k}$$

$$= x_1|_{\gamma=0} \, x_3|_{\gamma=0}^{a-t_1} + x_1|_{\gamma=0} \sum_{k=a-t_1+1}^{\infty} x_3|_{\gamma=0}^{k}$$

$$> \left. \frac{ds[t_0 + t_1]}{dm[t_0 + a]} \right|_{\text{temporary}}^{\gamma=0}.$$

Proof of Proposition 1 Firstly, substitute the conditions for money market equilibrium in (3)-(4) into the condition for PPP in (2):

$$s[t] = m[t] - m^*[t] - \alpha(y[t] - y^*[t]) + \beta(i[t] - i^*[t]), \tag{73}$$

and, secondly, substitute the condition for UIP in (1) into (73):

$$s[t] = m[t] - m^*[t] - \alpha(y[t] - y^*[t]) + \beta(s^e[t+1] - s[t]), \tag{74}$$

and, finally, solve (74) for the current exchange rate:

$$s[t] = \frac{1}{1+\beta} \cdot (m[t] - m^*[t] - \alpha(y[t] - y^*[t])) + \frac{\beta}{1+\beta} \cdot s^e[t+1], \quad (75)$$

which reduces to

$$s[t] = \frac{1}{1+\beta} \cdot f[t] + \frac{\beta}{1+\beta} \cdot s^e[t+1], \quad (76)$$

where (12) is substituted into (75), and we have the benchmark model summarized in one equation. Next, substitute the expectations formed by fundamental analysis and chartism in (7)-(8) into the market expectations in (5):

$$s^e[t+1] = \omega(\tau) E[s[t+1]] + (1-\omega(\tau))(s[t] + \gamma(s[t] - MA[t])). \quad (77)$$

Thereafter, substitute the long-period moving average in (9) into (77):

$$\begin{aligned} & s^e[t+1] \quad (78) \\ = \; & \omega(\tau) E[s[t+1]] + \\ & (1-\omega(\tau)) \left(s[t] + \gamma \left(s[t] - (1-\exp(-v)) \sum_{j=0}^{\infty} \exp(-jv) s[t-j] \right) \right). \end{aligned}$$

(78) is the market expectations summarized in one equation. Then, by substituting the market expectations in (78) into the benchmark model in (76), the expectational difference equation that describes the complete model is derived:

$$\begin{aligned} s[t] \;=\; & \frac{1}{1+\beta} \cdot f[t] + \quad (79) \\ & \frac{\beta}{1+\beta} \cdot \left(\begin{array}{l} \omega(\tau) E[s[t+1]] + \\ (1-\omega(\tau)) \left(\begin{array}{l} s[t] + \\ \gamma \left(\begin{array}{l} s[t] - \\ (1-\exp(-v)) \cdot \\ \sum_{j=0}^{\infty} \exp(-jv) s[t-j] \end{array} \right) \end{array} \right) \end{array} \right). \end{aligned}$$

Then, solve (79) for the current exchange rate, and substitute (13) as well as the weight function in (6) into the resulting equation, and the proof is completed.

Proof of Proposition 2 Firstly, let $j_0 = 1$ in the first equation in (33):

$$\beta_1 = \frac{\beta_2 x_3 - x_2 \exp(-v)}{1 - \beta_1 x_3} = \beta_2 \cdot \frac{x_3}{1-\beta_1 x_3} - x_2 \sum_{j=1}^{1} \frac{x_3^{j-1} \exp(-jv)}{(1-\beta_1 x_3)^j}. \quad (80)$$

Secondly, let $j_0 = 2$ in the first equation in (33), and substitute this equation into (80):

$$\begin{aligned} \beta_1 \;&=\; \frac{\beta_3 x_3 - x_2 \exp(-2v)}{1-\beta_1 x_3} \cdot \frac{x_3}{1-\beta_1 x_3} - x_2 \sum_{j=1}^{1} \frac{x_3^{j-1} \exp(-jv)}{(1-\beta_1 x_3)^j} \quad (81) \\ &=\; \beta_3 \cdot \left(\frac{x_3}{1-\beta_1 x_3} \right)^2 - x_2 \sum_{j=1}^{2} \frac{x_3^{j-1} \exp(-jv)}{(1-\beta_1 x_3)^j}, \end{aligned}$$

and repeat this procedure $j_{\max} - 3$ times:

$$\beta_1 = \beta_{j_{\max}} \cdot \left(\frac{x_3}{1-\beta_1 x_3}\right)^{j_{\max}-1} - x_2 \sum_{j=1}^{j_{\max}-1} \frac{x_3^{j-1} \exp(-jv)}{(1-\beta_1 x_3)^j}. \tag{82}$$

Finally, substitute the second equation in (33) into (82):

$$\begin{aligned} \beta_1 &= -\frac{x_2 \exp(-j_{\max} v)}{1-\beta_1 x_3} \left(\frac{x_3}{1-\beta_1 x_3}\right)^{j_{\max}-1} - \\ &\quad x_2 \sum_{j=1}^{j_{\max}-1} \frac{x_3^{j-1} \exp(-jv)}{(1-\beta_1 x_3)^j} \\ &= -x_2 \sum_{j=1}^{j_{\max}} \frac{x_3^{j-1} \exp(-jv)}{(1-\beta_1 x_3)^j}, \end{aligned} \tag{83}$$

and the proof is completed.

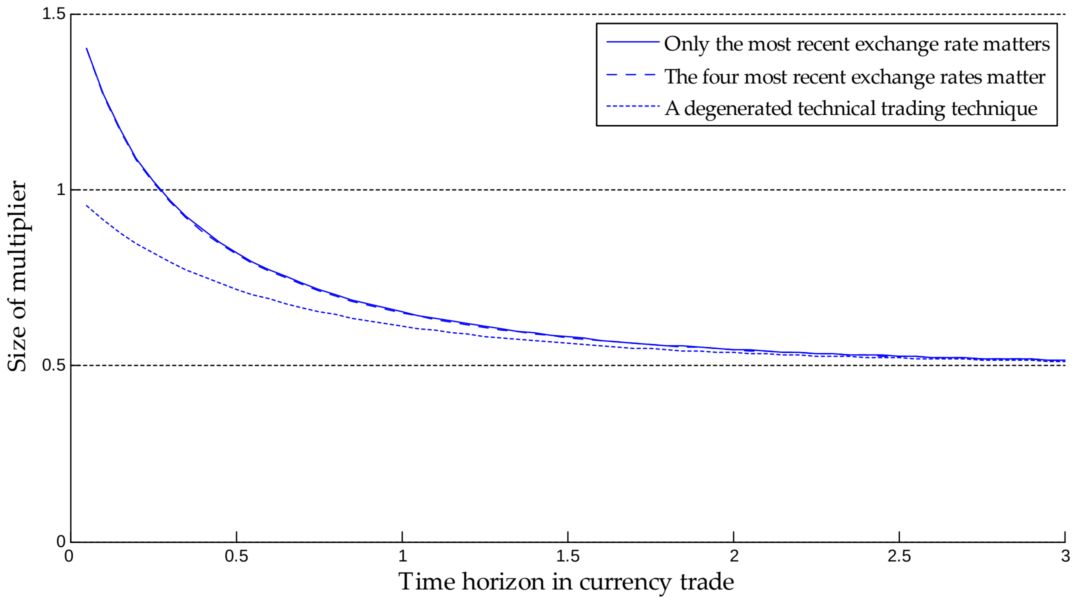

Figure 1: Graphs of the monetary policy multipliers of a *temporary* change in money supply, where the parameter values are $\beta = 1$, $\gamma = 1$ and $v = 1$. Note that the dashed and solid graphs are almost overlapping.

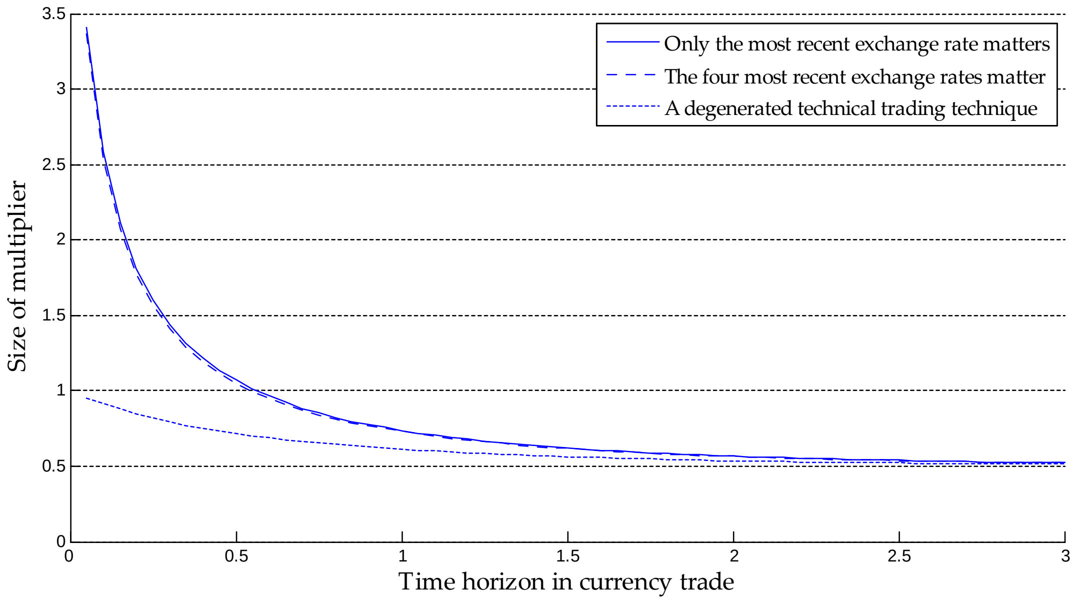

Figure 2: Graphs of the monetary policy multipliers of a *temporary* change in money supply, where the parameter values are $\beta = 1$, $\gamma = 1$ and $v = 0.2$. Note that the dashed and solid graphs are almost overlapping.

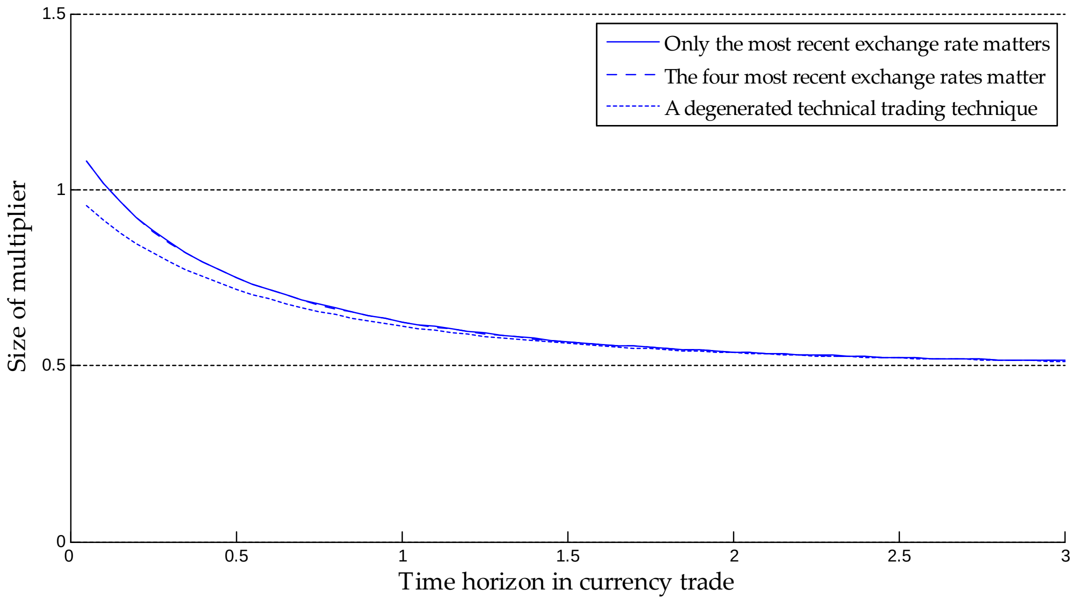

Figure 3: Graphs of the monetary policy multipliers of a *temporary* change in money supply, where the parameter values are $\beta = 1$, $\gamma = 1$ and $v = 2$. Note that the dashed and solid graphs are almost overlapping.

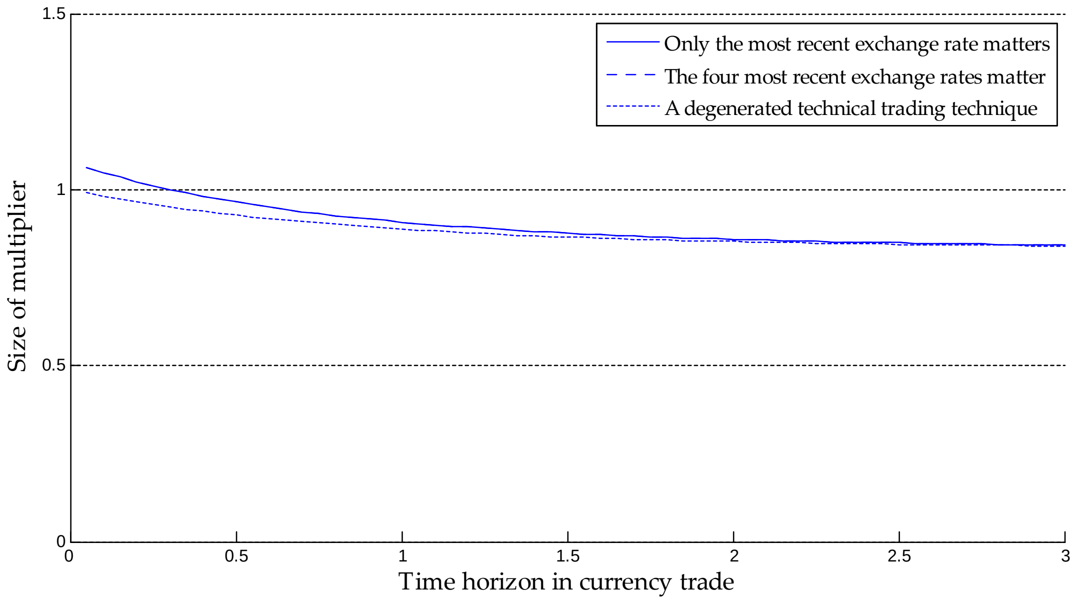

Figure 4: Graphs of the monetary policy multipliers of a *temporary* change in money supply, where the parameter values are $\beta = 0.2$, $\gamma = 1$ and $v = 1$. Note that the dashed and solid graphs are almost overlapping.

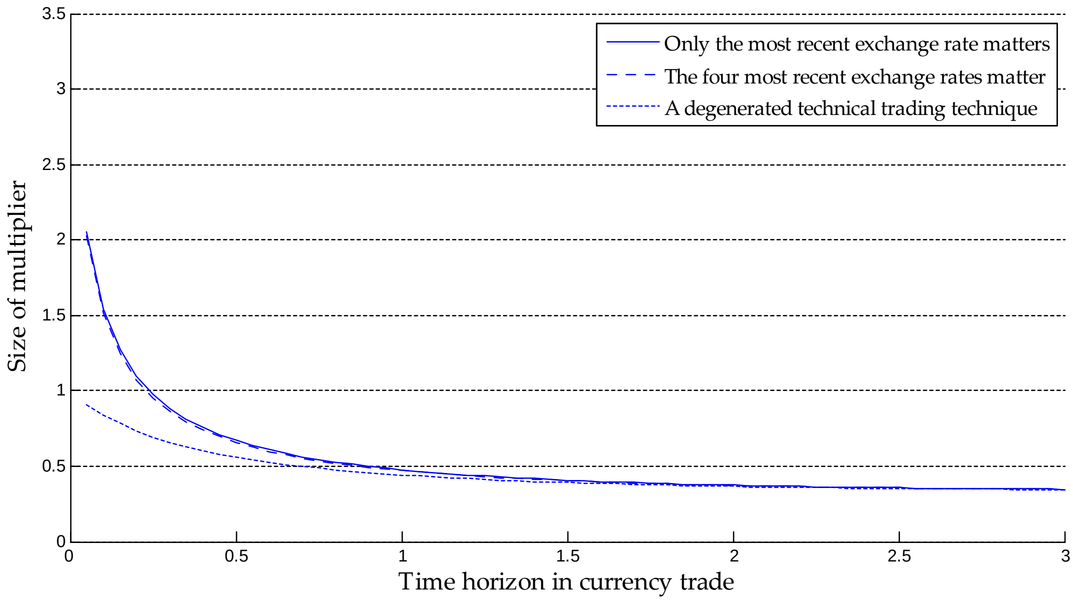

Figure 5: Graphs of the monetary policy multipliers of a *temporary* change in money supply, where the parameter values are $\beta = 2$, $\gamma = 1$ and $v = 1$. Note that the dashed and solid graphs are almost overlapping.

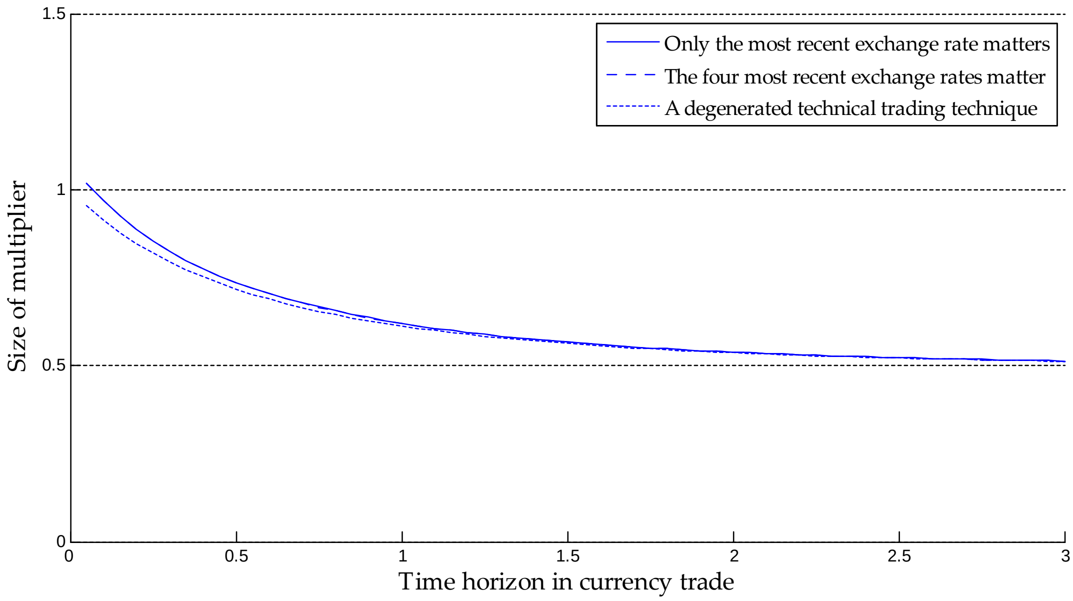

Figure 6: Graphs of the monetary policy multipliers of a *temporary* change in money supply, where the parameter values are $\beta = 1$, $\gamma = 0.2$ and $v = 1$. Note that the dashed and solid graphs are almost overlapping.

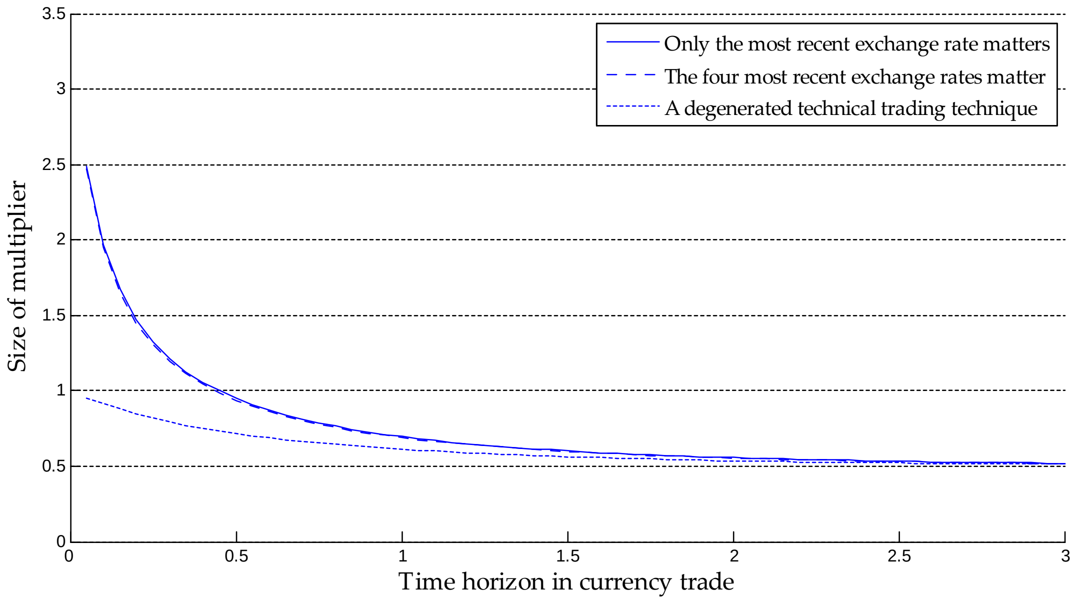

Figure 7: Graphs of the monetary policy multipliers of a *temporary* change in money supply, where the parameter values are $\beta = 1$, $\gamma = 2$ and $v = 1$. Note that the dashed and solid graphs are almost overlapping.

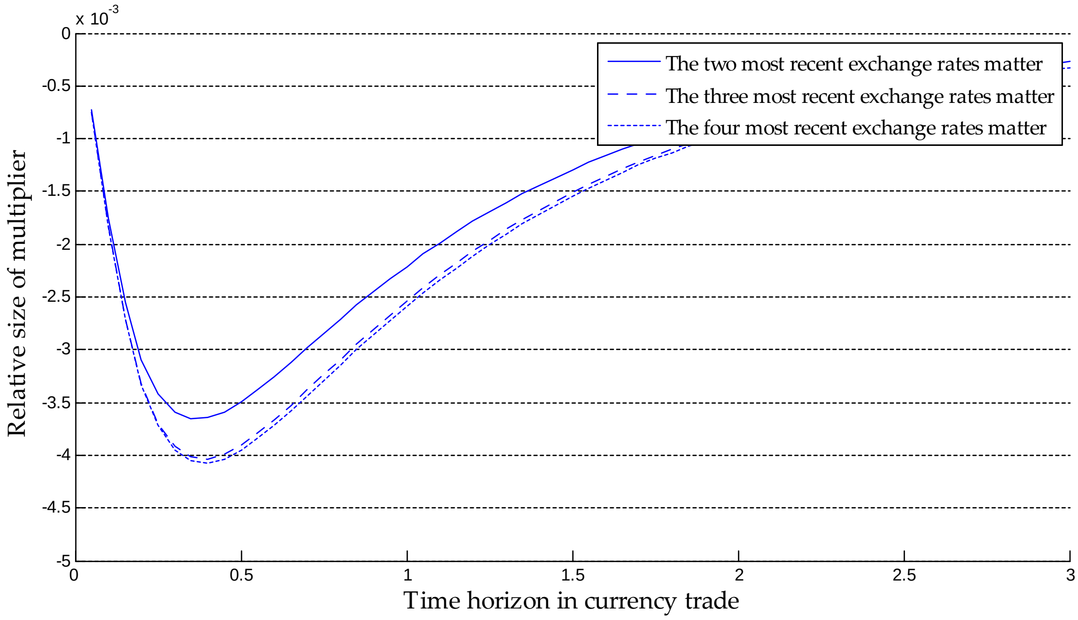

Figure 8: Graphs of the monetary policy multipliers of a *temporary* change in money supply, relative to the multiplier when only the most recent exchange rate matters in the technical trading rule, where the parameter values are $\beta = 1$, $\gamma = 1$ and $v = 1$. Note the scale at the vertical axis.

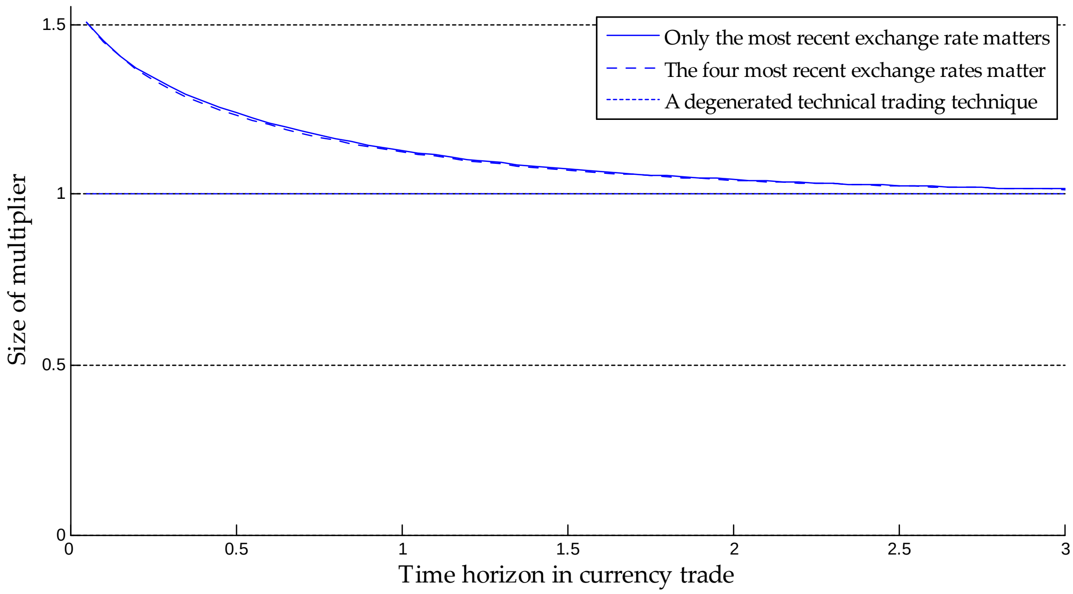

Figure 9: Graphs of the monetary policy multipliers of a *permanent* change in money supply, where the parameter values are $\beta = 1$, $\gamma = 1$ and $v = 1$. Note that the dashed and solid graphs are almost overlapping, and that the dotted graph is horizontal at the multiplier size 1.

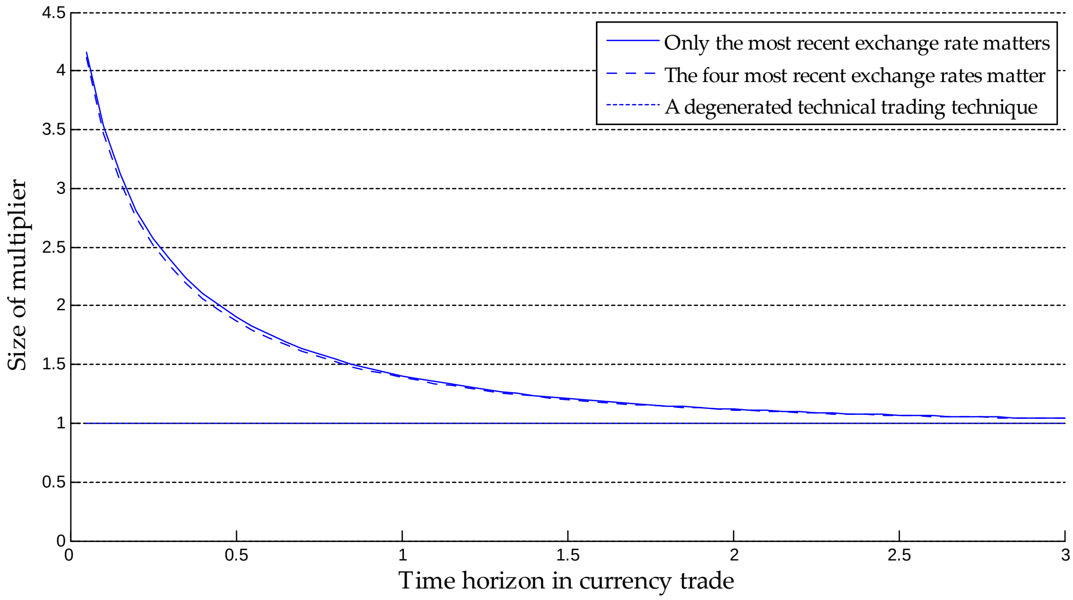

Figure 10: Graphs of the monetary policy multipliers of a *permanent* change in money supply, where the parameter values are $\beta = 1$, $\gamma = 1$ and $v = 0.2$. Note that the dashed and solid graphs are almost overlapping, and that the dotted graph is horizontal at the multiplier size 1.

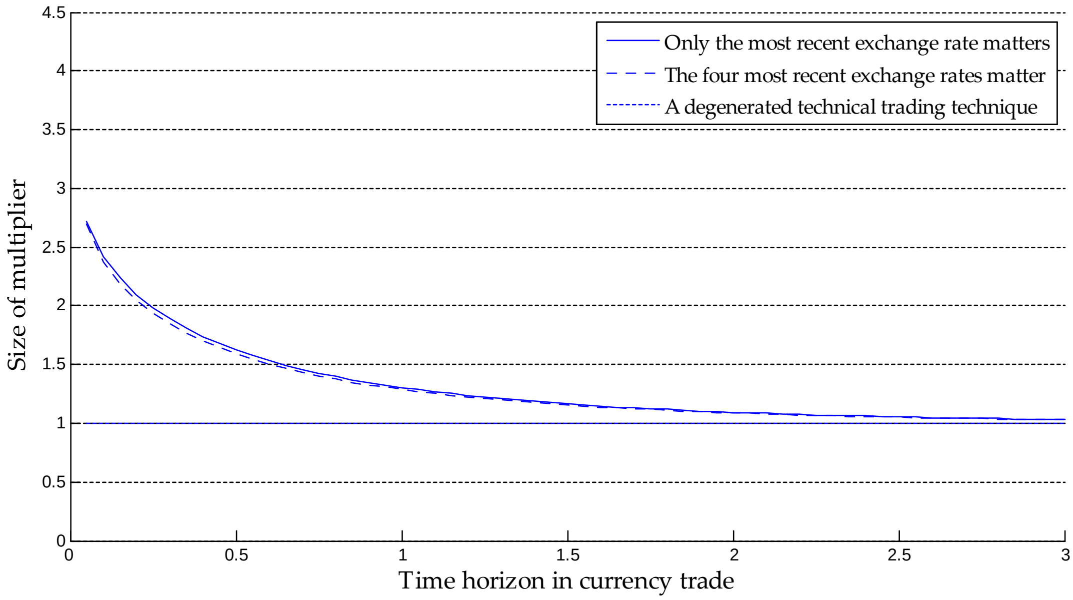

Figure 11: Graphs of the monetary policy multipliers of a *permanent* change in money supply, where the parameter values are $\beta = 2$, $\gamma = 1$ and $v = 1$. Note that the dashed and solid graphs are almost overlapping, and that the dotted graph is horizontal at the multiplier size 1.

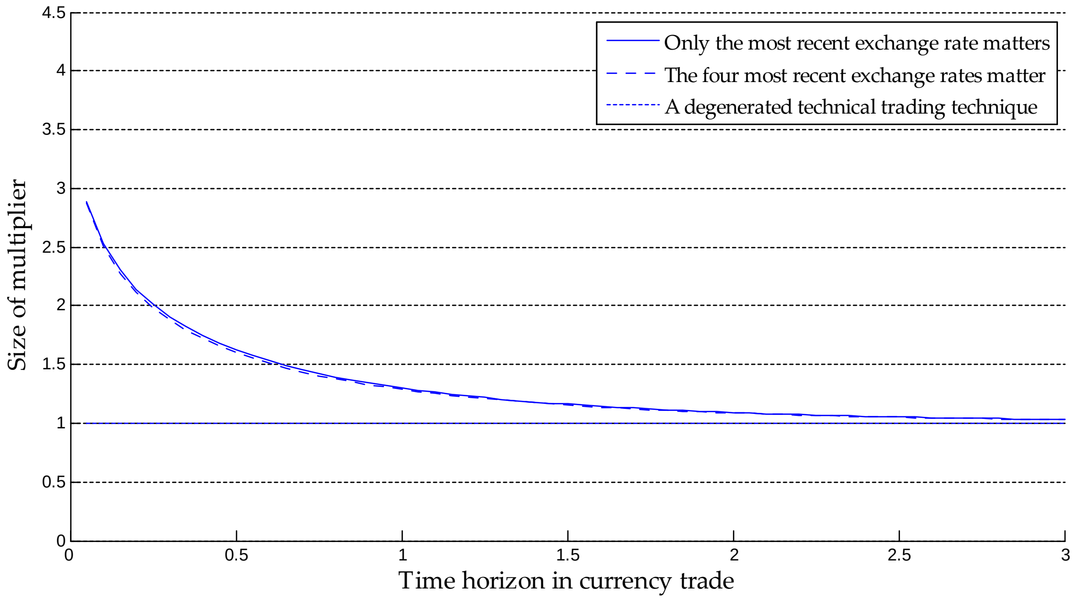

Figure 12: Graphs of the monetary policy multipliers of a *permanent* change in money supply, where the parameter values are $\beta = 1$, $\gamma = 2$ and $v = 1$. Note that the dashed and solid graphs are almost overlapping, and that the dotted graph is horizontal at the multiplier size 1.

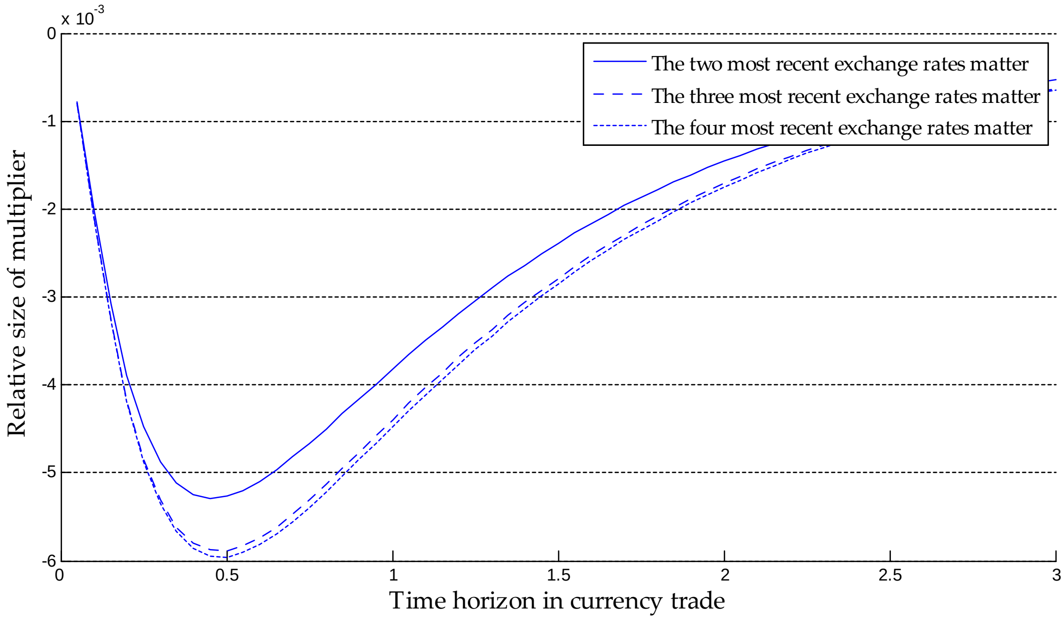

Figure 13: Graphs of the monetary policy multipliers of a *permanent* change in money supply, relative to the multiplier when only the most recent exchange rate matters in the technical trading rule, where the parameter values are $\beta = 1$, $\gamma = 1$ and $v = 1$. Note the scale at the vertical axis.